WAVE OPTICS

Chapter 17

REVIEW OF BASIC CONCEPTS

1. Wave Nature of Light

Light is an electromagnetic wave which does not require a material medium for propagation. The electric and magnetic fields vary in space and time resulting in the propagation of an electromagnetic wave even in free space.

The electric field varies in space and time as

$$E = A \sin(\omega t - kx)$$

which represents a wave travelling along the + x direction. A = amplitude, $\omega = 2\pi\nu$ (ω is angular frequency in rad s^{-1} and ν is frequency in Hz) and $k = \frac{2\pi}{\lambda}$; λ = wavelength. Also

$$v = \nu\lambda = \frac{\omega}{k}$$

where v is the wave velocity.

Phase

The phase ϕ of a wave at a point x and at time t is given by the argument of the harmonic function (sine or cosine) representing the wave, i.e.

$$\phi = \omega t - kx$$

Phase Difference

Suppose two waves meeting at a point P are represented by

$$E_1 = A_1 \sin(\omega_1 t - k_1 x_1)$$

and

$$E_2 = A_2 \sin(\omega_2 t - k_2 x_2)$$

where x_1 and x_2 are paths of the waves up to point P where they meet. The phase difference between them is

$$\begin{aligned}\Delta\phi &= \phi_1 - \phi_2 \\ &= (\omega_1 - \omega_2)t - (k_1 x_1 - k_2 x_2)\end{aligned}$$

$$\begin{aligned}\Delta\phi &= (\phi_1 - \phi_2) \\ &= 2\pi(\nu_1 - \nu_2)t - 2\pi\left(\frac{x_1}{\lambda_1} - \frac{x_2}{\lambda_2}\right)\end{aligned}$$

1. If the two waves have different frequencies, i.e., $\nu_1 \neq \nu_2$ then $\lambda_1 \neq \lambda_2$ and $\Delta\phi$ depends on time t.
2. If $\nu_1 = \nu_2$, then $\lambda_1 = \lambda_2$. In this case

$$\Delta\phi = \frac{2\pi}{\lambda}(x_2 - x_1)$$

or $\quad$ Phase difference = $\frac{2\pi}{\lambda} \times$ (path difference)

i.e., the phase difference is independent of time and depends only on the path difference $(x_2 - x_1)$. This holds only if the two sources of wave are 'coherent', i.e., they have a constant fixed phase relationship.

Intensity The intensity of a wave at any point in its path is proportional to the square of its amplitude at that point.

2. Reflection and Refraction of Light

When a light wave falls on a reflecting surface, it is reflected obeying the usual laws of reflection. When a wave travels from one medium into another, its velocity and wavelength undergo a change and the wave is said to suffer refraction. The frequency of the wave does not undergo any change in refraction (and reflection). If v_1 is the velocity of the wave in the medium in which the incident wave propagates and v_2 is the velocity of the wave in the medium in which the refracted wave propagates, then ${}^1\mu_2$, the reflective index of the second medium with respect to the first, is defined as

$${}^1\mu_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

where λ_1 and λ_2 are the wavelengths of the same wave in the two media. The frequency of the refracted wave remains the same as that of the incident wave.

When a wave, travelling in a rarer medium, is reflected at the boundary of a denser medium, the reflected wave suffers a phase change of 180° (or π radians) in relation to that of the incident wave. No phase change occurs if a wave, travelling in a denser medium, is reflected at the boundary of a rarer medium. The refracted wave, in both cases, does not undergo any phase change.

3. Interference of Light

When two or more light waves travelling in the same direction meet (or superpose) at a point in a medium, the electric field of the resultant wave can be obtained by using the *principle of superposition* which states that *the resultant electric field is given by the algebraic sum of the individual electric fields, at that point, due to the individual waves, i.e.,*

$$E = E_1 + E_2 + \cdots$$

resulting in a change in amplitude (and hence in intensity) at that point. The phenomenon in which the intensity of light at a point is modified by the superposition of two or more waves is known as *interference.*

If two waves of intensities I_1 and I_2, differing in phase by ϕ, superpose, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

Constructive Interference The resultant intensity I is maximum if $\cos\phi = +1$, i.e.

$$\phi = 2n\pi;$$

where $n = 0, 1, 2, 3, \ldots$ etc. is an integer

or $$\frac{2\pi\Delta}{\lambda} = 2n\pi$$

or $$\Delta = n\lambda$$

where Δ is the path difference between the interfering waves. Then

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

The interference is said to be constructive. If the two interfering waves have equal intensities $I_1 = I_2 = I_0$, then

$$I_{max} = 4I_0$$

Destructive Interference The resultant intensity I is minimum if $\cos\phi = -1$, i.e.

$$\phi = (2n - 1)\pi$$

and $$\Delta = \left(n - \frac{1}{2}\right)\lambda\ ;\ n = 1, 2, 3, \text{.....etc.}$$

Then

$$I_{min} = I_1 + I_2 - 2I_1 I_2 = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

At maxima, the waves reinforce each other and at minima they cancel out each other. These maxima and minima constitute the bright and dark fringes.

4. Coherent Light Sources

The resultant intensity of light at a point on the screen depends on the phase difference (ϕ) between the two interfering waves. This phase difference depends upon two factors—(1) the initial phase difference between the waves emitted by the two sources and (2) the phase difference resulting from the path difference for that point. The initial phase difference depends upon the time and remains constant only for about 10^{-8} to 10^{-10} second. Thus the resultant intensity changes so rapidly with time that, due to persistence of vision, we are unable to see the interference pattern. Thus, non-coherent sources cannot produce sustained interference effects. We conclude that, for a steady interference pattern, the following two conditions must be satisfied.

1. The sources must be coherent.
2. The wavelengths of the interfering waves must be the same. Thus, *only monochromatic coherent light sources produce observable interference pattern.*

5. Young's Double Slit Experiment

Monochromatic light from a source slit S illuminates two slits S_1 and S_2 which are very close together and equidistant from S (Fig. 17.1).

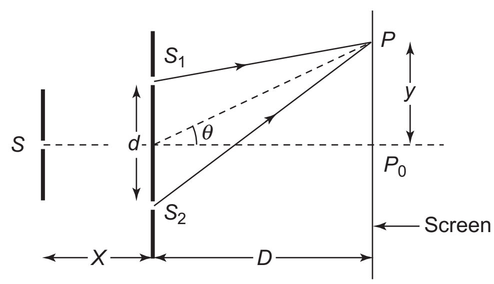


Fig. 17.1

Secondary waves from S_1 and S_2 interfere giving rise to bright and dark fringes on the screen. There is bright fringe at centre P_0 of the screen.

(i) The distance of the nth bright fringe from the centre of the fringe system is

$$y_n = \frac{n\lambda D}{d}\ ;\ n = 0, 1, 2, \text{... etc.}$$

where λ = wavelength of light used, d = seperation between slits S_1 and S_2 and D = distance between the screen and the plane of the two slits.

(ii) The distance of the nth dark fringe from the centre of the fringe system is

$$y_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d};\ n = 1, 2, 3, \ldots \text{ etc}$$

(iii) The separation between two consecutive bright or dark fringes is called fringe width (β) which is given by

$$\beta = \frac{\lambda D}{d}$$

(iv) Angular separation between nth bright fringe and the central fringe is

$$\theta_n = \frac{y_n}{D} = \frac{n\lambda}{d} \quad ; \quad \theta_n \text{ is in radian.}$$

(v) Angular separation between nth dark fringe and the central fringe is

$$\theta_n^* = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

6. Displacement of Fringes

If a transparent plate of thickness t and refractive index μ is introduced in the path of one of the interfering waves, the entire fringe pattern is shifted by a distance

$$y_0 = (\mu - 1)\frac{tD}{d}$$

$$\text{Number of fringes shifted} = \frac{(\mu - 1)t}{\lambda}$$

7. Diffraction at a Slit

When a parallel beam of monochromatic light falls normally on a narrow slit, the diffraction pattern on a screen has a bright central maximum bordered on both sides by secondary maxima of rapidly decreasing intensity.

If λ is the wavelength of light and a is the width of the slit, then

(i) For bright fringes : $\sin\theta = 0, \frac{3\lambda}{2a}, \frac{5\lambda}{2a}, \ldots$ etc

(ii) For dark fringes : $\sin\theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a} \ldots$ etc

(iii) Angular width of central maximum = $\frac{2\lambda}{a}$

(iv) Linear width of central maximum = $\frac{2f\lambda}{a}$, where f = focal length of the convex lens placed close to the slit.

8. Some Important Points about Interference of Light

1. In Young's double slit experiment, if monochromatic light is replaced by white light then central fringe will be white; all other fringes will be coloured. White light consists of colours between violet and rad (VIBGYOR). Wavelength λ is the shortest for violet light and longest for red light. At the central fringe, the path difference for all colours is zero. Hence at the central fringe, all colours superpose to give a white fringe. The first bright fringe after the central fringe will be violet colour.
2. In Young's double slit experiment, if one of the slits is covered with a transparent film or sheet of thickness t and refractive index μ, then
   (a) the path difference at the centre of the screen will not be zero, it will be equal to $(\mu - 1)t$.
   (b) the entire fringe pattern will shift by an amount $y_0 = \frac{(\mu - 1)tD}{d}$.
   (c) at the centre of the screen there will be a bright fringe if $(\mu - 1)\,t = n\lambda$; $n = 1, 2, 3, \ldots$ etc.
   (d) at the centre of the screen there will be a dark fringe if
   $$(\mu - 1)t = \left(n - \frac{1}{2}\right)\lambda;\ n = 1, 2, 3, \ldots \text{ etc.}$$
   (e) the fringe width will remain the same.
   (f) the intensity of light from the covered slit will decrease due to absorption by the film or sheet. Hence intensity of bright fringes will decrease and dark fringes will have some finite intensity (because the two interfering beam do not now have equal intensity). Hence the fringe pattern will become less distinct.
3. If one of the slits in Young's double slit experiment is closed (or covered with black paper), the interference pattern is replaced by single slit diffraction pattern which has a bright central fringe bordered on both sides by fringes of decreasing intensity.
4. If Young's interference experiment is performed in still water rather than in air, the fringe width will decrease. Since the refractive index of water is greater than that of the air, the speed of light in water (v) will be less than that in air (c). Since the frequency of light is the same in all media, $\lambda_w = \frac{v}{\nu}$ and $\lambda_a = \frac{c}{\nu}$ which give $\frac{\lambda_w}{\lambda_a} = \frac{v}{c} = \frac{1}{\mu_w}$.

Now $\mu_w = 4/3$. Hence $\lambda_w < \lambda_a$. Fringe width $\beta \propto \lambda$. Hence β in water < β in air.

5. In Young's interference experiment, if the beam of light has two wavelengths λ_1 and λ_2, their maxima will coincide if $n_1\lambda_1 = n_2\lambda_2$, where n_1 and n_2 are integers.
6. In an interference experiment if the two coherent light sources have intensities in the ratio $n : 1$, i.e. $\frac{I_1}{I_2} = n$, then the ratio of the intensity of maxima and minima in the interference pattern is

$$\frac{I_{\max}}{I_{\max}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

$$= \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2 = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

7. In an interference experiment with two coherent light sources, if the ratio of the intensities of maxima and minima in the interference pattern is $n : 1$, i.e. $\frac{I_{\max}}{I_{\min}} = n$, then ratio of the intensities of the coherent sources is

$$\frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2 \text{ because}$$

$$\frac{I_{\max}}{L_{\min}} = \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2$$

$$\Rightarrow \quad \sqrt{n} = \frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1} \Rightarrow \frac{I_1}{I_2} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

8. In an interference experiment, if the two coherent sources have intensities in the ratio $n : 1$, then in the interference pattern

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{n}}{(n+1)}$$

9. The intensity of light emerging from a slit is proportional to its width. If the two slits in Young's interference experiment have widths in the ratio $n : 1$, then $\frac{I_1}{I_2} = n$ and

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$$

10. In Young's double slit experiment (Fig. 17.1), if x is the width of the source slit S and X its distance from the plane of the slits, the interference fringes will not be seen (because the interference pattern becomes indistinct) if the condition

$$\frac{x}{X} < \frac{\lambda}{d}$$

is not satisfied.

9. Resolving Power

Resolving power of an optical instrument is its ability to produce distinctly separate images of two objects very close together.

(a) *Resolving power of a microscope*

$$R.P. = \frac{2\mu \sin\theta}{1.22\lambda}$$

where 2θ = angle of the cone of light rays entering the objective of the microscope (Fig. 17.2), μ = refractive index of the medium between the object and the objective and λ = wavelength of light used to illuminate the object.

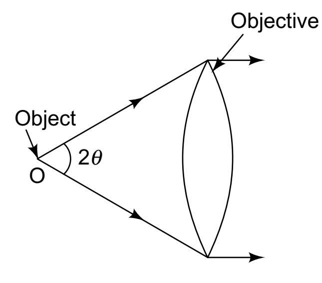


Fig. 17.2

(b) *Resolving power of a telescope*

$$R.P. = \frac{D}{1.22\lambda}$$

where D = diameter of objective and λ = wavelength of light

10. Polarization of Light

The phenomena of reflection, refraction, interference and diffraction are common on both transverse and longitudinal waves, mechanical as well as electromagnetic. The distinguishing feature is that only transverse waves can be *polarized*.

In an unpolarized light, the electric field vector has all the possible orientations in a plane perpendicular to the direction of propagation. When this light is passed through a specially cut crystal of calcite or quartz, called a polaroid, we obtain a plane polarized light.

Only transverse waves can be polarized. Longitudinal waves cannot be poarlized.

Polarization by Reflection: Brewster's Angle

In 1808, the French physicist Brewster discovered that when a beam of ordinary unpolarized light is incident at a particular angle i_p on the surface of a transparent medium, the reflected light is polarized.

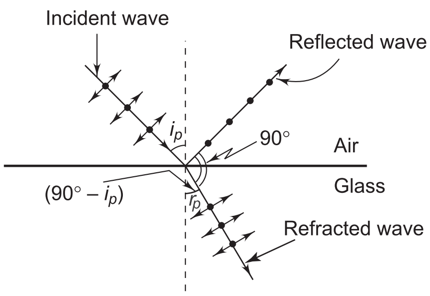


Fig. 17.3 *Polarization by reflection; i_p is Brewster's angle*

Figure 17.3 shows an unpolarized light incident at an angle i_p at the surface of glass. Brewster discovered that when $i = i_p$, the reflected and refracted rays are exactly 90° apart. The angle i_p when this happens is called the *polarizing* or the *Brewster angle*. If r_p is the corresponding angle of refraction, then from the geometry of Fig. 17.3,

$$i_p + r_p = 90° \text{ or } r_p = 90° - i_p$$

Now from Snell's law, the refractive index of glass is

$$n = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90° - i_p)}$$

$$= \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\therefore \quad n = \tan i_p$$

This equation is called the *Brewster law* and the angle i_p satisfying this equation is the *Brewster angle*.

EXAMPLE 1 In Young's double-slit experiment, the intesity of light at a point on the screen where the path difference is λ is K units. What is the intensity of light at a point where the path difference is $\lambda/3$; λ being the wavelength of light used?

SOLUTION Path difference $\Delta = \lambda$. Therefore, phase difference $\phi = \frac{2\pi}{\lambda}\Delta = 2\pi$. Hence intensity at a point where $\Delta = \lambda$ or $\phi = 2\pi$ is

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 2\pi$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= I + I + 2I = 4I = K \text{ units}$$

$$(\because\ I_1 = I_2 = I)$$

i.e. $I = K/4$. The intensity at a point where the path difference is

$$\Delta' = \frac{\lambda}{3} \text{ or } \phi' = \frac{2\pi}{\lambda}\Delta'$$

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} \text{ is}$$

$$I' = I + I + 2I\cos\frac{2\pi}{3}$$

$$= 2I - I = I = \frac{K}{4} \text{ units}$$

EXAMPLE 2 Two coherent sources of intensity ratio 100 : 1, interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?

SOLUTION Given $I_1/I_2 = 100$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \frac{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2} = \frac{\left(\sqrt{100} + 1\right)^2}{\left(\sqrt{100} - 1\right)^2} = \frac{121}{81}$$

EXAMPLE 3 In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the fourth bright fringe and the central bright fringe is measured to be 1.2 cm. What is the wavelength of light used in the experiment?

SOLUTION The position of the nth bright fringe with respect to the central fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

For the central bright fringe ($n = 0$), $y_0 = 0$. For the fourth bright fringe ($n = 4$), $y_4 = 4\,\lambda D/d$. Therefore

$$y_4 - y_0 = \frac{4\lambda D}{d} \quad \text{or } \lambda = (y_4 - y_0)\frac{d}{4D} \qquad \text{(i)}$$

It is given that $(y_4 - y_0) = 1.2$ cm $= 1.2 \times 10^{-2}$ m, $D = 1.4$ m and $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m. Substituting these values in (i) and solving, we get

$\lambda = 6 \times 10^{-7}$ m (or 600 nm or 6000 Å).

EXAMPLE 4 The ratio of the intensities of the maxima and minima in an interference pattern is 49 : 9. What is the ratio of the intensities of the two coherent sources employed in the interference experiment?

SOLUTION Given

$$\frac{I_{max}}{I_{min}} = \frac{49}{9}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}}$$

$$= \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{x+1}{x-1}\right)^2$$

where $$x = \sqrt{\frac{I_1}{I_2}}.$$

$$\Rightarrow \quad \frac{49}{9} = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow \frac{7}{3} = \frac{x+1}{x-1}$$

which gives $x = \frac{5}{2}$. Therefore

$$\frac{I_1}{I_2} = x^2 = \frac{25}{4}$$

EXAMPLE 5 In Young's double slit experiment, find the ratio of the intensities at points P and Q on the screen where the path difference between the interfering waves is (a) zero and (b) $\frac{\lambda}{4}$, where λ is the wavelength of light used.

SOLUTION

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

where $$\phi = \frac{2\pi\Delta}{\lambda}; \ \Delta = \text{path difference}$$

In Young's experiment $I_1 = I_2 = I_0$. Therefore,

$$I = I_0 + I_0 + 2I_0 \cos\phi$$

$$= 2I_0 (1 + \cos\phi)$$

(a) For $\Delta = 0$, $\phi = 0°$. Hence

$$I_1 = 2I_0 (1 + \cos 0°) = 4I_0$$

(b) For $\Delta = \frac{\lambda}{4}$, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$. Hence

$$I_2 = 2I_0\left(1 + \cos\frac{\pi}{2}\right) = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = 2$$

EXAMPLE 6 A beam of light consisting of two wavelengths 450 nm and 750 nm is used to obtain interference fringes in Young's double slit experiment. The separation between the slits is 1.0 mm and the distance between the plane of the slits and the screen is 100 cm. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

(a) 2.00 mm (b) 2.25 mm
(c) 2.50 mm (d) 2.75 mm

SOLUTION Let nth bright fringe of wavelength $\lambda_1 = 450$ nm coincide with the mth bright of $\lambda_2 = 750$ nm. Then

$$y = \frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d} \qquad \text{(i)}$$

$$\Rightarrow \quad n\lambda_1 = m\lambda_2$$

$$\Rightarrow \quad \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{750 \text{ nm}}{450 \text{ nm}} = \frac{5}{3} \qquad \text{(ii)}$$

The smallest integral values of n and m which satisfy (ii) are $n = 5$ and $m = 3$. From (i) we have

$$y_{min} = \frac{n\lambda_1 D}{d}$$

$$= \frac{5 \times (450 \times 10^{-9}) \times 1.0}{(1.0 \times 10^{-3})}$$

$$= 2.25 \times 10^{-3} \text{ m} = 2.25 \text{ mm}$$

EXAMPLE 7 Monochromatic light of wavelength 500 nm is used in Young's double slit experiment. One of the slits is covered by a glass sheet of thickness 2.0×10^{-2} mm and refractive index 1.5. The number of fringes shifted by the introduction of the sheet is

(a) 14 (b) 16
(c) 18 (d) 20

SOLUTION When light traverses a sheet of thickness t and refractive index μ, the optical path travelled $= \mu t$.

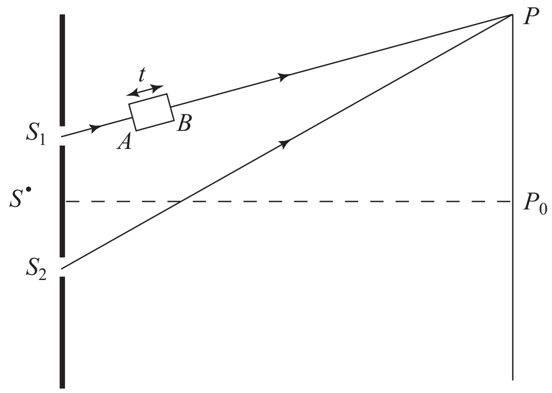


Fig. 17.4

Path difference in the absence of sheet is

$$\Delta_1 = S_2 P - S_1 P$$

Path difference when the sheet is introduced is

$$\Delta_2 = S_2 P - [S_1 A + \mu(AB) + BP]$$

$$= S_2 P - [S_1 A + BP + AB - AB + \mu(AB)]$$

$$= S_2 P - [S_1 P + (\mu - 1) AB]$$

$$= S_2 P - S_1 P - (\mu - 1)t$$

∴ Change in optical path is

$$\Delta = \Delta_1 - \Delta_2 = (\mu - 1)t$$

If the optical path changes by one wavelength, one fringe will shift. Therefore, the number of fringes shifted due to introduction of sheet is

$$\frac{(\mu - 1)t}{\lambda} = \frac{(1.5 - 1) \times (2.0 \times 10^{-5}\text{ m})}{(500 \times 10^{-9}\text{ m})} = 20$$

EXAMPLE 8 Monochromatic light of wavelength 640 nm falls normally on a glass plate of refractive index 1.6. The waves reflected from the upper and lower faces of the film will interfere constructively if the least thickness of the plate is

(a) 100 nm (b) 200 nm
(c) 300 nm (d) 400 nm

SOLUTION When light falls on a surface, it is partly reflected and partly refracted. Thus light from source S is reflected as wave 1 at *A* of the top surface of the plate, refracted into the plate and reflected as wave 2 at *B* of the bottom surface. On reaching *A*, the wave 2 has traversed an optical path length = $\mu t + \mu t = 2\,\mu t$ (Fig. 17.5).

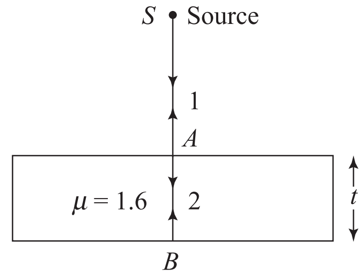


Fig. 17.5

Furthermore, on reflection from the denser medium, a wave suffers a phase change of π which implies a path change of $\lambda/2$. Hence on reaching *A* waves 1 and 2 will have a path change of

$$\Delta = 2\mu t - \frac{\lambda}{2}$$

For constructive interference, $\Delta = n\lambda$. Hence

$$2\mu t - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \quad 2\,\mu t = \left(n + \frac{1}{2}\right)\lambda$$

$$\text{or} \quad t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2\mu}$$

The minimum value of *t* corresponds to $n = 0$. Hence

$$t_{\min} = \frac{\lambda}{4\mu} = \frac{640\text{ nm}}{4 \times 1.6} = 100\text{ nm}$$

EXAMPLE 9 In Example 9 above the waves reflected from the upper and lower surfaces of the plate will interfere destructively if the least thickness of the plate is

(a) 100 nm (b) 200 nm
(c) 300 nm (d) 400 nm

SOLUTION For destructive interference,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda$$

$$\text{or} \quad 2\mu t + \frac{\lambda}{2} = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad 2\,\mu t = n\lambda$$

$$\Rightarrow \quad t = \frac{n\lambda}{2\mu}$$

The least integral value of *n* is 1. Hence

$$t_{\min} = \frac{\lambda}{2\mu} = \frac{640\text{nm}}{2 \times 1.6} = 200\text{ nm}$$

1 SECTION Multiple Choice Questions with One Correct Choice

Level A

1. How is the interference pattern in Young's double slit experiment affected if the sodium (yellow) light is replaced by red light of the same intensity?
   (a) The fringes will vanish
   (b) The fringes will become brighter
   (c) The fringe width will decrease
   (d) The fringe width will increase
2. In Young's double slit experiment, if the distance between the slits and the screen is doubled and the separation between the slits is reduced to half, the fringe width
   (a) is doubled (b) becomes four times
   (c) is halved (d) remains unchanged.
3. What happens if the monochromatic light used in Young's double slit experiment is replaced by white light?
   (a) All bright fringes become white
   (b) All bright fringes have colours between violet and red
   (c) Only the central fringe is white, all the other fringes are coloured
   (d) No fringes will be observed.
4. What will happen if one of the slits in Young's double slit experiment is covered with cellophane paper which absorbs a fraction of the intensity of light from the slit?
   (a) The fringe width will decrease
   (b) The fringes will become more distinct
   (c) The bright fringes will become less bright and the dark fringes will not be completely dark
   (d) No fringes will be observed
5. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same set up, the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that in the second case is
   (a) 1 : 1 (b) 1 : 2
   (c) 2 : 1 (d) $\sqrt{2}$: 1
6. How is the interference pattern affected if the Young's experiment was performed in still water than in air?
   (a) Fewer fringes will be visible
   (b) Fringes will be broader
   (c) Fringes will be narrower
   (d) No fringes will be observed.
7. How is the interference pattern in Young's experiment affected if one of the slits is covered with black opaque paper?
   (a) The bright fringes become fainter
   (b) The fringe width decreases
   (c) There will be uniform illumination all over the screen
   (d) There will be a bright central fringe bordered on both sides by fringes of decreasing intensity.
8. What happens to the interference pattern if the two slits S_1 and S_2 in Young's double-slit experiment are illuminated by two independent identical sources?
   (a) The intensity of the bright fringes is doubled
   (b) The intensity of the bright fringes becomes four times.
   (c) Two sets of interference fringes overlap
   (d) No interference pattern is observed.
9. What is the reason for your answer to Q.9?
   (a) The two sources do not emit light of the same wavelength
   (b) The two sources emit waves which travel with different speeds
   (c) The two sources emit light waves of different amplitude
   (d) There is no constant phase difference between the waves emitted by the two sources.
10. In Young's double–slit experiment, the intensity of light at a point on the screen where the path difference is λ is K units. What is the intensity of light at a point where the path difference is $\lambda/3$; λ being the wavelength of light used?
    (a) $K/4$ (b) $K/3$
    (c) $K/2$ (d) K
11. Two coherent sources of intensity ratio 100 : 1, interfere. What is the ratio of the intensity between the maxima and minima in the interference pattern?
    (a) 10 : 1 (b) 5 : 2
    (c) 121 : 81 (d) 11 : 9

12. In Young's double slit experiment the fringe width with light of wavelength 6000 Å is found to be 4.0 mm. What will be the fringe width if light of wavelength 4800 Å is used?

    (a) 2.8 mm (b) 3.2 mm
    (c) 4.0 mm (d) 4.8 mm

13. In Q. 13, what will be the fringe width using light of wavelength 6000 Å if the entire apparatus is immersed in a transparent liquid of refractive index 4/3?

    (a) 2.0 mm (b) 3.0 mm
    (c) 4.0 mm (d) 5.0 mm

14. In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the fourth bright fringe and the central bright fringe is measured to be 1.2 cm. What is the wavelength of light used in the experiment?

    (a) 200 nm (b) 400 nm
    (c) 600 nm (d) 800 nm

15. What is the effect on the interference fringes in Young's double slit experiment if the source slit is moved closer to the double slit plane?

    (a) The fringe width increases
    (b) The fringe width decreases
    (c) The fringes become more distinct
    (d) The fringes become less distinct.

16. What is the effect on the interference fringes in Young's double slit experiment if the width of the source slit is increased?

    (a) The fringe width increases
    (b) The fringe width decreases
    (c) The fringes become more distinct.
    (d) The fringes become less distinct.

17. What is the effect on the interference fringes in Young's double slit experiment if the widths of the two slits are increased?

    (a) The fringe width decreases
    (b) The fringe width increases
    (c) The bright fringes are equally bright and equally spaced
    (d) The bright fringes are no longer equally bright and equally spaced.

18. In Young's double slit experiment the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that the 9th bright fringe is at a distance of 9.0 mm from the second dark fringe from the centre of the fringe pattern. What is the wavelength of light used.

    (a) 2000 Å (b) 4000 Å
    (c) 6000 Å (d) 8000 Å

19. The ratio of the intensities of the maxima and minima in an interference pattern is 49 : 9. What is the ratio of the intensities of the two coherent sources employed in the interference experiment?

    (a) 7 : 3 (b) 49 : 9
    (c) 5 : 2 (d) 25 : 4

20. A screen is placed at a certain distance from a narrow slit which is illuminated by a parallel beam of monochromatic light. What will you observe if you scan the screen with the help of a microscope?

    (a) The whole screen is uniformly illuminated.
    (b) Equally spaced and equally bright fringes are observed.
    (c) One bright fringe is observed at the centre of the screen.
    (d) A bright central fringe bordered on both sides with fringes of rapidly decreasing intensity will be observed.

21. A single-slit diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?

    (a) There is no change in the diffraction pattern.
    (b) Diffraction fringes become narrower and crowded together.
    (c) Diffraction fringes become broader and farther apart.
    (d) The diffraction pattern disappears.

22. Which one of the following waves cannot be polarized?

    (a) radio waves
    (b) X-rays
    (c) transverse waves in a string
    (d) longitudinal waves in a gas.

23. The fact that light can be polarized establishes that light

    (a) travels in the form of particles
    (b) is an electromagnetic wave
    (c) is a transverse wave
    (d) is a longitudinal wave.

24. A parallel beam of light of wavelength 6000 Å is incident normally on a slit of width 0.2 mm. The diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. If the lens is placed close to the slit, the distance between the minima on both sides of the central maximum will be

    (a) 1 mm (b) 2 mm
    (c) 3 mm (d) 4 mm

25. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1.0 mm. What should the width of each slit be so as to obtain 10 maxima of the two slit interference pattern within the central maximum of the single slit diffraction pattern?
    (a) 0.1 mm (b) 0.2 mm
    (c) 0.3 mm (d) 0.4 mm
26. In a single slit diffraction experiment, the width of the slit is made double its original width. Then the central maximum of the diffraction pattern will become.
    (a) narrower and fainter
    (b) narrower and brighter
    (c) broader and fainter
    (d) broader and brighter.
27. The dispersion of light in a medium implies that
    (a) lights of different wavelengths travel with different speeds in the medium
    (b) lights of different frequencies travel with different speeds in the medium
    (c) the refractive index of the medium is different for different wavelengths of light
    (d) all of the above.
28. Monochromatic light is refracted from air into glass of refractive index μ. The ratio of the wavelengths of the incident and refracted waves is
    (a) 1 : 1 (b) 1 : μ
    (c) μ : 1 (d) μ^2 : 1
29. Light travels with a speed of 2×10^8 ms^{-1} in crown glass of refractive index 1.5. What is the speed of light in dense flint glass of refractive index 1.8?
    (a) 1.33×10^8 ms^{-1} (b) 1.67×10^8 ms^{-1}
    (c) 2.0×10^8 ms^{-1} (d) 3.0×10^8 ms^{-1}
30. In a vacuum, light travels at a speed of 3×10^8 ms^{-1}. What is the speed of light in glass of refractive index = 1.5?
    (a) 1.5×10^8 ms^{-1} (b) 2×10^8 ms^{-1}
    (c) 3×10^8 ms^{-1} (d) 4.5×10^8 ms^{-1}
31. When a ray of light goes from a denser into a rarer medium
    (a) the wavelength of light is decreased
    (b) the frequency of light is increased
    (c) the speed of light is increased
    (d) the light undergoes a phase change of π.
32. Which one of the following statements is correct? The refractive index of a given piece of glass is
    (a) less for violet than for red light
    (b) more for blue than for green light
    (c) less for green than for yellow light
    (d) the same for all colours of light.
33. A glass slab of thickness 8 cm contains the same number of waves as 10 cm of water when both are traversed by the same monochromatic light. If the refractive index of water is 4/3, the refractive index of glass is
    (a) 5/3 (b) 5/4
    (c) 16/15 (d) 3/2

Level B

34. In Young's double slit experiment, the intensity of the maxima is I. If the width of each slit is doubled the intensity of the maxima will be
    (a) $\frac{I}{2}$ (b) I
    (c) $2I$ (d) $4I$
35. In Young's double slit experiment, the 10th maximum of wavelength λ_1 is at a distance y_1 from its central maximum and the 5th maximum of wavelength λ_2 is at a distance y_2 from its central maximum. The ratio y_1/y_2 will be
    (a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
    (c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$
36. A ray of light is incident on glass at the polarising angle i_p. The angle between the reflected and refracted rays will be
    (a) between 0° and 90° (b) 90°
    (c) between 90° and 180° (d) $2\,i_p$
37. When a ray of light is incident on a glass slab at an angle of 60°, the angle between the reflected and refracted rays is 90°. The refractive index of glass is
    (a) 1.5 (b) $\sqrt{2}$
    (c) $\sqrt{3}$ (d) 2.0
38. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain region of the screen when light of wavelength 600 nm is used. If the light of wavelength 400 nm is used, the number of fringes observed in the same region of the screen will be
    (a) 12 (b) 18
    (c) 24 (d) 8
39. In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by a distance of 5×10^{-2} m towards the slits, the change in the fringe width is 3×10^{-5} m. If the separation

between the slits is 10^{-3} m, the wavelength of light used is

(a) 5×10^{-7} m (b) 6×10^{-7} m
(c) 7×10^{-7} m (d) 6×10^{-6} m

40. White light is used to illuminate the two slits in Young's double slit experiment. The distance between the slits is b and the screen is at a distance d ($>> b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of the missing wavelengths are

(a) $\lambda = \dfrac{b^2}{2d}$ (b) $\lambda = \dfrac{2b^2}{d}$
(c) $\lambda = \dfrac{b^2}{3d}$ (d) $\lambda = \dfrac{2b^2}{3d}$

41. Two waves of intensities I and $4I$ superpose, then the maximum and minimum intensities are

(a) $5I$, $3I$ (b) $9I$, I
(c) $9I$, $3I$ (d) $5I$, I

42. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is

(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

43. A parallel beam of monochromatic light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is

(a) zero (b) $\dfrac{\pi}{2}$
(c) π (d) 2π

44. Yellow light is used in a single slit diffraction experiment with a slit of width 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal

(a) that the central maximum is narrower
(b) more number of fringes
(c) less number of fringes
(d) no diffraction pattern

45. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other, then, in the interference pattern

(a) the intensities of both maxima and minima increase
(b) the intensity of the maxima increases and the minima have zero intensity
(c) the intensity of the maxima decreases and that of the minima increases
(d) the intensity of the maxima decreases and the minima have zero intensity.

46. In Young's double slit interference experiment the wavelength of light used is 6000 Å. If the path difference between waves reaching a point P on the screen is 1.5 microns, then at that point P:

(a) Second bright band occurs
(b) Second dark band occurs
(c) Third dark band occurs
(d) Third bright band occurs

47. The difference in the number of wavelengths, when yellow light (of wavelength 6000 Å in vacuum) propagates through air and vacuum columns of the same thickness is one. If the refractive index of air is 1.0003, the thickness of the air column is

(a) 1.8 mm (b) 2 mm
(c) 2 cm (d) 2.2 cm

48. In Young's double slit experiment, the fringe width is 2.0 mm, The separation between the 9th bright fringe and the second dark fringe from the centre of the fringe system will be

(a) 5.0 mm (b) 10 mm
(c) 15 mm (d) 20 mm

49. When one of the slits in Young's experiment is covered with a transparent sheet of thickness 3.6×10^{-3} cm the central fringe shifts to a position originally occupied by the 30th bright fringe. If $\lambda =$ 6000 Å, the refractive index of the sheet is

(a) 1.50 (b) 1.55
(c) 1.60 (d) 1.65

50. A beam of light, consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in Young's double slit experiment. The separation between the slits is 2.6 mm and the distance between the plane of the slits and the screen is 1.0 m. The least distance from the central maximum where the bright fringes due to both the wavelengths coincide is

(a) 1.0 mm (b) 1.5 mm
(c) 2.0 mm (d) 2.5 mm

51. Two coherent light sources of intensity ratio n are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\left(\dfrac{n+1}{n-1}\right)$ (b) $\left(\dfrac{n+1}{n-1}\right)^2$
(c) $\left(\dfrac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (d) $\left(\dfrac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$

52. Two coherent light sources are employed in an interference experiment. The ratio of the intensities of the maxima and minima in the interference pattern is n. The ratio of the intensities of the two coherent sources is

(a) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{n^2+1}{n^2-1}\right)$ (d) $\left(\frac{n+1}{n-1}\right)$

53. The two slits in Young's interference experiment have widths in the ratio $n:1$. The ratio of the intensities of the maxima and minima in the interference pattern is

(a) $\frac{\sqrt{n}+1}{\sqrt{n}}$ (b) $\frac{\sqrt{n}}{\sqrt{n}-1}$

(c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (d) $\left(\frac{n+1}{n}\right)^2$

54. Interference pattern is obtained with two coherent light sources of intensity ratio n. In the interference pattern, the ratio $\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$ will be

(a) $\frac{\sqrt{n}}{(n+1)}$ (b) $\frac{2\sqrt{n}}{(n+1)}$

(c) $\frac{\sqrt{n}}{(\sqrt{n}+1)^2}$ (d) $\frac{2\sqrt{n}}{(\sqrt{n}+1)^2}$

55. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0.1°. If the wavelength of light used is 628 nm, the spacing between the slits is

(a) 0.9×10^{-4} m (b) 1.8×10^{-4} m

(c) 3.6×10^{-4} m (d) 7.2×10^{-4} m

56. Interference pattern is obtained with two coherent light sources of intensities I and $4I$. The intensity at a point where the phase difference is $\pi/2$ is

(a) I (b) $2I$

(c) $3I$ (d) $5I$

57. Young's double slit expetiment is performed using light of wavelength λ. One of the slits is covered by a thin glass sheet of refractive index μ at this wavelength. The smallest thickness of the sheet to bring the adjacent minimum to the centre of the screen is

(a) $\frac{\lambda}{2(\mu-1)}$ (b) $\frac{\lambda}{(\mu-1)}$

(c) $\frac{\lambda}{2\mu}$ (d) $\frac{\lambda}{\mu}$

58. Monochromatic light of wavelength 500 nm is incident on two parallel slits separated by a distance of 5×10^{-4} m. The interference pattern is obtained on a screen at a distance of 1.0 m from the slits. The intensity of the central maximum is I_0. When one of the slits is covered by a glass sheet of thickness 1.5×10^{-6} m and refractive index 1.5, the intensity at the centre of the screen will be equal to

(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{3}$

(c) $\frac{I_0}{4}$ (d) zero

59. In Q. 58 above, the lateral shift of the central maximum is

(a) 5 mm (b) 4 mm

(c) 3 mm (d) 2 mm

60. In an interference experiment, 20th order maximum is observed at a point on the screen when light of wavelength 480 nm is used. If this light is replaced by light of wavelength 600 nm, the order of the maximum at the same point will be

(a) 16 (b) 14

(c) 12 (d) 10

61. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?

(a) Diffraction pattern is not observed on the screen in the case of electrons

(b) The angular width of the central maximum of the diffraction pattern will increase

(c) The angular width of the central maximum will decrease

(d) The angular width of the central maximum will remain the same.

62. In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a transparent sheet of thickness t and refractive index μ is introduced in the path of one of the interfering waves. The sheet is then removed and the distance between the screen and the slits is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift when the sheet was introduced. The wavelength of light used is

(a) $\lambda = (\mu - 1)t$ (b) $\lambda = \frac{1}{2}(\mu - 1)t$

(c) $\lambda = (\mu + 1)t$ (d) $\lambda = \frac{1}{2}(\mu + 1)t$

63. In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen 1 m away. The wavelength of light used is 6000 Å. What will be the angular width of a fringe if the entire experimental arrangement is immersed in water? Refractive index of water = $\frac{4}{3}$.

(a) 0.15° (b) 0.18°

(c) 0.2° (d) 0.27°

64. A coherent parallel beam of microwaves of wavelength 0.5 mm falls normally on Young's double slit apparatus. The separation between the slits is 1.0 mm and the screen is placed at a distance of 1.0 m from the slits. The number of minima in the interference pattern observed on the screen is

(a) 3 (b) 4

(c) 5 (d) much greater than 5.

65. In Young's double slit experiment sodium light composed of two wavelengths λ_1 and λ_2 close to each other (with λ_2 greater than λ_1) is used. The order n up to which the fringes can be seen on the screen is given by

(a) $n = \frac{\lambda_2}{\lambda_2 - \lambda_1}$ (b) $n = \frac{\lambda_1}{\lambda_2 - \lambda_1}$

(c) $n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}$ (d) $n = \frac{\lambda_1}{2(\lambda_2 - \lambda_1)}$

66. Monochromatic light of wavelength λ emerging from slit S illuminates slits S_1 and S_2 which are placed with respect to S as shown in Fig. 17.6. The distances x and D are large compared to the separation d between the slits. If $x = D/2$, the minimum value of d so that there is a dark fringe at the centre P of the screen is

(a) $\sqrt{\frac{\lambda D}{3}}$ (b) $\sqrt{\frac{2\lambda D}{3}}$

(c) $\sqrt{\lambda D}$ (d) $2\sqrt{\frac{\lambda D}{3}}$

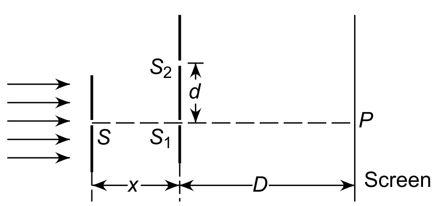


Fig. 17.6

67. Young's double slit experiment is performed by using two coherent light sources each of intensity I. In another experiment, with the same apparatus, the experiment is performed with incoherent sources each of intensity I and the same wavelength. The ratio of the intensity of light at the mid-point of the screen in the first experiment to that in the second experiment is

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) 4

68. A parallel beam of monochromatic light of wavelength 600 nm falls normally on a slit of width 0.5 mm. The resulting diffraction pattern is observed on a screen placed 50 cm from the slit. The linear separation between the first minimum and the first maximum on the same side of the central maximum is

(a) 0.003 mm (b) 0.03 mm

(c) 0.3 mm (d) 3 mm

69. The angular width $\Delta\theta$ of the central maximum in the diffraction pattern of a slit illuminated by light of wavelength 500 nm is measured. The angular width of the central maximum is found to increase by 25% when the same slit is illuminated by another light of wavelength λ'. Wavelength λ' (in nm) is

(a) 375 (b) 450

(c) 575 (d) 625

70. A parallel beam of monochromatic light of wavelength λ falls normally on a narrow slit of width a. The diffraction pattern is observed on a screen placed perpendicular to the direction of the incident beam. The phase difference between the waves coming from the edges of the slit at the position of the first maximum of the diffraction pattern is

(a) 3π (b) $\frac{3\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

71. Blue light is used in a single slit diffraction experiment with a slit of width a = 0.5 mm. If blue light is replaced by yellow light, then

(a) the central maximum becomes narrower

(b) the central maximum becomes broader

(c) more number of fringes are observed

(d) less number of fringes are observed.

72. Young's double slit experiment is performed using light of wavelength 600 nm. When a mica film of refractive index 1.6 and thickness t is introduced in

the path of one of the interfering beams, the intensity of light at the position where the central bright fringe previously appeared remains unchanged. The minimum thickness of the mica film is

(a) 10^{-6} m (b) 10^{-5} m
(c) 10^{-4} m (d) 1 mm

73. Young's double slit experiment is performed using light of wavelength λ. A thin glass plate of refractive index 1.5 and thickness t is now introduced in the path of one of the interfering beams. The value of t so that the central bright fringe is replaced by its neighbouring dark fringe is

(a) $\frac{\lambda}{2}$ (b) λ
(c) $\frac{3\lambda}{2}$ (d) 2λ

74. Monochromatic light of wavelength λ falls normally on a glass film of refractive index μ. The waves reflected from the upper and lower faces of the film will interfere constructively if the least thickness of the film is

(a) $\frac{\lambda}{2\mu}$ (b) $\frac{\lambda}{4\mu}$
(c) $\frac{\mu\lambda}{2}$ (d) $\frac{\mu\lambda}{4}$

75. A beam of light consisting of two wavelengths λ_1 = 750 nm and λ_2 = 450 nm is used in Young's double slit experiment. The separation between the slits is 2 mm and the distance of the screen from the plane of the slits is 100 cm. What is the minimum distance between two successive regions of maximum brightness of two wavelengths?

(a) 1.125 mm (b) 2.250 mm
(c) 3.525 mm (d) 4.750 mm

76. In Q. 75 above, what is the minimum distance between two successive regions of complete darkness.

(a) 1.125 mm (b) 2.250 mm
(c) 3.525 mm (d) 4.750 mm

77. In Young's double slit experiment, the intensity at a point P on the screen is 3/4 of the maximum intensity in the interference pattern. If d is the separation between the slits and λ is the wavelength of light, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ (b) $\cos^{-1}\left(\frac{\lambda}{3d}\right)$
(c) $\tan^{-1}\left(\frac{\lambda}{6d}\right)$ (d) $\sin^{-1}\left(\frac{2\lambda}{3d}\right)$

78. A beam of monochromatic light of intensity I is incident on a rectangular slab $ABCD$ as shown in Fig. 17.7. Each face AB and CD reflects 25% of light intensity incident on it and transmits the rest. The ratio of the maxima to minima in the interference of beams I_1 and I_2 is

(a) 49 : 1 (b) 25 : 16
(c) 16 : 3 (d) 36 : 1

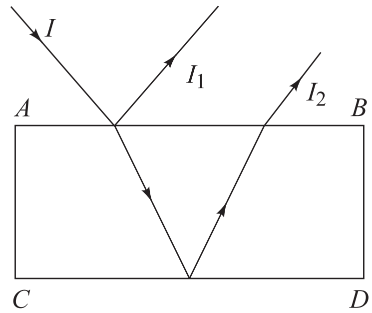


Fig. 17.7

79. In Q. 78, what is the ratio of the maxima and minima in the interference pattern of beams I_3 and I_4 shown in Fig. 17.8 ?

(a) $\frac{5}{3}$ (b) $\frac{16}{7}$
(c) $\frac{25}{9}$ (d) $\frac{39}{11}$

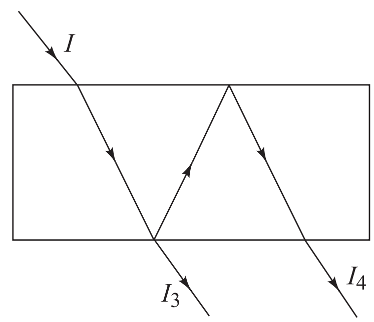


Fig. 17.8

80. In Young's double slit experiment, the separation between the two slits is 2.0 mm. What should be the width of each slit so as to accomodate 10 maxima of the double slit experiment within the central maximum of the single slit diffraction experiment?

(a) 0.1 mm (b) 0.2 mm
(c) 0.4 mm (d) 1.0 cm

Answers

Level A

1. (d)	2. (b)	3. (c)	4. (c)
5. (c)	6. (c)	7. (d)	8. (d)
9. (d)	10. (a)	11. (c)	12. (b)
13. (b)	14. (c)	15. (d)	16. (d)
17. (d)	18. (c)	19. (d)	20. (d)
21. (b)	22. (d)	23. (c)	24. (c)
25. (b)	26. (b)	27. (d)	28. (c)
29. (b)	30. (b)	31. (c)	32. (b)
33. (a)			

Level B

34. (c)	35. (a)	36. (b)	37. (c)
38. (b)	39. (b)	40. (c)	41. (b)
42. (d)	43. (d)	44. (a)	45. (a)
46. (c)	47. (b)	48. (c)	49. (a)
50. (a)	51. (d)	52. (a)	53. (c)
54. (b)	55. (c)	56. (b)	57. (a)
58. (d)	59. (a)	60. (a)	61. (c)
62. (b)	63. (a)	64. (b)	65. (c)
66. (a)	67. (b)	68. (c)	69. (d)
70. (a)	71. (b)	72. (a)	73. (b)
74. (b)	75. (a)	76. (a)	77. (c)
78. (a)	79. (c)	80. (a)	

Solutions

Level A

1. The wavelength of red light is greater than that of yellow light. Since $\beta = \lambda D/d$, the fringe width β will increase. Hence the correct choice is (d).
2. Since $\beta = \lambda D/d$, if D is doubled and d is halved, the fringe width β will become four times. Hence the correct choice is (b).
3. White light consists of colours between violet and red. The wavelength λ is the shortest for violet 1ight and the longest for red light. At the central fringe, the path difference for all colours is zero. Therefore, at the centre of the screen all colours superpose to give a white fringe. Hence the correct choice is (c).
4. The intensity (and hence the amplitude) of the light from the covered slit will decrease resulting in a difference in the intensities of the two virtual sources. Hence the correct choice is (c).
5. If the two sources are coherent, the resultant amplitude at the midpoint of the screen due to interference = $A + A = 2A$. Therefore, intensity is $I_1 \propto (2A)^2$ or $I_1 = k \times 4A^2$ where k is a constant of proportionality. But if the sources are not coherent, their intensities simply add up at the midpoint, i.e.
$$I_2 \propto (A^2 + A^2) \text{ or } I_2 = k \times 2A^2$$
$$\therefore \quad \frac{I_1}{I_2} = \frac{4kA^2}{2kA^2} = 2$$
Hence the correct choice is (c).
6. Since the refractive index of water is greater than that of the air, the speed of the light used in the experiment will be less in water than in air. Since the frequency of light is the same in water and in air, it follows from the relation $\lambda = v/\nu$ that the wavelength λ in water is less than in air. Since fringe width $\beta \propto \lambda$, the value of β will decrease. Hence the correct choice is (c).
7. We then obtain a single slit diffraction pattern on the screen. Hence the correct choice is (d).
8. Independent light sources are incoherent. Hence a permanent interference pattern is not obtained. Thus the correct choice is (d).
9. The correct choice is (d).
10. Path difference $\Delta = \lambda$. Therefore, phase difference $\phi = \frac{2\pi}{\lambda}\Delta = 2\pi$. Hence intensity at a point where $\Delta = \lambda$ or $\phi = 2\pi$ is
$$\begin{aligned} I &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 2\pi \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2} \\ &= I_0 + I_0 + 2I_0 = 4I_0 = K \text{ units} \end{aligned}$$
$$(\because I_1 = I_2 = I_0)$$
i.e. $I = K/4$. The intensity at a point where the path difference is
$$\Delta' = \frac{\lambda}{3} \text{ or } \phi' = \frac{2\pi}{\lambda}\Delta' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3} \text{ is}$$
$$I' = I_0 + I_0 + 2I_0 \cos\frac{2\pi}{3} = 2I_0 - I_0 = I_0 = \frac{K}{4} \text{ units}$$
11. Given $I_1/I_2 = 100$, i.e. $I_1 = 100$ units and $I_2 = 1$ unit. Intensity at maxima is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 100 + 1 + 2\sqrt{100 \times 1} = 121$$

Intensity at minima is

$$I' = I_1 + I_2 - 2\sqrt{I_1 I_2} = 81$$

$$\therefore \quad \frac{I}{I'} = \frac{121}{81} = 1.49$$

Hence the closest choice is (c).

12. Given $\beta = 4.0$ mm and $\lambda = 6000$ Å.

We know that the fringe width is given by

$$\beta = \frac{\lambda D}{d} \quad \text{(i)}$$

for $\lambda' = 4800$ Å, the fringe width will be

$$\beta' = \frac{\lambda' D}{d} \quad \text{(ii)}$$

From (i) and (ii) we have

$$\beta' = \beta \frac{\lambda'}{\lambda} = \frac{4.0\,\text{mm} \times 4800\,\text{Å}}{6000\,\text{Å}} = 3.2 \text{ mm}$$

13. Wavelength in air is $\lambda_a = 6000$ Å. Let its speed in air be v_a. When the apparatus is immersed in a liquid the frequency of the wave remains unchanged but its wavelength and speed both will change. Let λ_l be the wavelength and v_l be the speed in the liquid. Since $v = \nu\lambda$, we have

$$\nu = \frac{v_a}{\lambda_a} = \frac{v_l}{\lambda_l}$$

or
$$\lambda_l = \frac{\lambda_a v_l}{v_a} = \lambda_a \frac{v_l}{c}\frac{c}{v_a}$$

(c is the speed of light in vacuum). Now the refractive index of a medium is $n = c/v$. Hence

$$\lambda_l = \lambda_a \frac{n_a}{n_l} = \frac{\lambda_a}{n_l} \quad (\because n_a = 1)$$

Given $\lambda_a = 6000$ Å and $n_l = 4/3$. Therefore

$$\lambda_l = \frac{6000\,\text{Å}}{4/3} = 4500 \text{ Å}$$

Hence the fringe width in liquid will be

$$\beta_l = \frac{\beta_a \lambda_l}{\lambda_a} = \frac{4.0\,\text{mm} \times 4500\,\text{Å}}{6000\,\text{Å}} = 3.0 \text{ mm}$$

14. The position of the nth bright fringe with respect to the central fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

For the central bright fringe ($n = 0$), $y_0 = 0$. For the fourth bright fringe ($m = 4$), $y_4 = 4\,\lambda D/d$. Therefore

$$y_4 - y_0 = \frac{4\lambda D}{d}$$

or
$$\lambda = (y_4 - y_0)\frac{d}{4D} \quad \text{(i)}$$

It is given that $(y_4 - y_0) = 1.2$ cm $= 1.2 \times 10^{-2}$ m, $D = 1.4$ m and $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m. Substituting these values in (i) and solving, we get

$$\lambda = 6 \times 10^{-7} \text{ m (or 600 nm or 6000 Å)}.$$

15. and 16. Let x be the width of the source slit and X the distance between the source slit and the plane of the two slits. For interference fringes to be distinctly visible, the condition $x/X < \lambda/d$ should be satisfied. If x is too large (i.e. the source slit is too wide) or if X is too small (X is the distance between the source slit and the two slits) the requirement $x/X < \lambda/d$ may be violated and fringes will no longer be distinct. The reason is that the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and fringe pattern becomes indistinct. However, as long as the fringe pattern remains visible, a change in x or X has no effect on the fringe width β.

17. The single slit diffraction effects at the two slits becomes important and as a result, the interference fringe pattern will be modified. The bright fringes will not now be equally bright and equally spaced.

18. The distance of the mth bright fringe from the central fringe is

$$y_n = \frac{n\lambda D}{d} = n\beta$$

where $\beta = \lambda D/d$ is the fringe width.

$$\therefore \quad y_9 = 9\beta \quad \text{(i)}$$

The distance of the mth dark fringe from the central fringe is

$$y'_n = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(n - \frac{1}{2}\right)\beta$$

$$y'_2 = \frac{3}{2}\beta \quad \text{(ii)}$$

From (i) and (ii), we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$$

It is given that $y_9 - y'_2 = 9.0$ mm. Hence

$$\beta = \frac{9.0 \times 2}{15} = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$$

Now $\lambda = \beta d / D$. Substituting for β, d and D, we get

$$\lambda = 6 \times 10^{-7} \text{ m} = 6000 \text{ Å}$$

19. Given $\frac{I_{max}}{I_{min}} = \frac{49}{9}$. Therefore $\frac{A_{max}}{A_{min}} = \sqrt{\frac{49}{9}} = \frac{7}{3}$. Thus A_{max} = 7 units and A_{min} = 3 units. Now $A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$. Therefore, A_1 = 5 units and A_2 = 2 units. Hence

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

Hence the correct choice is (d).

20. A single slit diffraction pattern is characterised by a bright central fringe bordered on both sides with fringes of rapidly decreasing intensity.
21. The wavelength of blue light is less than that of the red light. Hence the angular width of the maxima will decrease which means that the fringes become narrower and crowded together.
22. Only transverse waves can be polarized. Radiowaves, X-rays and waves on strings are transverse and hence they can be polarized. Longitudinal waves such as sound waves or waves in springs cannot be polarized. Hence the correct choice is (d).
23. The correct choice is (c).
24. The angular separation of the minima on both sides of the central maximum is 2θ where θ is given by

$$\sin \theta = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 3 \times 10^{-3}$$

since θ is small, $\sin \theta \simeq \theta$. Therefore, $\theta = 3 \times 10^{-3}$ rad. If the lens is placed close to the slit then

$$x = f \tan \theta \simeq f\theta \quad (\because \theta \text{ is small}, \therefore \tan \theta \simeq \theta)$$

where x is the distance of the first minimum from the central maximum. Therefore, the distance between two minima on both sides of the central maximum is

$$2x = 2f\theta = 2 \times 0.5 \times 3 \times 10^{-3}$$
$$= 3 \times 10^{-3} \text{ m} = 3 \text{ mm}.$$

25. Let the width of each slit be a. The linear separation between m bright fringes in the double slit experiment is

$$y_m = \frac{n\lambda D}{d}$$

Since $y << D$, the angular separation between m bright fringes will be

$$\theta_m = \frac{y_n}{D} = \frac{n\lambda}{d}$$

For 10 bright fringes we have

$$\theta_{10} = \frac{10\lambda}{d} \qquad \text{(i)}$$

Now the angular width of the principal maximum in the diffraction pattern due to a slit of width a is

$$2\theta_1 = \frac{2\lambda}{a} \qquad \text{(ii)}$$

Equating (i) and (ii), we get

$$\frac{10\lambda}{d} = \frac{2\lambda}{a}$$

or $$a = \frac{d}{5} = \frac{1.0 \text{ mm}}{5} = 0.2 \text{ mm}$$

26. The angular width of the central maximum is $2\lambda / a$ where a is the width of the slit. If the value of a is doubled, the angular width of the central maximum decreases to half its earlier value. This implies that the central maximum becomes much sharper. Furthermore if a is doubled, the intensity of the central maximum becomes two times. Thus the central maximum becomes much sharper and brighter.
27. The correct choice is (d).
28. Since the frequency n of the light does not change as light travels from air into glass, we have

$$v_a = n\,\lambda_a \text{ and } v_g = n\,\lambda_g$$

Therefore $\frac{\lambda_a}{\lambda_g} = \frac{v_a}{v_g} = \mu$

Hence the correct choice is (c).

29. Refractive index $\mu = \frac{c}{v}$. Therefore, speed of light in crown glass is

$$v_c = \frac{c}{\mu_c}, \text{ where } c \text{ is the speed of light in vacuum,}$$

or $$c = \mu_c v_c = 1.5 \times 2 \times 10^8 = 3 \times 10^8 \text{ ms}^{-1}$$

$\therefore$ Speed of light in dense flint glass is

$$v_f = \frac{c}{\mu_f} = \frac{3 \times 10^8}{1.8} = 1.67 \times 10^8 \text{ ms}^{-1}$$

30. Speed of light in glass $= \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8$ ms^{-1}.
31. When a wave goes from a denser into a rarer medium, its speed increases. Since frequency n does not change, the wavelength $\lambda = v/n$ increases. Therefore is no phase change on refraction. Hence the correct choice is (c).
32. The refractive index of glass decreases with increase in wavelength. In VIBGYOR, the wavelength of violet light is the shortest and that of the red light is the longest. Hence the correct choice is (b).

33. Let λ_g (in cm) and λ_w (in cm) be the wavelengths in glass and water. By definition, in a distance λ there is one wave. Therefore,

Number of waves in 8 cm of glass = $\frac{8}{\lambda_g}$, and

Number of waves in 10 cm of water = $\frac{10}{\lambda_w}$. Thus

$$\frac{8}{\lambda_g} = \frac{10}{\lambda_w}$$

$$\frac{\lambda_w}{\lambda_g} = \frac{5}{4}$$

Now $\mu_g = \frac{c}{v_g}$ and $\mu_w = \frac{c}{v_w}$.

Therefore, $\frac{\mu_g}{\mu_w} = \frac{v_w}{v_g} = \frac{\nu\lambda_w}{\nu\lambda_g} = \frac{\lambda_w}{\lambda_g}$

or $\mu_g = \frac{\lambda_w}{\lambda_g} \times \mu_w = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}$

Level B

34. $I = I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

When the width of each slit is doubled, I_1 becomes $2I_1$ and I_2 becomes $2I_2$. Therefore,

$$I' = I'_{max} = 2I_1 + 2I_2 + 2\sqrt{2I_1 \times 2I_2}$$

$$= 2(I_1 + I_2 + 2\sqrt{I_1 I_2}) = 2\,I_{max} = 2I.$$

Hence the correct choice is (c).

35. We know that $y_m = \frac{m\lambda D}{d}$. Therefore, for wavelength λ_1,

$$y_1 = \frac{10\lambda_1 D}{d}$$

and for wavelength λ_2,

$$y_2 = \frac{5\lambda_2 D}{d}$$

$$\therefore \quad \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

Hence the correct choice is (a).

36. The correct choice is (b).

37. Angle of incidence = i, angle of refraction $r = 90 - i$. Hence the refractive index is

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90° - i)} = \frac{\sin i}{\cos i} = \tan i.$$

$\therefore \quad \mu = \tan 60° = \sqrt{3}$, which is choice (c).

38. Number of fringes = $\frac{\text{width of region}}{\text{fringes width}}$ or $n = \frac{L}{\beta}$.

Now, fringe width β is proportional to wavelength. Hence the new number of fringes will be

$$n' = n \times \frac{\beta}{\beta'} = 12 \times \frac{600}{400} = 18,$$ which is choice (b).

39. $\beta = \frac{\lambda D}{d}$. Therefore, $\Delta\beta = \frac{\lambda\Delta D}{d}$. Given $\Delta D = 5 \times 10^{-2}$ m, $\Delta\beta = 3 \times 10^{-5}$ m and $d = 10^{-3}$ m. Using these values, we have

$$3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{10^{-3}}$$

which gives $\lambda = 6 \times 10^{-7}$ m which is choice (b).

40. The missing wavelengths are

$$\lambda = \frac{b^2}{(2m+1)d}; \; m = 0, 1, 2, 3, \cdots \text{ etc.}$$

$$= \frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}, \cdots \text{ etc.}$$

Hence the correct choice is (c).

41. Given $I_1 = I$ and $I_2 = 4I$. Now

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= I + 4I + 2\sqrt{4I^2} = 9I$$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 5I - 4I = I$

Hence the correct choice is (b).

42. The angular separation between the mth dark fringe and the central bright fringe is given by

$$a \sin\theta_n = n\lambda$$

For the first dark fringe, $n = 1$. Therefore

$$a \sin\theta_1 = \lambda$$

or $\sin\theta_1 = \lambda/a$. Since $\lambda << a$, $\sin\theta_1 \simeq \theta_1$. Hence $\theta_1 = \lambda/a$. This is also the angular separation between the central bright fringe and the first dark fringe on the other side of the central bright fringe. Hence, the angular separation between the first dark fringes on either side of the central bright fringe = $2\theta_1 = 2\lambda/a$. Therefore, their separation at distance $d = 2$ m is

$$\frac{2\lambda d}{a} = \frac{2 \times (600 \times 10^{-9}\,\text{m}) \times 2\text{ m}}{1.0 \times 10^{-3}\,\text{m}}$$

$$= 2.4 \times 10^{-3}\text{ m} = 2.4\text{ mm},$$

which is choice (d).

43. It follows from Fig. 17.9 that the path difference at the first minimum between rays coming from A and B is

$$\Delta = BC = a \sin \theta_1$$

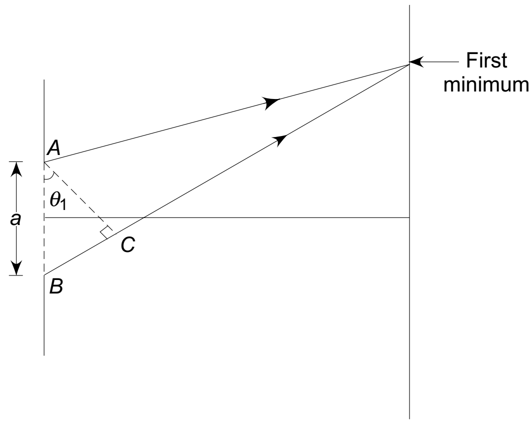


Fig. 17.9

But $\sin \theta_1 = \lambda/a$. Therefore, $\Delta = a \times \lambda/a = \lambda$. A path difference of λ corresponds to a phase difference of 2π, which is choice (d).

44. The wavelength of X-rays is of the order of $1 \text{ Å} \simeq 10^{-10}$ m. Now, the angular width of the central maximum is

$$\theta_0 = \frac{2\lambda D}{a}$$

where D is the distance of the screen from the slit. Since λ for X-rays is very small compared to that for yellow light, it follows that the angular width of the central maximum becomes extremely small. Hence, the central maximum is narrower. Thus the correct choice is (a).

45. In the case when the slits are of equal width, the intensity of light emerging from the two slits is the same, say, I_0. Then

$$I_{max} = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0$$

and $$I_{min} = I_0 + I_0 - 2\sqrt{I_0 I_0} = 0$$

When one slit say S_1 is made twice as wide as the order, the intensity of light from S_1 is doubled, i.e. $I_1 = 2I_0$ but $I_2 = I_0$. Hence, in this case

$$I'_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$
$$= 2I_0 + I_0 + 2\sqrt{2I_0 I_0}$$
$$= 3I_0 + 2\sqrt{2I_0} = 5.83\, I_0$$

and $$I'_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$
$$= 2I_0 + I_0 - 2\sqrt{2I_0 I_0}$$
$$= 3I_0 - 2\sqrt{2I_0} = 0.17I_0$$

Thus $I'_{max} > I_{max}$ and $I'_{min} > I_{min}$, which is choice (a).

46. Given $\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ m} = 6 \times 10^{-7}$ m and $\Delta = 1.5$ microns $= 1.5 \times 10^{-6}$ m. For bright fringes: $\Delta = n\lambda$; where n is an integer.

$n = \dfrac{\Delta}{\lambda} = \dfrac{1.5 \times 10^{-6}}{6 \times 10^{-7}} = \dfrac{5}{2}$, which is not an integer. Hence, path difference of 1.5×10^{-6} m does not correspond to a bright fringe. For dark fringes, we have

$$\Delta = \left(n - \frac{1}{2}\right) \lambda$$

or $$1.5 \times 10^{-6} = \left(n - \frac{1}{2}\right) \times (6 \times 10^{-7})$$

which gives $n - \dfrac{1}{2} = \dfrac{5}{2}$ or $n = 3$. Hence a path difference of 1.5×10^{-6} m corresponds to the third dark fringe. Thus the correct choice is (c).

47. Let λ_a and λ be the wavelengths of yellow light in air and vacuum respectively and v_a and c be their respective speeds in air and vacuum. Since the frequency of light is the same in both media, we have

$$\nu = \frac{v_a}{\lambda_a} = \frac{c}{\lambda}$$

or $\dfrac{c}{v_a} = \dfrac{\lambda}{\lambda_a}$. But $\dfrac{c}{v_a} = \mu_a$ (by defination)

$$\therefore \quad \mu_a = \frac{\lambda}{\lambda_a} \text{ or } \lambda_a = \frac{\lambda}{\mu_a} \quad (1)$$

Now, if t is the thickness of each column, then the number of wavelengths in the two media are $n_a = \dfrac{t}{\lambda_a}$ and $n = \dfrac{t}{\lambda}$. Given $(n_a - n) = 1$. Hence

$$1 = \frac{t}{\lambda_a} - \frac{t}{\lambda} = \frac{t}{\lambda}\left(\frac{\lambda}{\lambda_a} - 1\right) \quad (2)$$

Using (1) in (2), we have

$$1 = \frac{t}{\lambda}(\mu_a - 1) \quad (3)$$

Given $\mu_a = 1.0003$ and $\lambda = 6000 \text{ Å} = 6000 \times 10^{-8}$ cm. Using these values in (3), we have

$$1 = \frac{t}{6000 \times 10^{-8}}(1.0003 - 1)$$

or $t = \dfrac{6000 \times 10^{-8}}{0.0003} = 0.2$ cm = 2 mm

Hence the correct choice is (b).

48. The distance of the nth bright fringe from the central fringe is

$$y_n = n\lambda \frac{D}{d} = n\beta$$

where $\beta = \dfrac{\lambda D}{d}$ is the fringe width.

$$\therefore \quad y_n = 9\beta \qquad (1)$$

The distance of the mth dark fringe from the central fringe is

$$y'_n = \left(m - \frac{1}{2}\right)\frac{\lambda D}{d} = \left(m - \frac{1}{2}\right)\beta$$

$$y'_2 = \frac{3}{2}\beta \qquad (2)$$

From Eqs. (1) and (2) we get

$$y_9 - y'_2 = 9\beta - \frac{3}{2}\beta = \frac{15}{2}\beta$$

$$= \frac{15}{2} \times 2.0 \text{ mm} = 15 \text{ mm}$$

Hence the correct choice is (c).

49. The position of the 30th bright fringe is given by

$$y_{30} = 30\,\frac{\lambda D}{d}$$

Hence the shift of the central fringe is

$$y_0 = 30\,\frac{\lambda D}{d}$$

But $y_0 = \dfrac{D}{d}(\mu - 1)t$

$$\therefore \quad 30\,\frac{\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

or $(\mu - 1) = \dfrac{30\lambda}{t} = \dfrac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5$

$$\therefore \quad \mu = 1.5$$

50. Let the nth bright fringe of wavelength λ_n and the mth bright fringe of wavelength λ_m coincide at a distance y from the certral maximum, then

$$y = \frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d}$$

or $\dfrac{m}{n} = \dfrac{\lambda_n}{\lambda_m} = \dfrac{6500}{5200} = \dfrac{5}{4}$

The least integral values of m and n which satisfy the above condition are

$$m = 5 \text{ and } n = 4$$

i.e., the 5th bright fringe of wavelength 5200 Å coincides with the 4th bright fringe of wavelength 6500 Å. The smallest value of y at which this happens is

$$y_{\min} = \frac{m\lambda_m D}{d}$$

Substituting the values of m, λ_m, D and d, we get $y_{\min}$ = 1.0 mm, which is choice (a).

51. Given $\dfrac{I_1}{I_2} = n$. Therefore, the amplitude ratio is

$$\frac{A_1}{A_2} = \sqrt{n}$$

Now $I_{\max} = (A_1 + A_2)^2$ and $I_{\min} = (A_1 - A_2)^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\dfrac{A_1}{A_2} + 1\right)^2}{\left(\dfrac{A_1}{A_2} - 1\right)^2}$$

$$= \frac{\left(\sqrt{n} + 1\right)^2}{\left(\sqrt{n} - 1\right)^2}$$

Hence the correct choice is (d).

52. Given $\dfrac{I_{\max}}{I_{\min}} = n$. Hence

$$\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = n$$

or $\dfrac{(A_1 + A_2)}{(A_1 - A_2)} = \sqrt{n}$

which gives $\dfrac{A_1}{A_2} = \dfrac{\sqrt{n} + 1}{\sqrt{n} - 1}$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1}\right)^2$$

Hence the correct choice is (a).

53. The intensity of light emerging from a slit is proportional to its width. Since the amplitude is proportional to the square-root of the intensity, we have

$$\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{n}{1}} = \sqrt{n}$$

As shown in solution of Q. 51, the correct choice is (c).

54. The correct choice is (b). Refer to the solution of Q. 51.

55. Angular width of a fringe is given by

$$\theta = \frac{\lambda}{d} \quad \text{or} \quad d = \frac{\lambda}{\theta} \qquad (1)$$

Given λ = 628 nm = 628 × 10^{-9} m and θ = 0.1° = $\frac{0.1\times\pi}{180}$ rad. Using these values in Eq. (1), we find that $d = 3.6 \times 10^{-4}$ m. Hence the correct choice is (c).

56. The correct choice is (d). Use

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\phi$$

Put $\quad I_1 = I, I_2 = 4I \quad$ and $\quad \phi = \frac{\pi}{2}$.

57. If t is the thickness of the glass sheet, the fringes are displaced by an amount given by

$$\Delta y = (\mu - 1)\,\frac{tD}{d}$$

In order to bring the adjacent minimum to the centre of the screen (i.e. to bring the first dark fringe the central bright fringe), the fringes must be displaced by half the fringe width, i.e.

$$\Delta y = \frac{\beta}{2} = \frac{1}{2}\,\frac{\lambda D}{d}$$

Hence $(\mu - 1)\,\frac{tD}{d} = \frac{1}{2}\,\frac{\lambda D}{d}$

or $\quad t = \frac{\lambda}{2(\mu-1)}$, which is choice (a).

58. When a transparent plate of thickness t and refractive index μ is introduced in one of the interfering waves, the path difference at the centre of the screen is

$$\Delta = (\mu - 1)\,\frac{tD}{d}$$

$\therefore$ Phase difference $\quad \phi = \frac{2\pi\Delta}{\lambda}$

$$= \frac{2\pi}{\lambda}(\mu - 1)\,\frac{tD}{d} \qquad (1)$$

Given λ = 5000 Å = 5 × 10^{-7} m, μ = 1.5, t = 1.5 × 10^{-6} m, D = 1 m and $d = 5 \times 10^{-4}$ m. Using these values in Eq. (1). we get $\phi = 3\pi$. If I is the intensity of each interfering wave, the resultant intensity at the centre of the screen is

$$I_r = I + I + 2\sqrt{I \times I}\,\cos 3\pi$$

$$= 2I - 2I = 0 \qquad (\because \cos 3\pi = -1)$$

Hence the intensity at the centre is zero, i.e. there is a dark fringe at the centre.

Hence the correct choice is (d).

59. The correct choice is (a). The lateral shift is given by

$$\Delta y = (\mu - 1)\,\frac{tD}{d}$$

60. The position of the nth order maximum is given by

$$y_n = \frac{n\lambda D}{d}$$

For a given point y_n is fixed. Since D and d are also fixed, $n\lambda$ = constant, i.e. $n_1\lambda_1 = n_2\lambda_2$. Hence $n_2 = n_1\frac{\lambda_1}{\lambda_2} = \frac{20\times 480}{600}$ = 16, which is choice (a).

61. De Broglie wavelength of electron is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

If speed v of electron is increased, momentum $p(= mv)$ will increase. Hence wavelength λ will decrease. Now, the angular width of the central maximum of the diffraction pattern is 2θ where θ is given by

$$\sin\theta = \frac{\lambda}{a}$$

where a is the width of the slit. Thus, if λ decreases, θ and hence 2θ will decrease. Therefore, the correct statement is (c).

62. When a sheet of thickness t and refractive index μ is introduced in one of the interfering waves, the distance y_0 through which the fringes shift is given by

$$y_0 = (\mu - 1)t\,\frac{D}{d} \qquad (1)$$

The fringe width β, i.e., the distance between successive maxima (or minima) is given by

$$\beta = \frac{\lambda D}{d}$$

When the distance D between the slits and the screen is doubled, the new fringe width becomes

$$\beta' = 2\,\frac{\lambda D}{d} \qquad (2)$$

It is given that $y_0 = \beta'$. Equating Eqs. (1) and (2) we get

$$2\,\frac{\lambda D}{d} = (\mu - 1)\,t\frac{D}{d}$$

$$\lambda = \frac{1}{2}\,(\mu - 1)t$$

which is choice (b)

63. The angular width of a fringe is given by

$$\theta = \frac{\lambda}{d}$$

In air: $\quad \theta_a = \frac{\lambda_a}{d}$

In water: $\theta_w = \dfrac{\lambda_w}{d}$

$$\therefore \quad \frac{\theta_w}{\theta_a} = \frac{\lambda_w}{\lambda_a} \text{ or } \theta_w = \frac{\lambda_w \theta_a}{\lambda_a} \quad (1)$$

Now, refractive index is defined as

$$\mu_w = \frac{\text{speed of light in air}}{\text{speed of light in water}}$$

$$= \frac{v_a}{v_w} = \frac{\nu \lambda_a}{\nu \lambda_w} = \frac{\lambda_a}{\lambda_w} \quad (2)$$

where ν is the frequency of light which remains unchanged.

Using Eq. (2) in Eq. (1) we have

$$\theta_w = \frac{\theta_a}{\mu_w} = \frac{0.2°}{4/3} = 0.15°$$

So the correct choice is (a).

64. Refer to Fig. 17.10. When the incident beam falls normally on the slits S_1 and S_2, the path difference at the central point P_0 of the screen is zero. Hence we have the central maximum at P_0.

Let the minima appear along directions θ with respect to the incident direction. Coherent waves from S_1 and S_2 along this direction are brought to a focus at P. It is clear that the path difference between the waves from S_1 and S_2 on reaching P is

$$\Delta = d \sin \theta$$

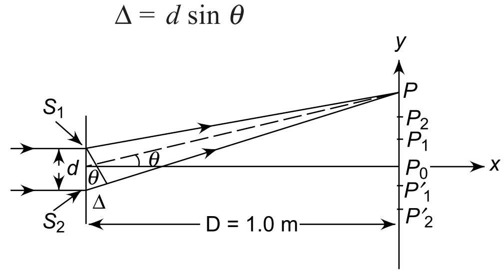


Fig. 17.10

The interference minima will appear on the screen if

$$\Delta = \left(n + \frac{1}{2}\right)\lambda$$

or $\quad d \sin \theta = \left(n + \frac{1}{2}\right)\lambda \; ; n = 0, \pm 1, \pm 2, \ldots$

Thus the directions of minima are given by

$$\sin \theta = \left(n + \frac{1}{2}\right) \times \left(\frac{\lambda}{d}\right)$$

Given $d = 1.0$ mm and $\lambda = 0.5$ mm. Therefore

$$\sin \theta = \left(n + \frac{1}{2}\right) \times \left(\frac{0.5}{1.0}\right) = \frac{1}{2}\left(n + \frac{1}{2}\right)$$

The allowed values of m are those integers for which $\sin \theta$ is not more than + 1 or less than – 1. These values are $n = 1, 0, -1$ and -2. Hence four minima will be observed. The correct choice is (b).

65. If an interference experiment is performed using two wavelengths close to each other, two interference patterns corresponding to the two wavelengths are obtained on the screen. The fringe system remains distinct upto a point on the screen where the nth order maximum of one wavelength, say $\lambda_1 = 5890$ Å falls on the nth order minimum of the other wavelength $\lambda_2 = 5895$ Å. Thus, interference pattern can be seen upto a distance y_n from the centre of the screen if

$$y_n = \frac{n \lambda_1 D}{d}; (n\text{th maximum}) \quad (1)$$

$$= \left(n - \frac{1}{2}\right)\frac{\lambda_2 D}{d}; (n\text{th minimum}) \quad (2)$$

or $\quad n\lambda_1 = \left(n - \frac{1}{2}\right)\lambda_2$ or $2n\lambda_1 = (2n - 1)\lambda_2$

which gives

$$n = \frac{\lambda_2}{2(\lambda_2 - \lambda_1)}, \text{ which is choice (c).}$$

66. Refer to Fig. 17.11. To reach point P, wave 1 has to travel a path $(SS_2 + S_2P)$ while wave 2 has to travel a path $(SS_1 + S_1P)$. Therefore, when the waves arrive at P, the path difference is

$$\Delta = (SS_2 + S_2P) - (SS_1 + S_1P) \quad (1)$$

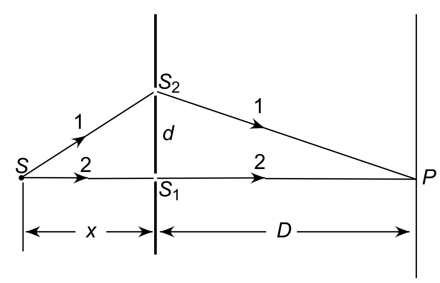


Fig. 17.11

Now, in triangle SS_2S_1, we have

$$SS_2 = (x^2 + d^2)^{1/2} = x\left(1 + \frac{d^2}{x^2}\right)^{1/2}$$

$$= x\left(1 + \frac{d^2}{2x^2}\right) \quad (\because d << x)$$

Similarly, $S_2P = (D^2 + d^2)^{1/2} = D\left(1 + \dfrac{d^2}{2D^2}\right)$

$(\because d << D)$

Also $(SS_1 + S_1P) = x + D$. Using these in Eq. (1), we have

$$\Delta = x\left(1+\frac{d^2}{2x^2}\right)+D\left(1+\frac{d^2}{2D^2}\right)-(x+D)$$

$$= x + \frac{d^2}{2x} + D + \frac{d^2}{2D} - x - D$$

or $$\Delta = \frac{d^2}{2}\left(\frac{1}{x}+\frac{1}{D}\right)$$

In order to have a dark fringe at P, $\Delta = \frac{\lambda}{2}$. Hence

$$\frac{\lambda}{2} = \frac{d^2}{2}\left(\frac{1}{x}+\frac{1}{D}\right) \quad (1)$$

or $$d = \left[\frac{\lambda xD}{(x+D)}\right]^{1/2} \quad (2)$$

Putting $x = \frac{D}{2}$ in Eq. (2), we find that the correct choice is (a).

67. The intensity at the mid-point of the screen is maximum. In the first experiment,

$$I_{max} = 4I$$

It the sources of incoherent, then the intensity at the mid-point = $I + I = 2I$. So the correct choice is (b).

68. For first minimum, (a = width of the slit)

$$a \sin \theta_1 = \lambda$$

$$\Rightarrow \quad \sin \theta_1 = \frac{\lambda}{a} \quad \Rightarrow \quad \theta_1 = \frac{\lambda}{a} \qquad (\because \quad \lambda << a)$$

For first maximum,

$$a \sin \theta_2 = \frac{3\lambda}{2} \quad \Rightarrow \quad \theta_2 = \frac{3\lambda}{2a}$$

Angular separation between them is

$$\Delta\theta = \theta_2 - \theta_1 = \frac{3\lambda}{2a} - \frac{\lambda}{a} = \frac{\lambda}{2a}$$

$\therefore$ Linear separation is

$$\Delta x = D\Delta\theta = \frac{D\lambda}{2a}$$

$$= \frac{0.5 \times (600 \times 10^{-9})}{2 \times (0.5 \times 10^{-3})} = 0.3 \text{ mm}$$

So the correct choice is (c).

69. $\Delta\theta = \frac{2\lambda}{a}$

$$\Delta\theta' = \frac{2\lambda'}{a}$$

Given $\Delta\theta' = \Delta\theta + 25\%$ of $\Delta\theta$

$$= \Delta\theta + \frac{25}{100}\Delta\theta = 1.25\,\Delta\theta$$

$$\therefore \quad \frac{\lambda'}{\lambda} = \frac{\Delta\theta'}{\Delta\theta} = 1.25$$

$$\Rightarrow \quad \lambda' = 1.25\,\lambda = 1.25 \times 500 \text{ nm} = 625 \text{ nm.}$$

So the correct choice is (d).

70. The path difference between the waves emerging from the edges of the slit on reaching the position of the first maximum is

$$\Delta x = \frac{3\lambda}{2}$$

Now phase difference $\Delta\phi$ is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{3\lambda}{2} = 3\pi$$

So the correct choice is (a).

71. The angular width of the central maximum is (since $a << \lambda$)

$$\Delta\theta = \frac{2\lambda}{a}$$

Since λ for yellow light is greater than that for blue light, $\Delta\theta$ increases. Hence the central maximum becomes broader. So the correct choice is (b).

72. Refer to Example 7 on page 17.6. When a transparent film is introduced in the path of one of the interfering beams, the entire fringe pattern shifts by an amount $\Delta y = (\mu - 1)\,t$. Since the path difference must change by λ for one bright fringe to be replaced by its neighbouring fringe, we have $\Delta y = \lambda \quad \Rightarrow \quad (\mu - 1)\,t = \lambda$ which gives

$$t_{min} = \frac{\lambda}{\mu - 1} = \frac{600 \times 10^{-9}}{(1.6 - 1)} = 10^{-6} \text{ m}$$

So the correct choice is (a).

73. In this case, $\Delta y = \frac{\lambda}{2}$ or

$$(\mu - 1)\,t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2\,(\mu - 1)} = \frac{\lambda}{2\,(1.5 - 1)} = \lambda$$

So the correct choice is (b).

74. Refer to Example 8 on page 17.7. The correct choice is (b).

75. Refer to Example 6 on page 17.6.

$$y_1 = \frac{n\lambda_1 \Delta}{d} \qquad (1)$$

$$y_2 = \frac{m\lambda_2 \Delta}{d} \qquad (2)$$

The waves will produce maximum brightness at the screen where $y_1 = y_2$, *i.e.*,

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d}$$

$$\Rightarrow \qquad \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{450 \text{ nm}}{750 \text{ nm}} = \frac{3}{5}$$

The minimum integral values of n and m which satisfy this condition are $n_1 = 3$ and $m_1 = 5$ and the next values are $n_2 = 6$ and $m_2 = 10$. So, we get the first region of maximum brightness when the 3rd bright fringe of λ_1 falls on the 5th bright fringe of λ_2.

$$(\Delta y)_1 = \frac{3\lambda_1 D}{d} = \frac{5\lambda_2 D}{d}$$

$$= \frac{3 \times (750 \times 10^{-9}) \times 1.0}{2 \times 10^{-3}} = 1.125 \text{ mm}$$

We get the next region of maximum brightness when the 6th bright fringe of λ_1 falls on the 10th bright fringe of λ_2

$$(\Delta y)_2 = \frac{6\lambda_1 D}{d} = \frac{10\lambda_2 D}{d}$$

$$= \frac{6 \times (750 \times 10^{-9}) \times 1.0}{2 \times 10^{-3}} = 2.25 \text{ mm}$$

$$(\Delta y)_{\min} = (\Delta y)_2 - (\Delta y)_1$$

$$= 2.25 - 1.125$$

$$= 1.125 \text{ mm, which is choice (a).}$$

76. In this case, for dark fringes,

$$y_1 = \left(n - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$y_2 = \left(m - \frac{1}{2}\right)\frac{\lambda_2 D}{d}$$

The waves will produce complete darkness at a point on the screen where $y_1 = y_2$, *i.e.*,

$$\left(n - \frac{1}{2}\right)\frac{\lambda_1 D}{d} = \left(m - \frac{1}{2}\right)\frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{\left(n - \frac{1}{2}\right)}{\left(m - \frac{1}{2}\right)} = \frac{\lambda_2}{\lambda_1} = \frac{450 \text{ nm}}{750 \text{ nm}} = \frac{3}{5}$$

$$\Rightarrow \qquad n = \frac{6m + 2}{10}$$

The minimum integral values of n and m which satisfy this condition are $m_1 = 3$ and $n_1 = 2$. The next values are $m_2 = 8$ and $n_2 = 5$. So we get the first region of complete darkness when 2nd dark fringe of λ_1 falls on the 3rd dark fringe of λ_2,

$$(\Delta y)_1 = \left(2 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

The next region of complete darkness occurs when the 5th dark fringe of λ_1 falls on 8 dark fringe of λ_2,

$$(\Delta y)_2 = \left(5 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$\therefore \quad (\Delta y)_{\min} = \left(5 - \frac{1}{2}\right)\frac{\lambda_1 D}{d} - \left(2 - \frac{1}{2}\right)\frac{\lambda_1 D}{d}$$

$$= 3\frac{\lambda_1 D}{d}$$

$$= \frac{3 \times (750 \times 10^{-9}) \times 1.0}{2 \times 10^{-3}}$$

$$= 1.125 \text{ mm}$$

So the correct choice is (a).

77. In Young's double slit experiment,

$$I = 4I_0 \cos^2 (\phi/2)$$

where ϕ is the phase difference between the interfering beams.

$$I_{\max} = 4I_0$$

Given $\qquad I = \frac{3}{4} I_{\max}$

or $\qquad 4I_0 \cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4} I_{\max}$

$$\Rightarrow \quad \cos^2\left(\frac{\phi}{2}\right) = \frac{3}{4}$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6} \quad \text{or } \phi = \frac{\pi}{3}$$

Now refer to Fig. 17.12.

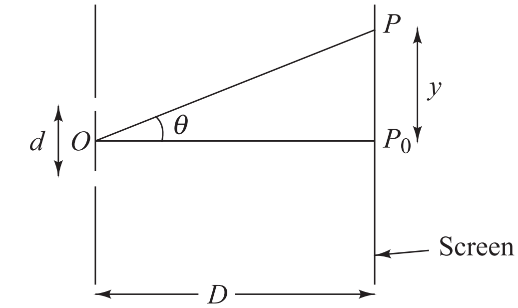


Fig. 17.12

If θ is the angular separation between P and P_0,

$$\tan\theta = \frac{y}{D}$$

If β is the fringe width between two consecutive maxima, than $\beta = 2\pi$. Hence for point P

$$\frac{y}{\beta} = \frac{\pi/3}{2\pi} = \frac{1}{6}$$

or $$\frac{yd}{D\lambda} = \frac{1}{6}$$

or $$\tan\theta \times \frac{d}{\lambda} = \frac{1}{6}$$

or $$\tan\theta = \frac{\lambda}{6d} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{\lambda}{6d}\right)$$

So the correct choice is (c).

78. As shown in Fig. 17.13, the intensity of the reflected beam at face AB is

$$I_1 = I \times \frac{25}{100} = \frac{I}{4}$$

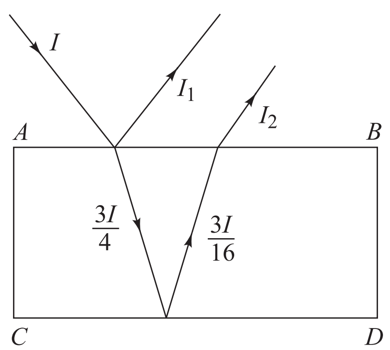


Fig. 17.13

The remaining intensity $\frac{3I}{4}$ falls on face CD which reflects 25% of this intensity incident on it which is equal to $\frac{3I}{4} \times \frac{25}{100} = \frac{3I}{16}$. This intensity falls on face AB which transmits 75% of it. Hence the intensity of I_2 is

$$I_2 = \frac{3I}{16} \times \frac{75}{100} = \frac{9I}{64}$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= \frac{I}{4} + \frac{9I}{64} + 2\sqrt{\frac{I}{4} \times \frac{9I}{64}} = \frac{49}{64}$$

and $$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= \frac{I}{4} + \frac{9I}{64} - 2\sqrt{\frac{I}{4} \times \frac{9I}{64}} = \frac{1}{64}$$

$\therefore$ $$\frac{I_{max}}{I_{min}} = 49, \text{ which is choice (a).}$$

79. Using the arguments given in Solution 78 above, we have

$$I_3 = \frac{9I}{16} \qquad (1)$$

and $$I_4 = \frac{9I}{254} \qquad (2)$$

$$I_{max} = I_3 + I_4 + 2\sqrt{I_3 I_4} \qquad (3)$$

$$I_{min} = I_3 + I_4 - 2\sqrt{I_3 I_4} \qquad (4)$$

Using (1) and (2) in (3) and (4) we get

$$I_{max} = \left(\frac{9 \times 25}{254}\right) I$$

and $$I_{min} = \left(\frac{9 \times 9}{254}\right) I$$

$\therefore$ $$\frac{I_{max}}{I_{min}} = \frac{25}{9}, \quad \text{which is choice (c).}$$

80. In Young's double slit experiment, the angular separation between 10 maxima is

$$\theta_1 = \frac{10\lambda}{d}$$

In single slit diffraction experiment, the angular width of the central maximum is

$$\theta_2 = \frac{2\lambda}{a}, \qquad (a = \text{width of each slit})$$

Given $\theta_1 = \theta_2$, which requires

$$\frac{10\lambda}{d} = \frac{2\lambda}{a}$$

$$\Rightarrow \quad a = \frac{2d}{10} = \frac{2 \times 2 \text{ mm}}{10} = 0.4 \text{ mm}$$

which is choice (c).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1 to 3 are based on the following passage.

Passage I

In a modified Young's double slit experiment, a monochromatic and parallel beam of light of wavelength 6000 Å and intensity $\frac{10}{\pi}$ Wm^{-2} is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength 6000 Å is placed in front of aperture A (Fig. 17.14).

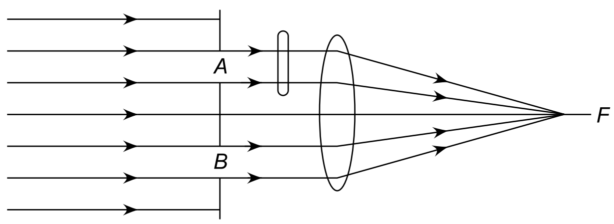


Fig. 17.14

1. The ratio of the powers received at aperture A to that at aperture B is
   (a) 1 : 2 (b) 1 : 4
   (c) 1 : 8 (d) 1 : 16
2. The phase difference between the interfering waves at point F is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
3. If 10% of the power received by each aperture goes in the original direction, the resultant power at point F will be
   (a) 5 μW (b) 6 μW
   (c) 7 μW (d) 8 μW

Solutions

1. Intensity of the beam (I) = $\frac{10}{\pi}$ Wm^{-2}

   Power received at aperture $A = I \times$ cross-sectional area of A

   $$= \frac{10}{\pi} \times \pi \times (0.001)^2 = 10^{-5} \text{ W}$$

   Power received at aperture $B = \frac{10}{\pi} \times \pi \times (0.002)^2$

   $$= 4 \times 10^{-5} \text{ W}$$

   So the correct choice is (b).
2. The phase difference at F is

   $$\delta = (\mu - 1) \times t \times \frac{2\pi}{\lambda}$$

   $$= \frac{(1.5 - 1) \times (2000 \times 10^{-8}) \times 2\pi}{(6000 \times 10^{-8})} = \frac{\pi}{3} \text{ rad}$$

   The correct choice is (b).
3. Since 10% the power received at each aperture goes in the original direction, the power at point F due to the two apertures respectively is

   $$P_A = 10\% \text{ of } 10^{-5} \text{ W} = 10^{-6} \text{ W}$$
   $$P_B = 10\% \text{ of } 4 \times 10^{-5} \text{ W} = 4 \times 10^{-6} \text{ W}$$

   Now, intensity (and hence power) is proportional the square of the amplitude. If A_1 and A_2 are the amplitudes at F due to the two sources, we have $P_A = kA^2{}_1$ and $P_B = kA^2{}_2$, where k is the proportionality constant. Thus

   $$A_1 = \sqrt{\frac{P_A}{k}} \text{ and } A_2 = \sqrt{\frac{P_B}{k}}$$

   Resulting amplitude

   $$A = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos \delta}$$

   Substituting the values of A_1, A_2 and δ, we get

   $$A = \sqrt{\frac{7 \times 10^{-6}}{k}}$$

   ∴ Resultant power at $F = kA^2$

   $$= k \times \frac{7 \times 10^{-6}}{k} = 7 \times 10^{-6} \text{ W}$$

   Hence the correct choices is (c).

Questions 4 to 6 are based on the following passage.

Passage II

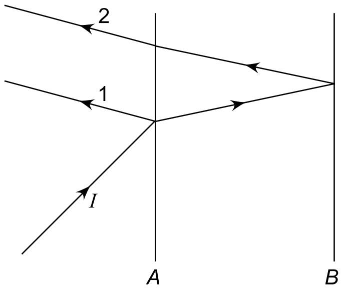


Fig. 17.15

A narrow monochromatic beam of light of intensity I is incident on a glass plate A as shown in Fig. 17.15. Another identical glass plate B is kept close to A and parallel to it. Each plate reflects 25% of the light intensity incident on it and transmits the remaining. Interference pattern is formed by beams 1 and 2 obtained after reflection at each plate.

4. The intensity of beam 2 is
   (a) $\frac{3I}{16}$ (b) $\frac{3I}{32}$
   (c) $\frac{9I}{32}$ (d) $\frac{9I}{64}$
5. The ratio of the intensities of beams 1 and 2 is
   (a) $\frac{16}{9}$ (b) $\frac{4}{3}$
   (c) $\frac{25}{16}$ (d) $\frac{5}{4}$
6. The ratio of the maximum and minimum intensities in the interference pattern is
   (a) 16 : 1 (b) 25 : 1
   (c) 36 : 1 (d) 49 : 1

Solutions

4. A beam of light of intensity I is incident on plate A. Since the plate reflects 25% of I, the intensity of the reflected beam 1 (see Fig.17.16) is

$$I_1 = I \times \frac{25}{100} = \frac{I}{4}$$

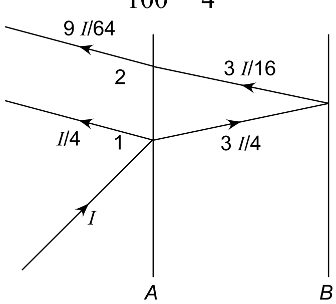


Fig. 17.16

The remaining intensity $3I/4$ falls on plate B which reflects 25% of the intensity incident on it. Hence intensity of beam reflected from B is

$$\frac{3I}{4} \times \frac{25}{100} = \frac{3I}{16}$$

A beam of intensity $3I/16$ falls on plate A which transmits 75% of this intensity. Hence the intensity of beam 2 is

$$I_2 = \frac{3I}{16} \times \frac{75}{100} = \frac{9I}{64}$$

So the correct choice is (d).

5. $\frac{I_1}{I_2} = \frac{I/4}{9I/64} = \frac{16}{9}$ which is choice (a).

6. The ratio of amplitudes is $\frac{a_1}{a_2} = \sqrt{\frac{16}{9}} = \frac{4}{3}$ Thus . a_1 = 4 units and a_2 = 3 units.

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+3}{4-3}\right)^3 = 49$$

Thus the correct choice is (d).

Questions 7 to 9 are based on the following passage.

Passage III

A monochromatic light of wavelength 5000 Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is observed on a screen placed at a distance of 1 m from the slits. A glass plate of thickness 1.5×10^{-6} m and refractive index 1.5 is introduced between one of the slits and the screen.

7. Due to the introduction of the glass plate, the phase difference between the interfering waves at the centre of the screen is equal to
   (a) $\pi/2$ (b) π
   (c) 2π (d) 3π
8. If I_0 is the intensity at the centre of the screen before the plate is introduced, the intensity at the centre after the plate is introduced
   (a) remains equal to I_0 (b) becomes less than I_0
   (c) becomes greater than I_0 (d) becomes equal zero.
9. The lateral shift of the central maximum is
   (a) 2 mm (b) 3 mm
   (c) 4 mm (d) 5 mm

Solutions

7. When a transparent plate of thickness t and refractive index μ is introduced in one of the interfering waves, the path difference at the centre of the screen is

$$\Delta = (\mu - 1)\ \frac{tD}{d}$$

$\therefore$ Phase difference $\phi = \dfrac{2\pi\Delta}{\lambda}$

$$= \frac{2\pi}{\lambda}\ (\mu - 1)\ \frac{tD}{d} \qquad (1)$$

Given $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m, $\mu = 1.5$, $t = 1.5 \times 10^{-6}$ m, $D = 1$ m and $d = 5 \times 10^{-4}$ m. Using these values in Eq. (1), we get $\phi = 3\pi$. So the correct choice is (d).

8. If I is the intensity of each interfering wave, the resultant intensity at the centre of the screen is

$$I_r = I + I + 2\sqrt{I \times I}\ \cos 3\pi$$
$$= 2I - 2I = 0 \qquad (\because \cos 3\pi = -1)$$

Hence the intensity at the centre is zero, i.e. there is a dark fringe at the centre.The correct choice is (d).

9. The lateral shift of the central maximum is given by

$$y_0 = (\mu - 1)\ \frac{tD}{d}$$

Substituting the values of μ, t, D and d, we get $y_0 = 5 \times 10^{-3}$ m $= 5$ mm,which is choice (d).

Questions 10 to 13 are based on the following passage.

Passage IV

A glass plate of refractive index $\mu_3 = 1.5$ is coated with a thin layer of thickness t and refractive index $\mu_2 = 1.8$. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere.

10. The two reflected waves interfere constructively if (n is an integer)

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2(\mu_2 - \mu_3)}$

(c) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_3}$

11. If $\lambda = 648$ nm, the least value of t for which the waves interface constructively is

(a) 90 nm (b) 180 nm
(c) 108 nm (d) 216 nm

12. The two reflected waves interfere destructively if

(a) $t = \dfrac{n\lambda}{2\mu_2}$ (b) $t = \dfrac{n\lambda}{2\mu_3}$

(c) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$ (d) $t = \dfrac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_3}$

13. If $\lambda = 648$ nm, the least value of t for which the waves interfere destructively is

(a) 90 nm (b) 180 nm
(c) 108 nm (d) 216 nm

Solutions

10. Refer to Fig. 17.17. A ray of light travelling in air ($\mu_1 = 1$) falls normally on a thin layer ($\mu_2 = 1.8$) of thickness t. It is partly reflected at point P as wave 1 and partly refracted as wave 2. Wave 2 on meeting the surface of the glass plate ($\mu_3 = 1.5$) is reflected at point Q and travels along QP.

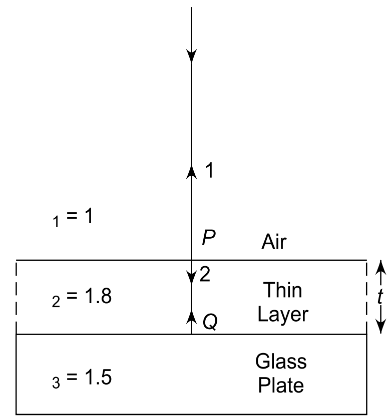


Fig. 17.17

Waves 1 and 2 meet at point P where they interfere. We know that when a wave is travelling in a rarer medium and gets reflected at the boundary of a denser medium, it undergoes a phase change of π or a path change of $\lambda/2$. Thus wave 1 has an optical path of $\Delta_1 = \lambda/2$. Wave 2 travelling from P to Q in the layer of refractive index 1.8 gets reflected at Q from the boundary of glass of refractive index 1.5. Thus wave 2 travelling in a denser medium is reflected from the boundary of a rarer medium undergoes no phase change due to reflection. Therefore,

Optical path for wave 2 from P to Q and from Q to P in the layer is

$$\Delta_2 = \text{refractive index of layer} \times 2(PQ)$$
$$= \mu_2 \times 2t = 2\mu_2 t$$

$\therefore$ Optical path difference between waves 1 and 2 at point p is

$$\Delta = \Delta_2 - \Delta_1 = 2\mu_2 t - \frac{\lambda}{2}$$

Now, for constructive interference, $\Delta = n\lambda$; $n = 0, 1, 2, \ldots$

or $$2\mu_2 t - \frac{\lambda}{2} = n\lambda \text{ or } 2\mu_2 t = \left(n + \frac{1}{2}\right)\lambda$$

or $$t = \frac{\left(n + \frac{1}{2}\right)\lambda}{2\mu_2}$$

So the correct choice is (c).

11. The minimum value of t corresponds to $n = 0$. Hence
$$t_{\min} = \frac{\lambda}{4\mu_2} = \frac{648 \text{ nm}}{4 \times 1.8} = 90 \text{ nm}.$$
So the correct choice is (a)

12. For destructive interference $\Delta = \left(n - \frac{1}{2}\right)\lambda$. Hence
$$2\mu_2 t - \frac{\lambda}{2} = \left(n - \frac{1}{2}\right)\lambda$$
which gives $t = \frac{n\lambda}{2\mu_2}$, which is choice (a).

13. The minimum value of t corresponds to $n = 1$. Hence
$$t_{\min} = \frac{\lambda}{2\mu_2} = \frac{648\text{nm}}{2 \times 1.8} = 180 \text{ nm}$$
Thus the correct choice is (b).

SECTION 3 Assertion-Reason Type Questions

In the following questions, Statement-1 (Assertion) is followed by Statement-2 (Reason). Each question has the following four choices out of which only one choice is correct.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

1. Statement-1

Red light travels faster in glass than green light.

Statement-2

The refractive index of glass is less for red light than for green light.

2. Statement-1

In Young's double slit experiment, if the width of the source slit is increased, the fringe pattern becomes indistinct.

Statement-2

The angular width of interference maxima increases if the width of the source slit is increased.

3. Statement-1

In a single slit diffraction experiment, if the width of the slit is increased, the diffraction maxima become sharper and brighter.

Statement-2

The angular width the diffraction maxima is inversely proportional to the width of the slit.

4. Statement-1

When light travels from a rarer to a denser medium, its speed decreases.

Statement-2

Energy carried by the refracted light is reduced.

5. Statement-1

When a light wave travels from one medium to another, its frequency remains unchanged.

Statement-2

The speed of the wave undergoes a change.

6. Statement-1

When a light wave is reflected from a mirror, it undergoes a phase change of π.

Statement-2

The direction of the propagation of light is changed due to reflection.

Solutions

1. The correct choice is (a). Refractive index of a medium is defined as
$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$$
The refractive index of glass is less for light of longer wavelength. The wavelength of red light is more than that of green light. Hence $\mu_R < \mu_G$ which implies that the speed of red light is more than that of green light in glass.

2. The correct choice is (c). If the source slit is wide, the interference pattern becomes indistinct because the interference patterns due to various parts of the source slit overlap. Consequently, the minima will not be totally dark and the fringe pattern becomes indistinct.

3. The correct choice is (a).
4. The correct choice is (b). The energy of a wave is determined by the square of its amplitude; it does not depend on the speed of the wave.
5. The correct choice is (c). The frequency of a wave does not depend on its speed or wavelength; it depends on the frequency of the source which produces that wave.
6. The correct choice is (c). The phase change is due to the reversal of amplitude of the wave on reflection from the mirror.

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. Lights of wavelengths λ_1 = 4000 Å and λ_2 = 5000 Å are used in an optical instrument. The ratio of the resolving powers of the instruments for wavelengths λ_1 and λ_2 is
   (a) 16 : 25 (b) 9 : 1
   (c) 4 : 5 (d) 5 : 4 **[2002]**
2. An astronomical telescope has a large aperture to
   (a) reduce spherical aberration
   (b) have high resolution
   (c) increase span of observation
   (d) have low dispersion **[2002]**
3. To demonstrate the phenomenon of interference, we require two sources which emit radiations of
   (a) nearly the same frequency
   (b) the same frequency
   (c) different wavelength
   (d) the same frequency and having a definite phase relationship. **[2003]**
4. The angle of incidence at which reflected light is polarised for reflection from air to glass of refractive index n is
   (a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$
   (c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$ **[2004]**
5. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double slit experiment is
   (a) infinite (b) 5
   (c) 3 (d) zero **[2004]**
6. In Young's double slit experiment using a monochromatic light, the shape of interference fringes formed on the screen is
   (a) hyperbola (b) circle
   (c) straight line (d) parabola **[2005]**
7. When an unpolarised light of intensity I_0 is incident normally on a polarising sheet, the intensity of light absorbed by the sheet is
   (a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$
   (c) zero (d) I_0 **[2005]**
8. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
   (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ **[2007]**
9. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of the known wavelength coincides with the 4th bright fringe of the unknown light. The wavelength of the unknown light is
   (a) 442.5 nm (b) 776.8 nm
   (c) 393.4 nm (d) 885.0 nm **[2009]**

Questions 10, 11 and 12 are based on following passage.

An initial parallel cylindrical beam travels in a medium of refractive index $\mu_1 = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

10. The speed of light in the medium is
    (a) directly proportional to the intensity I
    (b) maximum on the axis of the beam
    (c) minimum on the axis of the beam
    (d) the same everywhere in the beam **[2010]**
11. As the beam enters the medium, it will
    (a) diverge near the axis and coverage near the periphery
    (b) travel as a cylindrical beam
    (c) diverge
    (d) converge **[2010]**
12. The initial shape of the wavefront of the beam is
    (a) convex near the axis and concave near the periphery
    (b) planar
    (c) convex
    (d) concave **[2010]**
13. **Direction:** The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

    A thin air films is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

    Statement-1: When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

    Statement-2 : The centre of the interference pattern in dark

    (a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
    (b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1
    (c) Statement-1 is false, Statement-2 is true.
    (d) Statement-1 is true, Statement-2 is false. **[2011]**
14. In young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

    (a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$

    (c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ **[2012]**
15. Young double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are β_G, β_R and β_B respectively. Then,
    (a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
    (c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$ **[2012]**
16. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum (in kg ms^{-1}) of the object is
    (a) 0.3×10^{-17} (b) 1.0×10^{-17}
    (a) 3.0×10^{-17} (b) 9.0×10^{-17} **[2013]**
17. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is:

    (a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$

    (c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$ **[2013]**
18. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is:
    (a) 1m (b) 2m
    (c) 3 m (d) 6m **[2013]**
19. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45º relative to that of A. The intensity of the emergent light is:
    (a) $I_0/2$ (b) $I_0/4$
    (c) $I_0/2$ (d) $I_0/2$ **[2013]**

20. In Young's double slit experiment, the intensity at a point P on the screen is half the maximum intensity in the interference pattern. If the wavelength of ligh used is λ and d is the distance between the slits, the angular separation between point P and the centre of the screen is

(a) $\sin^{-1}\left(\dfrac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\dfrac{\lambda}{2d}\right)$

(c) $\sin^{-1}\left(\dfrac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\dfrac{\lambda}{4d}\right)$ **[2014]**

Answers

1. (d)	2. (b)	3. (d)	4. (d)
5. (b)	6. (d)	7. (a)	8. (d)
9. (a)	10. (c)	11. (d)	12. (b)
13. (a)	14. (d)	15. (d)	16. (b)
17. (b)	18. (c)	19. (b)	20. (d)

Solutions

1. Resolving power is inversely proportional to wavelength. Hence

$$\frac{(\text{R.P})_1}{(\text{R.P})_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = \frac{5}{4}$$

2. R.P. of telescope = $\dfrac{D}{1.22\lambda}$, where D = aperture.

Hence the correct choice is (2).

3. The correct choice is (4). Only coherent sources produce sustained interference.

4. From Brewster's law

$$\tan i_p = n$$

$$\Rightarrow \quad i_p = \tan^{-1}(n)$$

5. The angular separation θ of the nth maximum from the central maximum is given by

$$\sin\theta = \frac{n\lambda}{d}$$

Given $d = 2\lambda$. Therefore

$$\sin\theta = \frac{n}{2}$$

$$\Rightarrow \quad n = 2\sin\theta$$

$$\therefore \quad n_{max} = 2 \qquad \text{(maximum value of } \sin\theta = 1)$$

Thus there are two maxima on either side of the central maximum. Hence, maximum number of maxima = central maximum ($n = 0$) + 2 maxima ($n = \pm 1, \pm 2$) = 5.

6. The shape of fringes is general parabolic. In a small region of the screen near the centre, the fringes are straight.

7. The correct choice is (1). Half of the incident intensity is absorbed by the sheet and the remaining half is transmitted.

8. Let I be the intensity of each beam. Phase difference $\phi = \dfrac{2\pi}{\lambda} \times$ path difference $= \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{6} = \dfrac{\pi}{3}$ rad. The resultant intensity is given by

$$I_r = I + I + 2\sqrt{1 \times 1}\cos\phi$$

$$= 2I(1 + \cos\phi)$$

$I_r = I_{max} = I_0$ if $\cos\phi = +1$. Hence $I_0 = 4I$. When $\phi = \dfrac{\pi}{3} = 60°$, the resultant intensity is

$$I_r = 2I(1 + \cos 60°) = 2I(1 + 0.5) = 3I$$

$$\therefore \quad \frac{I_r}{I_0} = \frac{3I}{4I} = \frac{3}{4}$$

9. For bright fringes $y_n = \dfrac{n\lambda D}{d}$

Given y_3 (for λ_1 = 590 nm) = y_4 for λ_2

$$\Rightarrow \quad \frac{3\lambda_1 D}{d} = \frac{4\lambda_2 D}{d} \Rightarrow \lambda_2 = \frac{3\lambda_1}{4}$$

$$\Rightarrow \quad \lambda_2 = \frac{3 \times 590}{4} = 442.5 \text{ nm}$$

10. $v = \dfrac{c}{\mu_1} = \dfrac{c}{\mu_0 + \mu_2 I}$

Since the intensity I of the beam is decreasing with increasing radius, I will be maximum on the axis of the beam. Hence speed of light (v) is minimum on the axis of the beam, which is choice (c).

11. Since the intensity I is decreasing with increasing radius, the refractive index μ_1 of the medium is maximum on the axis of the beam decreasing to a minimum value at the periphery. Hence the beam will converge, which is choice (d).

12. Since the incident beam is parallel, the wavefronts are planar. So the correct choice is (b).

13. When light travelling in a rarer medium gets reflected from the boundary of a denser medium, the reflected wave undergoes a phase change of π. Hence the centre of the interference pattern is dark due to the destructive interference.

14. $I = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0}\ \cos\phi$

$I = I_0 + 4I_0 + \ 4I_0 \times \cos\ \phi$

$I_m = I_0 + 4I_0 + 4I_0 = 9I_0$

$\therefore\ I = \dfrac{I_m}{9}(5 + 4\cos\phi)$

$= \dfrac{I_m}{9}\left[5 + 4\left(2\cos^2\dfrac{\phi}{2} - 1\right)\right]$

$= \dfrac{I_m}{9}\left[1 + 8\cos^2\dfrac{\phi}{2}\right]$

15. Since $\lambda_R > \lambda_G > \lambda_B$ and $\beta = \dfrac{\lambda D}{d}$, $\beta_R > \beta_G > \beta_B$.

16. Momentum $= \dfrac{E}{c} = \dfrac{Power \times \Delta t}{c}$

$= \dfrac{(30 \times 10^{-3}) \times (100 \times 10^{-9})}{3 \times 10^8}$

$= 1.0 \times 10^{-17}$ kg ms^{-1}

17. $I = 4I_0 \cos^2\left(\dfrac{\Delta\phi}{2}\right)$; $\Delta\phi$ = phase difference

$I_{\max} = 4I_0$. For $I = \dfrac{1}{2}\ I_{\max} = 2I_0$

$2I_0 = 4I_0 \cos^2\left(\dfrac{\Delta\phi}{2}\right)$

$\Rightarrow \cos\left(\dfrac{\Delta\phi}{2}\right) = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \dfrac{\Delta\phi}{2} = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, .. = (2n+1)\ \dfrac{\pi}{4}$

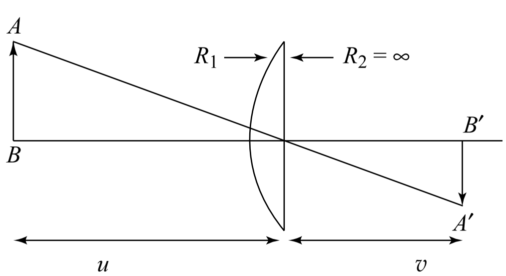
where $\quad n = 0, 1, 2, ...$

or $\quad \Delta\phi = (2n + 1)\dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{2\pi}{\lambda}\Delta x = (2n + 1)\ \dfrac{\pi}{2}$; Δx = path difference

$\Rightarrow \quad \Delta x = (2n + 1)\ \dfrac{\lambda}{4}$

18. Give $\dfrac{v}{u} = -\dfrac{1}{3} \Rightarrow u = -3v = -3 \times 8 = -24$ m



$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{8} - \dfrac{1}{-24} - \dfrac{1}{f} = \Rightarrow f = 6$ m

If λ is the wavelength of light in the lens and λ_0 in air, then

$v = \nu\lambda$

$c = \nu\lambda_0$

$\therefore \quad \mu = \dfrac{c}{v} = \dfrac{\lambda_0}{\lambda} \Rightarrow \lambda = \dfrac{\lambda_0}{\mu}$

Given $\lambda = \dfrac{2\lambda_0}{3\mu}$. Hence $\mu = \dfrac{3}{2} = 1.5$

$\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$= (1.5 - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{\infty}\right) = \dfrac{0.5}{R_1}$

$\Rightarrow \dfrac{1}{6} = \dfrac{0.5}{R_1} = R_1 = 3$ m

19. Intensity of light transmitted by $A = \dfrac{I_0}{2}$ According to Malus law, the intensity of light transmitted by B

$= \dfrac{I_0}{2}\cos^2\theta = \dfrac{I_0}{2}\cos^2(45^\circ) = \dfrac{I_0}{4}$

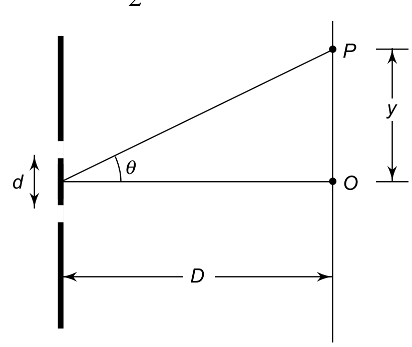
20. If δ is the phase difference between the interfering waves at point P, then the intensity at point P is given by

$I = I_{\max}\cos^2\left(\dfrac{\delta}{2}\right)$

Given $\quad I = \dfrac{I_{\max}}{2}$. Hence

$\cos^2\left(\dfrac{\delta}{2}\right) = \dfrac{1}{2}$ which gives $\dfrac{\delta}{2} = \dfrac{\pi}{4}$

or $\quad \delta = \dfrac{\pi}{2}$



The angular separation θ between points P and O is given by $\tan\theta = y/D$. Since θ is very small, $\tan\theta \simeq \sin\theta$. Hence

$$\sin\theta = \frac{y}{D} \qquad (1)$$

If β is the fringe width, then

$$\frac{y}{\beta} = \frac{\delta}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} \qquad (2)$$

This is so because phase difference δ between two consecutive maxima is 2. Now $\beta = \frac{\lambda D}{d}$.

Using this in Eq.(2), we get

$$\frac{yd}{\lambda D} = \frac{1}{4}$$

or

$$\frac{y}{D} = \frac{\lambda}{4d} \qquad (3)$$

Using Eq. (1) in Eq.(3), we have

$\sin\theta = \frac{\lambda}{4d}$ or $\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right)$, which is choice (d).