Sets

Question1

Let $A = \{1, 2, 3\}$. The number of relations on A, containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is _________.

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Answer: 3

Solution:

Transitivity

$(1, 2) \in \mathrm{R}, (2, 3) \in \mathrm{R} \Rightarrow (1, 3) \in \mathrm{R}$

For reflexive $(1, 1), (2, 2)(3, 3) \in R$

Now $(2, 1), (3, 2), (3, 1)$

$(3, 1)$ cannot be taken

(1) $(2, 1)$ taken and $(3, 2)$ not taken

(2) $(3, 2)$ taken and $(2, 1)$ not taken

(3) Both not taken

therefore 3 relations are possible.

Question2

Let $S = \{p_1, p_2 \ldots, p_{10}\}$ be the set of first ten prime numbers. Let $A = S \cup P$, where P is the set of all possible products of distinct elements of S. Then the number of all ordered pairs $(x, y), x \in S, y \in A$, such that x divides y, is ________ .

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Answer: 5120

Solution:

Let $\frac{y}{x} = \lambda$
$y = \lambda x$
$= 10 \times \left({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots . + {}^9C_9\right)$
$= 10 \times \left(2^9\right)$
10×512
5120

Question3

For $n \geq 2$, let S_n denote the set of all subsets of $\{1, 2, \ldots, n\}$ with no two consecutive numbers. For example $\{1, 3, 5\} \in S_6$, but $\{1, 2, 4\} \notin S_6$. Then $n\left(S_5\right)$ is equal to ________

JEE Main 2025 (Online) 7th April Morning Shift

Answer: 13

Solution:

To find $n(S_5)$, which is the number of subsets of $\{1, 2, 3, 4, 5\}$ with no consecutive numbers, we start by enumerating these subsets.

Let's denote the set $\{1, 2, 3, 4, 5\}$ as A. The subsets of A that meet the criteria are:

The empty set: $\{\}$

Single-element sets: $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$

Two-element sets with no consecutive numbers: $\{1, 3\}$, $\{1, 4\}$, $\{1, 5\}$, $\{2, 4\}$, $\{2, 5\}$, $\{3, 5\}$

Three-element set with no consecutive numbers: $\{1, 3, 5\}$

Counting these subsets, we have:

1 subset with zero elements

5 subsets with one element

6 subsets with two elements

1 subset with three elements

Adding these counts, there are $1 + 5 + 6 + 1 = 13$ subsets in total.

Thus, $n(S_5) = 13$.

Question4

The number of relations on the set $A = \{1, 2, 3\}$, containing at most 6 elements including $(1, 2)$, which are reflexive and transitive but not symmetric, is __________.

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Answer: 6

Solution:

Since relation needs to be reflexive the ordered pairs $(1,1),(2,2),(3,3)$ need to be there and $(1,2)$ is also to be included.

Let's call $R_0=\{(1,1),(2,2),(3,3),(1,2)\}$ the base relation.

$\because A\times A$ contain $3\times3=9$ ordered pairs, remaining 5 ordered are

$2,1),(1,3),(3,1),(2,3),(3,2)$

We have to add at most two ordered pairs to R_0 such that resulting relation is reflexive, transitive but not symmetric.

Following are the only possibilities.

$R=R_0U\{(1,3)\}$

OR $R_0U\{(3,2)\}$

OR $R_0U\{(1,3),(3,1)\}$

OR $R_0U\{(1,3),(3,2)\}$

OR $R_0U\{(3,1),(3,2)\}$

Question5

The number of non-empty equivalence relations on the set $\{1,2,3\}$ is :

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Options:

A. 7

B. 4

C. 5

D. 6

Answer: C

Solution:

An equivalence relation on a finite set is uniquely determined by its partition into equivalence classes. Hence, counting the number of equivalence relations on a set is equivalent to counting the number of ways to partition that set.

Step: Counting partitions of $\{1,2,3\}$

We want all possible ways to split the set $\{1,2,3\}$ into nonempty subsets (its "blocks").

3 blocks (each element in its own block)

$\{\{1\},\{2\},\{3\}\}$.

2 blocks

$\{\{1,2\},\{3\}\}$

$\{\{1,3\},\{2\}\}$

$\{\{2,3\},\{1\}\}$

1 block (all elements together)

$\{\{1,2,3\}\}$.

Counting these, there are a total of **5** distinct partitions, and thus **5** equivalence relations on the set $\{1,2,3\}$.

All equivalence relations are automatically nonempty (they include at least $(1,1),(2,2),(3,3)$ because they are reflexive), so the answer to "the number of nonempty equivalence relations" is also **5**.

Answer: Option C (5)

Question6

Let $A=\{1,2,3,\ldots,10\}$ and $B=\left\{\frac{m}{n}: m,n\in A, m<n \text{ and } \gcd(m,n)=1\right\}$. Then $n(B)$ is equal to :

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Options:

A. 29

B. 31

C. 37

D. 36

Answer: B

Solution:

To find the number of elements in set B, we consider pairs $\left(\frac{m}{n}\right)$ where $m,n\in A$ with $m<n$ and $\gcd(m,n)=1$.

Here's the breakdown for each possible m:

For $m=1$:

Possible values for n are $2,3,4,5,6,7,8,9,10$.

Total pairs: 9.

For $m=2$:

Possible values for n are $3,5,7,9$ (since these have $\gcd(2,n)=1$).

Total pairs: 4.

For $m=3$:

Possible values for n are $4,5,7,8,10$.

Total pairs: 5.

For $m=4$:

Possible values for n are $5,7,9$.

Total pairs: 3.

For $m = 5$:

Possible values for n are $6, 7, 8, 9$.

Total pairs: 4.

For $m = 6$:

Possible value for n is 7.

Total pairs: 1.

For $m = 7$:

Possible values for n are $8, 9, 10$.

Total pairs: 3.

For $m = 8$:

Possible value for n is 9.

Total pairs: 1.

For $m = 9$:

Possible value for n is 10.

Total pairs: 1.

Adding all these up, the total number of elements in set B is:

$9 + 4 + 5 + 3 + 4 + 1 + 3 + 1 + 1 = 31$

Question7

Let $\mathrm{R} = \{(1,2),(2,3),(3,3)\}$ be a relation defined on the set $\{1,2,3,4\}$. Then the minimum number of elements, needed to be added in R so that R becomes an equivalence relation, is:

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Options:

A. 9

B. 8

C. 7

D. 10

Answer: C

Solution:

$\mathrm{A} = \{1,2,3,4\}$

For relation to be reflexive

$\mathrm{R} = \{(1,2),(2,3),(3,3)\}$

Minimum elements added will be

$(1,1),(2,2),(4,4)(2,1)(3,2)(3,2)(3,1)(1,3)$
$\therefore$ Minimum number of elements $= 7$

Question8

Let $\mathrm{A} = \{(x,y) \in \mathbf{R} \times \mathbf{R} : |x+y| \geqslant 3\}$ and $\mathrm{B} = \{(x,y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$.If $\mathrm{C} = \{(x,y) \in \mathrm{A} \cap \mathrm{B} : x = 0$ or $y = 0\}$, then $\sum_{(x,y) \in \mathrm{C}} |x+y|$ is :

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Options:

A. 18

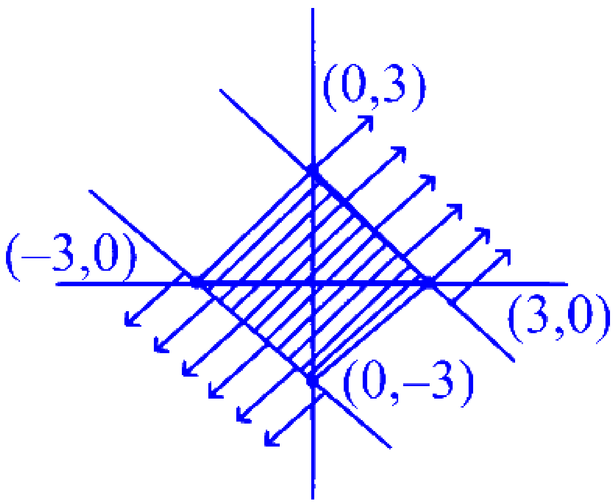
B. 24

C. 15

D. 12

Answer: D

Solution:



$\mathrm{C} = \{(3,0),(-3,0),(0,3),(0,-3)\}$
$\Sigma|\mathrm{x}+\mathrm{y}| = 12$

Question9

Let $\mathrm{X} = \mathbf{R} \times \mathbf{R}$. Define a relation R on X as :

$(a_1, b_1) R (a_2, b_2) \Leftrightarrow b_1 = b_2$

Statement I: R **is an equivalence relation.**

Statement II : For some $(\mathrm{a}, \mathrm{b}) \in \mathrm{X}$, the set $\mathrm{S}=\{(x, y) \in \mathrm{X}:(x, y) \mathrm{R}(\mathrm{a}, \mathrm{b})\}$ represents a line parallel to $y=x$.

In the light of the above statements, choose the correct answer from the options given below :

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Options:

A. Both Statement I and Statement II are true

B. Statement I is true but Statement II is false

C. Both Statement I and Statement II are false

D. Statement I is false but Statement II is true

Answer: B

Solution:

Statement - I :

Reflexive : $(a_1, b_1) R (a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $\left.\begin{array}{l}(\mathrm{a}_1, \mathrm{~b}_1) \mathrm{R}(\mathrm{a}_2, \mathrm{~b}_2) \Rightarrow \mathrm{b}_1=\mathrm{b}_2 \\ (\mathrm{a}_2, \mathrm{~b}_2) \mathrm{R}(\mathrm{a}_1, \mathrm{~b}_1) \Rightarrow \mathrm{b}_2=\mathrm{b}_1\end{array}\right\}$ True

Transitive: $\left.\begin{array}{l}(a_1, b_1) R (a_2, b_2) \Rightarrow b_1=b_2 \\ \&(a_2, b_2) R (a_3, b_3) b_2=b_3\end{array}\right\} b_1=b_3$

$\Rightarrow (\mathrm{a}_1, \mathrm{~b}_1) \mathrm{R}(\mathrm{a}_3 \cdot \mathrm{~b}_3) \Rightarrow$ True

Hence Relation R is an equivence relation Statement-I is true.

For statement - II $\Rightarrow \mathrm{y}=\mathrm{b}$ so False

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Question10

Let $\mathrm{A}=\left\{x \in(0, \pi)-\left\{\frac{\pi}{2}\right\}: \log_{(2 / \pi)}|\sin x|+\log_{(2 / \pi)}|\cos x|=2\right\}$ and $\mathrm{B}=\{x \geqslant 0: \sqrt{x}(\sqrt{x}-4)-3|\sqrt{x}-2|+6=0\}$. Then $\mathrm{n}(\mathrm{A} \cup \mathrm{B})$ is equal to :

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Options:

A. 4

B. 8

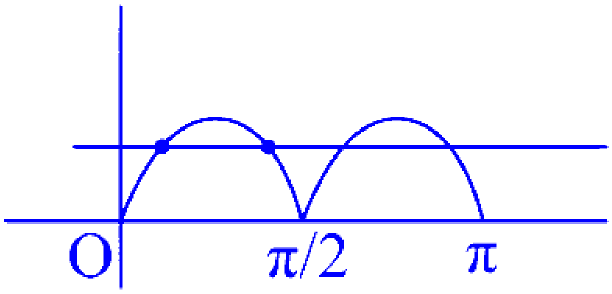
C. 6

D. 2

Answer: B

Solution:

$$\begin{aligned}&A: \log_{2/\pi}|\sin x| + \log_{2/\pi}|\cos x| = 2\\&\Rightarrow \log_{2/\pi}(|\sin x \cdot \cos x|) = 2\\&\Rightarrow |\sin 2x| = \frac{8}{\pi^2}\end{aligned}$$



Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}(\sqrt{x} - 4) + 3(\sqrt{x} - 2) + 6 = 0$

$$\begin{aligned}&\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0\\&\Rightarrow t^2 - t = 0, t = 0, t = 1\\&x = 0, x = 1\end{aligned}$$

again let $\sqrt{x} = t > 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$$\begin{aligned}&\Rightarrow t^2 - 7t + 12 = 0\\&\Rightarrow t = 3, 4\\&x = 9, 16\end{aligned}$$

Total number of solutions

$n(A \cup B) = 4 + 4 = 8$

Question11

The relation $R = \{(x, y) : x, y \in \mathbb{Z}$ and $x + y$ is even $\}$ is:

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Options:

A. reflexive and transitive but not symmetric

B. reflexive and symmetric but not transitive

C. an equivalence relation

D. symmetric and transitive but not reflexive

Answer: C

Solution:

$R = \{(x, y) : x, y \in z$ and $x + y$ is even $\}$

reflexive $x + x = 2x$ even

symmetric of $x + y$ is even, then $(y + x)$ is also even

transitive of $\mathrm{x} + \mathrm{y}$ is even $\& \mathrm{y} + \mathrm{z}$ is even then $x + z$ is also even

So, relation is an equivalence relation.

Question12

Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

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Options:

A.

both reflexive and symmetric but not transitive

B.

both reflexive and transitive but not symmetric

C.

reflexive but neither symmetric not transitive

D.

an equivalence relation

Answer: D

Solution:

$\sec^2 x - \tan^2 x = 1 \quad$ (on replacing y with x)
$\Rightarrow$ Reflexive
$\sec^2 x - \tan^2 y = 1$
$\Rightarrow 1 + \tan^2 x + 1 - \sec^2 y = 1$
$\Rightarrow \sec^2 y - \tan^2 x = 1$
$\Rightarrow$ symmetric
$\sec^2 x - \tan^2 y = 1$
$\sec^2 y - \tan^2 z = 1$

Adding both

$\Rightarrow \sec^2 x - \tan^2 y + \sec^2 y - \tan^2 z = 1 + 1$
$\sec^2 x + 1 - \tan^2 z = 2$
$\sec^2 x - \tan^2 z = 1$
$\Rightarrow$ Transitive

hence equivalence relation

Question13

Let $\mathrm{S} = \mathbf{N} \cup \{0\}$. Define a relation R from S to $\mathbf{R}$ by :

$\mathrm{R} = \left\{(x, y) : \log_{\mathrm{e}} y = x \log_{\mathrm{e}}\left(\frac{2}{5}\right), x \in \mathrm{S}, y \in \mathbf{R}\right\}.$

Then, the sum of all the elements in the range of R is equal to :

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Options:

A. $\frac{3}{2}$

B. $\frac{10}{9}$

C. $\frac{5}{2}$

D. $\frac{5}{3}$

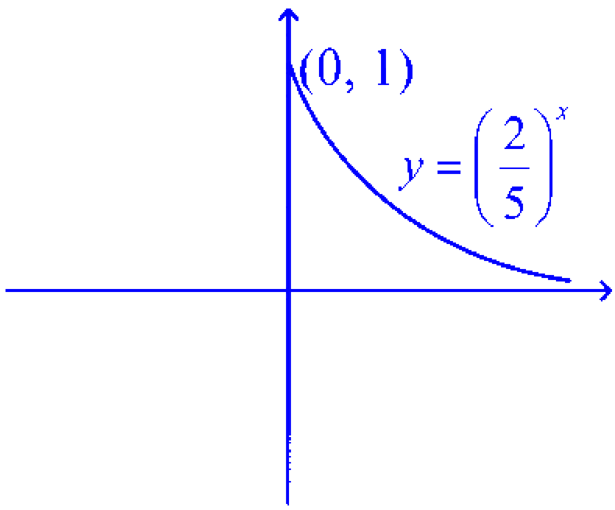
Answer: D

Solution:

$S = \{0, 1, 2, 3 \ldots\ldots\}$

$\log_{\mathrm{e}} \mathrm{y} = \log_{\mathrm{e}}\left(\frac{2}{5}\right)$

$\Rightarrow \mathrm{y} = \left(\frac{2}{5}\right)^{\mathrm{x}}$



Required

Sum $= 1 + \left(\frac{2}{5}\right)^1 + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \ldots\ldots - = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$

Question14

Let A be the set of all functions $f : \mathbf{Z} \rightarrow \mathbf{Z}$ and R be a relation on A such that $\mathrm{R} = \{(\mathrm{f}, \mathrm{g}) : f(0) = \mathrm{g}(1)$ and $f(1) = \mathrm{g}(0)\}$. Then R is :

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Options:

A. Symmetric and transitive but not reflective

B. Symmetric but neither reflective nor transitive

C. Transitive but neither reflexive nor symmetric

D. Reflexive but neither symmetric nor transitive

Answer: B

Solution:

For R to be reflexive, (f, f) must be in R.

The means $f(0) = f(1)$ and $f(1) = f(0)$ must be true for all f.

But $f(0) \neq f(1)$ always

Therefore, R is not reflexive

If $(f, g) \in R$, then $f(0) = g(1)$ and $f(1) = g(0)$

$\because \quad f(0) = g(1) \Rightarrow g(1) = f(0)$
and $f(1) = g(0) \Rightarrow g(0) = f(1)$

R is symmetric

If $(f, g) \in R$ and $(g, h) \in$ R, then $f(0) = g(1)$, $f(1) = g(0), g(0) = n(1) \& g(1) = h(0)$

Since, $f(0) = g(1)$ and $g(1) = h(0)$, then $f(0)$ is not necessarily equal to $h(0)$.

Therefore, R is not transitive.

$\therefore \quad$ The relation R is symmetric but not reflexive or transitive.

Question15

Let $A = \{1, 2, 3, \ldots., 100\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_2, a_3), (a_3, a_4), \ldots., (a_k, a_{k+1})$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k , for which such a sequence exists, is equal to :

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Options:

A. 6

B. 8

C. 7

D. 5

Answer: D

Solution:

The relation R is defined on the set $A = \{1, 2, 3, \ldots, 100\}$ such that $R = \{(a, b) : a = 2b + 1\}$. We need to find the largest integer k for which there exists a sequence of k ordered pairs from R where the second element of each pair is the first element of the next pair.

The sequence in terms of k is:

$(a_1, a_2), (a_2, a_3), \ldots, (a_k, a_{k+1})$

Here, each a_i satisfies the equation $a_i = 2a_{i+1} + 1$. Consequently, $a_1 = 2a_2 + 1$, making a_1 an odd number.

Let's examine the pattern:

$a_2 = 2a_3 + 1$, implying $a_1 = 2(2a_3 + 1) + 1 = 4a_3 + 3$.

$a_3 = 2a_4 + 1$, leading to $a_1 = 4(2a_4 + 1) + 3 = 8a_4 + 7$.

Continuing this pattern, we find:

$a_k = 2a_{k+1} + 1 \implies a_1 = 2^k \cdot a_{k+1} + (2^k - 1)$

where a_{k+1} needs to be in set A. This implies:

$a_{k+1} = \frac{a_1 + 1 - 2^k}{2^k}$

Thus, $2^k \mid (a_1 + 1)$. The task is to find the highest k where 2^k divides any e_i in $\{2, \ldots, 101\}$.

The largest power of 2 that divides an element within this range determines k.

After computation, we find that k can be a maximum of 6 because $2^6 = 64$ divides $95 + 1 = 96$, but $2^7 = 128$ does not divide any e_i for $e_i \in A$. Therefore, the maximum k is 6.

The sequence corresponding to this maximum k is:

$(95, 47), (47, 23), (23, 11), (11, 5), (5, 2)$

Question16

Let $\mathrm{A} = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by $x\mathrm{R}y$ if and only if $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then $l + m$ is equal to

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Options:

A. 18

B. 20

C. 17

D. 19

Answer: A

Solution:

The relation R is defined for the set A $= \{-3, -2, -1, 0, 1, 2, 3\}$ with the condition $0 \leq x^2 + 2y \leq 4$. Let's determine the pairs (x, y) that satisfy this condition.

For $y = -3$:

Solving $x^2 + 2(-3) \leq 4$, we find $x = \{3, -3\}$.

For $y = -2$:

Solving $x^2 + 2(-2) \leq 4$, we find $x = \{-2, 2\}$.

For $y = -1$:

Solving $x^2 + 2(-1) \leq 4$, we find $x = \{-2, 2\}$.

For $y = 0$:

Solving $x^2 + 0 \leq 4$, we find $x = \{-2, -1, 0, 1, 2\}$.

For $y = 1$:

Solving $x^2 + 2(1) \leq 4$, we find $x = \{-1, 0, 1\}$.

For $y = 2$:

Solving $x^2 + 2(2) \leq 4$, we find $x = \{0\}$.

The relation R consists of the following pairs:

$R = \{(3, -3), (-3, -3), (-2, -2), (2, -2), (-2, -1), (2, -1), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (-1, 1), (0, 1), (1, 1), (0, 2)\}$

Currently, R has $l = 15$ elements. To make R reflexive, it must include all pairs (x, x) for every $x \in$ A. We identify the missing reflexive pairs $(-1, -1)$, $(2, 2)$, and $(3, 3)$, which are required to satisfy reflexivity.

Thus, $m = 3$ more elements are needed. Therefore, the total $l + m = 15 + 3 = 18$.

Question17

Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by $x\text{R}y$ if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to

JEE Main 2025 (Online) 3rd April Evening Shift

Options:

A. 11

B. 12

C. 14

D. 13

Answer: B

Solution:

To solve the problem, we start by defining the set $A = \{-2, -1, 0, 1, 2, 3\}$ and the relation R on set A, where an element x is related to y (written as $x\,R\,y$) if and only if $y = \max\{x, 1\}$.

This leads us to the following pairs in the relation R:

For $x = -2$, $y = \max\{-2, 1\} = 1$, so $(-2, 1)$ is in R.

For $x = -1$, $y = \max\{-1, 1\} = 1$, so $(-1, 1)$ is in R.

For $x = 0$, $y = \max\{0, 1\} = 1$, so $(0, 1)$ is in R.

For $x = 1$, $y = \max\{1, 1\} = 1$, so $(1, 1)$ is in R.

For $x = 2$, $y = \max\{2, 1\} = 2$, so $(2, 2)$ is in R.

For $x = 3$, $y = \max\{3, 1\} = 3$, so $(3, 3)$ is in R.

Thus, the relation R consists of the pairs: $\{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3)\}$, and there are $l = 6$ elements in R.

Making the Relation Reflexive

A relation is reflexive if every element in the set A relates to itself. Therefore, the missing reflexive pairs are:

$(-2, -2)$

$(-1, -1)$

$(0, 0)$

Adding these three pairs will make the relation reflexive, so $m = 3$.

Making the Relation Symmetric

A relation is symmetric if whenever (x, y) is in R, (y, x) must also be in R. Therefore, the missing symmetric pairs are:

$(1, -2)$

$(1, -1)$

$(1, 0)$

Thus, we need to add these three pairs for symmetry, so $n = 3$.

Finally, we calculate the sum $l + m + n = 6 + 3 + 3 = 12$.

Question18

Consider the sets $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}, B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$ and $D = A \cap B$. The total number of one-one functions from the set D to the set C is:

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Options:

A. 15120

B. 18290

C. 17160

D. 19320

Answer: C

Solution:

$A=\{(x,y)\in R\times R: x^2+y^2=25\}, B=\{(x,y)\in\mathbb{R}\times$
$\mathbb{R}: x^2+9y^2=144\}$
$x^2+9y^2-(x^2+y^2)=144-25$
Plug in $y^2=\frac{119}{8}$ into either equation to find x.
$x^2=25-\frac{119}{8}$
$x^2=\frac{200-119}{8}$
$x^2=\frac{81}{8}$
$x=\pm\sqrt{\frac{81}{8}}, y=\pm\sqrt{\frac{119}{8}}$

Now, $C=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}: x^2+y^2\leq 4\}$

Valid points are $(-2,0),(-1,-1),(-1,0),(-1,1),(0,-2),(0,-1),(0,0),(0,1),(0,2),(1,-1),(1,0),(1,1)$

$\therefore$ Total valid points in $C=13$

$\Rightarrow$ There are 4 distinct real points in set D

$\therefore$ The number of one-one functions from D to C

$\Rightarrow$ $13P_4\Rightarrow\frac{13!}{(13-4)!}=\frac{13!}{9!}=17160$

Question19

Let $\mathrm{A}=\{-3,-2,-1,0,1,2,3\}$ and R be a relation on A defined by $x\mathrm{R}y$ if and only if $2x-y\in\{0,1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l+\mathrm{m}+\mathrm{n}$ is equal to:

JEE Main 2025 (Online) 4th April Evening Shift

Options:

A. 17

B. 18

C. 15

D. 16

Answer: A

Solution:

$xRy \Leftrightarrow 2x - y \in \{0,1\}$
$\Rightarrow \quad y = 2x$ or $y = 2x - 1$
$A = \{-3,-2,-1,0,1,2,3\}$
$\mathrm{R} = \{(-1,-2),(0,0),(1,2),(-1,-3),(0,-1),(1,1),$
$(2,3)\}$
$\Rightarrow \quad I = 7$
For R to be reflexive $(0,0),(1,1) \in R$

But other (a,a) such that $2a - a \in \{0,1\}$

$\Rightarrow \quad a \in \{0,1\}$

5 other pairs needs to be added $\Rightarrow m = 5$

$xRy \Rightarrow yRx$ to be symmetric

$(-1,-2) \Rightarrow (-2,-1)$

$(1,2) \Rightarrow (2,1)$

$(-1,-3) \Rightarrow (-3,-1)$

$(0,-1) \Rightarrow (-1,0)$

$(2,3) \Rightarrow (3,2) \Rightarrow$ 5 needs to be added, $n = 5$

$\Rightarrow \quad l + m + n = 17$

Question20

Let A = { $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$: |α - 1| ≤ 4 and |β - 5| ≤ 6 }

and B = { $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$: 16(α - 2)^2+ 9(β - 6)^2 ≤ 144 }.

Then

JEE Main 2025 (Online) 7th April Evening Shift

Options:

A.

A $\subset$ B

B.

B $\subset$ A

C.

neither A $\subset$ B nor B $\subset$ A

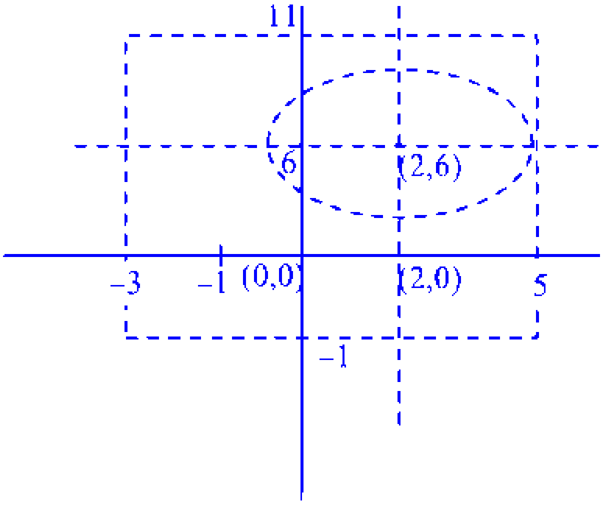
D.

$A \cup B = \{(x,y) : -4 \leqslant x \leqslant 4, -1 \leqslant y \leqslant 11\}$

Answer: B

Solution:

A: $|x-1| \le 4$ and $|y-5| \le 6$
$\Rightarrow -4 \le x-1 \le 4 \Rightarrow -6 \le y-5 \le 6$
$\Rightarrow -3 \le x \le 5 \quad \Rightarrow -1 \le y \le 11$
B : $16(x-2)^2 + 9(y-6)^2 \le 144$
B : $\frac{(x-2)^2}{9} + \frac{(y-6)^2}{16} \le 1$



From Diagram B ⊂ A

Question21

Let A = {0, 1, 2, 3, 4, 5}. Let R be a relation on A defined by (x, y) ∈ R if and only if max{x, y} ∈ {3, 4}. Then among the statements

(S_1): The number of elements in R is 18, and

(S_2): The relation R is symmetric but neither reflexive nor transitive

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Options:

A.

both are false

B.

only (S_1) is true

C.

only (S_2) is true

D.

both are true

Answer: C

Solution:

To evaluate the relation R on the set $A = \{0, 1, 2, 3, 4, 5\}$, we first need to understand the conditions for an element (x, y) to be in R. Specifically, $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3, 4\}$.

Considering this, let's list the pairs:

For $\max\{x, y\} = 3$, the possible pairs are:

$(0, 3), (3, 0), (1, 3), (3, 1), (2, 3), (3, 2), (3, 3)$

For $\max\{x, y\} = 4$, the possible pairs are:

$(0, 4), (4, 0), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)$

Combining these, the set R consists of the following elements:

$R = \{(0, 3), (3, 0), (1, 3), (3, 1), (2, 3), (3, 2), (3, 3), (0, 4), (4, 0), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4)\}$

This gives us a total of 16 elements in R, not 18 as initially claimed in statement S_1.

Next, we analyze the properties of the relation R:

Reflexivity: A relation is reflexive if $(x, x) \in R$ for all $x \in A$. For example, $(0, 0), (1, 1), (2, 2)$ are not in R, so R is not reflexive.

Symmetry: A relation is symmetric if whenever $(a, b) \in R$, then $(b, a) \in R$ as well. For all pairs (x, y) listed, both (x, y) and (y, x) are present. Thus, R is symmetric.

Transitivity: A relation is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$. An example where transitivity fails is $(0, 3)$ and $(3, 1)$ are in R but $(0, 1)$ is not in R. Therefore, R is not transitive.

In conclusion, statement S_2 is correct as R is symmetric but neither reflexive nor transitive.

Question22

Let S = {1, 2, 3,...,10}. Suppose M is the set of all the subsets of S, then the relation R = {(A, B) : A ∩ B ≠ φ ; A, B ∈ M} is :

[27-Jan-2024 Shift 1]

Options:

A.

symmetric and reflexive only

B.

reflexive only

C.

symmetric and transitive only

D.

symmetric only

Answer: D

Solution:

Solution:

Let $S = \{1, 2, 3, \ldots, 10\}$

$R = \{(A, B) : A \cap B \neq \phi; A, B \in M\}$

For Reflexive,

M is subset of ' S '

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $A \cap B \neq \phi$,

$\Rightarrow$BRA $\Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If $A = \{(1, 2), (2, 3)\}$

$B = \{(2, 3), (3, 4)\}$

$C = \{(3, 4), (5, 6)\}$

ARB & BRC but A does not relate to C

So it not transitive

Question23

Let R be a relation on Z × Z defined by (a, b) R (c, d) if and only if ad - bc is divisible by 5 . Then R is

[29-Jan-2024 Shift 1]

Options:

A.

Reflexive and symmetric but not transitive

B.

Reflexive but neither symmetric not transitive

C.

Reflexive, symmetric and transitive

D.

Reflexive and transitive but not symmetric

Answer: A

Solution:

Solution:

$(a, b)R(a, b)$ as $ab - ab = 0$

Therefore reflexive

Let $(a, b)R(c, d) \Rightarrow ad - bc$ is divisible by 5

$\Rightarrow bc - ad$ is divisible by $5 \Rightarrow (c, d)R(a, b)$

Therefore symmetric

Relation not transitive as $(3, 1)R(10, 5)$ and $(10, 5)R(1, 1)$ but $(3, 1)$ is not related to $(1, 1)$

Question24

If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1, 2), (1, 3)} ⊂ R, then the number of elements in R is

[29-Jan-2024 Shift 2]

Options:

A.

10

B.

12

C.

8

D.

15

Answer: A

Solution:

Given set {1, 2, 3, 4}

Minimum order pairs are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 1), (2, 1), (2, 3), (3, 2), (1, 3), (1, 2)

Thus no, of elements = 10

Question25

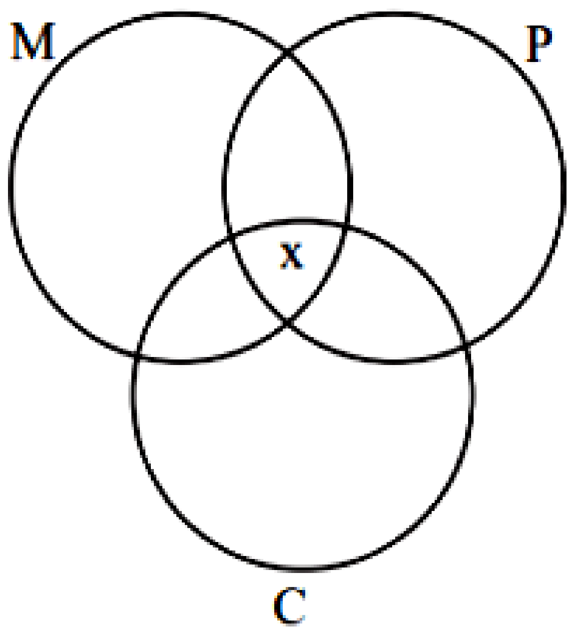
A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is____

[30-Jan-2024 Shift 1]

Answer: 10

Solution:

Solution:

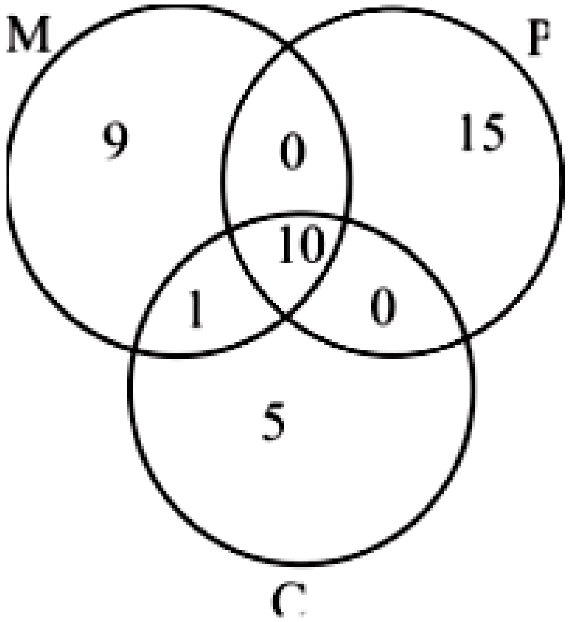


$11 - x \geq 0$ (Maths and Physics)

$x \leq 11$

x = 11 does not satisfy the data.

For x = 10



Hence maximum number of students passed in all the three subjects is 10 .

Question26

The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is____

[30-Jan-2024 Shift 2]

Answer: 960

Solution:

Total number of relation both symmetric and reflexive $= 2^{\frac{n^2-n}{2}}$

Total number of symmetric relation $= 2^{\left(\frac{n^2+n}{2}\right)}$

⇒ Then number of symmetric relation which are not reflexive

$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$

$\Rightarrow 2^{10} - 2^{6}$

$\Rightarrow 1024 - 64$

$= 960$

Question27

Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that R⊂S and the number of elements in S is n. Then, the minimum value of n is____

[31-Jan-2024 Shift 1]

Answer: 16

Solution:

All elements are included

Answer is 16

Question28

Let A = {1, 2, 3,........100}. Let R be a relation on A defined by (x, y) ∈ R if and only if 2x = 3y. Let R1 be a symmetric relation on A such that R⊂R1 and the number of elements in R1 is n. Then, the minimum value of n is

[31-Jan-2024 Shift 2]

Answer: 66

Solution:

R = {(3, 2), (6, 4), (9, 6), (12, 8),.........(99, 66)}

n(R) = 33

∴ 66

Question29

Let A = {1, 2, 3,...20}. Let R_1 and R_2 two relation on A such that
R1 = {(a, b) : b is divisible by a}
R2 = {(a, b) : a is an integral multiple of b}.
Then, number of elements in $R_1 - R_2$ is equal to_____

[1-Feb-2024 Shift 1]

Answer: 46

Solution:

$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 \ \ + 2 + [\underbrace{1 + \ldots + 1}_{10\text{ times}}]$

$n(R_1) = 66$

$R_1 \cap R_2 = \{(1, 1), (2, 2), \ldots (20, 20)\}$

$n(R_1 \cap R_2) = 20$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= n(R_1) - 20$

$= 66 - 20$

$R_1 - R_2 = 46$ Pair

Question30

The number of elements in the set

S = {(x, y, z) : x, y, z ∈ Z, x + 2y + 3z = 42, x, y, z ≥ 0} equals ____

[1-Feb-2024 Shift 1]

Answer: 169

Solution:

$x+2y+3z=42, \quad x,y,z \geq 0$

$z=0 \quad x+2y=42 \Rightarrow 22$

$z=1 \quad x+2y=39 \Rightarrow 20$

$z=2 \quad x+2y=36 \Rightarrow 19$

$z=3 \quad x+2y=30 \Rightarrow 17$

$z=4 \quad x+2y=30 \Rightarrow 16$

$z=5 \quad x+2y=27 \Rightarrow 14$

$z=6 \quad x+2y=24 \Rightarrow 13$

$z=7 \quad x+2y=21 \Rightarrow 11$

$z=8 \quad x+2y=18 \Rightarrow 10$

$z=9 \quad x+2y=15 \Rightarrow 8$

$z=10 \quad x+2y=12 \Rightarrow 7$

$z=11 \quad x+2y=9 \Rightarrow 5$

$z=12 \quad x+2y=6 \Rightarrow 4$

$z=13 \quad x+2y=3 \Rightarrow 2$

$z=14 \quad x+2y=0 \Rightarrow 1$

Total : 169

Question31

Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2+b^2=1$ for all $a,b, \in R$ and (a, b) R_2 $(c, d) \Leftrightarrow a+d=b+c$ for all $(a, b), (c, d) \in N \times N$. Then

[1-Feb-2024 Shift 2]

Options:

A.

Only R_1 is an equivalence relation

B.

Only R_2 is an equivalence relation

C.

R_1 and R_2 both are equivalence relations

D.

Neither R_1 nor R_2 is an equivalence relation

Answer: B

Solution:

$aR_1b \Leftrightarrow a^2 + b^2 = 1;\ a, b \in R$

$(a, b)R_2(c, d) \Leftrightarrow a + d = b + c\ ;(a, b), (c, d) \in N$

for R_1 : Not reflexive symmetric not transitive

for R_2 : R_2 is reflexive, symmetric and transitive

Hence only R_2 is equivalence relation.

Question32

The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d } so that it is an equivalence relation, is__ [24-Jan-2023 Shift 2]

Answer: 13

Solution:

Solution:
Given R = {(a, b), (b, c), (b, d)}
In order to make it equivalence relation as per given set, R must be
{ (a, a), (b, b), (c, c), (d , d), (a, b), (b, a), (b, c), (c, b), (b, d), (d , b), (a, c), (a, d), (c, d), (d , c), (c, a), (d , a) }
There already given so 13 more to be added.

Question33

In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is :

[6-Apr-2023 shift 2]

Options:

A.

$\frac{\sqrt{129}}{12}$

B.

$\frac{\sqrt{117}}{12}$

C.

$$\frac{\sqrt{119}}{12}$$

D.

$$\frac{3\sqrt{15}}{12}$$

Answer: C

Solution:

Solution:

$n(A \cap B) = n(A) + n(B) - n(A \cap B)$

$n(ABB) = 75 + 40 - 100$

$n(A \cap B) = 15$

Only $E \rightarrow 60\ \alpha = 60$

Only $H \rightarrow 25\ \beta = 25$

Both $= 15$

$$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1$$

$$\frac{25x^2}{(60)^2} + \frac{(25y^2)}{(25)^2} = 1$$

$$e^2 = 1 - \left[\frac{25 \times 25}{(60)^2}\right]$$

$$e^2 = \frac{(60)^2 - (25)^2}{(60)^2}$$

$$e^2 = \frac{(60 - 25)(60 + 25)}{60 \times 60}$$

$$e^2 = \frac{(35)(85)}{60 \times 60} = \frac{119}{144}$$

$$e = \frac{\sqrt{119}}{12}$$

Question34

Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

[8-Apr-2023 shift 1]

Options:

A.

752

B.

772

C.

782

D.

792

Answer: D

Solution:

Solution:

$n(A \times B) = 10$

${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

Question35

The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is

[10-Apr-2023 shift 1]

Answer: 6

Solution:

Solution:

$-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$(n-5)^2 > 0$ $5 - 3\sqrt{2} < n < 5 + 3\sqrt{2}$

$N \in Z - \{5\}$ $n = \{2, 3, 4, 5, 6, 7, 8\}$

$\ldots(i) \ldots (ii)$

From (i) $\cap (ii)$

$N = \{2, 3, 4, 5, 6, 8,\}$

Number of values of $n = 6$

Question36

The number of elements in the set $S = \{\theta \in [0, 2\pi] : 3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0\}$ is :

[11-Apr-2023 shift 1]

Options:

A.

10

B.

9

C.

8

D.

12

Answer: B

Solution:

Solution:

$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$

$\Rightarrow 3\cos^2\theta(\cos^2\theta - 1) + 2\sin^2\theta(\sin^4\theta - 1) = 0$

$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta(1 + \sin^2\theta)\cos^2\theta - 1$

$\Rightarrow \sin^2\theta\cos^2\theta(2 + 2\sin^2\theta - 3) = 0$

$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta - 1) = 0$

(C1) $\sin^2\theta = 0 \rightarrow 3$ solution: $\theta = \{0, \pi, 2\pi\}$

(C2) $\cos^2\theta = 0 \rightarrow 2$ solution: $\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \frac{1}{2} \rightarrow 4$ solution; $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

No. of solution $= 9$

Question37

An organization awarded 48 medals in event ' A ', 25 in event ' B ' and 18 in event ' C '. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

[11-Apr-2023 shift 1]

Options:

A.

15

B.

9

C.

21

D.

10

Answer: C

Solution:

Solution:

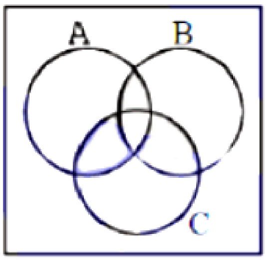
$|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cap B \cap C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$

$= 36$

No. of men who received exactly 2 medals

$\Rightarrow \sum |A \cap B| - 3|A \cap B \cap C|$

$= 36 - 15$

$= 21$

Question38

The number of the relations, on the set {1,2,3} containing (1,2) and (2,3), which are reflexive and transitive but not symmetric, is ________.

[12-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:

$A = \{1, 2, 3\}$

For Reflexive $(1, 1)(2, 2), (3, 3) \in R$

For transitive : $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$

Not symmetric : $(2, 1)$ and $(3, 2) \notin R$

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

Question39

The number of elements in the set $\{n \in N : 10 \leq n \leq 100.$ and $3^n - 3$ is a multiple of $7\}$ is ________

[15-Apr-2023 shift 1]

Answer: 15

Solution:

Solution:

$n \in [10, 100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

$n = 1, 7, 13, 20, \ldots\ldots 97$

Number of possible values of $n = 15$

Question40

Let $A = \{z \in C : 1 \leq |z - (1 + i)| \leq 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B : [24-Jun-2022-Shift-1]

Options:

A. is an empty set

B. contains exactly two elements

C. contains exactly three elements

D. is an infinite set

Answer: D

Solution:

Solution:

Let, $z = x + iy$

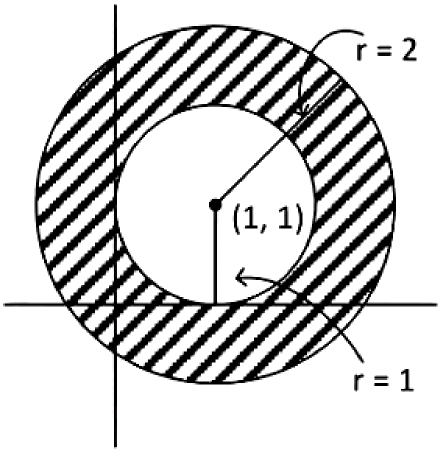
Given, $1 \leq |z-(1+i)| \leq 2$

$\Rightarrow 1 \leq |x+iy-1-i| \leq 2$

$\Rightarrow 1 \leq |(x-1)+i(y-1)| \leq 2$

$\Rightarrow 1 \leq \sqrt{(x-1)^2+(y-1)^2} \leq 2$

It represent two concentric circle both have center at $(1, 1)$ and radius 1 and 2 .



Also given,

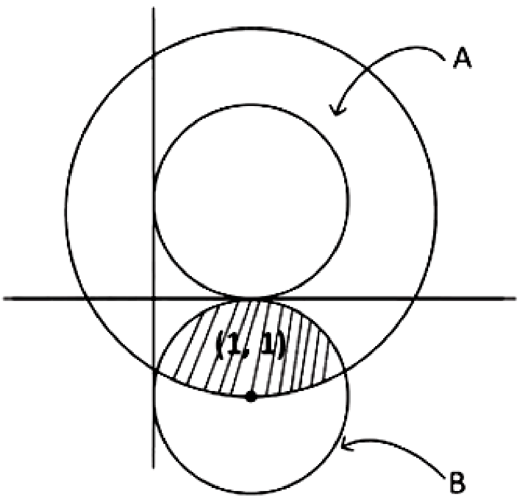
$|z-(1-i)| = 1$

$\Rightarrow |x+iy-1+i| = 1$

$\Rightarrow |(x-1)+i(y+1)| = 1$

$\Rightarrow \sqrt{(x-1)^2+(y+1)^2} = 1$

This represent a circle with center at $(1, -1)$ and radius $= 1$.



Question41

Let A = {x ∈ R: | x + 1 | <2} and B = {x ∈ R: | x − 1 | ≥2}. Then which one of the following statements is NOT true?
[25-Jun-2022-Shift-2]

Options:

A. $A - B = (-1, 1)$

B. $B - A = R - (-3, 1)$

C. $A \cap B = (-3, -1]$

D. $A \cup B = R - [1, 3)$

Answer: B

Solution:

Solution:

$A = (-3, 1)$ and $B = (-\infty, -1] \cup [3, \infty)$

So, $A - B = (-1, 1)$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

$A \cap B = (-3, -1]$

and $A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

So Option B is not True

Question42

Let A = { n ∈ N : H.C.F. (n, 45) = 1 } and
Let B = {2k : k ∈ {1, 2, ……, 100}}. Then the sum of all the elements of A ∩ B is______
[26-Jun-2022-Shift-1]

Answer: 5264

Solution:

Solution:

Sum of all elements of A∩B=2 [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$$= 2\left[\frac{100 \times 101}{2} - 3\left(\frac{33 \times 34}{2}\right) - 5\left(\frac{20 \times 21}{2}\right) + 15\left(\frac{6 \times 7}{2}\right)\right]$$

$$= 10100 - 3366 - 2100 + 630$$

$$= 5264$$

Question43

Let a set $A = A_1 \cup A_2 \cup ..... \cup A_k$, where $A_i \cap A_j = \varphi$ for $i \neq j$, $1 \leq j, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$. if and only if .$x \in A_i, 1 \leq i \leq k\}$. Then, R is :
[29-Jun-2022-Shift-1]

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Options:

A. reflexive, symmetric but not transitive.

B. reflexive, transitive but not symmetric.

C. reflexive but not symmetric and transitive.

D. an equivalence relation.

Answer: D

Solution:

Solution:
$R = \{(x, y) : y \in A_i, \quad \text{iff } x \in A_i 1 \leq i \geq k\}$
(1) Reflexive
$(a, a) \Rightarrow a \in A_i$ iff $a \in A_i$
(2) Symmetric
$(a, b) \Rightarrow a \in A_i$ iff $b \in A_i$
(b, a) $\in$R as $b \in A_i$ iff $a \in A_i$
(3) Transitive
$(a, b) \in R \& (b, c) \in R$
$\Rightarrow a \in A_i$ iff $b \in A_i \& b \in A_i$ iff $c \in A_i$
$\Rightarrow a \in A_i$ iff $c \in A_i$
$\Rightarrow (a, c) \in R.$
⇒ RElation is equivalnece.

Question44

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A :$ either $1 \notin T$ or $2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B \cup C$ is____
[25-Jul-2022-Shift-2]

Answer: 107

Solution:

Solution:
$\because (B \cup C)' = B' \cap C'$
B' is a set containing sub sets of A containing element 1 and not containing 2.
And C' is a set containing subsets of A whose sum of elements is not prime.
So, we need to calculate number of subsets of $\{3, 4, 5, 6, 7\}$ whose sum of elements plus 1 is composite.
Number of such 5 elements subset $= 1$
Number of such 4 elements subset $= 3$ (except selecting 3 or 7)
Number of such 3 elements subset $= 6$ (except selecting $\{3, 4, 5\}, \{3, 6, 7\}, \{4, 5, 7\}$ or $\{5, 6, 7\}$)
Number of such 2 elements subset $= 7$ (except selecting $\{3, 7\}, \{4, 6\}, \{5, 7\}$)
Number of such 1 elements subset $= 3$ (except selecting $\{4\}$ or $\{6\}$)
Number of such 0 elements subset $= 1$
$n(B' \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$

Question45

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \varphi\}$ is _______.
[26-Jul-2022-Shift-2]

Answer: 112

Solution:

Solution:

$A = \{7, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$

Total subset of $A = 2^7 = 128$

$C \cap B = \varphi$ when set C contains the element $1, 2, 4, 5$

$\therefore S = \{C \subseteq A; C \cap B \neq \varphi\}$

$= \text{Total} - (C \cap B = \varphi)$

$= 128 - 2^4 = 112$

Question46

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $aR_1b \Leftrightarrow ab \geq 0$ and $aR_2b \Leftrightarrow a \geq b$ Then,
[27-Jul-2022-Shift-1]

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Options:

A. R_1 is an equivalence relation but not R_2

B. R_2 is an equivalence relation but not R_1

C. both R_1 and R_2 are equivalence relations

D. neither R_1 nor R_2 is an equivalence relation

Answer: D

Solution:

Solution:
$R_1 = \{xy \geq 0, x, y \in R\}$
For reflexive $x \times x \geq 0$ which is true.
For symmetric
If $xy \geq 0 \Rightarrow yx \geq 0$
If $x = 2, y = 0$ and $z = -2$
Then $x \cdot y \geq 0 \& y \cdot z \geq 0$ but $x \cdot z \geq 0$ is not true
$\Rightarrow$ not transitive relation.
$\Rightarrow R_1$ is not equivalence
R_2 if $a \geq b$ it does not implies $b \geq a$
$\Rightarrow R_2$ is not equivalence relation

Question47

For $\alpha \in N$, consider a relation R on N given by $R. = \{(x, y) : 3x + \alpha y$ is a multiple of $7\}$. The relation R is an equivalence relation if and only if : [28-Jul-2022-Shift-1]

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Options:

A. $\alpha = 14$

B. α is a multiple of 4

C. 4 is the remainder when α is divided by 10

D. 4 is the remainder when α is divided by 7

Answer: D

Solution:

Solution:
$R = \{(x, y) : 3x + \alpha y$ is multiple of $7\}$, now R to be an equivalence relation
(1) R should be reflexive : $(a, a) \in R \ \forall a \in N$
$\therefore 3a + a\alpha = 7k$
$\therefore (3 + \alpha)a = 7k$
$\therefore 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3$
$= 7k_1 + 4$
(2) R should be symmetric : $aRb \Leftrightarrow bRa$
$aRb : 3a + (7k - 3)b = 7m$

$\Rightarrow 3(a-b)+7kb=7m$
$\Rightarrow 3(b-a)+7ka=7m$
So, $aRb \Rightarrow bRa$
$\therefore$ R will be symmetric for $a=7k_1-3$
(3) Transitive : Let $(a, b) \in R, (b, c) \in R$
$\Rightarrow 3a+(7k-3)b=7k_1$ and
$3b+(7k_2-3)c=7k_3$
Adding $3a+7kb+(7k_2-3)c=7(k_1+k_3)$

$3a+(7k_2-3)c=7m$
$\therefore (a, c) \in R$
$\therefore$R is transitive
$\therefore \alpha=7k-3=7k+4$

Question48

Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that R = { (a, b) : b = pq, where p, q ≥ 3 are prime numbers }. Then, the number of elements in R is:
[29-Jul-2022-Shift-1]

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Options:

A. 600

B. 660

C. 540

D. 720

Answer: B

Solution:

Solution:
b can take its values as 9, 15, 21, 33, 39, 51, 57, 25, 35, 55, 49
b can take these 11 values and a can take any of 60 values
So, number of elements in $R=60\times 11=660$

Question49

Let $A=\{n \in N$: n is a 3 -digit number $\}$ $B=\{9k+2 : k \in N\}$
and $C=\{9k+I : k \in N\}$ for some I $(0<1<9)$
If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to
[2021, 24 Feb. Shift-1]

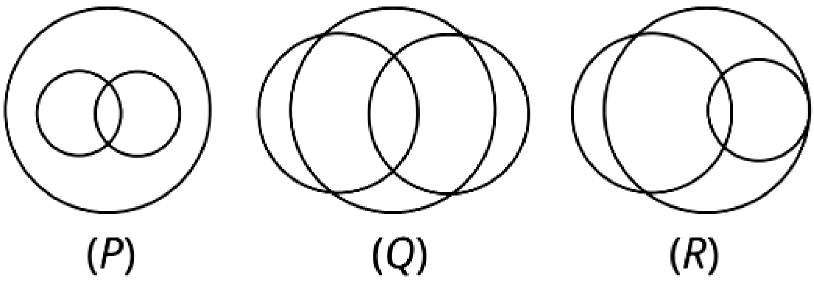
Answer: 5

Solution:

Solution:
Given, $A = \{n \in N : n$ is a 3 -digit number $\}$
$B = \{9k + 2 : k \in N\}$
$C = \{9k + 1 : k \in N\}$
$\because$ 3 digit number of the form $3k + 2$ are $\{101, 109, \ldots 992\}$
$\Rightarrow \text{Sum} = \frac{100}{2}[101 + 992] = \frac{100 \times 1093}{2}$
Similarly, 3-digit number of the form $9k + 5$ is
$\frac{100}{2}[104 + 995] = \frac{100 \times 1099}{2}$
[$\because$ numbers are $104, 113, \ldots, 995$] Their sum $= \frac{100 \times 1093}{2} + \frac{100 \times 1099}{2}$
$= 100 \times 1096 = 400 \times 274$
Hence, we can say the value of $I = 5$ as the second series of numbers obtained by set C is of the form $9k + 5$.
$\therefore$ Required value of $I = 5$

Question50

In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagram can justify the above statement?



[2021, 17 March Shift-1]

Options:

A. P and Q

B. P and R

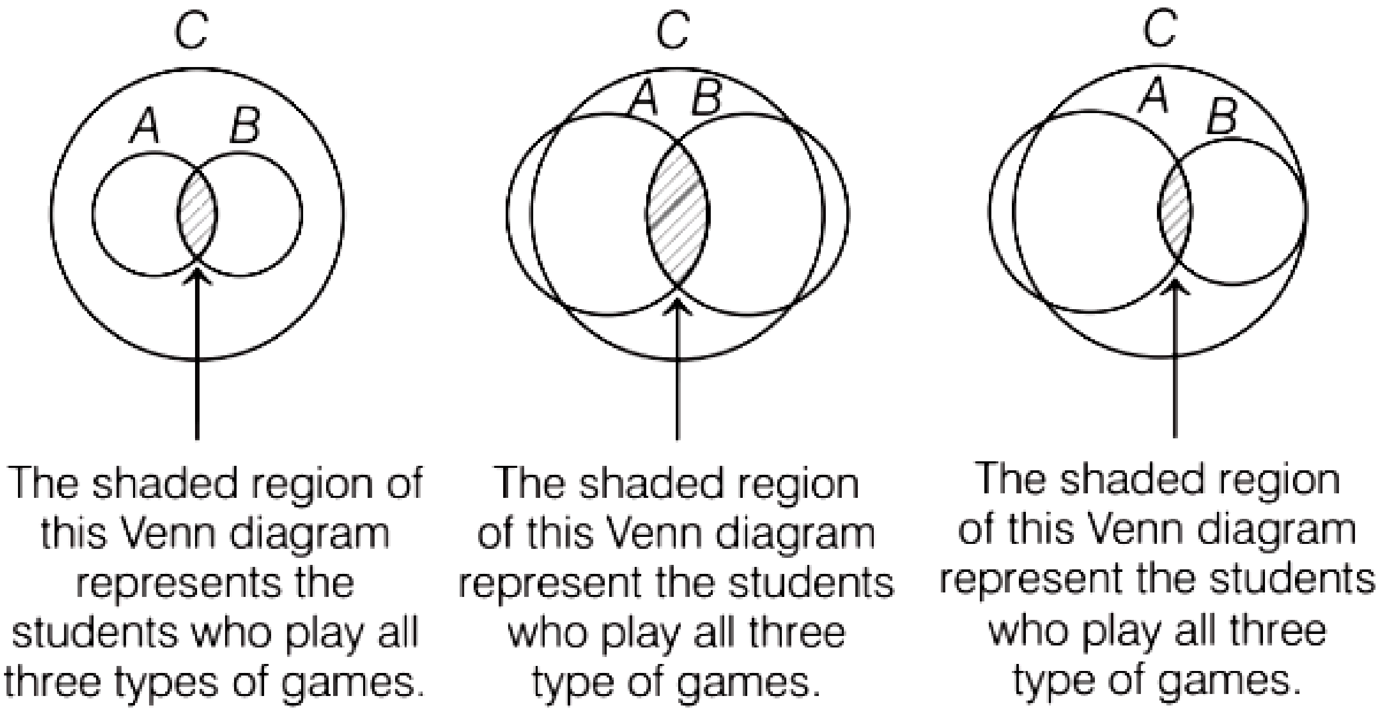
C. None of these

D. 0 and R

Answer: C

Solution:

Solution:



Question51

Let $A = \{n \in N \mid n^2 \leq n + 10,000\}$,
$B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to
[2021, 27 July Shift-II]

Answer: 832

Solution:

Solution:
Let $A = \{n \in N \mid n^2 \leq n + 10000\}$
$n^2 \leq n + 10000$
$n^2 - n \leq 10000$
$\Rightarrow n(n-1) \leq 100 \times 100$
$\Rightarrow A = \{1, 2, 3, \ldots\ldots, 100\}$
Now, $B = \{3k + 1 \mid k \in N\}$
$B = \{4, 7, 10, 13, \ldots\}$
and $C = \{2k \mid k \in N\}$
$C = \{2, 4, 6, 8, \ldots\}$
So, $B - C = \{7, 13, 19, \ldots\ldots, 97, \ldots\}$
So, $A \cap (B - C) = \{7, 13, 19, \ldots\ldots, 97\}$
This form an AP with common difference (d=6)
$\Rightarrow 97 = 7 + (n-1)6$
$n = \frac{97-7}{6} + 1 = 16 \quad [\because a_n = a + (n-1)d]$
Hence, sum $= \frac{16}{2}[7 + 97]$
$= 832 \quad \left\{\because S_n = \frac{n}{2}(a + 1)\right\}$

Question52

If $A = \{x \in R : |x - 2| > 1\}$,
$B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$ and
$C = \{x \in R : |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is
[2021, 27 Aug. Shift-I]

Answer: 256

Solution:

Solution:
$A = \{x \in R : |x - 2| > 1\}$
$\Rightarrow A = (-\infty, 1) \cup (3, \infty)$
$B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$
$\Rightarrow B = (-\infty, -2) \cup (2, \infty)$
$C = \{x \in R : |x - 4| \geq 2\}$
$\Rightarrow C = (-\infty, 2] \cup [6, \infty)$
$\Rightarrow A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$
$\Rightarrow (A \cap B \cap C)^C = [-2, 6)$
$\therefore (A \cap B \cap C)^C \cap Z = \{-2, -1, 0, 1, 2, 3, 4, 5\}$
Number of subsets of $(A \cap B \cap C)^C \cap Z$
$= 2^8 = 256$

Question53

Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K % of them are suffering from both ailments, then K can not belong to the set
[2021, 26 Aug. Shift-1]

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Options:

A. $\{80, 83, 86, 89\}$

B. $\{84, 86, 88, 90\}$

C. $\{79, 81, 83, 85\}$

D. $\{84, 87, 90, 93\}$

Answer: C

Solution:

Solution:
Let $A =$ Patient suffering from heart ailment and $B =$ Set of patient suffering from lungs infection
Given, $n(A) = 89\%$ and $n(B) = 98\%$
$n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$
$\Rightarrow \ 100 \geq 89 + 98 - n(A \cap B)$
$\Rightarrow \ 87 \leq n(A \cap B)$
Also, $n(A \cap B) = \min\{n(A), n(B)\}$
$\Rightarrow \ n(A \cap B) \leq 89$
$\therefore \ 87 \leq n(A \cap B) \leq 89$
So, $n(A \cap B) \notin \{79, 81, 83, 85\}$.

Question54

Let $X = \{n \in N : 1 \leq n \leq 50\}$. If $A = \{n \in X : n$ is a multiple of $2\}$ and $B = \{n \in X : n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is
[Jan. 7, 2020 (II)]

Answer: 29

Solution:

Solution:
From the given conditions, $n(A) = 25, n(B) = 7$ and $n(A \cap B) = 3$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 25 + 7 - 3 = 29$

Question55

Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is
[Sep. 06, 2020 (I)]

Answer: 28

Solution:

Solution:
$2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$
$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$
$\therefore m = 7, n = 4 \Rightarrow mn = 28$

Question56

A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:
[Sep. 05, 2020 (I)]

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Options:

A. 63

B. 36

C. 54

D. 38

Answer: B

Solution:

Solution:
Given, $n(C) = 73, n(T) = 65, n(C \cap T) = x$
$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$
$\Rightarrow 65 \geq x \geq 38 \Rightarrow x \neq 36$

Question57

A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:
[Sep. 04, 2020 (I)]

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Options:

A. 29

B. 37

C. 65

D. 55

Answer: D

Solution:

Solution:
Let $n(U) = 100$, then $n(A) = 63, n(B) = 76$ $n(A \cap B) = x$
Now, $n(A \cup B) \quad = n(A) + n(B) - n(A \cap B) \leq 100$
$\quad = 63 + 76 - x \leq 100$
$\Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$
$\because n(A \cap B) \leq n(A)$

$\Rightarrow x \leq 63$
$\therefore 39 \leq x \leq 63$

Question58

Let $\cup_{i=1}^{50} X_i = \cup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets $X_i{}'$s and exactly 6 of sets $Y_i{}'$s, then n is equal to
[Sep. 04, 2020 (II)]

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Options:

A. 15

B. 50

C. 45

D. 30

Answer: D

Solution:

Solution:
$\cup_{i=1}^{50} X_i = \cup_{i=1}^{n} Y_i = T$
$\because\ n(X_i) = 10,\ n(Y_i) = 5$
So, $\cup_{i=1}^{50} X_i = 500,\ \cup_{i=1}^{n} Y_i = 5n$
$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$

Question59

Let S = {1, 2, 3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :
[Jan. 12, 2019 (I)]

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Options:

A. $2^{100} - 1$

B. $2^{50}(2^{50} - 1)$

C. $2^{50} - 1$

D. $2^{50} + 1$

Answer: B

Solution:

Solution:
∵ Product of two even number is always even and product of two odd numbers is always odd.
∴ Number of required subsets
= Total number of subsets − Total number of subsets having only odd numbers
$= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

Question60

Let Z be the set of integers. If $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is :
[Jan. 12, 2019 (II)]

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Options:

A. 2^{15}

B. 2^{18}

C. 2^{12}

D. 2^{10}

Answer: A

Solution:

Solution:
(a) Let $x \in A$, then
∵ $2^{(x+2)(x^2-5x+6)} = 1 \Rightarrow (x+2)(x-2)(x-3) = 0$
$x = -2, 2, 3$
$A = \{-2, 2, 3\}$
Then, $n(A) = 3$
Let $x \in B$, then
$-3 < 2x - 1 < 9$
$-1 < x < 5$ and $x \in Z$
∴ $B = \{0, 1, 2, 3, 4\}$
$n(B) = 5$
$n(A \times B) = 3 \times 5 = 15$
Hence, Number of subsets of $A \times B = 2^{15}$

Question61

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:
[Jan. 10, 2019 (II)]

Options:

A. 102

B. 42

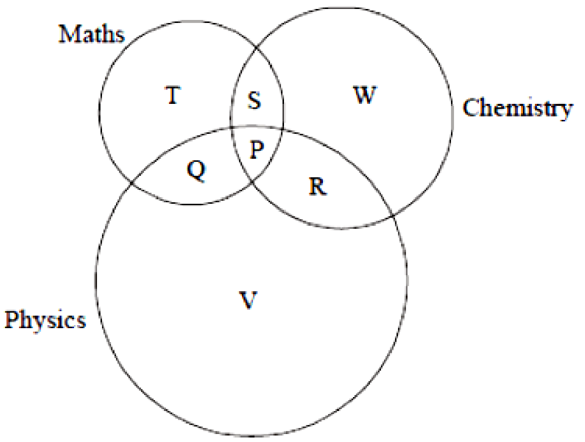
C. 1

D. 38

Answer: D

Solution:

Solution:



$P = \{30, 60, 90, 120\}$
$\Rightarrow n(P) = 4$
$Q = \{6n : n \in N, 1 \le n \le 23\} - P$
$\Rightarrow n(Q) = 19$
$R = \{15n : n \in N, 1 \le n \le 9\} - P$
$\Rightarrow n(R) = 5$
$S = \{10n : n \in N, 1 \le n \le 14\} - P$
$\Rightarrow n(S) = 10$
$n(T) = 70 - n(P) - n(Q) - n(S) = 70 - 33 = 37$
$n(V) = 46 - n(P) - n(Q) - n(R) = 46 - 28 = 18$
$n(W) = 28 - n(P) - n(R) - n(S) = 28 - 19 = 9$
$\Rightarrow$ Number of required students $= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$
$= 140 - 102 = 38$

Question62

Let A, B and C be sets such that $\varphi \ne A \cap B \subseteq C$. Then which of the following statements is not true? [April 12, 2019 (II)]

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Options:

A. $B \cap C \ne \varphi$

B. If $(A - B) \subseteq C$, then $A \subseteq C$

C. $(C \cup A) \cap (C \cup B) = C$

D. If $(A - C) \subseteq B$, then $A \subseteq B$

Answer: D

Solution:

Solution:
(1),(2) and (4) are always correct
In (3) option,
If $A = C$ then $A - C = \varphi$
Clearly, $\varphi \subset eqB$ but $A \subset eqB$ is not always true.

Question63

Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: [April. 09, 2019 (II)]

Options:

A. 13.9

B. 12.8

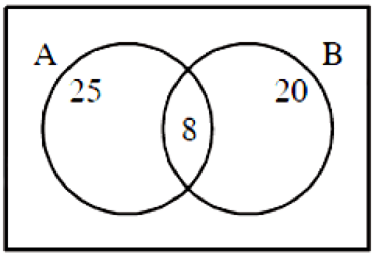
C. 13

D. 13.5

Answer: A

Solution:

Solution:



% of people who reads A only $= 25 - 8 = 17\%$
% of people who read B only $= 20 - 8 = 12\%$
% of people from A only who read advertisement $= 17 \times 0.3 = 5.1\%$
% of people from B only who read advertisement $= 12 \times 0.4 = 4.8\%$
% of people from A&B both who read advertisement $= 8 \times 0.5 = 4\%$
$\therefore$ total % of people who read advertisement $= 5.1 + 4.8 + 4 = 13.9\%$

Question64

Let $S = \{x \in R : x \geq 0$ and $2 \mid \sqrt{x} - 3 \mid + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S [2018]

©

Options:

A. contains exactly one element.

B. contains exactly two elements.

C. contains exactly four elements.

D. is an empty set

Answer: B

Solution:

Solution:
Case-I: $x \in [0, 9]$
$2(3-\sqrt{x})+x-6\sqrt{x}+6=0$
$\Rightarrow x-8\sqrt{x}+12=0 \Rightarrow \sqrt{x}=4, 2$
$\Rightarrow x=16, 4$
Since $x \in [0, 9]$
$\therefore\ x=4$
Case-II: $x \in [9, \infty]$
$2(\sqrt{x}-3)+x-6\sqrt{x}+6=0$
$\Rightarrow x-4\sqrt{x}=0 \Rightarrow x=16, 0$
Since $x \in [9, \infty]$
$\therefore x=16$
Hence, $x=4\&16$

Question65

If $f(x)+2f\left(\frac{1}{x}\right)=3x, x \neq 0$ and $S=\{x \in R : f(x)=f(-x)\}$; then S [2016]

©

Options:

A. contains exactly two elements.

B. contains more than two elements.

C. is an empty set.

D. contains exactly one element.

Answer: A

Solution:

Solution:
$f(x)+2f\left(\frac{1}{x}\right)=3x \quad \ldots..(1)$
$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$
Adding (1) and (2)
$\Rightarrow f(x)+f\left(\frac{1}{x}\right)=x+\frac{1}{x}$
Substracting (1) from (2)
$\Rightarrow f(x)-f\left(\frac{1}{x}\right)=\frac{3}{x}-3x\ldots$
On adding (3) and (4)

$\Rightarrow f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$

$x^2 = 2$ or $x = \sqrt{2}, -\sqrt{2}$

Question66

Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then:
[Online April 10, 2016]

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Options:

A. $P \subset Q$ and $Q - P \neq \varphi$

B. Q not $\subset$ P

C. $P = Q$

D. P not $\subset$ Q

Answer: C

Solution:

Solution:

$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

$= (\sqrt{2} + 1)\cos\theta = \left(\frac{2-1}{\sqrt{2}-1}\right)\cos\theta$

$\Rightarrow (\sqrt{2} - 1)\sin\theta = \cos\theta$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\therefore\ P = Q$

Question67

In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements:
(A) 5% families own both a car and a phone
(B) 35% families own either a car or a phone
(C) 40,000 families live in the town
Then,
[Online April 10, 2015]

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Options:

A. Only (A) and (C) are correct.

B. Only (B) and (C) are correct.

C. All (A), (B) and (C) are correct.

D. Only (A) and (B) are correct.

Answer: C

Solution:

Solution:
$n(P) = 25\%$
$n(C) = 15\%$
$n(P' \cup C') = 65\%$
$\Rightarrow n(P \cup C)' = 65\%$
$n(P \cup C) = 35\%$
$n(P \cap C) = n(P) + n(C) - n(P \cup C)$
$25 + 15 - 35 = 5\%$
$x \times 5\% = 2000$
$x = 40,000$

Question68

A relation on the set $A = \{x : |x| < 3, x \in Z\}$ where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is:
[Online April 12, 2014]

©

Options:

A. 32

B. 16

C. 8

D. 64

Answer: B

Solution:

Solution:
(b) $A = \{x : |x| < 3, x \in Z\}$
$A = \{-2, -1, 0, 1, 2\}$
$R = \{(x, y) : y = |x|, x \neq -1\}$
$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}$
R has four elements Number of elements in the power set of R
$= 2^4 = 16$

Question69

Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is :
[2012]

©

Options:

A. 5^2

B. 3^5

C. 2^5

D. 5^3

Answer: B

Solution:

Solution:
Let $X = \{1, 2, 3, 4, 5\}$ $n(x) = 5$
Each element of x has 3 options. Either in set Y or set Z or none.
$(\because Y \cap Z = \varphi)$
So, number of ordered pairs $= 3^5$

Question70

If A, B and C are three sets such that A ∩ B = A ∩ C and A ∪ B = A ∪ C, then [2009]

©

Options:

A. $A = C$

B. $B = C$

C. $A \cap B = \varphi$

D. $A = B$

Answer: B

Solution:

Solution:
Finding the value:
$A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$
$\Rightarrow (A \cap C) \cup (B \cap C) = C$
$\Rightarrow (A \cap B) \cup (B \cap C) = C \; \ldots .(i) \; (\because A \cap C = A \cap B)$
$\Rightarrow A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$
$\Rightarrow B = (A \cap B) \cup (C \cap B)$
$= (A \cap B) \cup (B \cap C) \; \cdots\cdots (ii)$
From (i) and (ii)
$B = C$
