UNIT – III : CALCULUS
CHAPTER-8
DIFFERENTIAL EQUATIONS

Topic-1

Basic Concepts and Variable Separable Methods

Concepts covered: Definition of differential equation, degree, order, general and particular solutions of a differential equation and solution of differential equations by method of separation of variables.

Revision Notes

- **Differential Equation :**
  An equation consisting of an independent variable, dependent variable and differential coefficients of dependent variable with respect to the independent variable is known as differential equation.

  e.g. : **(i)** $\frac{d^2y}{dx^2} = -a^2y$, **(ii)** $\frac{dy}{dx} = \frac{x+y}{x^2}$, **(iii)** $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = p\frac{dy}{dx}$

- **Order of Differential Equation :** The order of a differential equation is the order of the highest derivative appearing in the differential equation.

  e.g. : $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^3 + 2 = 0$ is the differential equation of order 3 because highest order derivative of y w.r.t. x is $\frac{d^3y}{dx^3}$.

- **Degree of Differential Equation :** The degree of the differential equation is the degree (power) of the highest order derivative, when the differential coefficient has been made free from the radicals and fractions.

  e.g. : $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{2}{3}} = 3\frac{d^2y}{dx^2}$ is the differential equation of degree 3, because the power of highest order derivative $\frac{d^2y}{dx^2}$ is 3 (after cubing).

- **Formation of Differential Equation :**
  If the equation of the family of curves is given then its differential equation is obtained by **eliminating arbitrary constants** occurring in equation with the help of equation of the curve and the equations obtained by differentiating the equation of the curve.

- **Algorithm for the Formation of the Differential Equation :**
  Step 1 : Write down the given equation of the curve.
  Step 2 : Differentiate the given equation with respect to the independent variable as many times as the number of arbitrary constants.
  Step 3 : Eliminate the arbitrary constants by using given equation and the equations obtained by the differentiation in step2.

- **Solution of Differential Equations :**
  (a) General solution : The solution which contains as many as arbitrary constants as the order of the differential equations, *e.g.*, $y = \alpha \cos x + \beta \sin x$ is the general solution of $\frac{d^2y}{dx^2} + y = 0$.
  Here, the differential equation is of second order and there are two arbitrary constants *i.e.*, α and β in the general solution.
  (b) Particular solution: Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular solution *e.g.*, $y = 3 \cos x + 2 \sin x$ is a particular solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.

(c) Solution of Differential Equation by Variable Separable Method : A variable separable form of the differential equation is the one which can be expressed in the form of $f(x)\,dx = g(y)\,dy$. The solution is given by

$\int f(x)dx = \int g(y)dy + k$, or $\int g(y)\,dy = \int f(n)\,dn + k$, where k is the constant of integration.

Topic-2

Linear Differential Equations

Concepts covered: Solution of linear differential equation in y and solution of linear differential equation in x.

Revision Notes

➢ **Linear Differential Equation :**

A differential equation is said to be linear if dependent variable (say y) and its derivative occurs in the first degree.

(a) Linear differential equation in y : It is of the form $\frac{dy}{dx} + P(x)y = Q(x)$, where $P(x)$ and $Q(x)$ are functions of x only.

- **Solution of Linear Differential Equation in y :**

  Step 1 : Write the given differential equation in the form $\frac{dy}{dx} + P(x)y = Q(x)$.

  Step 2 : Find the **integration Factor** (I.F.) $= e^{\int P(x)dx}$.

  Step 3 : The solution is given by, $y(I.F.) = \int Q(x).(I.F.)dx + C$, where C is the constant of integration.

(b) Linear Differential equation in x : It is of the form $\frac{dx}{dy} + P(y)x = Q(y)$ where $P(y)$ and $Q(y)$ are functions of x only.

- **Solution of Linear Differential Equation in x :**

  Step 1 : Write the given differential equation in the form $\frac{dx}{dy} + P(y)x = Q(y)$.

  Step 2 : Find the **integration Factor** (I.F.) $= e^{\int P(y)dy}$.

  Step 3 : The **solution** is given by, $x(I.F.) = \int Q(y).(I.F.)dy + \lambda$, where λ is the constant of integration.

Mnemonics

Concept : Linear Differential equation $\frac{dy}{dx} + Py = Q$

Mnemonics : WHY IF KYON IF

Interpretation : Its solution can be remember as :

$$y \times IF = \int (Q \times IF)dx + C$$

WHY IF KYON IF

Topic-3

Homogeneous Differential Equations

Concepts covered: Homogeneous differential equations and their solutions.

Revision Notes

➢ **Homogeneous Differential Equations and their Solutions**

- **Identifying a Homogeneous Differential Equation :**

  Step 1 : Write down the given differential equation in the form $\frac{dy}{dx} = F(x, y)$.

  Step 2 : If $f(kx, ky) = k^n f(x, y)$, then the given differential equation is homogeneous of degree 'n'.

- **Solving a homogeneous differential equation :**

Case I: If $\frac{dy}{dx} = f(x, y)$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Case II: If $\frac{dx}{dy} = f(x, y)$

Put $x = vy$

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

Then, we separate the variables to get the required solution.