UNIT – 4: QUADRATIC EQUATIONS

1. Roots of the equation

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ are } \alpha \text{ and } \beta$$

2. Sum and product of the roots:

$$S = \alpha + \beta = -\frac{b}{a}; \qquad P = \alpha\beta = \frac{c}{a}.$$

3. To find the equation whose roots are α and β

$$x^2 - Sx + P = 0$$

where S is sum and P is product of roots.

4. Nature of the roots

$D = b^2 - 4ac$ where D is called discriminant.

(a) If $b^2 - 4ac \geq 0$, roots are real

i) $b^2 - 4ac > 0$, roots are real and unequal.

ii) $b^2 - 4ac = 0$, roots are real an equal. In this case, each root = $-\frac{b}{2a}$

iii) $b^2 - 4ac$ = perfect square, roots are rational.

iv) $b^2 - 4ac$ = not a perfect square, roots are irrational.

(b) if $b^2 - 4ac < 0$, i.e., -ve, then $\sqrt{b^2 - 4ac}$ is imaginary.

Therefore the roots are imaginary and unequal.

5. Symmetric function of the roots

If α and β are the roots of $ax^2 + bx + c = 0$, then

i) $\alpha+\beta=-\dfrac{b}{a}$ and $\alpha\beta=\dfrac{c}{a}$

ii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

iii) $\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

iv) $\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta)$

v) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

vi) $\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$

6. Condition for Common roots

Equations: $a_1x^2 + b_1x + c_1 = 0$

Roots: $a_2x^2 + b_2x + c_2 = 0$

Condition for one common root:

$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$

Condition for both the roots to be common:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

⇒ **Note:** If the coefficient of x^2 in both the equations be unity, then the value of common root is obtained by subtracting their equations.

7. Inequalities

a) $x^2 > a^2$ is positive if $<-a$ or $x> a$

b) $x^2 < a^2$ if $-a<x<a$

c) $a^2 < x^2 < b^2$ double inequality $\therefore$ $a<x<b$ or $-b<x<-a$.

8. In an inequality you can always multiply or divide by a +ve quantity but not by a –ve quantity. Multiplying by a –ve quantity or taking reciprocal will reverse the inequality.

e.g., $a > b \Rightarrow -a < -b$ or $\dfrac{1}{a} < \dfrac{1}{b}$.