Chapter 13

Permutations and Combinations

CHAPTER HIGHLIGHTS

- *Permutations*
- *Combinations*
- *Total Number of Combinations*
- *Dividing Given Items into Groups*
- *Circular Permutations*
- *Rank of a Word*
- *Arrangements*

Introduction

Permutations and combinations is one of the important areas in many exams because of two reasons. The first is that solving questions in this area is a measure of students' reasoning ability. Secondly, solving problems in areas like probability requires thorough knowledge of permutations and combinations.

Before discussing permutations and combinations, let us look at what is called as the 'fundamental rule'.

'If one operation can be performed in 'm' ways and (when, it has been performed in any one of these ways), a second operation then can be performed in 'n' ways, the number of ways of performing the two operations will be $m \times n$'.

This can be extended to any number of operations.

If there are three cities A, B, and C such that there are 3 roads connecting A and B and 4 roads connecting B and C, then the number of ways one can travel from A to C is 3×4, i.e. 12.

This is a very important principle, and we will be using it extensively in permutations and combinations. Because we use it very extensively, we do not explicitly state every time that the result is obtained by the fundamental rule but directly write down the result.

Permutations

Each of the arrangements which can be made by taking some or all of a number of items is called a permutation. Permutation implies 'arrangement' or that 'order of the items' is important.

The permutations of three items a, b, and c, taken two at a time are ab, ba, ac, ca, cb, and bc. Since the order in which the items are taken is important, ab and ba are counted as two different permutations. The words 'permutation' and 'arrangement' are synonymous and can be used interchangeably.

The number of permutations of n things taking r at time is denoted by ^nP_r (and read as '^nP_r').

Combinations

Each of the groups or selections which can be made by taking some or all of a number of items is called a combination. In combinations, the order in which the items are taken is not considered as long as the specific things are included.

The combination of three items a, b, and c taken two at a time are ab, bc, and ca. Here, ab and ba are not considered separately because the order in which a and b are taken is not important but it is only required that a combination including a and b is what is to be counted. The words 'combination' and 'selection' are synonymous.

The number of combinations of n things taking r at time is denoted by ^nC_r (and read as '^nC_r').

Number of linear permutations of 'n' dissimilar items taken 'r' at a time without repetition (^nP_r)

Consider r boxes each of which can hold one item. When all the r boxes are filled, what we have is an arrangement of r items taken from the given n items. So, each time we fill up the r boxes with items taken from the given n items, we have an arrangement of r items taken from the given n items without repetition. Hence, the number of ways in which we can fill up the r boxes by taking things from the given n things is equal to the number of permutations of n things taking r at a time.

Boxes □ □ □ □ .... □
1 2 3 4 r

The first box can be filled in n ways (because any one of the n items can be used to fill this box). Having filled the first box, to fill the second box, we now have only $(n-1)$ items; any one of these items can be used to fill the second box, and, hence. the second box can be filled in $(n-1)$ ways; similarly, the third box in $(n-2)$ ways and so on the r^{th} box can be filled in $\{n-(r-1)\}$ ways, i.e. $[n-r+1]$ ways. Hence, from the Fundamental Rule, all the r boxes together be filled up in

$$n \times (n-1) \times (n-2) \ldots (n-r+1) \text{ ways}$$

So, $${}^nP_r = n \times (n-1) \times (n-2) \ldots (n-r+1)$$

This can be simplified by multiplying and dividing the right hand side by $(n-r)(n-r-1) \ldots 3.2.1$ giving us ${}^nP_r = n(n-1)(n-2) \ldots [n-(r-1)]$

$$= \frac{(n-1)(n-2)\ldots[n-(r-1).(n-r)\ldots3.2.1]}{(n-r)\ldots\ldots\ldots3.2.1} = \frac{n!}{(n-r)!}$$

The number of permutations of n distinct items taking r items at a time is

$$\boxed{{}^n p_r = \frac{n!}{(n-r)!}}$$

If we take n items at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the r boxes above, we can find that ${}^nP_n$ is equal to $n!$

The first box can be filled in n ways, the second one in $(n-1)$ ways, the third one in $(n-2)$ ways, and so on, then the n^{th} box in 1 way; hence, all the n boxes can be filled in $n(n-1)(n-2) \ldots 3.2.1$ ways, i.e., $n!$ ways. Hence,

$$\boxed{{}^n\boldsymbol{P}_n = \boldsymbol{n!}}$$

But if we substitute $r = n$ in the formula for ${}^nP_r$, then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$, we can conclude that $0! = 1$

Number of combinations of n dissimilar things taken r at a time.

Let the number of combinations ${}^nC_r$ be x. Consider one of these x combinations. Since this is a combination, the order of the r items is not important. If we now impose the condition that order is required for these r items, we can get $r!$ arrangements from this one combination. So, each combination can give rise to $r!$ permutations. x combinations will thus give rise to $x \,.\, r!$ permutations. But, since these are all permutations of n things taken r at a time, this must be equal to ${}^nP_r$. So,

$$x.r! = {}^nP_r = \frac{n!}{(n-r)!}$$

$$\Rightarrow \quad {}^nC_r = \frac{n!}{r!.(n-r)!}$$

The number of combinations of n dissimilar things taken all at a time is 1.

Out of n things lying on a table, if we select r things and remove them from the table, we are left with $(n-r)$ things on the table — that is, whenever r things are selected out of n things, we automatically have another selection of the $(n-r)$ things. Hence, the number of ways of making combinations taking r out of n things is the same as selecting $(n-r)$ things out of n given things, i.e.

$${}^n\boldsymbol{C}_r = {}^n\boldsymbol{C}_{n-r}$$

When we looked at ${}^nP_r$, we imposed two constraints which we will now release one by one and see how to find out the number of permutations.

Number of arrangements of *n* items of which *p* are of one type, *q* are of a second type, and the rest are distinct

When the items are all not distinct, then we **cannot** talk of a general formula for ${}^n\boldsymbol{P}_r$ for any r but we can talk of only ${}^nP_n$ (which is given below). If we want to find out ${}^nP_r$ for a specific value of r in a given problem, we have to work on a case to case basis (this has been explained in one of the solved examples).

The number of ways in which n things may be arranged taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest all distinct is $\frac{n!}{p!\,q!\,r!}$.

Number of arrangements of n distinct items where each item can be used any number of times (i.e. repetition allowed)

You are advised to apply the basic reasoning given while deriving the formula for ${}^nP_r$ to arrive at this result also. The first box can be filled up in n ways; the second box can be filled in again n ways (even though the first box is filled with one item, the same item can be used for filling the second box also because repetition is allowed); the third box can also be filled in n ways, and so on; the r^{th} box can be filled in n ways. Now, all the r boxes together can be filled in $\{n.n.n.n \ldots r \text{ times}\}$ ways, i.e. $\boldsymbol{n^r}$ ways.

The number of permutations of n things, taken r at a time when each item may be repeated once, twice, up to r times in any arrangement is $\boldsymbol{n^r}$.

What is important is not this formula by itself but the reasoning involved. So, even while solving problems of this type, you will be better off if you go from the basic reasoning and not just apply this formula.

Total number of combinations: Out of n given things, the number of ways of selecting **one or more** things is where we can select 1 or 2 or 3 and so on n things at a time; hence the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$

This is called 'the total number of combinations' and is equal to $2^n - 1$ where n is the number of things.

The same can be reasoned out in the following manner also.

There are n items to select from. Let each of these be represented by a box.

	1	2	3	4		n
No. of ways of dealing	☐	☐	☐	☐	....	☐
with the boxes	2	2	2	2		2

The first box can be dealt with in two ways. In any combination that we consider, this box is **either** included **or** not included. These are the two ways of dealing with the first box. Similarly, the second box can be dealt with in two ways, the third one in two ways, and so on; the n^{th} box in two ways. By the Fundamental Rule, the number of ways of dealing with all the boxes together in $2 \times 2 \times 2 \times \cdots n$ times ways, i.e. in 2^n ways. But out of these, there is one combination where we 'do not include the first box, do not include the second box, do not include the third box and so on, do not include the n th box'. That means, no box is included. But this is not allowed because we have to select **one or more** of the items (i.e. at least one item). Hence, this combination of no box being included is to be subtracted from the 2^n ways to give the result of

> *Number of ways of selecting one or more items from n given items is* $2^n - 1$

Dividing given items into groups: Dividing $(p + q)$ items into two groups of p and q items, respectively.

Out of $(p + q)$ items, if we select p items (which can be done in ${}^{p+q}C_p$ ways), then we will be left with q items, i.e. we have two groups of p and q items, respectively. So, the number of ways of dividing $(p + q)$ items into two groups of p and q items, respectively, is equal to ${}^{p+q}C_p$ which is equal to $\dfrac{(p+q)!}{p!\,q!}$

> *The number of ways of dividing (p + q) items into two groups of p and q items respectively is* $\dfrac{(p+q)!}{p!\cdot q!}$.

If $p = q$, i.e. if we have to divide the given items into two EQUAL groups, then two cases arise

1. When the two groups have distinct identity and
2. When the two groups do not have distinct identity.

In the first case, we just have to substitute $p = q$ in the aforementioned formula which then becomes

> *The number of ways of dividing 2p items into two equal groups of p each is* $\dfrac{(2p)!}{(p!)^2}$ *where the two groups have distinct identity.*

In the second case, where the two groups do not have distinct identity, we have to divide the above result by 2!, i.e. it then becomes

> *The number of ways of dividing 2p items into two equal groups of p each is* $\dfrac{(2p)!}{2!(p!)^2}$ *where the two groups do not have distinct identity.*

Dividing $(p + q + r)$ items into three groups consisting of p, q, and r items, respectively

> *The number of ways in which (p + q + r) things can be divided into three groups containing p, q, and r things, respectively, is* $\dfrac{(p+q+r)!}{p!\,q!\,r!}$.

If $p = q = r$, i.e. if we have to divide the given items into three EQUAL groups, then we have two cases where the three groups are distinct and where the groups are not distinct.

When the three groups are distinct, the number of ways is $\dfrac{(3p)!}{(p!)^3}$.

When the three groups are not distinct, then the number of ways is $\dfrac{(3p)!}{3!(p!)^3}$.

Circular Permutations

When n distinct things are arranged in a straight line taking all the n items, we get $n!$ permutations. However, if these n items are arranged in a circular manner, then the number of arrangements will not be $n!$ but it will be less than that. This is because in a straight line manner, if we have an arrangement ABCDE and if we move every item one place to the right (in cyclic order), the new arrangement that we get EABCD is not the same as ABCDE and this also is counted in the $n!$ permutations that we talked of. However, if we have an arrangement ABCDE in a circular fashion, by shifting every item by one place in the clockwise direction, we still get the same arrangement ABCDE. So, if we now take $n!$ as the number of permutations, we will be counting the same arrangement more than once.

The number of arrangements in circular fashion can be found out by first fixing the position of one item. Then the remaining $(n - 1)$ items can be arranged in $(n - 1)!$ ways. Now, even if we move these $(n - 1)$ items by one place in the clockwise direction, then the arrangement that we get will not be the same as the initial arrangement because one item is fixed and it does not move.

Hence, the number of ways in which n distinct things can be arranged in a circular arrangement is $(n - 1)!$

*The number of **circular arrangements** of **n distinct items** is **(n – 1)!** if there is **DIFFERENCE** between clockwise and anticlockwise arrangements and **(n – 1)!/2** if there is **NO DIFFERENCE** between clockwise and anticlockwise arrangements.*

The number of diagonals in an *n*-sided regular polygon

An n-sided regular polygon has n vertices. Joining any two vertices, we get a line of the polygon that are ${}^nC_2$ in number. Of these ${}^nC_2$ lines, n of them are sides. Hence, diagonals are ${}^nC_2 - n = \frac{n(n-3)}{2}$.

Number of integral solution of the equation

$$x_1 + x_2 + \cdots + x_n = s$$

Consider the equation $x_1 + x_2 + x_3 = 10$.

If we consider all possible integral solutions of this equation, there are infinitely many. But, the number of positive (or non-negative) integral solutions is finite.

We would like the number of positive integral solutions of this equation, i.e. values of (x_1, x_2, x_3) such that each $x_i > 0$.

We imagine 10 identical objects arranged on a line. There are 9 gaps between these 10 objects. If we choose any two of these gaps, we are effectively splitting the 10 identical objects into 3 parts of distinct identity. Conversely, every split of these 10 objects corresponds to a selection of 2 gaps out of the 9 gaps.

Therefore, the number of positive integral solutions is ${}^9C_2$. In general, if $x_1 + x_2 + \cdots + x_n = s$ where $s \geq n$, the number of positive integral solutions is ${}^{s-1}C_{n-1}$.

If we need the number of non negative integral solutions, we proceed as follows. Let $a_1, a_2, \ldots$ be a non-negative integral solution. Then, $a_1 + 1, a_2 + 1, \ldots, a_n + 1$ is a positive integral solution of the equation $x_1 + x_2 + \cdots + x_n = s + n$. Therefore, the number of non-negative integral solutions of the given equation is equal to the number of positive integral solutions of $x_1 + x_2 + \cdots + x_n = s + n$, which is ${}^{s+n-1}C_{n-1}$.

*For $x_1 + x_2 + x_3 + \cdots + x_n = s$ where $s \geq 0$, the number of **positive integral solutions** (when $s \geq n$) is ${}^{s-1}C_{n-1}$ and the number of **non-negative integral solutions** is ${}^{n+s-1}C_{n-1}$*

Some additional points

1. Suppose there are n letters and n corresponding addressed envelopes. The numbers of ways of placing these letters into the envelopes such that no letter is placed in its corresponding envelope is often referred as derangements. The number of derangements of n objects is given by

$$D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right]$$

For example, when $n = 3$, the number of derangements is $D(3) = 3!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right] = 2$ and when $n = 4$,

$$D(4) = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$$

2. The total number of ways in which a selection can be made by taking some or all out of $p + q + r + \cdots$ things where p are alike of one kind, q alike of a second kind, r alike of a third kind, and so on is

$$[\{(p+1)(q+1)(r+1)\ldots\} - 1].$$

3. ${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$ and ${}^nP_r = r.{}^{n-1}P_{r-1} + {}^{n-1}P_r$

Solved Examples

Example 1

Consider the word PRECIPITATION. Find the number of ways in which

(i) a selection
(ii) an arrangement

of 4 letters can be made from the letters of this word.

Solution

The word PRECIPITATION has 13 letters I, I, I, P, P, T, T, E, R, C, A, O, N of 9 different sorts.

In taking 4 letters, the following are the possibilities to be considered.

(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

Selections

(a) 4 distinct letters can be selected from 9 distinct letters (I, P, T, E, R, C, A, O, N) in ${}^9C_4 = 126$ ways.
(b) As 3 letters have to be alike, the only possibility is selecting all the *I*'s. Now the 4th letter can be selected from any of the remaining 8 distinct letters in ${}^8C_1 =$ 8 ways.
(c) Two pairs of two alike letters can be selected from I's, *Q*'s, and *T*'s in ${}^3C_2 = 3$ ways.
(d) The two alike letters can be selected in ${}^3C_1 = 3$ ways and the two distinct letters can now be selected from the 8 distinct letters in ${}^8C_2 = 28$ ways. Hence, required number of ways are $3 \times 28 = 84$.

Hence, the total selections are $126 + 8 + 3 + 84 = 221$.

Arrangements: For arrangements, we find the arrangements for each of the aforementioned selections and add them up.

(a) As the 4 letters are distinct, there are 4! arrangements for each selection. Hence required arrangements are $126 \times 4! = 3024$.

(b) Since 3 of the 4 letters are alike, there are $\frac{4!}{3!}$ arrangements for each of the selection. Hence, required arrangements are $8 \times \frac{4!}{3!} = 32$.

(c) The required arrangements here are $3 \times \frac{4!}{2!\,2!} = 18$

(d) The required arrangements are $84 \times \frac{4!}{2!} = 1008$.

Total number of arrangements are 3024 + 32 + 18 + 1008 = 4082.

Example 2

How many four letter words can be formed using the letters of the word 'ROAMING'?

Solution

None of the letters in the word are repeated.

$\therefore$ The number of four letter words that can be formed $= {}^7P_4$

$= \frac{7!}{3!} = (7)\,(6)\,(5)\,(4) = 840.$

Example 3

In a party, each person shook hands with every other person present. The total number of hand shakes was 28. Find the number of people present in the party.

Solution

Let the number of people present in the party be n.

Method 1: The first people shakes hands with a total of $(n - 1)$ persons, the second with $(n - 2)$ other people, and so on.

The total number of hand shakes is

$$(n-1) + (n-2) + \cdots + 2 + 1$$

$$\frac{n(n-1)}{2} = 28 \quad \text{(given)}$$

$$\Rightarrow \quad n = 8$$

Method 2: Number of hand shakes = Number of ways of selecting 2 people out of $n = {}^nC_2$.

$${}^nC_2 = 28$$

$$\frac{n(n-1)}{2!} = 28$$

$$\Rightarrow \quad n = 8$$

Direction for examples 3 to 7: The following examples are based on the data below.

The letters of NESTLE are permuted in all possible ways.

Example 4

How many of these words begin with T?

Solution

NESTLE has 6 letters of which the letter E occurs two times. Therefore, the required number of words = Number of ways of filling $N, E, S, E,$ and L in the second to sixth positions $= \frac{5!}{2!} = 60$.

Example 5

How many of these words begin and end with E?

Solution

The required number of words = The number of ways of filling $N, S, T,$ and L in the second to fifth positions = 4! = 24.

Example 6

How many of these words begin with S and end with L?

Solution

The required number of words = The number of ways of filling $N, E, T,$ and E in the second to fifth positions $= \frac{4!}{2!} = 12$.

Example 7

How many of these words neither begin with S nor end with L?

Solution

The required number of words = The total number of words which can be formed using the letters $N, E, S, T,$ and E – (Number of words which begin with S or end with L) $= \frac{6!}{2!}$ – (Number of words beginning with S + Number of words ending with L – Number of words beginning with S and ending with L)

$$= \frac{6!}{2!} - \left(\frac{5!}{2!} + \frac{5!}{2!} - \frac{4!}{2!}\right)$$

$$= 360 - (60 + 60 - 12) = 252.$$

Example 8

How many of these words begin with T and do not end with N?

Solution

The required number of words = The number of words beginning with T – The number of words beginning with T and ending with $N = \frac{5!}{2!} - \frac{4!}{2!} = 48$.

Direction for examples 8 to 11: The following examples are based on the data below.

The letters of FAMINE are permuted in all possible ways.

Example 9

How many of these words have all the vowels occupying odd places?

Solution

FAMINE has 3 vowels and 3 consonants.

The vowels can be arranged in the odd places in 3! or 6 ways.

The consonants would have to be arranged in even places. This is possible in 3! or 6 ways as well.

∴ The required number of words = $6^2 = 36$.

Example 10

How many of these words have all the vowels together?

Solution

If all the vowels are together, the vowels can be arranged in 3! ways among themselves.

Considering the vowels as separate a unit and each of the other letters as a unit, we have a total of 4 units that can be arranged in 4! ways.

∴ The required number of words = 4! 3! = 144

Example 11

How many of these words have at least two of the vowels separated?

Solution

The required number of words = The total number of words which can be formed using the letters *F, A, M, I, N,* and *E* – The number of words with all the vowels together = 6! – 4! 3! = 576.

Example 12

How many of these words have no two vowels next to each other?

Solution

To ensure that no two vowels are together, we first arrange the 3 consonants say $-c_1 - c_2 - c_3 -$ and place the vowels in the gaps between the consonants or the initial or final position. For each arrangement of the consonants, there are 4 places where the vowels can go. The vowels can be dealt with in 4 (3) (2) ways.

∴ The total number of words is 3! 4! = 144.

Direction for examples 17 and 18: The following examples are based on the data below.

A committee of 5 is to be formed from 4 women and 6 men.

Example 18

In how many ways can it be formed if it consists of exactly 2 women?

Solution

The committee must have 2 women and 3 men.

∴ The required number of ways = ${}^4C_2\,{}^6C_3 = 120$.

Example 19

In how many ways can it be formed if it consists of more women than men?

Solution

The committee must have either 4 women and 1 man or 3 women and 2 men.

∴ The required number of ways

$$= {}^4C_4\,{}^6C_1 + {}^4C_3\,{}^6C_2 = 6 + 60 = 66.$$

Example 20

Find the number of four-digit numbers that can be formed using four of the digits 0, 1, 2, 3, and 4 without repetition.

Solution

The first digit has 4 possibilities (1, 2, 3, and 4).

The second digit has 4 possibilities (0 and any of the three digits not used as the first digit).

The third digit has 3 possibilities.

The last digit has 2 possibilities.

∴ The required number of numbers = (4) (4) 3 (2) = 96.

Example 21

The number of diagonals of a regular polygon is four times the number of its sides. How many sides does it have?

Solution

Let the number of sides in the polygon be n.

$$\frac{n(n-3)}{2} = 4n$$

$$n(n-11) = 0;\ n > 0$$

$$\therefore \quad n - 11 = 0;$$

$$\therefore \quad n = 11.$$

Exercises

Direction for questions 1 to 25: Select the correct alternative from the given choices.

1. A man has 12 blazers, 10 shirts, and 5 ties. Find the number of different possible combinations in which he can wear the blazers, shirts, and ties.
(A) 27 (B) 300 (C) 240 (D) 600

2. How many different words can be formed by using all the letters of the word INSTITUTE?
(A) $\frac{9!}{2!}$ (B) 9 ! (C) $\frac{9!}{3!}$ (D) $\frac{9!}{3!2!}$

3. In how many ways can a cricket team of 11 members be selected from 15 players, so that a particular player is included and another particular player is left out?
(A) 216 (B) 826 (C) 286 (D) 386

4. A group contains n persons. If the number of ways of selecting 6 persons is equal to the number of ways of selecting 9 persons, then the number of ways of selecting four persons from the group is
(A) 1365 (B) 273 (C) 455 (D) 285

5. The number of ways of arranging 10 books on a shelf such that two particular books are always together is
(A) 9! 2! (B) 9! (C) 10! (D) 8

6. Find the number of ways of inviting at least one among 6 people to a party.
(A) 2^6 (B) $2^6 - 1$ (C) 6^2 (D) $6^2 - 1$

7. An eight-letter word is formed by using all the letters of the word 'EQUATION'. How many of these words begin with a consonant and end with a vowel?
(A) 3600 (B) 10800
(C) 2160 (D) 720

8. A committee of 5 members is to be formed from a group of 6 men and 4 women. In how many ways can the committee be formed such that it contains more men than women?
(A) 180 (B) 186 (C) 126 (D) 66

9. In how many ways can 10 boys and 10 girls be arranged in a row so that all the girls sit together?
(A) 10! (B) 11!
(C) 20! (D) 10! 11!

10. In how many ways can 6 boys and 5 girls be arranged in a row so that boys and girls sit alternately?
(A) $(6!)^2$ (B) $(5!)^2$
(C) 6! 5! (D) 2.5! 6!

11. There are seven letters and corresponding seven addressed envelopes. All the letters are placed randomly into the envelopes—one in each envelope. In how many ways can exactly two letters be placed into their corresponding envelopes?
(A) 44 (B) 924 (C) 308 (D) 189

12. We are given 3 different green dyes, 4 different red dyes, and 2 different yellow dyes. The number of ways in which the dyes can be chosen so that at least one green dye and one yellow dye is selected is
(A) 336 (B) 335 (C) 60 (D) 59

13. Prahaas attempts a question paper that has 3 sections with 6 questions in each section. If Prahaas has to attempt any 8 questions, choosing at least two questions from each section, then in how many ways can he attempt the paper?
(A) 18000 (B) 10125
(C) 28125 (D) 9375

14. Find the number of selections that can be made by taking 4 letters from the word INKLING.
(A) 48 (B) 38 (C) 28 (D) 18

15. A man has $(2n + 1)$ friends. The number of ways in which he can invite at least $n + 1$ friends for a dinner is 4096. Find the number of friends of the man.
(A) 11 (B) 15 (C) 17 (D) 13

16. How many four-digit numbers are there between 3200 and 7300, in which 6, 8, and 9 together or separately do not appear?
(A) 1421 (B) 1420
(C) 1422 (D) 1077

17. Raju has forgotten his six-digit ID number. He remembers the following: the first two digits are either 1, 5 or 2, 6, the number is even and 6 appears twice. If Raju uses a trial and error process to find his ID number at the most, how many trials does he need to succeed?
(A) 972 (B) 2052
(C) 729 (D) 2051

18. A matrix with four rows and three columns is to be formed with entries 0, 1, or 2. How many such distinct matrices are possible?
(A) 12 (B) 36 (C) 3^{12} (D) 2^{12}

19. In how many ways can 4 postcards be dropped into 8 letter boxes?
(A) ${}^8P_4$ (B) 4^8 (C) 8^4 (D) 24

20. In how many ways can 12 distinct pens be divided equally among 3 children?
(A) $\frac{12!}{(3!)^4}$ (B) $\frac{12!}{(4!)^3\,3!}$
(C) $\frac{12!}{3!4!}$ (D) $\frac{12!}{(4!)^3}$

21. If all possible five-digit numbers that can be formed using the digits 4, 3, 8, 6, and 9 without repetition are arranged in the ascending order, then the position of the number 89634 is
(A) 91 (B) 93 (C) 95 (D) 98

22. Manavseva, a voluntary organization, has 50 members who plan to visit 3 slums in an area. They decide to divide themselves into 3 groups of 25, 15, and 10. In how many ways can the group division be made?

(A) 25! 15! 10! (B) $\frac{50!}{25!\,15!\,10!}$

(C) 50! (D) 25! + 15! + 10!

23. In how many ways is it possible to choose two white squares so that they lie in the same row or same column on an 8×8 chessboard?

(A) 12 (B) 48 (C) 96 (D) 60

24. The number of four digit telephone numbers that have at least one of their digits repeated is

(A) 9000 (B) 4464 (C) 4000 (D) 3986

25. There are 4 identical oranges, 3 identical mangoes, and 2 identical apples in the basket. The number of ways in which we can select one or more fruits from the basket is

(A) 60 (B) 59 (C) 57 (D) 55

Answer Keys

1. D	**2.** D	**3.** C	**4.** A	**5.** A	**6.** B	**7.** B	**8.** B	**9.** D	**10.** C
11. B	**12.** A	**13.** C	**14.** D	**15.** D	**16.** D	**17.** B	**18.** C	**19.** C	**20.** D
21. C	**22.** B	**23.** C	**24.** B	**25.** B					