Continuity and Differentiability

Question 1: The function $f(x) = [\ln(1+ax)-\ln(1-bx)]/x$, not defined at $x=0$. The value should be assigned to f at $x=0$, so that it is continuous at $x=0$, is

(a) a+b

(b) a-b

(c) b-a

(d) ln a+ ln b

Question 2: If $x \sin(a+y) = \sin y$, then dy/dx is equal to

(a) $[\sin^2(a+y)]/\sin a$

(b) $\sin a /[\sin^2(a+y)]$

(c) $[\sin(a+y)]/\sin a$

(d) $\sin a /[\sin(a+y)]$

Question 3: The function $f(x) = [x]$, where [x] denotes the greatest integer function is continuous at:

(a) 4

(b) -2

(c) 1

(d) 1.5

Question 4: Consider the following in respect of the function $f(x) = 10^x$:

1. **Its domain is (∞, ∞)**
2. **It is a continuous function**
3. **It is differentiable at $x = 0$**

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Question 5: Let f(x) = |sin x|. Then

(a) f is everywhere differentiable

(b) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.

(c) f is everywhere continuous but not differentiable at $x = (2n + 1)$, $n \in Z$.

(d) none of these

Question 6: If the function $f(x) = (2x-\sin^{-1}x)/(2x+\tan^{-1}x)$ is continuous at each point of its domain, then the value of f(0) is

(a) 1/3

(b) -1/3

(c) 2/3

(d) 2

Question 7:

If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}$ is equal to:

(a) $\frac{4x^3}{1-x^4}$

(b) $\frac{-4x}{1-x^4}$

(c) $\frac{1}{4-x^4}$

(d) $\frac{-4x^3}{1-x^4}$

Answer: (b) $\frac{-4x}{1-x^4}$

Question 8:

If y = x tan y, then $\frac{dy}{dx}$ =

(a) $\frac{tanx}{x-x^2-y^2}$

(b) $\frac{y}{x-x^2-y^2}$

(c) $\frac{tany}{y-x}$

(d) $\frac{tanx}{x-y^2}$

Answer: (b) $\frac{y}{x-x^2-y^2}$

Question 9: The value of c in Rolle's theorem for the function, f(x) = sin 2x in [0, π/2] is

(a) π/4

(b) π/6

(c) π/2

(d) π/3

Question 10:

If $\sec(\frac{x^2-2x}{x^2+1})$ – y then $\frac{dy}{dx}$ is equal to

(a) $\frac{y*2}{x^2}$

(b) $\frac{2y\sqrt{y^2-1}(x^2+x-1)}{(x^2+1)^2}$

(c) $\frac{(x^2+x-1)}{y\sqrt{y^2-1}}$

(d) $\frac{x^2-y^2}{x^2+y^2}$

Answer: (b) $\frac{2y\sqrt{y^2-1}(x^2+x-1)}{(x^2+1)^2}$

Question Number	Answers
1	(a) a+b

2	(a) $[\sin^2(a+y)]/\sin a$
3	(d) 1.5
4	(d) 1, 2 and 3
5	(b) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$.
6	(a) 1/3
7	(b)
8	(b)
9	Option (a) $\pi/4$
10	(b)