Algebraic Expressions

Factors, Coefficients and Terms of Algebraic Expressions

Shiva and Somesh are brothers. Shiva's age is 3 years less than Somesh's age.

Now, how can we represent this situation with an algebraic expression?

Let us assume Somesh's age as x years. Therefore, Shiva's age = $(x - 3)$ years

Here, $(x - 3)$ is an algebraic expression that represents Shiva's age.

Here, we can notice one thing. The ages of both Somesh and Shiva can vary, but the difference between the ages, i.e. 3 years, is always constant. In this algebraic expression $(x - 3)$, x can vary but the number 3 does not. Hence, x is known as a **variable (or algebraic number)** and 3 is called a **constant (absolute term**).

Let us consider some algebraic expressions given below.

(i) $3x + 5$

(ii) $4x^2 - 21$

Here, the first expression $(3x + 5)$ is formed by adding $3x$ and 5. In this case, $3x$ and 5 are called **algebraic terms or simply terms** of the expression. The terms are always added to form an algebraic expression. They are never subtracted to form an algebraic expression. However, an expression may have positive or negative terms. In the expression $(3x - 5)$, the terms of the expression are $3x$ and (-5), and not $3x$ and 5. Thus, we added the terms $3x$ and (-5) to get the expression $(3x - 5)$.

In expression (ii), $4x^2$ and (-21) are added to form $4x^2 - 21$. Therefore, $4x^2$ and (-21) are terms of the expression $4x^2 - 21$.

Let us again consider the expression $3x + 5$. Here, the term $3x$ is a product of 3 and x. We cannot factorise 3 and x further. Hence, 3 and x are called **factors** of the term $3x$.

The term 5 cannot be expressed as the product of variables and constant. Therefore, 5 is itself a factor of 5.

In expression (ii), the term $4x^2$ can be written as

$4x^2 = 4 \times x^2$

$4x^2 = 4x \times x$

But x^2 and $4x$ cannot be the factors of $4x^2$ as they can be factorised further.

$x^2 = x \times x$ and $4x = 4\times x$

We can write $4x^2$ as the product of 4, x, and x as shown below.

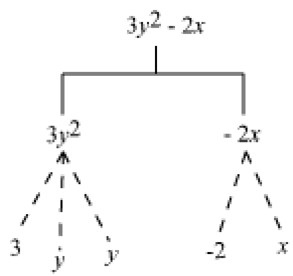
$4x^2 = 4 \times x \times x$

Therefore, 4, x and x are called factors of $4x^2$.

We can also represent the factors and terms of an algebraic expression by a tree diagram.

Let us understand the concept of a tree diagram with the help of an example in the given video.

Similarly, we can represent the tree diagram of an expression ($3y^2 - 2x$) as shown below.



In the expression $3y^2 - 2x$, we can see that the term $3y^2$ is the product of a numerical, i.e. 3, and other variables. This numerical, i.e. 3, is known as the **numerical coefficient** of the term, $3y^2$. Similarly, the coefficient of the term $-2x$ is -2. Generally, we define the numerical coefficient or efficient as

The numerical factor of a term is called the numerical coefficient (or constant coefficient) of the term.

Using this definition, we can say that the numerical coefficient of $-15xy$ is -15, since

$-15xy = -15 \times x \times y$

We can also write $-15xy$ as $-15xy = x \times (-15y) = y \times (-15x)$

Thus, we can say that the coefficient of x is $-15y$ and the coefficient of y is $-15x$.

Can we find the numerical coefficients of x in the expression $(x - 5)$ and that of xy in the expression $(7 - xy)$?

In case of $(x - 5)$, 1 is the numerical coefficient of x. In case of $(7 - xy)$, −1 is the numerical coefficient of xy.

Note: In the expression $-15xy$, xy is said to be the algebraic coefficient.

Thus, we can say that

If the coefficient of a term is 1, then it is not written before the term. If the coefficient of the term is −1, then only the '−' sign is put before the term.

Let us look at the factorization of the terms $5x^3yz^2$ and $-23x^3yz^2$.

$$5x^3yz^2 = 5\underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

$$-23x^3yz^2 = -23 \times \underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

Here, we can see that the two terms have different numerical factors 5 and −23, but same algebraic factors (each of these term contains the same variable, i.e. x, y, and z. Also, powers of these variables of each term are the same, i.e. power of x, y, and z are 3, 1, and 2 respectively). These terms are known as **like terms.** We can define them as

The terms having the same algebraic factors are called like terms. Like terms may have different numerical factors.

Let us consider the terms $6xy$ and $6x$. Now, $6xy = 6 \times x \times y$ and $6x = 6 \times x$

Here, we can see that the two terms have the same numerical factor 6. Their algebraic factors xy and x are different. Such type of terms having different algebraic factors are said to be **unlike terms.** We define them as

The terms having different algebraic factors are called unlike terms.

Let us discuss some examples to understand these concepts better.

Example 1:

Find the terms in the algebraic expression $\left(-\frac{xy}{7}+14xy^2-3\right)$**.**

Solution:

The terms of the expression are $-\frac{xy}{7}, 14xy^2$, and −3.

Example 2:

Find the factors of $\left(-3x^2yz^3\right)$**.**

Solution:

$-3x^2yz^3 = -3 \times x \times x \times y \times z \times z \times z$

Therefore, the factors of $-3x^2yz^3$ are −3, *x*, *x*, *y*, *z*, *z*, and *z*.

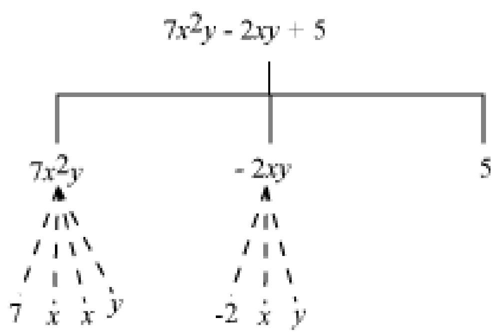
Example 3:

Represent the terms and factors of the algebraic expression $7x^2y - 2xy + 5$ through a

tree diagram.

Solution:

The tree diagram representation of the algebraic expression, $7x^2y - 2xy + 5$ is



Example 4:

Find the like terms in the algebraic expression $51x^2y - 21x^2y^2 + \frac{x^2y}{2} + 31 - 6xy - x^2y$.

Solution:

Here, the like terms are $51x^2y$, $\frac{x^2y}{2}$, and $-x^2y$.

Example 5:

Find the coefficients of *pq* in the following terms.

pq^2, $-3pq$, $15p^2q^2$, $-\frac{31}{5}p^2q$

Solution:

Terms	Coefficients of pq
pq^2	q
$-3pq$	-3
$15p^2q^2$	$15pq$
$-\frac{31}{5}p^2q$	$-\frac{31}{5}p$

Concept of Polynomials

Polynomials

Consider this situation involving trains.

The speed of an express train is ten less than twice that of a passenger train. If each travels for as many hours as its speed, then what is the difference between the distances travelled by them?

Let the speed of the passenger train be x km/hr.

Then, travelling time of the train = x hours

Distance travelled by it = Speed × Time = $x \times x = x^2$ km

Now, speed of the express train = $(2x - 10)$ km/hr

Its travelling time = $(2x - 10)$ hours

Distance travelled by it = $(2x - 10)(2x - 10) = (4x^2 - 40x + 100)$ km

Thus, required difference = $4x^2 - 40x + 100 - x^2 = (3x^2 - 40x + 100)$ km

The expression $3x^2 - 40x + 100$ is an example of a polynomial. Different real-life problems such as the one given above can be expressed in the form of polynomials. Go through this lesson to familiarize yourself with these useful expressions.

Topics to be covered in this lesson:

- Identifying polynomials
- Constant polynomials
- Classification of polynomials according to the number of terms

Did You Know?

Ancient Babylonians developed a unique system to calculate things using formulae. These formulae consisted of letters, mathematical operators (+, −, ×, ÷) and numbers. It was this system that led to the development of algebra. The word 'algebra' is derived from the Arabic word 'al-jabr' meaning 'the reunion of broken parts'. Another Arabian connection with algebra is the Arab mathematician Muhammad ibn Musa al-Khwarizmi, whose theories greatly influenced this branch of mathematics.

Solved Examples

Easy

Example 1:

Which of the following expressions are polynomials? Justify your answers.

i) $2x^{1/2} + 3x + 4$

ii) $8x^3 + 7 + x$

iii) $2x^3 - \sqrt{2}y^5$

iv) $\sqrt{121}x - \frac{2}{3}y^2 + x^3$

v) $5\left(\sqrt{x}\right)^2 + 9x^2$

vi) $\frac{14}{x^2} - 9x^2$

Solution:

i) $2x^{1/2} + 3x + 4$

This expression is not a polynomial because the exponent of the first term is 1/2, which is not a whole number.

ii) $8x^3 + 7 + x$

This expression is a polynomial as all the coefficients are real numbers and the exponents of the variable *x* are whole numbers.

iii) $2x^3 - \sqrt{2}y^5$

This expression is a polynomial as all the coefficients are real numbers and the exponents of the variables *x* and *y* are whole numbers.

iv) $\sqrt{121}x - \frac{2}{3}y^2 + x^3 = 11x - \frac{2}{3}y^2 + x^3$

This expression is a polynomial as all the coefficients are real numbers and the exponents of the variables x and y are whole numbers.

v) $5\left(\sqrt{x}\right)^2 + 9x^2$

This expression is a polynomial because it can be written as $5x + 9x^2$, in which all the coefficients are real numbers and the exponents of the variable x are whole numbers.

vi) $\frac{14}{x^2} - 9x^2$

This expression is not a polynomial because it can be written as$14x^{-2} - 9x^2$, in which the variable in the first term has a negative exponent.

Example 2:

For each of the given polynomials, state whether it is a monomial, binomial or trinomial.

i) $4x^3$

ii) $13y^5 - y$

iii) $29t^3 + 14t - 9$

iv) $x - x^2$

Solution:

i) $4x^3$ is a monomial as it has only one term.

ii) $13y^5 - y$ is a binomial as it has two terms.

iii) $29t^3 + 14t - 9$ is a trinomial as it has three terms.

iv) $x - x^2$ is a binomial as it has two terms.

Medium

Example 1:

For each of the given polynomials, state whether it is a monomial, binomial or trinomial.

i) $(4x^3 + 3y) - (3x^3 + x^2) + (y - x^3)$

ii) $(t^2 + 1)^2$

iii) $[(m - 1)(m + 1)] + 2m^2 + 1$

Solution:

i) $(4x^3 + 3y) - (3x^3 + x^2) + (y - x^3)$

$= 4x^3 + 3y - 3x^3 - x^2 + y - x^3$

$= (4x^3 - 3x^3 - x^3) - x^2 + (3y + y)$

$= - x^2 + 4y$

The polynomial can be reduced to $-x^2 + 4y$, which has two terms; so, it is a binomial.

ii) $(t^2 + 1)^2$

= $(t^2)^2 + 2t^2 + 1^2$[$\because (a + b)^2 = a^2 + 2ab + b^2$]

= $t^4 + 2t^2 + 1$[$\because (a^m)^n = a^{m \times n}$]

The polynomial can be reduced to $t^4 + 2\,t^2 + 1$, which has three terms; so, it is a trinomial.

iii) $[(m - 1)(m + 1)] + 2m^2 + 1$

$= (m^2 - 1) + 2m^2 + 1$ [$\because (a - b)(a + b) = a^2 - b^2$]

$= m^2 + 2m^2 - 1 + 1$

$= 3m^2$

The polynomial can be reduced to $3m^2$, which has only one term; so, it is a monomial.

Hard

Example 1:

State whether or not the following expressions are polynomials. Justify your answers.

i) $\dfrac{x^3}{xy^{-2}}+\dfrac{xy}{10}-\dfrac{1}{7}-\dfrac{\sqrt{2}y^2}{y^{-1}}$

ii) $\dfrac{y^2}{3}+\dfrac{14x^{-4}}{x^2}-5xy+\dfrac{x}{2}$

Solution:

i) $\dfrac{x^3}{xy^{-2}}+\dfrac{xy}{10}-\dfrac{1}{7}-\dfrac{\sqrt{2}y^2}{y^{-1}}$

$$=x^3(x^{-1}y^2)+\frac{1}{10}xy-\frac{1}{7}-\sqrt{2}y^2(y)$$
$$=x^{3-1}y^2+\frac{1}{10}xy-\frac{1}{7}-\sqrt{2}y^{2+1}$$
$$=x^2y^2+\frac{1}{10}xy-\frac{1}{7}-\sqrt{2}y^3$$

In the reduced form of the given expression, all the coefficients are real numbers and the exponents of the variables *x* and *y* are whole numbers. Hence, the given expression is a polynomial.

ii) $\dfrac{y^2}{3}+\dfrac{14x^{-4}}{x^2}-5xy+\dfrac{x}{2}$

$$=\frac{1}{3}y^2+14x^{-4}(x^{-2})-5xy+\frac{1}{2}x$$
$$=\frac{1}{3}y^2+14x^{-4-2}-5xy+\frac{1}{2}x$$
$$=\frac{1}{3}y^2+14x^{-6}-5xy+\frac{1}{2}x$$

In the reduced form of the given expression, the exponent of the second term (i.e., −6) is not a whole number. Hence, the given expression is not a polynomial.

Generalized Form of a Polynomial

Did You Know?

The word ‘polynomial’ is a combination of the Greek words ‘poly’ meaning ‘many’ and ‘nomos’ meaning ‘part or portion’. Thus, a polynomial is an algebraic expression having many parts.

Solved Examples

Easy

Example 1:

Find the coefficient of *x* in the following polynomials.

i) $\frac{\pi}{2}x^3 - 3x^2$

ii) $(2x^2 - x) + 7 + 3x$

Solution:

i) $\frac{\pi}{2}x^3 - 3x^2$

This expression can also be written as $\frac{\pi}{2}x^3 - 3x^2 + 0.x$.

Thus, in the given polynomial, the coefficient of *x* is 0.

ii) $(2x^2 - x) + 7 + 3x$

$= 2x^2 - x + 7 + 3x$

$= 2x^2 + 2x + 7$

Thus, in the given polynomial, the coefficient of *x* is 2.

Medium

Example 1:

For each of the following polynomials, write the constant term and the coefficient of the variable having the highest exponent.

i) $-\left(x^4\right)^{\frac{1}{2}}$ **− 3** $\left(\sqrt{x^3}\right)^2$ **+ $(x + 2) - 4$**

ii)3 (y + 2)^2 − 5y (y^2 + 2y2y**) + y^3**

Solution:

i) $-\left(x^4\right)^{\frac{1}{2}}$ − 3$\left(\sqrt{x^3}\right)^2$ + (x + 2) − 4

= $-x^{4\times\frac{1}{2}}$ − 3x^3 + (x + 2) − 4 [∵ $(a^m)^n = a^{m \times n}$ and $\left(\sqrt{a}\right)^2 = a$]

= $-x^2 - 3x^3 + x - 2$

= $- 3x^3 - x^2 + x - 2$

In this reduced form of the given polynomial, we have:

Constant term = −2

Term with the highest exponent = $-3x^3$

So, coefficient of the variable having the highest exponent = −3

ii)3 (y + 2)^2 − 5y (y^2 + $\frac{2}{y}$) + y^3

= 3 (y^2+ 4y + 4) − 5y^3 − 10 + y^3

= 3y^2+ 12y + 12 − 4y^3 − 10

= $-4y^3 + 3y^2 + 12y + 2$

In this reduced form of the given polynomial, we have:

Constant term = 2

Term with the highest exponent = $-4y^3$

So, coefficient of the variable having the highest exponent = −4

Hard

Example 1:

Find the coefficients of x^3, x^2, x, y^3, y^2 and y in the following polynomials. Also, find the real constants.

i) $17x^4 - 3y - x(2x^3 + x) + 3(2y - 4x^2 + 1) - 4$

ii) $y^4\left(\frac{3}{2y} - 2\right) - \frac{1}{2}(6x^2 + y + 1) + y - 4x^2$

Solution:

i) $17x^4 - 3y - x(2x^3 + x) + 3(2y - 4x^2 + 1) - 4$

$= 17x^4 - 3y - 2x^4 - x^2 + 6y - 12x^2 + 3 - 4$

$= 15x^4 - 13x^2 + 3y - 1$

This reduced form of the given polynomial can be further written as:

$15x^4 + 0.x^3 - 13x^2 + 0.x + 0.y^3 + 0.y^2 + 3y - 1$

Therefore, we have:

Coefficient of $x^3 = 0$ Coefficient of $x^2 = -13$ Coefficient of $x = 0$

Coefficient of $y^3 = 0$ Coefficient of $y^2 = 0$ Coefficient of $y = 3$

Real constant $= -1$

ii) $y^4\left(\frac{3}{2y} - 2\right) - \frac{1}{2}(6x^2 + y + 1) + y - 4x^2$

$$= \frac{3}{2y} \times y^4 - 2y^4 - \frac{1}{2} \times 6x^2 - \frac{1}{2} \times y - \frac{1}{2} + y - 4x^2$$

$$= \frac{3}{2}y^3 - 2y^4 - 3x^2 - \frac{1}{2}y + y - 4x^2 - \frac{1}{2}$$

$$= \frac{3}{2}y^3 - 2y^4 - 7x^2 + \frac{1}{2}y - \frac{1}{2}$$

$$= -7x^2 - 2y^4 + \frac{3}{2}y^3 + \frac{1}{2}y - \frac{1}{2}$$

This reduced form of the given polynomial can be further written as:

$$0.x^3 - 7x^2 + 0.x - 2y^4 + \frac{3}{2}y^3 + 0.y^2 + \frac{1}{2}y - \frac{1}{2}$$

Therefore, we have:

Coefficient of x^3 = 0 Coefficient of x^2 = −7 Coefficient of x = 0

Coefficient of y^3 = $\frac{3}{2}$ Coefficient of y^2 = 0 Coefficient of y = $\frac{1}{2}$

Real constant = $-\frac{1}{2}$

Different Forms of a Polynomial

A polynomial can found and written in different forms. These forms are explained below.

Standard form: If the terms of a polynomial are written in descending order or ascending order of the powers of the variables then the polynomial is said to be in the standard form.

For example, the polynomial $3x + 15x^4 - 1 - 13x^2$ is not in the standard form. It can be written in the standard form as $15x^4 - 13x^2 + 3x - 1$ or $-1 + 3x - 13x^2 + 15x^4$.

Index form: Observe the polynomial $x^6 - 2x^4 - 10x^3 + 5$. In this polynomial, terms having x^5, x^2 and x are missing. These terms can be added to the polynomial with coefficient 0. Thus, the obtained polynomial will be $x^6 + 0x^5 - 2x^4 - 10x^3 + 0x^2 + 0x + 5$.

The polynomial obtained on adding the missing terms is said to be in the index form.

Coefficient form: When the coefficients of all the terms of a polynomial are written in a bracket by separating with comma then the polynomial is said to be written in the coefficient form.

It should be noted that if a term is missing then its coefficient is taken as 0. So, it is better to write the given polynomial in the index form before writing it in the coefficient form.

For example, to write the polynomial $x^6 - 2x^4 - 10x^3 + 5$ in the coefficient form, we will first write it in the index form as $x^6 + 0x^5 - 2x^4 - 10x^3 + 0x^2 + 0x + 5$.

Now, it can be written in the coefficient form as (1, 0, −2, −10, 0, 0, 5).

Solved Examples

Easy

Example 1:

Express the given polynomials in the standard form.

(i) $-2y^3 + 5y^5 - 2 + y$

(ii) $11a - a^6 - 2a^3 + a^2 - 1$

Solution:

We know that if the terms of a polynomial are written in descending order or ascending order of the powers of the variables then the polynomial is said to be in the standard form.

Given polynomials can be written in the standard form as follows:

(i) **Given form:** $-2y^3 + 5y^5 - 2 + y$

Standard form: $5y^5 - 2y^3 + y - 2$ or $-2 + y - 2y^3 + 5y^5$

(ii) **Given form:** $11a - a^6 - 2a^3 + a^2 - 1$

Standard form: $-a^6 - 2a^3 + a^2 + 11a - 1$ or $-1 + 11a + a^2 - 2a^3 - a^6$

Example 2:

Express the given polynomials in the index form and coefficient form.

(i) $2m^7 + 12m^5 - 7m^2 - m$

(ii) $-4p^6 + 3p^3 - 2p + 7$

Solution:

We know that the polynomial obtained on adding the missing terms is said to be in the index form.

Also, when the coefficients of all the terms of a polynomial are written in a bracket by separating with comma then the polynomial is said to be written in the coefficient form.

Given polynomials can be written in the index form and coefficient form as follows:

(i) **Given form:** $2m^7 + 12m^5 - 7m^2 - m$

Index form: $2m^7 + 0m^6 + 12m^5 + 0m^4 + 0m^3 - 7m^2 - m$

Coefficient form: (2, 0, 12, 0, 0, −7, −1)

(ii) **Given form:** $-4p^6 + 3p^3 - 2p + 7$

Index form: $-4p^6 + 0p^5 + 0p^4 + 3p^3 + 0p^2 - 2p + 7$

Coefficient form: (−4, 0, 0, 3, 0, −2, 7)

Example 3:

Express the given coefficient forms in the index forms by taking x as the variable.

(i) (10, 0, −2, 1, 0, 0, 7)

(ii) (−1, 2, 0, 3, 0, 6)

Solution:

Given polynomials can be written in the index form as follows:

(i) **Given form:** (10, 0, −2, 1, 0, 0, 7)

Index form: $10x^6 + 0x^5 - 2x^4 + x^3 + 0x^2 + 0x + 7$

(ii) **Given form:** (−1, 2, 0, 3, 0, 6)

Index form: $-x^5 + 2x^4 + 0x^3 + 3x^2 + 0x + 6$

Degree of Polynomial

More about Polynomials

We know that a polynomial comprises a number of terms, which may have variables or numbers or both. Also, each term can be represented with a variable having some **exponent**. Exponents of the variables in a given polynomial can be the same or different.

Let us consider a polynomial $2x^5 + 4x^2 + 9$.

The terms of this polynomial and their exponents are as follows:

First term = $2x^5$; exponent in the first term = 5

Second term = $4x^2$; exponent in the second term = 2

Third term = 9 = $9x^0$; exponent in the third term = 0

Note that all the exponents in the above polynomial are different. These exponents help us to identify the degrees of polynomials. Polynomials are categorized based on their degrees.

In this lesson, we will learn about the degrees of polynomials and the classification of polynomials based on the same.

The Degree of a Polynomial

Whiz Kid

When a polynomial has an equals sign (=), then it becomes an equation. The maximum number of solutions of an equation is less than or equal to the degree of that equation.

Solved Examples

Easy

Example 1:

Find the degree of each term of the polynomial $3x^6 + 3x^4 - 6x + 3$. Also find the degree of the polynomial.

Solution:

The degree of the term $3x^6$ is 6.

The degree of the term $3x^4$ is 4.

The degree of the term $-6x$ is 1.

The degree of the term 3 is 0.

Here, the highest degree is 6. Hence, the degree of the polynomial is 6.

Medium

Example 1:

Write the degree of each of the following polynomials.

i) $\frac{x^2}{2x} - 9x^7 + \frac{1}{x^{-4}} + 7$

ii) $\frac{x^2}{3} + \frac{4x^{-1}}{x^{-2}} - 5x^2 + \frac{x}{2} - 9$

Solution:

i) $\frac{x^2}{2x} - 9x^7 + \frac{1}{x^{-4}} + 7$

$= \frac{x^{2-1}}{2} - 9x^7 + x^4 + 7 \qquad \left(\because \frac{a^m}{a^n} = a^{m-n}, \text{where } m > n; \ \frac{1}{a^{-m}} = a^m\right)$

$= \frac{x}{2} - 9x^7 + x^4 + 7$

In the given polynomial, the highest degree is 7. Hence, the degree of the polynomial is 7.

ii) $\frac{x^2}{3} + \frac{4x^{-1}}{x^{-2}} - 5x^2 + \frac{x}{2} - 9$

$= \frac{x^2}{3} + 4x^{2-1} - 5x^2 + \frac{x}{2} - 9 \qquad \left(\because \frac{a^m}{a^n} = a^{n-m}, \text{where } m < n\right)$

$= \frac{x^2}{3} + 4x - 5x^2 + \frac{x}{2} - 9$

$= -\frac{14x^2}{3} + \frac{9x}{2} - 9$

In the given polynomial, the highest degree is 2. Hence, the degree of the polynomial is 2.

The Degree of a Polynomial in more than one Variable

In case of the polynomials in one variable, the degree of a polynomial is the highest exponent of the variable in the polynomial, but what about the degree of the polynomial in more than one variable?

In this case, the sum of the powers of all variables in each term is obtained and the highest sum among all is the degree of the polynomial.

For example, find the degree of the polynomial $2xy + 3y^2z + 4x^2yz^2 – xyz – 2x^3$. Let us find the sum of the powers of all variables in each term of this polynomial.

Sum of the powers of all variables in the term $2xy = 1 + 1 = 2$

Sum of the powers of all variables in the term $3y^2z = 2 + 1 = 3$

Sum of the powers of all variables in the term $4x^2yz^2 = 2 + 1 + 2 = 5$

Sum of the powers of all variables in the term $–xyz = 1 + 1 + 1 = 3$

Sum of the powers of all variables in the term $–2x^3 = 3$

Among all the sums, 5 is the highest and thus, the degree of the polynomial $2xy + 3y^2z + 4x^2yz^2 – xyz – 2x^3$ is 5.

Similarly, we can find the degree of any polynomial in more than one variable.

Solved Examples

Easy

Example 1:

Write the degree of each of the following polynomials.

(i) $24a^2b – abc + 11abc^2 + ab^2c^3 – 7a^2b^2$

(ii) $5p^5 + 10p^2qr – 9p^2qr^2 – p^2r^2 + 2pq^2 – 2p^3q^2r$

Solution:

(i) Let us find the sum of the powers of all variables in each term of $24a^2b – abc + 11abc^2 + ab^2c^3 – 7a^2b^2$.

Sum of the powers of all variables in the term $24a^2b = 2 + 1 = 3$

Sum of the powers of all variables in the term $–abc = 1 + 1 + 1 = 3$

Sum of the powers of all variables in the term $11abc^2 = 1 + 1 + 2 = 4$

Sum of the powers of all variables in the term $ab^2c^3 = 1 + 2 + 3 = 6$

Sum of the powers of all variables in the term $–7a^2b^2 = 2 + 2 = 4$

Among all the sums, 6 is the highest and thus, the degree of the polynomial $24a^2b - abc + 11abc^2 + ab^2c^3 - 7a^2b^2$ is 6.

(ii) Let us find the sum of the powers of all variables in each term of $5p^5 + 10p^2qr - 9p^2qr^2 - p^2r^2 + 2pq^2 - 2p^3q^2r$.

Sum of the powers of all variables in the term $5p^5 = 5$

Sum of the powers of all variables in the term $10p^2qr = 2 + 1 + 1 = 4$

Sum of the powers of all variables in the term $-9p^2qr^2 = 2 + 1 + 2 = 5$

Sum of the powers of all variables in the term $-p^2r^2 = 2 + 2 = 4$

Sum of the powers of all variables in the term $2pq^2 = 1 + 2 = 3$

Sum of the powers of all variables in the term $-2p^3q^2r = 3 + 2 + 1 = 6$

Among all the sums, 6 is the highest and thus, the degree of the polynomial $5p^5 + 10p^2qr - 9p^2qr^2 - p^2r^2 + 2pq^2 - 2p^3q^2r$ is 6.

Classification of Polynomials According to Their Degrees

Whiz Kid

If all the terms in a polynomial have the same exponent, then the expression is referred to as a **homogenous polynomial**.

Did You Know?

The graphs oflinear polynomials are always straight lines. This is why these polynomials are called ‘linear’ polynomials.

Solved Examples

Easy

Example 1:

Classify each of the given polynomials according to its degree.

i) $11x^3 + 7x + 3$

ii) $8x^2 + 3x$

iii) $x + 5$

iv) $9t^3$

Solution:

i) $11x^3 + 7x + 3$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

ii) $8x^2 + 3x$

The degree of this polynomial is 2. Hence, it is a quadratic polynomial.

iii) $x + 5$

The degree of this polynomial is 1. Hence, it is a linear polynomial.

iv) $9t^3$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

Example 2:

Give an example of each of the following polynomials.

i) A monomial of degree 50

ii) A binomial of degree 17

iii) A trinomial of degree 99

Solution:

i) A monomial of degree 50 means a polynomial having one term and 50 as the highest exponent. An example of such a polynomial is $23y^{50}$.

ii) A binomial of degree 17 means a polynomial having two terms and 17 as the highest exponent. An example of such a polynomial is $41t^{17} + 53t$.

iii) A trinomial of degree 99 means a polynomial having three terms and 99 as the highest exponent. An example of such a polynomial is $p^{99} + 5p - 12$.

Medium

Example 1:

Classify each of the given polynomials according to its degree.

i) $\dfrac{x^2}{3}+4x^3-(5x^2+4x^3)+\dfrac{x}{2}-9$

ii) $x + 3x^2 + (x + 2)(x^2 + 4 - 2x) + 54$

Solution:

i) $\dfrac{x^2}{3}+4x^3-(5x^2+4x^3)+\dfrac{x}{2}-9$

$$=\frac{x^2}{3}+4x^3-5x^2-4x^3+\frac{x}{2}-9$$
$$=\frac{x^2}{3}-5x^2+\frac{x}{2}-9$$
$$=-\frac{14x^2}{3}+\frac{x}{2}-9$$

The degree of this polynomial is 2. Hence, it is a quadratic polynomial.

ii) $x + 3x^2 + (x + 2)(x^2 + 4 - 2x) + 54$

$= x + 3x^2 + (x^3 + 2^3) + 54$ $\quad [\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$

$= x + 3x^2 + x^3 + 8 + 54$

$= x + 3x^2 + x^3 + 62$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

Addition and Subtraction of Polynomials

Just like natural numbers, we can even perform mathematical operations on algebraic expressions. The process is very much similar to the process required for natural numbers. You can understand the process by going through the following video.

The most important point to remember in this topic is as follows.

We can add and subtract like terms. In case of addition or subtraction of like terms, only their numerical coefficients are added or subtracted. The algebraic part of the terms remains as it is.

In the video given above, we learned adding polynomials by arranging them horizontally. We can add polynomials by arranging them vertically as well.

Adding and subtracting like monomials by arranging them vertically:
In this method, we have to write given like monomials one below other to perform the addition or subtraction. Also, write the coefficients for pure variable terms as 1. To add or subtract like monomials, we just need to add or subtract the coefficients and write the result with the variable.

For example, let us add monomials a^2, $-2a^2$ and $5a^2$. These monomials can be added by arranging vertically as follows:

$$\begin{array}{rr} & 1a^2 \\ + & -2a^2 \\ + & 5a^2 \\ \hline & 4a^2 \\ \hline \end{array}$$

Here, the sum of 1 and –2 is obtained as –1. Further, the sum of –1 and 5 is obtained as 4. So, we wrote 4 with a^2 and obtained the sum of given polynomials as $4a^2$.

Similarly, we can perform subtraction for the given monomials.

$$\begin{array}{rr} & 1a^2 \\ - & -2a^2 \\ - & 5a^2 \\ \hline & -2a^2 \\ \hline \end{array}$$

When we subtracted –2 from 1, we got 3 [1 – (–2) = 3]. Further, on subtracting 5 from 3, we got –2 (3 – 5 = –2). So, we wrote –2 with a^2 and obtained the result as $-2a^2$.

Adding and subtracting polynomials by arranging them vertically:

To add the polynomials, we can arrange them vertically such that each term of lower polynomial is written below its like term in the upper polynomial. Also, write the coefficients for pure variable terms as 1.

Let us add the polynomials $-3x^2 + 4xy - z$ and $2xy + x^2 - 3z$ to learn the concept.
We can observe that $-3x^2$ and x^2 are like terms as they have same variable having same powers. Similarly, $4xy$ and $2xy$, and $-z$ and $-3z$ are other pairs of like terms.

These polynomials can be arranged vertically as follows:

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad 1x^2 + 2xy - 3z \\ \hline \\ \hline \end{array}$$

Now, we just need to add the coefficients of like terms and write the variables as they are.

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad 1x^2 + 2xy - 3z \\ \hline -2x^2 + 6xy - 4z \\ \hline \end{array}$$

Thus, the sum of the given polynomials is $-2x^2 + 6xy - 4z$.

Let us now study the concept of subtraction of polynomials by subtracting $2xy + x^2 - 3z$ from $-3x^2 + 4xy - z$.

Let us arrange them vertically first as we have done before.

$$\begin{array}{r} -3x^2 + 4xy - z \\ - \quad 1x^2 + 2xy - 3z \\ \hline \\ \hline \end{array}$$

To subtract a polynomial from other, we add its opposite.

Now, we get

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad -1x^2 - 2xy + 3z \\ \hline \\ \hline \end{array}$$

Now, we perform the addition as we have done before.

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad -1x^2 - 2xy + 3z \\ \hline -4x^2 + 2xy + 2z \\ \hline \end{array}$$

Thus, the required difference is $-4x^2 + 2xy + 2z$.

Let us discuss some more examples to understand the concept better.

Example 1:

Add the following monomials by arranging them horizontally as well as vertically.

(a) $-3p^2$, $6p^2$ **and** $-11p^2$

(b) $8x^2y$, $-10x^2y$ **and** $-2x^2y$

Solution:

Addition by arranging horizontally:

(a) $-3p^2 + 6p^2 + (-11p^2) = (-3 + 6 - 11)p^2 = -8p^2$

(b) $8x^2y + (-10x^2y) + (-2x^2y) = (8 - 10 - 2)x^2y = -4x^2y$

Addition by arranging vertically:

(a)

$$\begin{array}{rr} & -3p^2 \\ + & 6p^2 \\ + & -11p^2 \\ \hline & -8p^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{rr} & 8x^2y \\ + & -10x^2y \\ + & -2x^2y \\ \hline & -4x^2y \\ \hline \end{array}$$

Example 2:

Subtract the following monomials by arranging them horizontally as well as vertically.

(a) $25mn^2$ from the sum of $14mn^2$ and $-mn^2$

(b) $(-x^2y^2 + 12x^2y^2)$ from $19x^2y^2$

Solution:

Subtraction by arranging horizontally:

(a) $(14mn^2 - mn^2) - 25mn^2 = 13mn^2 - 25mn^2 = -12mn^2$

(b) $19x^2y^2 - (-x^2y^2 + 12x^2y^2) = 19x^2y^2 - 11x^2y^2 = 8x^2y^2$

Subtraction by arranging vertically:

(a)

$$\begin{array}{lr} & 14m^2n^2 \\ + & -1m^2n^2 \\ \hline & 13m^2n^2 \\ \hline \end{array} \qquad \begin{array}{lr} & 13m^2n^2 \\ - & 25m^2n^2 \\ \hline & -12m^2n^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{lr} & -x^2y^2 \\ + & 12x^2y^2 \\ \hline & 11x^2y^2 \\ \hline \end{array} \qquad \begin{array}{lr} & 19x^2y^2 \\ - & 11x^2y^2 \\ \hline & 8x^2y^2 \\ \hline \end{array}$$

Example 3:

Add the expressions $5x^2 + 6xy - 11$, $7x^2y - 3y$, and $12x^2y - 3xy + 4$.

Solution:

$(5x^2 + 6xy - 11) + (7x^2y - 3y) + (12x^2y - 3xy + 4)$

$= 5x^2 + 6xy - 11 + 7x^2y - 3y + 12x^2y - 3xy + 4$

$= 5x^2 + 6xy - 3xy + 7x^2y + 12x^2y - 3y - 11 + 4$ (Rearranging the terms)

$=5x^2 + (6 - 3)xy + (7 + 12)x^2y - 3y + (-11 + 4)$

$= 5x^2 + 3xy + 19x^2y - 3y - 7$

Example 4:

Which expression when subtracted from the expression ($7x – 3y + 45xy + 7$) gives

($2x – 21y – 42xy$)?

Solution:

To get the required expression, we have to subtract ($2x – 21y – 42xy$) from ($7x – 3y + 45xy + 7$).

$(7x – 3y + 45xy + 7) – (2x – 21y – 42xy)$

$= 7x – 3y + 45xy + 7 – 2x + 21y + 42xy$

$= 7x – 2x – 3y + 21y + 45xy + 42xy + 7$

$= (7 – 2)x + (–3 + 21)y + (45 + 42)xy + 7$

$= 5x + 18y + 87xy + 7$

Example 5:

Subtract the sum of ($4y^2 – 6y$) and ($–2y^2 + 3y – 3$) from the sum of ($5y + 7$) and

($3y^2 – 9y + 2$).

Solution:

$(5y + 7) + (3y^2 – 9y + 2)$

$= 5y + 7 + 3y^2 – 9y + 2$

$= 3y^2 + 5y – 9y + 7 + 2$ [Rearranging the terms]

$= 3y^2 + (5y – 9y) + (7 + 2)$

$= 3y^2 + (5 – 9)\ y + 9$

$= 3y^2 + (–4)\ y + 9$

$= 3y^2 – 4y + 9$

$(4y^2 – 6y) + (–2y^2 + 3y – 3)$

$= 4y^2 – 6y – 2y^2 + 3y – 3$

$= 4y^2 – 2y^2 – 6y + 3y – 3$ [Rearranging the terms]

$= (4y^2 – 2y^2) + (–6y + 3y) – 3$

$= (4 – 2) y^2 + (–6 + 3) y – 3$

$= 2y^2 –3y – 3$

Now, subtracting the sum of $(4y^2 – 6y)$ and $(–2y^2 + 3y – 3)$ from the sum of $(5y + 7)$and $(3y^2 – 9y + 2)$ is the same as subtracting $(2y^2 – 3y – 3)$ from $(3y^2 – 4y + 9)$.

This can be done as

$(3y^2 – 4y + 9) – (2y^2 – 3y – 3)$

$= 3y^2 – 4y + 9 – 2y^2 + 3y + 3$

$= 3y^2 – 2y^2 – 4y + 3y + 9 + 3$ [Rearranging the terms]

$= (3 – 2) y^2 + (–4 + 3) y + (9 + 3)$

$= y^2 – y + 12$

Addition and Subtraction of Linear Algebraic Expressions

Just like on natural numbers, mathematical operations are also performed on algebraic expressions.

We can add and subtract like terms. For adding and subtracting like terms, only their numerical coefficients are added or subtracted; the algebraic part of the terms remains the same.

Consider the addition of two like terms, $8xy$ and $5xy$.

The numerical coefficient of $8xy$ and $5xy$ are respectively 8 and 5; algebraic part of both the terms is xy.

Therefore, for adding $8xy$ and $5xy$, we will add 8 and 5, keeping xy as it is.

$\therefore 8xy + 5xy = (8 + 5)xy = 13xy$

Similarly, $5xy$ can be subtracted from $8xy$ in the following manner:

$8xy - 5xy = (8 - 5)xy = 3xy$

As by now we have learnt how to add and subtract like terms, let us learn how to simplify the algebraic expressions.

Let us first recall the two properties satisfied by all the numbers, x, y and z, which will be used to simplify the algebraic expressions.

Property I: $x - (y + z) = (x - y) - z$

Property II: $x - (y - z) = x - y + z$

Now, let us learn the method of simplification of expressions with the help of some examples.

Consider $(2a + 5b) - (a + b)$.

For simplifying this, let us take $x = (2a + 5b)$, $y = a$ and $z = b$.

So, the given expression reduces to $x - (y + z)$.

On using property I, we get:

$x - (y + z) = (x - y) - z$

i.e. $(2a + 5b) - (a + b) = (2a + 5b - a) - b$

$= (2a - a + 5b) - b$

$= a + 5b - b$

$= a + 4b$

Similarly, $(3a + 2b) - (2a + b)$ can be simplified using property II, which is shown in the following manner:

$(3a + 2b) - (2a + b)$

$= 3a + 2b - 2a - b$ [Here, $x = (3a + 2b)$, $y = 2a$ and $z = b$]

$= 3a - 2a + 2b - b$

$= a + b$

Value of an expression at a point:

Length and breadth of a rectangle are $(3x + 2y)$ units and $(2x - y)$ units respectively. Express the perimeter of the rectangle with the help of an algebraic expression.

We know that perimeter of a rectangle = 2 (Length + Breadth)

$= 2\,[(3x + 2y) + (2x - y)]$ units

$= 2\,[3x + 2y + 2x - y]$ units

$= 2\,[(3x + 2x) + (2y - y)]$ units

$= 2\,(5x + y)$ units

$= (10x + 2y)$ units

If we put $x = 1$ and $y = 1$, then the length of the rectangle = [3 (1) + 2 (1)] units = 5 units.

Breadth of the rectangle = [2 (1) – 1] units = 1 unit

Also, perimeter of the rectangle = [10 (1) + 2 (1)] units = 12 units

Similarly, if we put $x = 2$ and $y = 1$, then the length of the rectangle = 8 units.

Breadth of the rectangle = 3 units

Perimeter of the rectangle = 22 units

As seen here, the perimeter of the rectangle changes with the change in its length and breadth. In this way, we can find the value of an expression, at a given point, by substituting the values of the variables.

The value of an algebraic expression depends upon the value of its variables.

Using this concept, some of the properties of numbers can be verified.

Let us verify that the same for all positive numbers a, b, $(a - b) \neq (b - a)$.

In fact, $\mathbf{(a - b) = -(b - a)}$.

This can be verified in the following manner:

A	***b***	***a* – *b***	***b*– *a***	**–(*b* – *a*)**
1	2	–1	1	–1
1	–2	3	–3	3

–1	2	–3	3	–3
–1	–2	1	–1	1
2	1	1	–1	1
2	–1	3	–3	3
–2	1	–3	3	–3
–2	–1	–1	1	–1
1	0	1	–1	1
–1	0	–1	1	–1
0	1	–1	1	–1
0	–1	1	–1	1

It can be observed from the table that the value of $(a - b)$ is same as the value of $-(b - a)$, for different values of a and b. In general, this result is true.

Thus, $(a - b) = -(b - a)$ for all the values of a and b.

However, $(a - b) \neq (b - a)$

Similarly, **properties I and II** and those given below can also be verified:

- $(a + b) + c = a + (b + c)$
- $(a - b) + c = a - (b - c)$
- $(a + b) - c = a + (b - c)$

Let us go through some examples to understand the concept in a better way.

Example 1:

Simplify the following expressions:

(i) $(2x + 5y) + (3x + 7y)$

(ii) $(x + 7y) + (2x - 3y)$

(iii) $(5x - 4y) + (9x - y)$

Solution:

(i) $(2x + 5y) + (3x + 7y)$

= $(2x + 3x) + (5y + 7y)$

$= 5x + 12y$

(ii) $(x + 7y) + (2x - 3y)$

= $(x + 7y + 2x) - 3y$ $\{a + (b - c) = (a + b) - c\}$

$= (3x + 7y) - 3y$

$= 3x + (7y - 3y)$ $\{(a + b) - c = a + (b - c)\}$

$= 3x + 4y$

(iii) $(5x - 4y) + (9x - y)$

$= 5x - 4y + 9x - y$

$= 5x + 9x - 4y - y$

$= 14x - 4y - y$

$= 14x - (4y + y)$

$= 14x - 5y$

Example 2:

Simplify the following expressions:

(i) $(3a - 11b) - (2a - 9b)$

(ii) $\{(6a - 7b) + (5a + 9b)\} - (a + 8b)$

Solution:

(i) $(3a - 11b) - (2a - 9b)$

$= 3a - 11b - 2a + 9b$ [Using property II]

$= (3a - 2a) - (11b - 9b)$ [Rearranging the terms]

$= a - 2b$

(ii) $\{(6a - 7b) + (5a + 9b)\} - (a + 8b)$

$= \{6a - 7b + 5a + 9b\} - (a + 8b)$

$= \{(6a + 5a) – (7b – 9b)\} – (a + 8b)$ [Rearranging the terms]

$= (11a + 2b) – (a + 8b)$

$= (11a + 2b – a) – 8b$ [Using property I]

$= \{(11a – a) + 2b\} – 8b$ [Rearranging the terms]

$= (10a + 2b) – 8b$

$= 10a + (2b – 8b)$ [Rearranging the terms]

$= 10a – 6b$

Example 3:

If $a = 7$ and $b = 11$, then what is the value of the expression obtained on subtracting $\{(2a – 3b) – (a – b)\}$ from $\{(11a + 7b) – 5b\}$?

Solution:

The expression $(2a – 3b) – (a – b)$ can be simplified in the following way:

$(2a – 3b) – (a – b)$

$= 2a – 3b – a + b$

$= (2a – a) + (–3b + b)$

$= a – 2b$

In the same way, the expression $(11a + 7b) – 5b$ can be simplified in the following way:

$(11a + 7b) – 5b$

$= 11a + 7b – 5b$

$= 11a + (7b – 5b)$

$= 11a + 2b$

Now, $(a – 2b)$ can be subtracted from $(11a + 2b)$ in the following way:

$(11a + 2b) – (a – 2b)$

$= 11a + 2b - a + 2b$

$= (11a - a) + (2b + 2b)$

$= 10a + 4b$

When $a = 7$ and $b = 11$, then the value of the expression $10a + 4b$ is:

$10 (7) + 4 (11) = 70 + 44 = 114$

Thus, the required value is 114.

Example 4:

Add the expressions $(5a + 6b - 2c) - (a + b + 3c)$ and $(3a + 2b) + (2a + b - c)$.

Solution:

The expression $(5a + 6b - 2c) - (a + b + 3c)$ can be simplified in the following way:

$(5a + 6b - 2c) - (a + b + 3c)$

$= 5a + 6b - 2c - a - b - 3c$

$= (5a - a) + (6b - b) - (2c + 3c)$

$= 4a + 5b - 5c$

In the same way, the expression $(3a + 2b) + (2a + b - c)$ can be simplified in the following manner:

$(3a + 2b) + (2a + b - c)$

$= 3a + 2b + 2a + b - c$

$= (3a + 2a) + (2b + b) - c$

$= 5a + 3b - c$

Now, the two expressions can be added in the following way:

$(4a + 5b - 5c) + (5a + 3b - c)$

$= 4a + 5b - 5c + 5a + 3b - c$

$= (4a + 5a) + (5b + 3b) – (5c + c)$

$= 9a + 8b – 6c$

Thus, the required expression is $9a + 8b – 6c$.

Multiplication of Monomials with Polynomials

Let us discuss another example based on the above concept.

Suppose we have two monomials $4x$ and $5y$. By multiplying them, we get

$4x \times 5y = (4 \times x) \times (5 \times y)$

$= (4 \times 5) \times (x \times y)$

$= 20xy$

Hence, we can say that $4x \times 5y = 20xy$

Now, $10x \times 5x^2z = (10 \times x) \times (5 \times x^2 \times z)$

This becomes,

$(10 \times 5) \times (x \times x^2 \times z) = 50x^3z$

Now, what if you have three monomials and you want to multiply them? How will you do so?

Suppose you want to multiply $10x$, $2xy$, and $5z$.

$10x \times 2xy \times 5z = (10 \times x) \times (2 \times x \times y) \times (5 \times z)$

First, we multiply the first two monomials.

$= \{(10 \times 2) \times (x \times x \times y)\} \times (5 \times z)$

$= (20 \times x^2 \times y) \times (5 \times z)$ $[\because x \times x = x^2]$

$= (20 \times 5) \times (x^2 \times y \times z)$

$= 100x^2yz$

This method of multiplication of three monomials can be extended to find out the product of any number of monomials.

Let us discuss some more examples based on multiplication of monomials.

Example 1:

Find the product of the following:

(a) $-2x^2y$ **and** $15\ xy^2z^3$

(b) $7ap,\ 2qa^2x^3$ **and** $-\ 5rx$

(c) $ab,\ -2bc,\ -3cd$ **and** $4ad$

Solution:

(a) $-2x^2y$ and $15\ xy^2z^3$

$= \left(-2x^2y\right) \times \left(15\ xy^2z^3\right)$

$= (-2 \times 15) \times \left(x^2y \times xy^2z^3\right)$

$= -30x^3y^3z^3 \qquad \left\{\text{as } x^2 \times x = x^3 \text{ and } y \times y^2 = y^3\right\}$

(b) $7ap,\ 2qa^2x^3$, and $-\ 5rx$

$= (7ap) \times \left(2qa^2x^3\right) \times (-\ 5rx)$

$= \left\{(7 \times 2) \times \left(ap \times qa^2x^3\right)\right\} \times (-\ 5rx)$

$= \left(14a^3pqx^3\right) \times (-\ 5rx)$

$= (14 \times -\ 5) \times \left(a^3pqx^3 \times rx\right)$

$= -70a^3pqrx^4 \qquad \left[\text{as } x^3 \times x = x^4\right]$

(c) $ab,\ -\ 2bc,\ -\ 3cd$, and $4ad$

$= (ab) \times (-\ 2bc) \times (-\ 3cd) \times (4ad)$

$$= \left[(1 \times -2) \times (ab \times bc)\right] \times \left[(-3 \times 4) \times (cd \times ad)\right]$$

{Multiplying the 1st to 2nd and 3rd to 4th term}

$$= \left(-2ab^2c\right) \times \left(-12cd^2a\right)$$

$$= (-2 \times -12) \times \left(ab^2c \times cd^2a\right)$$

$$= 24a^2b^2c^2d^2$$

Example 2.

If the side of a square is 4*x* cm, what is its area?

Answer:

Side of square = 4*x* cm

∴ Area of square = side × side = 4*x* × 4*x* = $16x^2$ cm^2

Example 3:

The length, breadth, and height of three cuboids are given below in the table. Find the volume and area of the base of these cuboids.

	Length	Breadth	Height
(i)	$3ab$	$2bx$	$5xy$
(ii)	a^2b	b^2c	c^2a
(iii)	$3x$	$9x^2$	$27x^3$

Solution:

We know that, area of the base = Length × Breadth

Volume of cuboid = Length × Breadth × Height

(i) Area of the base = $3ab \times 2bx = (3 \times 2) \times (ab \times bx) = 6ab^2x$

Volume of the cuboid = $3ab \times 2bx \times 5xy$

$= \{(3 \times 2) \times (ab \times bx)\} \times (5xy)$
$= (6ab^2x) \times (5xy)$
$= (6 \times 5) \times (ab^2x \times xy)$
$= 30\ ab^2x^2y$

(ii) Area of the base = $a^2b \times b^2c = a^2b^3c$

Volume of the cuboid = $(a^2b \times b^2c) \times c^2a = a^2b^3c \times c^2a = a^3\,b^3\,c^3$

(iii) Area of the base = $3x \times 9x^2 = (3 \times 9) \times (x \times x^2) = 27x^3$

Volume of the cuboid = $3x \times 9x^2 \times 27x^3$

$= [(3 \times 9) \times (x \times x^2)] \times 27x^3$
$= 27x^3 \times 27x^3$
$= (27 \times 27)(x^3 \times x^3)$
$= 729\ x^6$

So far, we know how two multiply any number of monomials. But, what if we need to multiply a monomial with a binomial or a trinomial, etc.?

Can we multiply them?

We can multiply them easily.

To understand the method, look at the following video.

The method discussed in the above video shows the **horizontal arrangement** of multiplying monomials with polynomials.

Let us now learn about the **vertical arrangement** for the same by performing the multiplication of $(4x^2 + 2x)$ and $3x$.

This is similar to vertical method of multiplication of whole numbers.

Here, we will first multiply $3x$ with $2x$ and write the product with sign at the bottom. After doing this, we will multiply $3x$ with $4x^2$ and write the product with sign at the bottom. The expression obtained at the bottom will be the required product.

This can be done as follows:

$$\begin{array}{l} 4x^2+2x \\ \underline{\times \quad\quad 3x} \\ \underline{12x^3+6x^2} \end{array}$$

Similarly, we can multiply a trinomial with monomial as follows:

$$\begin{array}{r} 2y^3-5y+1 \\ \underline{\times \quad\quad\quad\quad 2y} \\ \underline{4y^4-10y^2+2y} \end{array}$$

Let us discuss some more examples based on the multiplication of a monomial with polynomials.

Example 4:

Multiply the following in horizontal and vertical arrangements:

(a) $\frac{3}{5}p$ and $p - 6q$

(b) $(1.5x + y + 3z)$ and $7z$

Also find the values of the above expressions if $p = -5$, $q = -1$, $x = 2$, $y = -3$, and $z = -1$.

Solution:

(a) Horizontal arrangement:

$$\frac{3}{5}p \times (p - 6q) = \frac{3}{5}p \times p - \frac{3}{5}p \times 6q = \frac{3}{5}p^2 - \frac{18}{5}pq$$

Vertical arrangement:

$$
\begin{array}{r}
p - 6q \\
\times \qquad \frac{3}{5}p \\
\hline
\frac{3}{5}p^2 - \frac{18}{5}pq \\
\hline
\end{array}
$$

Substituting the values of *p* and *q*, we get

$= \frac{3}{5} \times (-5)^2 - \frac{18}{5} \times (-5)(-1)$

$= \frac{3}{5} \times 25 - 18$

$= 15 - 18$

$= -3$

(b) Horizontal arrangement:

$(1.5x + y + 3z) \times 7z = 1.5x \times 7z + y \times 7z + 3z \times 7z = 10.5xz + 7yz + 21z^2$

Vertical arrangement:

$$
\begin{array}{r}
1.5x + y + 3z \\
\times \qquad\qquad 7z \\
\hline
10.5xz + 7yz + 21z^2 \\
\hline
\end{array}
$$

Substituting the values of *x*, *y*, and *z*, we get

$10.5xz + 7yz + 21z^2$

$= 10.5 \times 2 \times (-1) + 7 \times (-3) \times (-1) + 21 \times (-1)^2$

$= -21 + 21 + 21$

= 21

Example 5:

(a) Add $a(b-c), b(c-a), \text{ and } c(a-b)$

(b) Subtract $5x(x-y+z)-2z(-3x+4y+5z)$ from $3y(4x+3y-2z)$

Solution:

(a) Addition of *a* (*b* − *c*), *b* (*c* − *a*), and *c* (*a* − *b*)

$$
\begin{aligned}
&= a(b-c)+b(c-a)+c(a-b) \\
&= ab-ac+bc-ab+ac-bc \\
&= 0
\end{aligned}
$$

(b) Subtraction of $5x(x-y+z)-2z(-3x+4y+5z)$ from $3y(4x+3y-2z)$

$$= 3y(4x+3y-2z)-\{5x(x-y+z)-2z(-3x+4y+5z)\}$$

$$= (3y)(4x)+(3y)(3y)+(3y)(-2z)-\begin{Bmatrix}(5x)(x)+(5x)(-y)+(5x)(z) \\ -2z(-3x)-2z(4y)-2z(5z)\end{Bmatrix}$$

$$= 12xy+9y^2-6yz-\{5x^2-5xy+5zx+6zx-8yz-10z^2\}$$

$$= 12xy+9y^2-6yz-\{5x^2-5xy+11zx-8yz-10z^2\}$$

$$= 12xy+9y^2-6yz-5x^2+5xy-11zx+8yz+10z^2$$

$$= -5x^2+9y^2+10z^2+17xy+2yz-11zx$$

Multiplication of Two Polynomials

Suppose you want to buy (2*x* + *y*) metres of rope at the rate of Rs (*a* − 3*b*) per metre.

Can you calculate the amount of money you require?

The amount you require is (2*x* + *y*) × (*a* − 3*b*).

Now, how will you carry out this type of multiplication?

The above expression is the multiplication of a binomial with a binomial. Let us see how we will multiply a binomial with a binomial with the help of the following video.

In the video, we have multiplied binomial with binomial in **horizontal arrangement**. Let us now multiply the binomials ($3x - y$) and ($x + 3y$) in **vertical arrangement**.

Here, first multiply $3x - y$ with $3y$ and then multiply $3x - y$ with x. After doing so, add the like terms as shown below:

$$\begin{array}{r} 3x - y \\ \times \quad x + 3y \\ \hline 9xy - 3y^2 \\ +3x^2 - xy \quad\quad \\ \hline 3x^2 + 8xy - 3y^2 \\ \hline \end{array}$$

The process of multiplying a binomial with a trinomial is not too different from that of multiplying two binomials. Let us learn more about it.

In the video, we have multiplied binomial with trinomial in **horizontal arrangement**. Let us now multiply the binomial ($x + y$) with trinomial ($2x + 3y + 1$) in **vertical arrangement**.

$$\begin{array}{r} 2x + 3y + 1 \\ \times \quad x + y \\ \hline 2xy + 3y^2 + y \\ +2x^2 + 3xy + x \quad\quad \\ \hline 2x^2 + 5xy + 3y^2 + x + y \\ \hline \end{array}$$

Thus, we can perform multiplication of binomials with binomials and trinomials using any of the horizontal or vertical arrangement method.

Let us now solve examples based on the above concepts.

Example 1:

Multiply the following using horizontal and vertical arrangement:

(a) 2 ($x + y$) and $x - 3y$

(b) ($l + 3m$) and ($l + 6m + 7n$)

Solution:

(a) Horizontal arrangement:

2 (x + y) = 2x + 2y

Now, we have to multiply (2x + 2y) and x − 3y.

(2x + 2y) × (x − 3y) = 2x × (x − 3y) + 2y × (x − 3y)

(Using distributive property)

= 2 × x × x − 2 × 3 × x × y + 2 × x × y − 2 × 3 × y × y

= $2x^2 - 6xy + 2xy - 6y^2$

= $2x^2 - 4xy - 6y^2$ (Combining the like terms)

Vertical arrangement:

$$\begin{array}{r} x + y \\ \times \quad 2 \\ \hline 2x + 2y \\ \hline \end{array} \qquad \begin{array}{r} x - 3y \\ \times \quad 2x + 2y \\ \hline 2xy - 6y^2 \\ +2x^2 - 6xy \qquad \\ \hline 2x^2 - 4xy - 6y^2 \\ \hline \end{array}$$

(b) Horizontal arrangement:

(l + 3m) × (l + 6m + 7n)

= l × (l + 6m + 7n) + 3m × (l + 6m + 7n)

(Using distributive property)

= l × l + l × 6m + l × 7n + 3m × l + 3m × 6m + 3m × 7n

= $l^2 + 6lm + 7ln + 3ml + 18m^2 + 21mn$

= $l^2 + 9ml + 7ln + 21mn + 18m^2$

[Combining the like terms 6lm and 3ml]

Vertical arrangement:

$$\begin{array}{rl} & l + 6m + 7n \\ \times & l + 3m \\ \hline & 3lm + 18m^2 + 21mn \\ & +l^2 + 6lm + 7nl \\ \hline & l^2 + 9lm + 18m^2 + 21mn + 7nl \\ \hline \end{array}$$

Example 2:

Simplify the following:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2\,(z^2 + 5z)$

(b) $(l - m)\,(l + m) + (m - n)\,(m + n) - (l - n)\,(n + l)$

(c) $(x - 4)\,(y - 4) - 16$

(d) $(a + b + c)\,(a - b + c)$

Solution:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2\,(z^2 + 5z)$

$= x^2\,(x^3 + y + z^2) + y^2\,(x^3 + y + z^2) + 2(z^2 + 5z)$

(Using distributive property)

$= x^2 \times x^3 + x^2 \times y + x^2 \times z^2 + y^2 \times x^3 + y^2 \times y + y^2 \times z^2 + 2 \times z^2 + 2 \times 5z$

$= x^5 + x^2y + x^2z^2 + x^3y^2 + y^3 + y^2z^2 + 2z^2 + 10z$

(b) $(l - m)\,(l + m) + (m - n)\,(m + n) - (l - n)\,(n + l)$

$= l\,(l + m) - m\,(l + m) + m\,(m + n) - n\,(m + n) - l\,(n + l) + n\,(n + l)$

(Using distributive property)

$= l^2 + lm - ml - m^2 + m^2 + mn - nm - n^2 - ln - l^2 + n^2 + nl$

$= (l^2 - l^2) + (lm - lm) + (-m^2 + m^2) + (mn - mn) + (-n^2 + n^2) + (-ln + ln)$

{*lm* and *ml*, *ln* and *nl*, *mn* and *nm* are like terms}

= 0

(c) $(x - 4) \times (y - 4) - 16$

$= x \times (y - 4) - 4(y - 4) - 16$ (Using distributive property)

$= xy - 4x - 4y + 16 - 16$

$= xy - 4x - 4y$

(d) $(a + b + c)\ (a - b + c)$

$= a\ (a - b + c) + b\ (a - b + c) + c\ (a - b + c)$ (Using distributive property)

$= a^2 - ab + ac + ba - b^2 + bc + ca - cb + c^2$ (Combining the like terms)

$= a^2 + c^2 - b^2 + 2ac$

Division of Polynomials by Monomials Using Factorization Method

Division is exactly the opposite of multiplication. For example, if 4 × 5 = 20, then it is also correct to say that 20 ÷ 5 = 4 and 20 ÷ 4 = 5.

We can use the same concept to divide algebraic expressions.

Let us try to divide the expression $3x$ by 3 and x.

We can factorize $3x$ as $3 \times x$.

This means that $3x$ is a product of 3 and x.

$\therefore 3x \div 3 = x$ and $3x \div x = 3$

Each of the expressions i.e., 3, x, and $3x$ is a monomial. Hence, these were examples of division of monomials by monomials.

When we divide a monomial by another monomial, we first need to factorise each monomial. Next, we divide the monomial by cancelling the common factors.

To understand the concept better, look at the following video.

How do we divide a polynomial by a monomial? For this, we have two methods.

To understand both the methods, look at the following video.

Let us discuss some more examples based on the two methods we just discussed.

Example 1:

Divide the following expressions:

(i). $\boldsymbol{27x^2y^2z \div 27xyz}$

(ii). $\boldsymbol{144pq^2r \div (-48qr)}$

Solution:

(i). $27x^2y^2z \div 27xyz$

Dividend = $27x^2y^2z$

$= 3 \times 3 \times 3 \times x \times x \times y \times y \times z$

Divisor = $27xyz$

$= 3 \times 3 \times 3 \times x \times y \times z$

$$\Rightarrow \frac{27x^2y^2z^2}{27xyz} = \frac{3\times3\times3\times x\times x\times y\times y\times z}{3\times3\times3\times x\times y\times z}$$

$= xy$

$\therefore 27x^2y^2z \div 27xyz = xy$

(ii). $144pq^2r \div (-48qr)$

Dividend = $144\,pq^2r$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times p \times q \times q \times r$

Divisor = $-48\,qr$

$= -2 \times 2 \times 2 \times 2 \times 3 \times q \times r$

$$\Rightarrow \frac{144pq^2r}{-48qr} = \frac{2\times2\times2\times2\times3\times3\times p\times q\times q\times r}{-2\times2\times2\times2\times3\times q\times r}$$
$$= -3\times p\times q$$
$$= -3pq$$

$\therefore$ $144pq^2r \div (-48qr) = -3pq$

Example 2:

Carry out the following divisions:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

(ii). $26xy\ (x + 5) \div 13xy$

(iii). $27\ (-a^2bc + ab^2c - abc^2) \div (-3abc)$

Solution:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

Dividend = $x^3y^6 - x^6y^3$

= $x^3y^3\ (y^3 - x^3)$

Divisor = x^3y^3

$$\Rightarrow \frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^3\left(y^3 - x^3\right)}{x^3y^3}$$

= $y^3 - x^3$

Another method of simplifying this expression is

$$\frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^6}{x^3y^3} - \frac{x^6y^3}{x^3y^3}$$
$$= y^3 - x^3$$

$\therefore$ $(x^3y^6 - x^6y^3) \div x^3y^3 = y^3 - x^3$

(ii). Dividend = $26xy\ (x + 5)$

= $2 \times 13 \times x \times y \times (x + 5)$

Divisor = $13xy$

= $13 \times x \times y$

$$\therefore \frac{26xy(x+5)}{13xy} = \frac{2\times 13\times x\times y\times (x+5)}{13\times x\times y}$$

= 2 (x + 5)

= 2x + 10

∴ 26xy (x + 5) ÷ 13xy = 2x + 10

(iii). $27\left(-a^2bc + ab^2c - abc^2\right) \div \left(-3abc\right)$

$$= \frac{27\left(-a^2bc + ab^2c - abc^2\right)}{\left(-3abc\right)}$$

$$= \frac{-27a^2bc + 27ab^2c - 27abc^2}{-3abc}$$

$$= \left(\frac{-27a^2bc}{-3abc}\right) + \left(\frac{27ab^2c}{-3abc}\right) + \left(\frac{-27abc^2}{-3abc}\right)$$

$$= 9a - 9b + 9c$$

Values of Algebraic Expressions at Different Points

A car is moving at a speed of 5 km/h more than twice the speed of a bus. Now, how can we represent the speed of the car with the help of an algebraic expression?

As seen here, the speed of the car changes with a change in the speed of the bus. In this way, we can find the value of an expression at a given point by substituting the value of the variable.

Let us discuss some more examples based on values of algebraic expressions at different points.

Example 1:

Find the value of the following expressions.

(i) $x^3 + \frac{x^2}{25} - 8$ **at** $x = 5$

(ii) $4k^2 - 2k + 1$ at $k = \frac{1}{2}$

(iii) $2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)$ when $x = 5$, $y = -2$

Solution:

(i) Putting $x = 5$ in the expression $x^3 + \frac{x^2}{25} - 8$, we obtain

$$x^3 + \frac{x^2}{25} - 8 = 5^3 + \frac{5^2}{25} - 8 = 125 + \frac{25}{25} - 8 = 125 + 1 - 8 = 118$$

(ii) Putting $k = \frac{1}{2}$ in the expression $4k^2 - 2k + 1$, we obtain

$$4k^2 - 2k + 1 = 4\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1 = 4 \times \frac{1}{4} - 2 \times \frac{1}{2} + 1 = 1 - 1 + 1 = 1$$

(iii) Putting $x = 5$, $y = -2$ in the expression $2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)$, we

obtain

$2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)$

$= 2 \times 5^3 \times (-2)^2 - 4 \times 5 - 3 \times 5 \times (-2) - 5\ [5^2 \times (-2)^2 - 5^2]$

$= 2 \times 125 \times 4 - 20 + 30 - 5(25 \times 4 - 25)$

$= 1000 - 20 + 30 - 5(100 - 25)$

$= 1000 - 20 + 30 - 5 \times 75$

$= 1000 - 20 + 30 - 375$

$= (1000 + 30) - (20 + 375)$

$= 1030 - 395$

$= 635$

Example 2:

At *p* = −2, the value of the expression $3p^2 - \frac{p}{2} + k - 11$ is 12. Find the value of *k*.

Solution:

Putting *p* = −2 in the given expression, we obtain

$$\begin{aligned}&3p^2 - \frac{p}{2} + k - 11\\&= 3(-2)^2 - \frac{(-2)}{2} + k - 11\\&= 3\times 4 + 1 + k - 11\\&= 12 + 1 + k - 11\\&= 2 + k\end{aligned}$$

But the value of the given expression at *p* = −2 is given as 12.

∴ 2 + *k* = 12

⇒ *k* = 12 − 2 = 10

Therefore, the value of *k* is 10.

Example 3:

Time taken by a car to cover a distance is given by the expression $\frac{2}{x} - \frac{3}{x^2} + \frac{1}{k}$. Find the value of *k* when *x* = 2 and the time taken by the car is half an hour.

Solution:

The given expression $\frac{2}{x} - \frac{3}{x^2} + \frac{1}{k}$ represents the time taken by the car to cover a particular distance.

When *x* = 2, we have

Time taken by the car = $\frac{1}{2}$ hour

$$\Rightarrow \frac{2}{2} - \frac{3}{2^2} + \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{3}{4} + \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{4} + \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{k} = \frac{1}{2} - \frac{1}{4}$$

$$\Rightarrow \frac{1}{k} = \frac{1}{4}$$

$$\Rightarrow k = 4$$

Example 4:

Numeric value of area of a figure is given by the expression $2\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right)$**. When** $a = \frac{5}{2} \text{ and } c = \frac{5}{8}$ **the numeric value of area is** $\frac{8}{5}$**. Find the value of *b.***

Solution:

Expression for numeric value of area of a figure **=** $2\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right)$

When $a = \frac{5}{2} \text{ and } c = \frac{5}{8}$, we have

Numeric value of area of figure = $\frac{8}{5}$

$$\Rightarrow 2\left(\frac{1}{\left(\frac{5}{2}\right)b}+\frac{1}{b\left(\frac{5}{8}\right)}+\frac{1}{\left(\frac{5}{8}\times\frac{5}{2}\right)}\right)=\frac{8}{5}$$

$$\Rightarrow 2\left(\frac{2}{5b}+\frac{8}{5b}+\frac{16}{25}\right)=\frac{8}{5}$$

$$\Rightarrow 2\left(\frac{10}{5b}+\frac{16}{25}\right)=\frac{8}{5}$$

$$\Rightarrow \frac{4}{b}+\frac{32}{25}=\frac{8}{5}$$

$$\Rightarrow \frac{4}{b}=\frac{8}{5}-\frac{32}{25}$$

$$\Rightarrow \frac{4}{b}=\frac{40-32}{25}$$

$$\Rightarrow \frac{4}{b}=\frac{8}{25}$$

$$\Rightarrow b=\frac{25}{2}$$

Simplification of Expressions Involving Brackets

An algebraic expression may contain some brackets, namely line bracket, common bracket, curly bracket, or rectangular brackets, and some mathematical operations.

An expression enclosed within a bracket is considered as a single quantity even though it may consist of many terms.

Therefore, for simplifying an expression, we remove the bracket by the following rules.

(i) If ‘+’ sign occurs before a bracket, then the signs of all the terms inside the bracket do not change.

(ii) If ‘−’ sign occurs before a bracket, then the signs of all the terms inside the bracket change.

Brackets are removed in order of

(a) line brackets (b) common brackets

(c) curly brackets and lastly (d) rectangular brackets

It can be noted that the above rules apply when we insert a bracket.

Let us see how we simplify an algebraic expression by taking an example.

$$x-\left[2y+2\left\{y-\left(z-\overline{x+y}\right)\right\}\right]$$

Change the signs of the terms inside the line bracket as '−' sign occurs before the line bracket.

$$=x-\left[2y+2\{y-(z-x-y)\}\right]$$

Similarly, change the signs of terms inside the common bracket as '−' sign occurs before the common bracket.

$$=x-\left[2y+2\{y-z+x+y\}\right]=x-\left[2y+2\{2y-z+x\}\right]$$

Signs of terms inside the curly bracket remain unchanged as '+' sign occurs before it.

$$=x-[2y+4y-2z+2x]=x-[6y-2z+2x]$$

Change the signs of terms inside rectangular bracket as '−' sign occurs before it.

$$=x-6y+2z-2x$$
$$=-x-6y+2z$$

Example 1:

Simplify the following:

(a) $2y-\left\{y-\left(x-\overline{y+z}\right)\right\}$

(b) $4(2p-q)-3\left(p-\overline{q+2p}\right)$

(c) $3e^2-\left[d^2-4\left\{f^2-\left(2e^2-\overline{f^2+d^2}\right)\right\}\right]$

Solution:

(a) $2y-\left\{y-\left(x-\overline{y+z}\right)\right\}$

$= 2y - \{y - (x - y - z)\}$[Line bracket is removed]

$= 2y - \{y - x + y + z\}$ [Line bracket is removed]

$= 2y - \{2y - x + z\}$

$= 2y - 2y + x - z = x - z$

(b) $4\left(2p-q\right)-3\left(p-\overline{q+2p}\right)$

$= 8p - 4q - 3\ (p - q - 2p)$ [One common bracket is removed and line bracket is removed in the other common bracket]

$= 8p - 4q - 3\ (- q - p)$

$= 8p - 4q + 3q + 3p$ [Common bracket is removed]

$= 11p - q$

(c) $3e^2-\left[d^2-4\left\{f^2-\left(2e^2-\overline{f^2+d^2}\right)\right\}\right]$

$= 3e^2 - [d^2 - 4\ \{f^2 - (2e^2 - f^2 - d^2)\}]$ [Line bracket is removed]

$= 3e^2 - [d^2 - 4\ \{f^2 - 2e^2 + f^2 + d^2\}]$ [Common bracket is removed]

$= 3e^2 - [d^2 - 4\ \{2f^2 - 2e^2 + d^2\}]$

$= 3e^2 - [d^2 - 8f^2 + 8e^2 - 4d^2]$ [Curly bracket is removed]

$= 3e^2 - [-3d^2 - 8f^2 + 8e^2]$

$= 3e^2 + 3d^2 + 8f^2 - 8e^2$[Rectangular bracket is removed]

$= 3d^2 - 5e^2 + 8f^2$

Mathematical Expressions Of Word Problems

Suppose Rahul has Rs 2100 with him. He goes to a market and purchases five shirts. Now the money left with him is Rs 100.

How can we write this situation mathematically?

Let us look at some more examples now.

Example:

Write the following statements in the form of a linear equation.

1. The sum of two consecutive even numbers is 46.

2. One-fourth of a number plus 5 is 30.

3. When 20 is subtracted from *m*, the result is 16.

4. The perimeter of an equilateral triangle is 27 cm.

5. Mohit is 5 years older than Rohit and the sum of their ages is 35.

Solution:

1. Let one even number be $2x$.
   Then, the other even number will be $(2x + 2)$.
   The linear equation is
   $2x + (2x + 2) = 46$
   $\Rightarrow 4x + 2 = 46$

2. Let the number be z.
   One-fourth of the number = z4z4
   According to the given statement,
   z4+5=30z4+5=30

3. The difference between m and 20 is 16.
   Therefore,
   $m - 20 = 16$

4. We know that in an equilateral triangle, all sides are equal in length.
   Let the length of one side be x.
   The perimeter is the sum of all sides of the triangle.
   $\Rightarrow x + x + x = 27$
   $\Rightarrow 3x = 27$

5. Let the age of Rohit be x years.
   Mohit's age will be $(x + 5)$ years.
   The sum of their ages is 35,
   $\therefore x + (x + 5) = 35$
   $\Rightarrow 2x + 5 = 35$