DPP - Daily Practice Problems

Date : ______ **Start Time :** ______ **End Time :** ______

MATHEMATICS CM08

SYLLABUS : Binomial Theorem

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. If $x = 99^{50} + 100^{50}$ and $y = (101)^{50}$ then

(a) $x = y$ (b) $x < y$

(c) $x > y$ (d) None of these

2. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + ... =$

(a) $\frac{2^{n+1}}{n+1}$ (b) $\frac{2^{n+1}-1}{n+1}$

(c) $\frac{2^n}{n+1}$ (d) None of these

3. If P_n denotes the product of the binomial coefficients in the expansion of $(1 + x)^n$, then $\frac{P_{n+1}}{P_n}$ equals

(a) $\frac{n+1}{n\,!}$ (b) $\frac{n^n}{n\,!}$

(c) $\frac{(n+1)^n}{(n+1)\,!}$ (d) $\frac{(n+1)^{n+1}}{(n+1)\,!}$

4. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{55}C_4$ (b) ${}^{55}C_3$

(c) ${}^{56}C_3$ (d) ${}^{56}C_4$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. If C_0, C_1, C_2, ..., C_{15} are binomial coefficients in $(1 + x)^{15}$, then $\frac{C_1}{C_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+.....+15\frac{C_{15}}{C_{14}}=$

(a) 60 (b) 120
(c) 64 (d) 124

6. The value of the term independent of x in the expansion of $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x \neq 0$ is equal to

(a) 1 (b) – 6
(c) –5 (d) 6

7. The coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-7/2}$ is

(a) 15 (b) 21
(c) 12 (d) 30

8. A set contains (2n + 1) elements. If the number of subsets of this set which contain at most n elements is 4096, then the value of n is

(a) 6 (b) 15
(c) 21 (d) None of these

9. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

10. Coefficient of x^{25} in expansion of expression

$\sum_{r=0}^{50} {}^{50}C_r\,(2x-3)^r\,(2-x)^{50-r}$ is

(a) ${}^{50}C_{25}$ (b) $-{}^{50}C_{30}$
(c) ${}^{50}C_{30}$ (d) $-{}^{50}C_{25}$

11. If $(1 + x)^{2n} = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then

(a) $a_0 + a_2 + a_4 + ... = \frac{1}{2}(a_0 + a_1 + a_2 + a_3 + ...)$
(b) $a_{n+1} < a_n$
(c) $a_{n-3} = a_{n+3}$
(d) All of these

12. One value of α for which the coefficients of the middle terms in the expansion of $(1 + \alpha x)^4$ and $(1 - \alpha x)^6$ are equal, is $\frac{-3}{10}$. Other value of 'α' is

(a) 0 (b) 1 (c) 2 (d) 3

13. Number of ways of selection of 8 letters from 24 letters of which 8 are a, 8 are b and the rest unlike, is given by

(a) 2^7 (b) 8.2^8
(c) 10.2^7 (d) None of these

14. The expression

$$(\sqrt{2x^2+1}+\sqrt{2x^2-1})^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$$ is *a* polynomial of degree:

(a) 5 (b) 6
(c) 7 (d) 8

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. The value of

$$\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}.....+\binom{30}{20}\binom{30}{30}$$

is where $\binom{n}{r} = {}^nC_r$

(a) $\binom{30}{10}$ (b) $\binom{30}{15}$

(c) $\binom{60}{30}$ (d) $\binom{31}{10}$

16. If 'n' is positive integer and three consecutive coefficient in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110, then n is equal to :

(a) 9 (b) 6

(c) 12 (d) 16

17. If the coefficient of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

(a) $a - b = 1$ (b) $a + b = 1$

(c) $\frac{a}{b} = 1$ (d) $ab = 1$

18. If $7^9 + 9^7$ is divided by 64 then the remainder is

(a) 0 (b) 1

(c) 2 (d) 63

19. For natural numbers m, n if $(1-y)^m(1+y)^n$ $= 1 + a_1y + a_2y^2 + \ldots$ and $a_1 = a_2 = 10$, then (m, n) is

(a) (20,45) (b) (35,20)

(c) (45,35) (d) (35,45)

20. If $(1 + x)^n = \sum_{r=0}^{n} C_r x^r$, then the value of

$$C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + ...$$
$$+ (C_0 + C_1 + C_2 + ... + C_n) \text{ is}$$

(a) $n \cdot 2^{n-1}$ (b) $(n + 2) \cdot 2^n$

(c) 2^n (d) $(n + 2) \cdot 2^{n-1}$

21. Let a = $3^{\frac{1}{223}} + 1$ and for all $n \geq 3$, let $f(n) = {}^nC_0 . a^{n-1} - {}^nC_1 a^{n-2} + {}^nC_2 a^{n-3} - .... + (-1)^{n-1} . {}^nC_{n-1} a^0$. If the value of $f(2007) + f(2008) = 9k$, where $k \in$ N, then find k.

(a) 2187 (b) 1987

(c) 3232 (d) 4187

22. Find the value of ${}^{4n}C_0 + {}^{4n}C_4 + {}^{4n}C_8 + ... + {}^{4n}C_{4n}$

(a) $(-1)^n 2^{2n-1} + 2^{4n-2}$ (b) $(-1)^n 2^{2n-1}$

(c) $(-1)^n 2^{4n-1}$ (d) None of these

23. The sum of the series

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... -.....+ {}^{20}C_{10} \text{ is}$$

(a) 0 (b) ${}^{20}C_{10}$

(c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

24. If 7^{103} is divided by 25, then the remainder is

(a) 20 (b) 16

(c) 18 (d) 15

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If x is very small in magnitude compared with a, then $\left(\frac{a}{a+x}\right)^{\frac{1}{2}}+\left(\frac{a}{a-x}\right)^{\frac{1}{2}}$ can be approximately equal to

(a) $1+\frac{1}{2}\frac{x}{a}$ (b) $\frac{x}{a}$

(c) $1+\frac{3}{4}\frac{x^2}{a^2}$ (d) $2+\frac{3}{4}\frac{x^2}{a^2}$

26. If $C_0, C_1, C_2, ..., C_n$ be the coefficients in the expansion of $(1 + x)^n$, then $\frac{2^2 . C_0}{1.2}+\frac{2^3 . C_1}{2.3}+.....+\frac{2^{n+2}.C_n}{(n+1)(n+2)}$ is equal to

(a) $\frac{3^{n+1}-2n-5}{(n+1)(n+2)}$ (b) $\frac{3^{n+2}-2n-5}{(n+1)(n+2)}$

(c) $\frac{3^{n+2}+2n-5}{(n+1)(n+2)}$ (d) None of these

27. The coefficient of x^n in the expansion of $(1-9x+20x^2)^{-1}$ is

(a) $5^n - 4^n$ (b) $5^{n+1} - 4^{n+1}$

(c) $5^{n-1} - 4^{n-1}$ (d) None of these

28. If $T_0, T_1, T_2....T_n$ represent the terms in the expansion of $(x+a)^n$, then $(T_0-T_2+T_4-.......)^2+(T_1-T_3+T_5-.....)^2=$

(a) (x^2+a^2) (b) $(x^2+a^2)^n$

(c) $(x^2+a^2)^{1/n}$ (d) $(x^2+a^2)^{-1/n}$

29. The coefficient of x^n in the polynomial $(x+{}^nC_0)(x+3 . {}^nC_1)(x+5 . {}^nC_2)....(x+(2n+1)\,{}^nC_n)$ is

(a) $n . 2^n$ (b) $n . 2^{n+1}$

(c) $(n+1) . 2^n$ (d) $n . 2^n+1$

30. In the expansion of $(1+x)^{18}$, if the coefficients of $(2r+4)^{th}$ and $(r-2)^{th}$ terms are equal, then the value of r is :

(a) 12 (b) 10

(c) 8 (d) 6

RESPONSE GRID	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ
	30. ⓐⓑⓒⓓ				

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 8 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	38	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(b)** $(101)^{50} - (99)^{50} = (100+1)^{50} - (100-1)^{50}$

$= 2[\,^{50}C_1(100)^{49} + \,^{50}C_3(100)^{47} + ...+ \,^{50}C_{49}(100)]$

$> 2\,.\,^{50}C_1\,.\,(100)^{49} = 2\times 50(100)^{49} = (100)^{50}$

$\Rightarrow (101)^{50} > (99)^{50} + (100)^{50} \Rightarrow y > x \Rightarrow x < y$.

2. **(c)** Putting the value of $C_0, C_2, C_4.....$, we get

$$= 1 + \frac{n(n-1)}{3.2!} + \frac{n(n-1)(n-2)(n-3)}{5.4!} + ... = \frac{1}{n+1}$$

$$\left[(n+1) + \frac{(n+1)n(n-1)}{3!} + \frac{(n+1)n(n-1)(n-2)(n-3)}{5!} + ...\right]$$

Put $n + 1 = N$

$$= \frac{1}{N}\left[N + \frac{N(N-1)(N-2)}{3!} + \frac{N(N-1)(N-2)(N-3)(N-4)}{5!} + ...\right]$$

$$= \frac{1}{N}\left\{^N C_1 + \,^N C_3 + \,^N C_5 + ...\right\}$$

$$= \frac{1}{N}\left\{2^{N-1}\right\} = \frac{2^n}{n+1} \qquad \{\because N = n+1\}$$

3. **(d)** Here, $P_n = \,^nC_0\,.\,^nC_1\,.\,^nC_2....\,^nC_n$

and $P_{n+1} = \,^{n+1}C_0\,.\,^{n+1}C_1\,.\,^{n+1}C_2....\,^{n+1}C_{n+1}$

$$\therefore \frac{P_{n+1}}{P_n} = \frac{^{n+1}C_0\,.\,^{n+1}C_1\,.\,^{n+2}C_2....\,^{n+1}C_{n+1}}{^nC_0\,.\,^nC_1\,.\,^nC_2\,....\,^nC_n}$$

$$= \left(\frac{^{n+1}C_1}{^nC_0}\right)\left(\frac{^{n+1}C_2}{^nC_1}\right)\left(\frac{^{n+1}C_3}{^nC_0}\right).....\left(\frac{^{n+1}C_{n+1}}{^nC_n}\right)$$

$$= \left(\frac{n+1}{1}\right)\left(\frac{n+1}{2}\right)\left(\frac{n+1}{3}\right).....\left(\frac{n+1}{n+1}\right) = \frac{(n+1)^{n+1}}{(n+1)\,!}$$

4. **(d)** $^{50}C_4 + \sum_{r=1}^{6} \,^{56-r}C_3$

$$= \,^{50}C_4 + \left[\begin{matrix} ^{55}C_3 + \,^{54}C_3 + \,^{53}C_3 + \,^{52}C_3 \\ + \,^{51}C_3 + \,^{50}C_3 \end{matrix}\right]$$

We know $\left[\,^nC_r + \,^nC_{r-1} = \,^{n+1}C_r\right]$

$= (^{50}C_4 + \,^{50}C_3) + \,^{51}C_3 + \,^{52}C_3 + \,^{53}C_3 + \,^{54}C_3 + \,^{55}C_3$

$= (^{51}C_4 + \,^{51}C_3) + \,^{52}C_3 + \,^{53}C_3 + \,^{54}C_3 + \,^{55}C_3$

Proceeding in the same way, we get

$^{55}C_4 + \,^{55}C_3 = \,^{56}C_4$.

5. **(b)** General term of the given series is

$$r\frac{^nC_r}{^nC_{r-1}} = n + 1 - \text{r}$$

By taking summation over n, we get

$$\sum_1^{15} r\frac{^nC_r}{^nC_{r-1}} = \sum_{n=1}^{15}(n+1-r) = \sum_1^{15}(16-r)$$

$$= 16 \times 15 - \frac{1}{2}\cdot 15\times 16$$

By using sum of n natural numbers $= \dfrac{n(n+1)}{2}$

$= 240 - 120 = 120$

6. **(c)** $\left(1 + \dfrac{x}{2} - \dfrac{2}{x}\right)^4$

$$= \,^4C_0 + \,^4C_1\left(\frac{x}{2} - \frac{2}{x}\right) + \,^4C_2\left(\frac{x}{2} - \frac{2}{x}\right)^2$$

$$+ \,^4C_3\left(\frac{x}{2} - \frac{2}{x}\right)^3 + \,^4C_4\left(\frac{x}{2} - \frac{2}{x}\right)^4$$

$$= \,^4C_0 + \,^4C_1\left(\frac{x}{2} - \frac{2}{x}\right) + \,^4C_2\left[\frac{x^2}{4} - 2 + \frac{4}{x^2}\right]$$

$$+ \,^4C_3\left[\,^3C_0\left(\frac{x}{2}\right)^3 - \,^3C_1\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right) + \,^3C_2\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^2 - \,^3C_3\left(\frac{2}{x}\right)^3\right]$$

$$+ \,^4C_4\left[\,^4C_0\left(\frac{x}{2}\right)^4 - \,^4C_1\left(\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + \,^4C_2\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - \,^4C_3\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + \,^4C_4\left(\frac{2}{x}\right)^4\right]$$

The term independent of x in above

$= \,^4C_0 + \,^4C_2(-2) + \,^4C_4.\,^4C_2 = 1 - 12 + 6 = -5$

7. **(b)** $1 + 2x + 3x^2 + ... = (1+x)^{-2}$

$$\Rightarrow \left(1 + 2x + 3x^2 + ...\right)^{-3/2} = \left\{(1+x)^{-2}\right\}^{-7/2} = (1+x)^7$$

$\therefore$ The coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-7/2}$

$=$ Coefficient of x^5 in $(1+x)^7$

$= \,^7C_5 = 21$

8. **(a)** The number of subsets of the set which contain at most n elements is

$^{2n+1}C_0 + \,^{2n+1}C_1 + \,^{2n+1}C_2 + ... + \,^{2n+1}C_n = K$ (say)

We have

$2K = 2\,(^{2n+1}C_0 + \,^{2n+1}C_1 + \,^{2n+1}C_2 + ... + \,^{2n+1}C_n)$

$= (^{2n+1}C_0 + \,^{2n+1}C_{2n+1}) + (^{2n+1}C_1 + \,^{2n+1}C_{2n})$

$+ ... + (^{2n+1}C_n + \,^{2n+1}C_{n+1}) \qquad (\because \,^nC_r = \,^nC_{n-r})$

$= \,^{2n+1}C_0 + \,^{2n+1}C_1 + \,^{2n+1}C_2 + ... + \,^{2n+1}C_{2n+1}$

$= 2^{2n+1} \Rightarrow K = 2^{2n}$

9. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$$

$$= (1-x)^{\frac{-1}{2}}\left[\left(1 + \frac{3}{2}x + \frac{\frac{3}{2}\cdot\frac{1}{2}}{2!}x^2\right) - \left(1 + \frac{3x}{2} + \frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right]=\frac{-3}{8}x^2$$

(as x^3 and higher powers of x can be neglected)

10. (d) $\sum_{r=0}^{50} {}^{50}C_r\,(2x-3)^r(2-x)^{50-r}$

$= [(2-x)+(2x-3)]^{50}$
$= (x-1)^{50}$
$= (1-x)^{50}$
$= {}^{50}C_0 - {}^{50}C_1\,x\,\ldots\ldots - {}^{50}C_{25}\,x^{25} + \ldots\ldots$

Coefficient of x^{25} is $-{}^{50}C_{25}$

11. (d) $a_0 + a_1 + a_2 + \ldots = 2^{2n}$ and $a_0 + a_2 + a_4 + \ldots = 2^{2n-1}$

$a_n = {}^{2n}C_n$ = the greatest coefficient, being the middle coefficient

$a_{n-3} = {}^{2n}C_{n-3} = {}^{2n}C_{2n-(n-3)} = {}^{2n}C_{n+3} = a_{n+3}$

12. (a) In the expansion of $(1+\alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

In the expansion of $(1-\alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

It is given that

Coefficient of the middle term in $(1+\alpha x)^4$ = Coefficient of the middle term in $(1-\alpha x)^6$

$\Rightarrow 6\alpha^2 = -20\alpha^3$

$\Rightarrow \alpha = 0, \alpha = -\frac{3}{10}$

13. (c) The number of selection = coefficient of x^8 in $(1+x+x^2+\ldots+x^8)(1+x+x^2+\ldots\ldots+x^8).(1+x)^8$

= coefficient of x^8 in $\frac{(1-x^9)^2}{(1-x)^2}(1+x)^8$

= coefficient of x^8 in $(1+x)^8$ in $(1+x^8)(1-x)^{-2}$

= coefficient of x^8 in

$({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + \ldots\ldots + {}^8C_8x^8)$
$\times(1+2x+3x^2+4x^3+\ldots\ldots+9x^8+\ldots)$

$= 9.\,{}^8C_0 + 8\cdot{}^8C_1 + 7.\,{}^8C_2 + \ldots + 1.\,{}^8C_8$

$= C_0 + 2C_1 + 3C_2 + \ldots + 9C_8$ $[C_r = {}^8C_r]$

Now $C_0x + C_1x^2 + \ldots + C_8x^9 = x(1+x)^8$

Differentiating with respect to x, we get

$C_0 + 2C_1x + 3C_2x^2 + \ldots\ 9C_8x^8 = (1+x)^8 + 8x(1+x)^7$

Putting $x = 1$, we get $C_0 + 2C_1 + 3C_2 + \ldots\ldots + 9C_8$

$= 2^8 + 8.2^7. = 2^7(2+8) = 10.2^7.$

14. (b) $\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}} = \sqrt{2x^2+1}-\sqrt{2x^2-1}$

$\therefore$ given expression

$= (\sqrt{2x^2+1}+\sqrt{2x^2-1})^6 + (\sqrt{2x^2+1}-\sqrt{2x^2-1})^6$

we know that,

$$(a+b)^6+(a-b)^6 = 2[a^6 + {}^6C_2a^4b^2 + {}^6C_4a^2b^4 + {}^6C_6b^6]$$

$$\therefore (\sqrt{2x^2+1}+\sqrt{2x^2-1})^6 + (\sqrt{2x^2+1}-\sqrt{2x^2-1})^6$$

$$= 2[(2x^2+1)^3 + 15(2x^2+1)^2(2x^2-1) + 15(2x^2+1)(2x^2-1) + (2x^2-1)^3]$$

Which is a polynomial of degree 6.

15. (a) To find

${}^{30}C_0{}^{30}C_{10} - {}^{30}C_1{}^{30}C_{11} + {}^{30}C_2{}^{30}C_{12} - \ldots + {}^{30}C_{20}{}^{30}C_{30}$

We know that

$(1+x)^{30} = {}^{30}C_0 + {}^{30}C_1x + {}^{30}C_2x^2 + \ldots + {}^{30}C_{20}x^{20} + \ldots\ {}^{30}C_{30}x^{30}$ ....(1)

$(x-1)^{30} = {}^{30}C_0x^{30} - {}^{30}C_1x^{29} + \ldots + {}^{30}C_{10}x^{20} - {}^{30}C_{11}x^{19} + {}^{30}C_{12}x^{18} + \ldots\ {}^{30}C_{30}x^0$ ....(2)

Multiplying eq^n (1) and (2) and equating the coefficients of x^{20} on both sides, we get

${}^{30}C_{10} = {}^{30}C_0{}^{30}C_{10} - {}^{30}C_1{}^{30}C_{11} + {}^{30}C_2{}^{30}C_{12} - \ldots + {}^{30}C_{20}{}^{30}C_{30}$

$\therefore$ Req. value is ${}^{30}C_{10}$

16. (c) Let the consecutive coefficient of $(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

From the given condition,

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

Now ${}^nC_{r-1} : {}^nC_r = 6 : 33$

$\Rightarrow \frac{n!}{(r-1)!\,(n-r+1)!}\times\frac{r!(n-r)!}{n!} = \frac{6}{33}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{11} \Rightarrow 11r = 2n - 2r + 2$

$\Rightarrow 2n - 13r + 2 = 0$ ....(i)

and ${}^nC_r : {}^nC_{r+1} = 33 : 110$

$\Rightarrow \frac{n!}{r!(n-r)!}\times\frac{(r+1)!\,(n-r-1)!}{n!} = \frac{33}{110} = \frac{3}{10}$

$\Rightarrow \frac{(r+1)}{n-r} = \frac{3}{10} \Rightarrow 3n - 13r - 10 = 0$ ...(ii)

Solving (i) & (ii), we get n = 12

17. (d) T_{r+1} in the expansion

$$\left[ax^2+\frac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$$

$= {}^{11}C_r\,(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the coefficient of x^7, we have

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^7 = {}^{11}C_5(a)^6\ (b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$$\left[ax-\frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$$

$= {}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r}(x)^{11-r}$

For the coefficient of x^{-7}, we have

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6\,a^5\times 1\times(b)^{-6}$

$\therefore$ Coefficient of x^7 = Coefficient of x^{-7}

$\Rightarrow {}^{11}C_5\,(a)^6(b)^{-5} = {}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab = 1.$

18. **(a)** We have

$7^9+9^7=(8-1)^9+(8+1)^7=(1+8)^7-(1-8)^9$

$=[1+{}^7C_1 8+{}^7C_2 8^2+.....+{}^7C_7 8^7]$

$-[1-{}^9C_1 8+{}^9C_2 8^2-.....-{}^9C_9 8^9]$

$={}^7C_1 8+{}^9C_1 8+[{}^7C_2+{}^7C_3.8+....-{}^9C_2+{}^9C_3.8-.....]8^2$

$=8(7+9)+64k=8..16+64k=64q$, where $q=k+2$

Thus, 7^9+9^7 is divisible by 64.

19. **(d)** $(1-y)^m(1+y)^n$

$=[1-{}^mC_1 y+{}^mC_2 y^2-......][1+{}^nC_1 y+{}^nC_2 y^2+.....]$

$=1+(n-m)y+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+.....$

By comparing coefficients with the given expression, we get

$\therefore\ a_1=n-m=10$ and

$a_2=\frac{m^2+n^2-m-n-2mn}{2}=10$

So, $n-m=10$ and $(m-n)^2-(m+n)=20$

$\Rightarrow\ m+n=80\quad \therefore\ m=35, n=45$

20. **(d)** We have

$S=C_0+(C_0+C_1)+(C_0+C_1+C_2)+$

$....+(C_0+C_1+...+C_n)$

$=(C_0+C_0+....n+1\text{ times})+(C_1+C_1+....n\text{ times})$

$(C_2+C_2+....\ n-1\text{ times})+.....+(C_{n-1}+C_{n-1})+C_n$

$=(n+1)C_0+nC_1+(n-1)C_2+....+2C_{n-1}+C_n$

$=C_0+2C_1+3C_2+.....+(n+1)C_n\quad [\because C_r=C_{n-r}]$

General Term $T_{r+1}=(r+1)C_r$

$T_{r+1}=r\,{}^nC_r+{}^nC_r=n.{}^{n-1}C_{r-1}+{}^nC_r$

$\therefore S=\sum_{r=0}^{n}T_{r+1}=n[{}^{n-1}C_0+{}^{n-1}C_1+.....+{}^{n-1}C_{n-1}]$

$+[{}^nC_0+{}^nC_1+.....+{}^nC_n]$

$=n.2^{n-1}+2^n=(n+2)2^{n-1}$

21. **(a)** We know that,

$(a-1)^n={}^nC_0.a^n-{}^nC_1.a^{n-1}+{}^nC_2a^{n-2}-$
$........+(-1)^{n-1}.{}^nC_{n-1}a+(-1)^n\,{}^nC_n$

$\therefore\ \frac{(a-1)^n}{a}={}^nC_0a^{n-1}-{}^nC_1a^{n-2}+{}^nC_2a^{n-3}-$

$........+(-1)^{n-1}.{}^nC_{n-1}+\frac{(-1)^n}{a}\,{}^nC_n$

$\therefore f(n)=\frac{(a-1)^n-(-1)^n}{a}$

Now, $f(2007)+f(2008)=\frac{(a-1)^{2007}+1}{a}+\frac{(a-1)^{2008}-1}{a}$

$=\frac{(a-1)^{2007}(1+a-1)}{a}=(a-1)^{2007}$

$=\left(3^{\frac{1}{223}}\right)^{2007}=3^9=3^2\cdot 3^7=9\,(2187)$

$\therefore\ k=2187$

22. **(a)** $(1+x)^{4n}={}^{4n}C_0+{}^{4n}C_1x+{}^{4n}C_2x^2+{}^{4n}C_3x^3$
$+{}^{4n}C_4x^4+.....+{}^{4n}C_{un}x^{4n}$

Put $x=1$ and $x=-1$, then adding.

$2^{4n-1}={}^{4n}C_0+{}^{4n}C_2+{}^{4n}C_4+.....+{}^{4n}C_{4n}$ .....(i)

Now put, $x=i$

$(1+i)^{4n}={}^{4n}C_0+{}^{4n}C_1 i-{}^{4n}C_2-{}^{4n}C_3 i+{}^{4n}C_4+$
$........+{}^{4n}C_{4n}$

Compare real and imaginary part, we get

$(-1)^n(2)^{2n}={}^{4n}C_0-{}^{4n}C_2+{}^{4n}C_4-{}^{4n}C_6+....+{}^{4n}C_{4n}$... (ii)

Adding (i) and (ii), we get

$\Rightarrow\ {}^{4n}C_0+{}^{4n}C_4+....+{}^{4n}C_{4n}=(-1)^n(2)^{2n-1}+2^{4n-2}$

23. **(d)** We know that, $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x+{}^{20}C_2x^2$
$+......{}^{20}C_{10}x^{10}+.....{}^{20}C_{20}x^{20}$

Put $x=-1$, $(0)={}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......$
$+{}^{20}C_{10}-{}^{20}C_{11}....+{}^{20}C_{20}$

$\Rightarrow 0=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-{}^{20}C_9]+{}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10}=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3$
$+......-{}^{20}C_9+{}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+....+{}^{20}C_{10}=\frac{1}{2}\,{}^{20}C_{10}$

24. **(c)** We have, $7^{103}=7(49)^{51}=7(50-1)^{51}$

$=7(50^{51}-{}^{51}C_1 50^{50}+{}^{51}C_2 50^{49}-...-1)$

$=7(50^{51}-{}^{51}C_1 50^{50}+{}^{51}C_2 50^{49}-...)-7+18-18$

$=7(50^{51}-{}^{51}C_1 50^{50}+{}^{51}C_2 50^{49}-...)-25+18$

$=k+18$ (say) where k is divisible by 25,

$\therefore$ remainder is 18.

25. **(d)** $\left(\frac{a}{a+x}\right)^{\frac{1}{2}}+\left(\frac{a}{a-x}\right)^{\frac{1}{2}}=\left(\frac{a+x}{a}\right)^{-\frac{1}{2}}+\left(\frac{a-x}{a}\right)^{-\frac{1}{2}}$

$=\left(1+\frac{x}{a}\right)^{-\frac{1}{2}}+\left(1-\frac{x}{a}\right)^{-\frac{1}{2}}$

$=\left[1-\frac{1}{2}\frac{x}{a}+\frac{3}{8}\frac{x^2}{a^2}\right]+\left[1+\frac{1}{2}\frac{x}{a}+\frac{3}{8}\frac{x^2}{a^2}\right]$

$\left[\because x<<a, \therefore \frac{x}{a}<<1\right]=2+\frac{3}{4}\cdot\frac{x^2}{a^2}$

26. **(b)** We have,

$t_{r+1}=\frac{2^{r+2}\,{}^nC_r}{(r+1)(r+2)}=\frac{2^{r+2}}{r+2}\cdot\frac{1}{r+1}\,{}^nC_r$

$=\frac{2^{r+2}}{r+2}\cdot\frac{1}{n+1}\,{}^{n+1}C_{r+1}$

$=\frac{2^{r+2}}{n+1}\cdot\left(\frac{1}{r+2}\,{}^{n+1}C_{r+1}\right)$

$$= \frac{2^{r+2}}{n+1} \cdot \frac{1}{n+2} \, {}^{n+2}C_{r+2}$$

$$\left[\because \frac{1}{r+1} \, {}^{n}C_r = \frac{1}{n+1} \, {}^{n+1}C_{r+1}\right]$$

Putting r = 0, 1, 2, ..., n and adding we get,
The given expression

$$= \frac{1}{(n+1)(n+2)} \{2^2 . {}^{n+2}C_2 + 2^3 . {}^{n+2}C_3 + ... + 2^{n+2} . {}^{n+2}C_{n+2}\}$$

$$= \frac{1}{(n+1)(n+2)} \{(1+2)^{n+2} - {}^{n+2}C_0 - 2. {}^{n+2}C_1\}$$

$$= \frac{3^{n+2} - 2(n+2) - 1}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

27. **(b)** $(1 - 9x + 20x^2)^{-1} = [(1-4x)(1-5x)]^{-1}$

$$= \frac{1}{x}\left[\frac{(1-4x)-(1-5x)}{(1-4x).(1-5x)}\right] = \frac{1}{x}[(1-5x)^{-1} - (1-4x)^{-1}]$$

$$= \frac{1}{5}[(5-4)x + (5^2-4^2)x^2 + (5^3-4^3)x^3 + ...+ (5^n-4^n)x^n + ...]$$

$\therefore$ coeff. of $x^n = 5^{n+1} - 4^{n+1}$

28. **(b)** From the given condition, replacing a by ai and $-ai$ respectively, we get

$$(x+ai)^n = (T_0 - T_2 + T_4 - ......) + i(T_1 - T_3 + T_5 - ....) \quad ....(i)$$

and

$$(x-ai)^n = (T_0 - T_2 + T_4 - ......) - i(T_1 - T_3 + T_5 - ....) \quad ....(ii)$$

Multiplying (ii) and (i) we get required result

i.e., $(x^2 + a^2)^n = (T_0 - T_2 + T_4 - ....)^2 + (T_1 - T_3 + T_5 - ....)^2$

29. **(c)** $(x + {}^nC_0)(x + 3.{}^nC_1)(x + 5.{}^nC_2).....(x + (2n+1).{}^nC_n)$

$$= x^{n+1} + x^n\{{}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + ... + (2n+1).{}^nC_n\} + ...$$

Coeff. of $x^n = {}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + ... + (2n+1).{}^nC_n$

$$= 1 + ({}^nC_1 + 2.{}^nC_1) + ({}^nC_2 + 4.{}^nC_2) + ... + ({}^nC_n + 2n.{}^nC_n)$$

$$= (1 + {}^nC_1 + ... + {}^nC_n) + 2({}^nC_1 + 2\,{}^nC_2 + ... + n.{}^nC_n)$$

$$= 2^n + 2\left[n + 2.\frac{n(n-1)}{2!} + 3.\frac{n(n-1)(n-2)}{3!} + ... + n.1\right]$$

$$= 2^n + 2n[1 + {}^{n-1}C_1 + {}^{n-1}C_2 + ... + {}^{n-1}C_{n-1}]$$

$$= 2^n + 2n.2^{n-1} = 2^n(1+n) = (n+1).2^n$$

30. **(d)** Since the coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1+x)^{18}$

coefficient of $(2r+4)^{th}$ term $= {}^{18}C_{2r+3}$,

Similarly, coefficient of $(r-2)^{th}$ term in the expansion of $(1+x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$

So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18$

$\Rightarrow 3r = 18 \Rightarrow r = 6.$