Pair of Linear Equations in Two Variables 3

Objective Section (1 mark each)

Multiple Choice Questions

Q. 1. The value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is [CBSE OD, Set 1, 2020]

(a) $-\frac{14}{3}$ **(b)** $\frac{2}{5}$

(c) 5 **(d)** 10

Ans. For the system of linear equations to be inconsistent, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Here $a_1 = 1$, $b_1 = 2$, $c_1 = -3$ and $a_2 = 5$, $b_2 = k$, $c_2 = 7$

So, $$\frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$$

$$\Rightarrow \quad k = 10$$

$\therefore$ Option (d) is correct. **Ans.**

Q. 2. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is [CBSE Delhi, Set 1, 2020]

(a) -2 **(b)** $\neq 2$

(c) 3 **(d)** 2

Ans. Given,

$$x + y - 4 = 0$$
$$2x + ky - 3 = 0$$
$$a_1 = 1, b_1 = 1, c_1 = -4$$
$$a_2 = 2, b_2 = k, c_2 = -3$$

For no solution, the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} \text{ and } \frac{1}{k} \neq \frac{4}{3}$$

$$\Rightarrow \quad k = 2 \text{ and } k \neq \frac{3}{4}$$

$$\Rightarrow \quad k = 2$$

$\therefore$ Option (d) is correct. **Ans.**

Short Answer Type Questions-I (2 marks each)

Q. 1. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution. [CBSE OD, Set 1, 2019]

Ans. Given, $$x + 2y = 5$$
$$3x + ky + 15 = 0$$

Comparing above equations with

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

We get,

$$a_1 = 1, b_1 = 2, c_1 = -5$$
$$a_2 = 3, b_2 = k, c_3 = 15$$

Condition for the pair of equations to have unique solution is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$
$$\frac{1}{3} \neq \frac{2}{k}$$
$$k \neq 6$$

k can have any value except 6.

Q. 2. The larger of two supplementary angles exceeds the smaller by 18°. Find the angles. [CBSE OD, Set 1, 2019]

Ans. Let two angles A and B are supplementary.

$\therefore$ $$A + B = 180° \quad ...(i)$$

Given, $$A = B + 18°$$

On putting $A = B + 18°$ in equation (i), we get

$$B + 18° + B = 180°$$
$$2B + 18° = 180°$$
$$2B = 162°$$
$$B = 81°$$
$$A = B + 18°$$
$$\Rightarrow \quad A = 99°$$

Q. 3. Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present? [CBSE OD, Set 1, 2019]

Ans. Let age of Sumit be x years and age of his son be y years. Then, according to question we have,

$$x = 3y \qquad ...(i)$$

Five years later,

$$x + 5 = 2\frac{1}{2}(y + 5) \qquad ...(ii)$$

On putting $x = 3y$ in equation (ii)

$$3y + 5 = \frac{5}{2}(y + 5)$$

$$3y + 5 = \frac{5y}{2} + \frac{25}{2}$$

$$\frac{y}{2} = \frac{15}{2}$$

$$y = 15 \text{ years}$$

Then, present age of sumit is

$$3 \times y = 3 \times 15$$

$$= 45 \text{ years}$$

Q. 4. For what value of k, will the following pair of equations have infinitely many solutions:

$2x + 3y = 7$ and $(k + 2)x - 3(1 - k)y = 5k + 1$

[CBSE OD, Set 2, 2019]

Ans. Given, the system of equations is
$2x + 3y = 7$ and $(k + 2)x - 3(1 - k)y = 5k + 1$
Since, the given system of equations have infinitely many solutions.

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{k+2} = \frac{3}{-3(1-k)} = \frac{-7}{-(5k+1)}$$

$$\frac{2}{k+2} = \frac{3}{-3(1-k)} \text{ and } \frac{3}{-3(1-k)} = \frac{7}{(5k+1)}$$

$$-6(1-k) = 3k + 6 \text{ and } 3(5k+1) = -21(1-k)$$

$$-6 + 6k = 3k + 6 \qquad 15k + 3 = -21 + 21k$$

$$3k = 12 \qquad 6k = 24$$

$$k = 4 \qquad k = 4$$

Hence, the given system of equations has infinitely many solutions when $k = 4$.

Q. 5. Find c if the system of equations $cx + 3y + (3 - c) = 0$, $12x + cy - c = 0$ has infinitely many solutions?

[CBSE Delhi, Set 1, 2019]

Ans. The given equations are

$$cx + 3y + (3 - c) = 0$$

and $\quad 12x + cy - c = 0$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or $\quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\frac{c}{12} = \frac{3}{c}$		or	$\frac{3}{c} = \frac{3-c}{-c}$
$\Rightarrow \quad c^2 = 36$		$\Rightarrow$	$-3c = 3c - c^2$
$\Rightarrow \quad c = \pm 6$		$\Rightarrow$	$-6c = -c^2$
		$\Rightarrow$	$c^2 - 6c = 0$
		$\Rightarrow$	$c(c - 6) = 0$
		$\Rightarrow$	$c = 0$ or $c = 6$

So, from both the above cases

$$c = 6$$

Q. 6. Find the value of k for which the following pair of linear equations have infinitely many solutions.

$2x + 3y = 7$, $(k + 1)x + (2k - 1)y = 4k + 1$

[CBSE Delhi, Set 2, 2019]

Ans. Given,

$2x + 3y = 7$ and $(k + 1)x + (2k - 1)y = 4k + 1$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)}$$

$$\Rightarrow \quad 2(2k - 1) = 3(k + 1)$$

$$4k - 2 = 3k + 3$$

$$k = 5$$

or $\quad 3(4k + 1) = 7(2k - 1)$

$$\Rightarrow \quad k = 5$$

Hence, $\quad k = 5.$

Q. 7. In fig. 1, $ABCD$ is a rectangle. Find the values of x and y. [CBSE 2018]

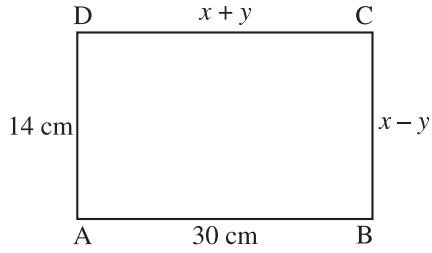


Figure 1

Ans. Given, $ABCD$ is a rectangle.

$\therefore \quad AB = CD$

$\Rightarrow \quad 30 = x + y$

or $\quad x + y = 30 \quad ...(i)$

Similarly, $\quad AD = BC$

$\Rightarrow \quad 14 = x - y$

or $\quad x - y = 14 \quad ...(ii)$

On adding eqs. (i) and (ii), we get

$2x = 44$

$\Rightarrow \quad x = 22$

Putting the value of x in eq. (i), we get

$22 + y = 30$

$\Rightarrow \quad y = 30 - 22$

$\Rightarrow \quad y = 8$

So, $x = 22$, $y = 8$.

Q. 8. Given the linear equation $x - 2y - 6 = 0$, write another linear equation in these two variables, such that the geometrical representation of the pair so formed is:
(i) coincident lines
(ii) intersection lines

[CBSE Term 1, 2016]

Ans. **(i)** Given, $x - 2y - 6 = 0$

For line to be coincident

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus one possible option will be

$2x - 4y - 12 = 0$

Here, $\quad a_1 = 1, b_1 = -2, c_1 = -6$

$a_2 = 2, b_2 = -4, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2};$$

$$\frac{c_1}{c_2} = \frac{-6}{-12} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, it is showing coincident lines.

(ii) Given, $\quad x - 2y - 6 = 0$

For intersecting lines

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Thus, one possible option will be,

$2x - 7y - 13 = 0$

Here, $\quad a_1 = 1, b_1 = -2, c_1 = -6$

$a_2 = 2, b_2 = -7, c_2 = -13$

Here, $\quad \frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{-2}{-7} = \frac{2}{7}$

$$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, it is representing intersecting lines.

Q. 9. Find whether the following pair of linear equations is consistent or inconsistent:

$$3x + 2y = 8$$

$$6x - 4y = 9$$

[CBSE Term 1, Set 1, 2015]

Ans. Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$

$$\frac{1}{2} \neq \frac{-1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which will give a unique solution.

Hence, given pair of linear equations is consistent.

Short Answer Type Questions-II (3 marks each)

Q. 1. If $2x + y = 23$ and $4x - y = 19$, find the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$.

[CBSE OD, Set 1, 2020]

Ans. Given : $\quad 2x + y = 23 \quad ...(i)$

and $\quad 4x - y = 19 \quad ...(ii)$

From equation (i), we have

$y = 23 - 2x$

Putting this value in equation (ii), we get

$4x - (23 - 2x) = 19$

$\Rightarrow \quad 4x - 23 + 2x = 19$

$\Rightarrow \quad 6x = 19 + 23$

$\Rightarrow \quad 6x = 42$

$x = 7$

So, $y = 23 - 2 \times 7 = 23 - 14 = 9$

So, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31$

And $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = -\frac{5}{7}$ **Ans.**

Q. 2. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$. [CBSE Delhi, Set 1, 2020]

Ans. Given the equations,

$2y - x = 8$

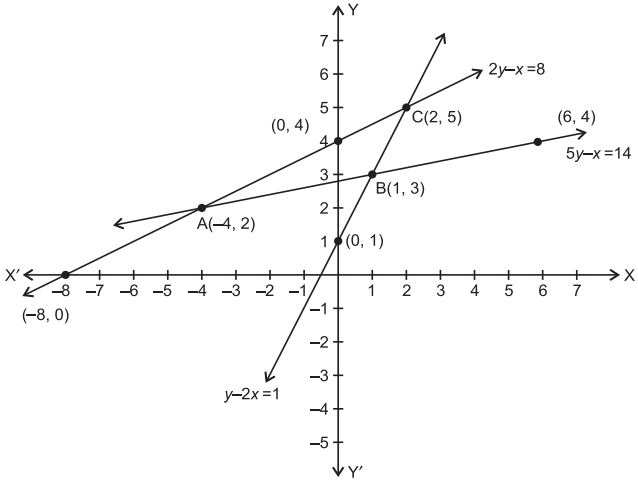
x	0	-8	2
y	4	0	5

$5y - x = 14$

x	1	6	-4
y	3	4	2

$y - 2x = 1$

x	1	0	2
y	3	1	5



Therefore, the coordinates of the vertices of the triangle are A(– 4, 2), B(1, 3) and C(2, 5).

Ans.

Q. 3. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father. [CBSE Delhi, Set 1, 2019]

Ans. Let the present age of father be x years and sum of ages of his two children be y years

According to question,

$$x = 3y \quad ...(i)$$

After 5 years,

Father's age $= (x + 5)$ years

Sum of ages of two children $= (y + 5 + 5)$ years

$= (y + 10)$ years

In 2nd case

According to question,

$$x + 5 = 2(y + 10)$$

or $x + 5 = 2y + 20$

or $x - 2y = 15$

or $3y - 2y = 15$ [Using equation (i)]

$y = 15$

Now from equation (i),

$x = 3y$ (Put $y = 15$)

or $x = 3 \times 15$

$x = 45$

So, Present age of father = 45 years.

Q. 4. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

[CBSE Delhi, Set 1, 2019]

Ans. Let the fraction be $\frac{x}{y}$

According to question,

$$\frac{x-2}{y} = \frac{1}{3}$$

or $3(x - 2) = y$

or $3x - y = 6 \quad ...(i)$

again, According to question,

$$\frac{x}{y-1} = \frac{1}{2}$$

or $2x = y - 1$

or $2x - y = -1 \quad ...(ii)$

On solving equations (i) and (ii), we get

$x = 7, y = 15$

$\therefore$ The required fraction is $\frac{7}{15}$.

Q. 5. Solve the following pair of equations by reducing them to a pair of linear equations:

$$\frac{1}{x} - \frac{4}{y} = 2$$

$$\frac{1}{x} + \frac{3}{y} = 9$$

[CBSE Term 1, 2016]

Ans. Given, $\frac{1}{x}-\frac{4}{y}=2$

and $\frac{1}{x}+\frac{3}{y}=9$

Let $\frac{1}{x}=u,\ \frac{1}{y}=v$

So, $u-4v=2$ ...(i)

$u+3v=9$ ...(ii)

On solving eqs. (i) and eq. (ii),

$u-4v=2$

$u+3v=9$

$-\quad -\quad -$

$-7v=-7$

$\Rightarrow \quad v=1$

Putting the value of v in eq. (i),

$\Rightarrow \quad u-4\times 1=2$

$\Rightarrow \quad u=6$

So, $v=1 \Rightarrow \frac{1}{y}=1,\ y=1$

and $u=6 \Rightarrow \frac{1}{x}=6,\ x=\frac{1}{6}$

Hence, $x=\frac{1}{6}$ and $y=1$

Q. 6. Solve by elimination:

$$3x-y=7$$

$$2x+5y+1=0$$

[CBSE Term 1, 2015]

Ans. $3x-y=7$ ...(i)

$2x+5y=-1$ ...(ii)

Multiplying equation (i) by 5 and solving it with equation (ii), we get

$2x+5y=-1$

$15x-5y=35$

$17x=34$ [On adding]

$\Rightarrow \quad x=\frac{34}{17}=2$

Putting the value of x in (i), we have

$3(2)-y=7$

$\Rightarrow \quad 6-y=7$

$\Rightarrow \quad y=-1$

$\therefore\ x=2, y=-1$

Q. 7. The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.

[CBSE Term 1, Set 1, 2015]

Ans. Let unit digit = x

Tens digit = y

So, original number = unit digit + 10 × tens digit

$= x+10y$

According to question,

Sum of digits = 8

So, $x+y=8$

On reversing the digits, unit digit = y

Tens digit = x

so, New number = $10x+y$

According to question,

Difference = 18

$\Rightarrow \quad x+10y-(10x+y)=18$

$\Rightarrow \quad x+10y-10x-y=18$

$\Rightarrow \quad 9y-9x=18$

$\Rightarrow \quad y-x=2$ ...(ii)

By adding eq. (i) and (ii),

$2y=10$

$\Rightarrow \quad y=\frac{10}{2} \Rightarrow y=5$

Put the value of y in eq. (i),

$\Rightarrow \quad x+5=8$

$\Rightarrow \quad x=3$

$\therefore$ Original number = $10y+x$

$= 10\times 5+3$

$= 50+3$

$= 53$

Long Answer Type Questions (4 marks each)

Q. 1. Draw graph of following pair of linear equations:

$$y=2(x-1)$$

$$4x+y=4$$

Also write the coordinate of the points where these lines meets x-axis and y-axis.

[CBSE Term 1, 2016]

Ans. $y=2(x-1)$

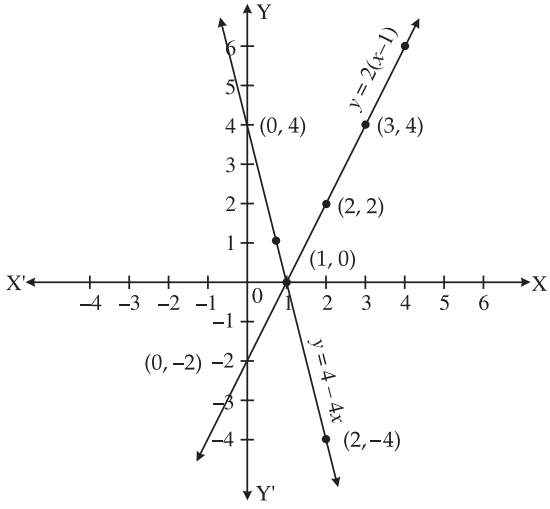
So,

x	1	2	3	4
y	0	2	4	6

And for, $4x+y=4$

$\Rightarrow \quad y=4-4x$

x	1	2	1/2
y	0	– 4	2



Co-ordinate of point where lines meets coordinates axes:

Line $y = 2(x - 1)$: x-axis = (1, 0)

and y-axis = (0, – 2)

Line $4x + y = 4$: x-axis = (1, 0)

y-axis = (0, 4)

Q. 2. Draw the graph of the following pair of linear equations:

$x + 3y = 6$ and $2x - 3y = 2$

Find the ratio of the areas of the two triangles formed by first line, $x = 0$, $y = 0$ and second line, $x = 0$, $y = 0$.

[CBSE Term 1, Set 1, 2015]

Ans.

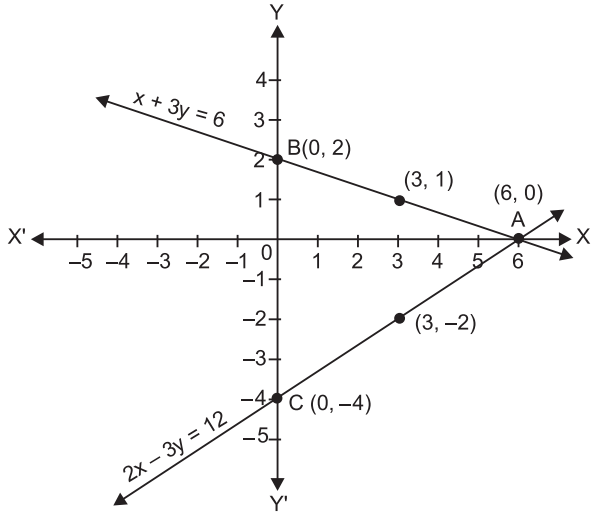
	First Line	**Second Line**
	$x + 3y = 6$	$2x - 3y = 12$
$\Rightarrow$	$x = 6 - 3y$	$2x = 12 + 3y$
$\Rightarrow$		$x = \dfrac{12 + 3y}{2}$

x	6	3	0
y	0	1	2

(6, 0), (3, 1), (0, 2)

x	6	3	0
y	0	–2	–4

(6, 0), (3, –2), (0, – 4)



Area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{corresponding altitude}$$

$$\therefore \quad \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta AOC} = \frac{1/2 \times OA \times OB}{1/2 \times OA \times OC}$$

$$\Rightarrow \quad \frac{OB}{OC} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ Required ratio = 1 : 2