Chapter 1

Limit State Method

CHAPTER HIGHLIGHTS

- *Introduction*
- *Methods of design of reinforced concrete*
- *Characteristic load*
- *Limit state of serviceability*

Introduction

In this chapter, the various methods used for design of reinforced concrete structures are discussed. The principles of limit state of design and serviceability are also discussed.

Methods of Design of Reinforced Concrete

- The aim of design is to decide the size of member and amount of reinforcement required, so that the structure will satisfactorily perform during its lifetime at minimum cost.
- The following three methods were developed for the design of reinforced structures.
  1. Working stress method
  2. Ultimate load method
  3. Limit state method

Working Stress Method (WSM)

- In this method, the design is based on elastic theory. Hence, the reinforced concrete is assumed as an elastic material.
- In working stress method, the structural members are designed for working loads and design stresses or permissible stresses in material are obtained by dividing the ultimate stress by a factor called 'factor of safety'.
- For concrete, a factor of safety of 3 is used. For steel, it is 1.78.
- In working stress method, the failure criterion is the stress.
- This method is simple and reasonably reliable.

The drawbacks of this method are as follows:

- This method gives uneconomical sections.
- Stress strain curve for concrete is assumed as linear, which is not true.
- Factor of safety does not predict the true margin of safety.
- The failure criteria assumed is stress, but strain criteria is the reliable.

Ultimate Load Method (ULM) or Load Factor Method (LFM)

- In ultimate load method, the structural members are designed for ultimate loads which are obtained by multiplying the working loads with a factor called 'load factor'.
- This method uses the actual stress—strain curve of concrete.
- In ultimate load method, the failure criterion is based on ultimate strain.
- This method gives economical sections and also true margin of safety.

- The only drawback of this method is excessive cracking and deformation, fails to satisfy the serviceability and durability criteria.

Limit State Method (LSM)

- In limit state method, the structural elements are designed for ultimate load and checked for serviceability (i.e., deflections, cracking, etc.) at working loads, so that the structure is fit for using throughout its life period.
- The acceptable limit for safety against strength and serviceability requirements before failure occurs is called 'limit state'.
- The loads and strength of materials, i.e., the characteristic values are estimated by probabilistic approach.
- The design loads and design strengths are obtained from characteristic values through the use of partial safety factors.
- Two important limit states to be considered in the design are limit state of collapse and limit state of serviceability.

Limit State of Collapse

A structure is said to have collapsed if the material ruptures at one or more critical sections or loss of overall stability due to buckling or overturning.

This limit state may correspond to:

1. Flexure
2. Shear
3. Compression
4. Torsion

Limit State of Serviceability

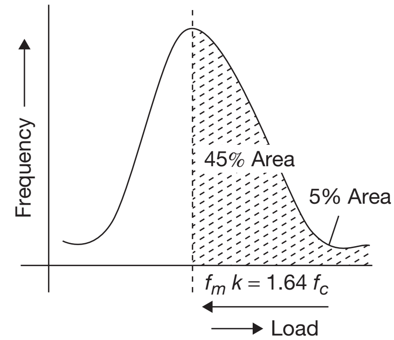
This limit state relates to the performance of structure at working loads.

This limit state may correspond to:

1. Deflection
2. Cracking
3. Other limit states (i.e., vibration, fire-resistance, durability)

Characteristic Load

- The maximum working load that the structure has to withstand and, for which it has to be designed is called 'characteristic load'.
- The characteristic loads are calculated based on statistical analysis and follows normal distribution.
- As per the code, the characteristic load is defined as the value of load which has 95% of probability of not being exceeded during the life of structure.
- Normal distribution means, distribution symmetric about mean value are as follows:



- Characteristic load $(f_c) = f_m + k\,S$

$$\boxed{f_c = f_m + 1.64S}$$

Where

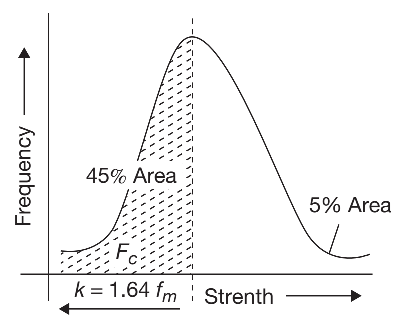
$$f_m : \text{Mean load} = \frac{\sum F_i}{n}$$

$$S : \text{Standard deviation} = \sqrt{\frac{\Sigma(f_m - f_i)^2}{n-1}}$$

- As statistical data of loads is not available, dead loads given in IS:875 (part-1), imposed loads given in IS:875 (part-2), wind loads given in IS:875 (part-3), snow loads given in IS:875 (part-4) and seismic forces in IS:1893 shall be assumed as characteristic load as per IS:456–2000.

Characteristic Strength of Materials

- Characteristic strength is defined as the strength below which not more than 5% of the test results are expected to fall.
- It is based on statistical analysis, and it also follows normal distribution.



Characteristic strength

Characteristic strength (F_c) = mean strength (f_m) – 1. 64S

$$\boxed{F_c = F_m - 1.64S}$$

Where

F_m: Mean strength

S : Standard deviation

- For concrete, characteristic strength is compressive strength of 150 mm cubes of 28 days in N/mm^2, below which not more than 5% of test results are expected to fall.
- Concrete grades are specified based on this strength.
- For steel, the minimum yield strength or 0.2% proof strength is taken as the characteristic strength of steel.

Design Values and Partial Safety Factors

Material

The design strength of materials is obtained by dividing the characteristic strength by a factor called 'partial safety factor'.

$$\text{Design strength}(f_d) = \frac{\text{Characteristic strength } (f)}{\text{Partial safety factor}}$$

The values of partial safety factors recommended by IS:456–2000 are given in the following table.

Partial Safety Factors for Material Strengths, γ_m (as per IS:456–2000)

Material	Limit State of Collapse	Limit State of Serviceability
Steel	1.15	1.0
Concrete	1.5	1.0

Loads

Design loads or factored loads, for which the structure is to be designed, are obtained by multiplying the characteristic load with a factor called 'partial safety factor'.

Design load (f_d) = Characteristic load (f_c) × Partial safety factor(γ_f)

The values of partial safety factors recommended by IS:456–2000 are given in the following table.

Partial Safety Factor for Loads, γ_f

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL	DL	LL	WL
DL + LL	1.5	1.5	-	1.0	1.0	-
DL + WL	1.5 or 0.9*	-	1.5	1.0	-	1.0
DL + LL + WL	1.2	1.2	1.2	1.0	0.8	0.8

- This value is to be used when stability against overturning or stress reversal is critical.
- While considering earthquake load (EL), substitute EL for WL.
- If EL and WL, both are acting as structure, consider maximum of two.

SOLVED EXAMPLE

Example 1

Un-factored maximum bending moments at a section of a reinforced concrete beam resulting from a frame analysis are 50, 80, 120 and 180 kN-m under dead, live, wind and earthquake loads. The design moment (kN-m) as per IS:456–2000, for limit state of collapse (in flexure), is:

(A) 195 (B) 250

(C) 345 (D) 372

[GATE, 2008]

Solution

Given:

Dead Load (DL) = 50 kN-m

Live Load (LL) = 80 kN-m

Wind Load (WL) = 120 kN-m

Earthquake Load (EL) = 180 kW-m

If both wind load and earthquake load are acting considering maximum of two. Therefore, consider earthquake load.

Design moment (kN-m) = (characteristic load) × γf

γ_f: Depends on load combination .

For DL + LL:

$$f_d = 1.5\text{DL} + 1.5\text{LL}$$
$$= 1.5(50) + 1.5(80)$$
$$f_d = 195 \text{ kN-m} \quad (1)$$

For DL + EL:

$$f_d = 1.5\text{DL} + 1.5\text{EL}$$
$$= 1.5(50) + 1.5(180)$$
$$f_d = 345 \text{ kN-m} \quad (2)$$

For DL + LL + EL:

$$f_d = (1.2)(\text{DL}) + 1.2)(\text{LL}) + (1.2)(\text{EL})$$
$$= (1.2)(50) + (1.2)(80) + (1.2)(180)$$
$$f_d = 372 \text{ kN-m} \quad (3)$$

∴ Design moment (f_d) is maximum of (1), (2), (3).

∴ f_d = 372 kN-m.

Hence, the correct answer is Option (D).

Limit State of Serviceability

Deflection Limits as per IS:456–2000

The following criteria are adopted in IS:456, for ensuring proper performance of beams and slabs.

- The final deflection due to all loads (including the effects of temperature, creep and shrinkage) should not exceed span/250. This limitation is to control the cracks.
- The deflection occurring after the construction of finishes and partitions (including the effects of temperature, creep and shrinkage) should not exceed span/350 or 20 mm whichever is less. This limit is intended to avoid damage of partitions and finishes.

Deflection in RC members may be divided into two types. These can be described as follows.

1. Short-term deflection
2. Long-term deflection

Short-term Deflection

Short-term or instantaneous deflection caused by the service loads may be calculated by using the elastic theory using short-term modulus of elasticity and effective moment of inertia (I_{eff}).

For example, a simply supported beam is subjected to a central point load (P), the maximum deflection at center is given by: $(\delta) = \dfrac{PL^3}{48EI}$.

To get short-term deflection, replace E by E_c and I by $I_{eff.}$ This is a theoretical method of calculating short term-deflection.

Long-term Deflection

- It mainly consists of deflection due to shrinkage and creep.
- Deflection due to shrinkage depends on support conditions, shrinkage strain, overall depth of section, percentage of tension and compression reinforcement and length of beam.
- Approximate value of shrinkage strain of concrete is 0.0003.
- Deflection due to creep depends only upon the permanent loads (perm).
- Deflection due to creep may be calculated from the following equation.

$$\boxed{a_{cc}(\text{perm}) = a_{i,cc}(\text{perm}) - a_i(\text{perm})}$$

Where

a_{cc}(perm) = Creep deflection due to permanent loads.

$a_{i,cc}$(perm) = Initial plus creep deflection due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity (E_{ce}).

E_{ce} = Long-term modulus of elasticity.

$$\boxed{E_{ce} = \frac{E_C}{1+\theta}}$$

θ = Creep coefficient depends on age of loading.

a_i(perm) = Short-term deflection due to permanent load using E_c.

Alternate Method of Ensuring Limit State Requirement of Deflection

Code provisions

1. Basic values of span to effective depth ratios for rectangular beams and slabs:

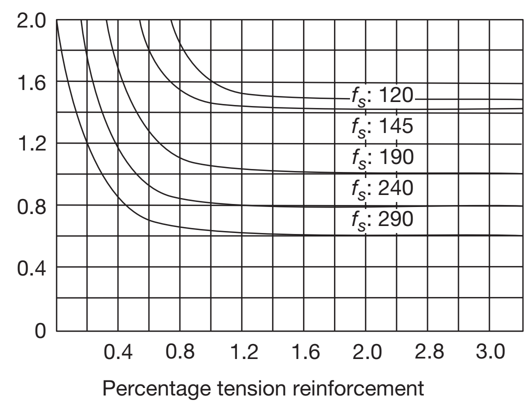
Support Conditions	l_{eff}/d	
	Span ≯ 10 m	Span > 10 m
Cantilever	7	Deflection calculation should be made
Simply supported	20	(20 × 10)/span
Continuous	26	(26 × 10)/span

2. For two way slabs, the basic values of l/D are given as under for spans up to 3.5 m and for maximum live load up to 3 kN-m².

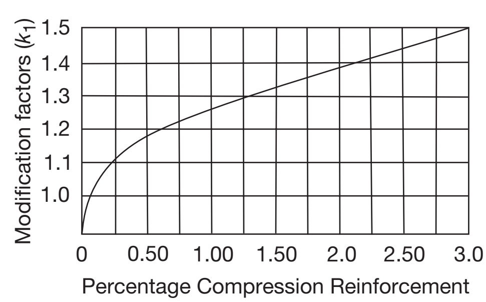
Types of Slabs	Type of Reinforcement	
	Fe250	HYSD
Simply supported slabs	35	0.8 × 35
Continuous slabs	40	0.8 × 40

3. Depending upon the area and the stress in tension or compression reinforcement, the above values are to be modified with some factors called ‘modification factor’.

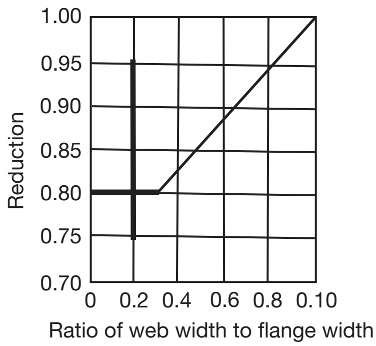
The graphs for various modification factors (K_1, K_2, K_3) are shown below.



Modification factor for tension reinforcement (K_1)



Modification factor for compression reinforcement (K_2)



Modification factor for flanged beams (K_3)

NOTES

1. The value of modification factor (K_1) decreases with increase in stress for the same percentage tension reinforcement. Thereafter, the deflection will increase with increase in stress.
2. As the value of percentage compression reinforcement increases, the modification factor (K_2) value increases and, thus, decreases the deflection.
3. In case of flanged beams, the modification factor (K_3) is less compared to value of modification factor for rectangular beams. Thus, from deflection point of view, flanged beams are more effective compared to rectangular beams.
4. However, from overall point of view, flanged beams are more efficient compared to rectangular beams.

Limit State of Serviceability—Cracking

IS:456–2000 specifies the following acceptable limits of crack widths keeping in mind that cracking should not affect the appearance or durability of the structure.

- 0.3 mm in members, where cracking is not harmful to the durability of the strcture.
- 0.2 mm in members continuously exposed to moisture or in contact with soil or water.
- 0.1 mm in members exposed to aggressive environment.

Important Points

- Concrete and steel exhibits reduced strength after being subjected to high temperature.
- Durability depends on condition of exposure, quality of concrete, cover to steel reinforcement and width of cracks.

Exercises

1. The factored loads at the limit state of collapse for DL + LL, DL + WL, and DL + LL + WL combinations, according to IS:456–2000 are respectively
   (A) 1.5DL + 1.5LL, 1.2DL + 1.2WL, 1.5DL + 1.5LL + 1.5WL
   (B) (0.9 or 1.5)DL + 1.5LL, 1.5DL + 1.5WL, 1.2DL + 1.2LL + 1.2WL
   (C) 1.2DL + 1.2LL, 1.5DL + 1.5WL, 1.5DL + 1.5LL + 1.5WL
   (D) 1.5DL + 1.5LL, (0.9 or 1.5)DL + 1.5WL, 1.2DL + 1.2LL + 1.2WL
2. The characteristic strength of concrete is defined as that compressive strength below which not more than
   (A) 10% of results fall
   (B) 5% of results fall
   (C) 2% of results fall
   (D) None of these
3. For avoiding the limit state of the collapse, the safety of RC structures is checked for appropriate combination of Dead Load (DL), imposed load or live load (IL), wind load (WL) and earthquake load (EL). Which of the following load combination is not considered?
   (A) 0.9DL + 1.5WL
   (B) 1.5DL + 1.5WL
   (C) 1.5DL + 1.5WL + 1.5EL
   (D) 1.2DL + 1.2IL + 1.2WL
4. The partial factor of safety for concrete as per IS:456–2000 is
   (A) 1.50 (B) 1.15
   (C) 0.87 (D) 0.446
5. In a random sampling procedure for cube strength of concrete, one sample consists of X number of specimens. These specimens are tested at 28 days and average strength of these X specimens is considered as test result of the sample, provided the individual variation in the strength of specimens is not more than Y percent of the average strength. The value of X and Y as per IS:456–2000 are

(A) 4 and 10 respectively
(B) 3 and 10 respectively
(C) 4 and 15 respectively
(D) 3 and 15 respectively

6. If the characteristic strength of concrete f_{ck} is defined as the strength below which not more than 50% of the test result are expected to fall, the expression for f_{ck} in terms of mean strength f_m and standard deviation s would be

(A) $f_m - 0.16455$ (B) $f_m - 1.645S$
(C) f_m (D) $f_m + 1.645S$

7. The stress-stain curve of the concrete as per IS:456 is
(A) a perfect straight line upto failure.
(B) straight line upto 0.002 strain value and then parabolic upto failure.
(C) parabolic upto 0.002 strain value and then uniform upto failure.
(D) linear upto 0.002 strain and uniform upto failure.

8. The characteristic strength of concrete is
(A) higher than the average cube strength.
(B) lower than the average cube strength.
(C) the same as the average cube strength.
(D) higher than 90% of the average cube strength.

9. The span to depth ratio limit is specified in IS:456–2000 for the reinforced concrete beams, in order to ensure that the
(A) tensile crack width is below a limit.
(B) shear failure is avoided.
(C) stress in the tension reinforcement is less than the allowable value.
(D) deflection of the beam is below a limiting value.

10. The final deflection due to all including effects of temperature, creep and shrinkage measured from a cast level of the supports of floors, roofs and all other horizontal members of reinforced concrete should not normally exceed
(A) span/350
(B) span/250
(C) span/350 or 20 mm whichever is less
(D) 5/384 of span

11. In the limit state design of serviceability the deflection after erection of partitions and application of finishes is restricted to
(A) span/350 (B) span/250
(C) span/325 (D) span/150

12. Which one of the following set of values gives the minimum clear cover (in mm) for the main reinforcements in the slab, beam, column and footing respectively, according to IS:456–2000?
(A) 20, 25, 40, 50 (B) 5, 15, 25, 50
(C) 15, 25, 40, 75 (D) None of these

13. IS 456–2000 recommends providing certain minimum steel in a RCC beam
(A) to ensure compression failure.
(B) to avoid rupture of steel in case a flexural failure occurs.
(C) to hold the stirrup steel in position.
(D) to provide enough ductility to the beam.

14. The working stress method of design specifies the value of modular ratio, $m = \frac{280}{(3\sigma_{cbc})}$, where σ_{cbc} is the allowable stress in bending compression in concrete. To what extent does the above value of 'm' make any allowance for the creep of concrete?
(A) No compensation
(B) Full compensation
(C) Partial compensation
(D) The two are unrelated

15. Minimum grade of concrete for structural purpose
(A) M15 (B) M20
(C) M25 (D) M30

16. Minimum percentage of steel in both directions in a slab when HYSD bars are used is _____% of (bD).
(A) 0.1% (B) 0.15%
(C) 0.12% (D) 0.2%

17. In the limit state design of serviceability the deflection after erection of partitions and erection of finishes is limited to
(A) span/250 (B) span/325
(C) span/350 (D) span/150

18. Calcium lignosulphate is an example of
(A) retarder (B) accelerator
(C) dispersal agent (D) hardness agent

19. Match List I with List II and select the correct answer from the codes given below.

	List I (Admixtures)		List II (Example)
P.	Retarder	1.	Volcanic Tuff
Q.	Accelerator	2.	Natural wood resins
R.	Pozzolona	3.	Calcium sulphate
S.	Air entraining agent	4.	Calcium chloride

Codes:

	P	Q	R	S		P	Q	R	S
(A)	1	2	3	4	(B)	1	2	4	3
(C)	3	4	1	2	(D)	2	1	3	4

20. Consider the following statements regarding the Portland Puzzolana Cement:
I. It produces less heat of hydration.
II. Addition of Pozzolano does not contribute to the strength at early stages.
III. Strength of this cement at any time is always less than the strength of the Portland cement.
IV. It is particularly useful in marine and hydraulic construction.
(A) I, II, III are correct
(B) II, III, IV are correct

(C) I, III, IV are correct
(D) I, II, IV are correct

21. Unfactored maximum bending moments at a section of a reinforced concrete beam resulting from a frame analysis are 330, 420 and 150 kN-m under dead, live and wind loads respectively. The design moment (kN-m) as per IS:456: 2000 for the limit state of collapse is
(A) 720 kN-m (B) 840 kN-m
(C) 1125 kN-m (D) 1530 kN-m

22. In under reinforced concrete beam, which of the following statements are correct?
I. Actual depth of neutral axis is less than the critical depth of neutral axis.
II. Concrete reaches ultimate stress prior to steel reaching the ultimate stress.
III. Moment of resistance is less than that of balanced section.
IV. Lever arm of resisting couple is less than the balanced section.
(A) I and II only (B) I and III only
(C) II, III and IV (D) I, II and IV

23. Maximum depth of Neutral axis, $x_{u,\max}$ depends on ______.
(A) grade of concrete
(B) grade of steel
(C) grade of concrete and steel
(D) independent of material grades

24. For an axially loaded short column, the maximum compressive strain in concrete is _______.
(A) 0.0035
(B) 0.002
(C) $0.002 + \frac{f_y}{1.15E_s}$
(D) 0.035

25. In limit state method of design of concrete structures; stress is assumed to be linear up to ______.
(A) failure
(B) proportionality limit
(C) elastic limit
(D) None of these

26. For a given concrete of M20 mix and creep coefficient of 1.6; Consider the following statements.
I. Flexural strength of the concrete is 3.13 N/mm^2
II. Short term modulus of elasticity of the concrete is 22.36 kN/mm^2
III. Long term modulus of elasticity of the concrete is 8.6 kN/mm^2

27. Un-factored maximum bending moments at a section of *RC* beam under dead, live wind and earthquake loads are given as 60, 90, 130 and 190 kN-m respectively. Then find the design moment (in kN-m) as per IS:456–2000 for the limit state of collapse? ______.
(A) 308 (B) 408
(C) 508 (D) 608

28. Factor of safety (FOS) adopted for concrete and steel as per working stress method of design is ________.
(A) 3 and 1.15 (B) 1.5 and 1.15
(C) 1.5 and 1.78 (D) 3 and 1.78

29. The main reinforcement of a *RC* slab consisting of 10 mm bars at 10 cm spacing. It is desired to replace 10 mm bars by 12 mm bars, then the spacing of 12 mm bars in mm would be _______.
(A) 123 mm (B) 143 mm
(C) 163 mm (D) 183 mm

30. Reinforced concrete slab is 75 mm thick. The maximum size of reinforcement bar that can be used is of
(A) 6 mm diameter (B) 8 mm diameter
(C) 10 mm diameter (D) 12 mm diameter

Previous Years' Questions

1. Unfactored maximum bending moments at a section of a reinforced concrete beam resulting from a frame analysis are 50, 80, 120 and 180 kN-m under Dead, Live, Wind and Earthquake loads respectively. The design moment (kN-m) as per IS:456–2000 for the limit state of collapse (flexure) is **[GATE, 2008]**
(A) 195 (B) 250
(C) 345 (D) 372

2. For limit state of collapse, the partial safety factors recommended by IS:456–2000 for estimating the design strength of concrete and reinforcing steel are respectively **[GATE, 2009]**
(A) 1.15 and 1.5 (B) 1.0 and 1.0
(C) 1.5 and 1.15 (D) 1.5 and 1.0

3. As per IS:456–2000, in the limit state design of a flexural member, the strain in reinforcing bars under tension at ultimate state should not be less than **[GATE, 2012]**
(A) $\frac{f}{E}$ (B) $\frac{f}{E} + 0.002$
(C) $\frac{f}{1.15E}$ (D) $\frac{f}{1.15E} + 0.002$

4. The creep strains are **[GATE, 2013]**
(A) caused due to dead loads only
(B) caused due to live loads only
(C) caused due to cyclic loads only
(D) independent of loads

5. The target mean strength f_{cm} for concrete mix design obtained from the characteristic strength f_{ck} and standard deviation σ, as defined in IS:456–2000 is **[GATE, 2014]**
   (A) $f_{ck} + 1.35\sigma$ (B) $f_{ck} + 1.45\sigma$
   (C) $f_{ck} + 1.55\sigma$ (D) $f_{ck} + 1.65\sigma$
6. The modulus of elasticity, $E = 5000\sqrt{fck}$ where f_{ck} is the characteristic compressive strength of concrete, specified in IS:456–2000 is based on **[GATE, 2014]**
   (A) tangent modulus
   (B) initial tangent modulus
   (C) secant modulus
   (D) chord modulus
7. According to the concept of Limit State Design as per IS:456–2000, the probability of failure of a structure is ________. **[GATE, 2015]**
8. For M25 concrete with creep coefficient of 1.5, the long-term static modulus of elasticity (expressed in MPa) as per the provisions of IS:456–2000 is ________. **[GATE, 2016]**

Answer Keys

Exercises

1. D	**2.** B	**3.** C	**4.** A	**5.** D	**6.** C	**7.** C	**8.** B	**9.** B	**10.** B
11. A	**12.** A	**13.** D	**14.** C	**15.** B	**16.** C	**17.** C	**18.** A	**19.** C	**20.** D
21. C	**22.** B	**23.** B	**24.** B	**25.** B	**26.** D	**27.** B	**28.** D	**29.** B	**30.** B

Previous Years' Questions

1. D	**2.** C	**3.** D	**4.** A	**5.** D	**6.** B	**7.** 0.09 to 0.10	**8.** 10000