Chapter 8
Electromagnetic Waves

Displacement Current

The current due to changing electric field or electric flux is called called displacement current or Maxwell's displacement current.

$$\text{Electric flux} \qquad \phi_E = \frac{q}{\varepsilon_0}$$

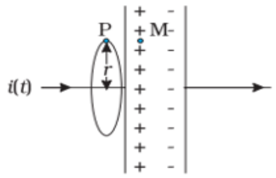
$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt}$$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0} i_d$$

$$\text{Displacement current} \qquad \mathbf{i_d = \varepsilon_0 \frac{d\phi_E}{dt}}$$

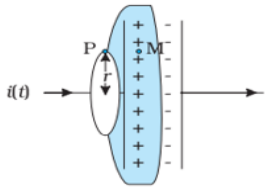
Inconsistency in the Ampere's circuital law and Displacement current

While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time-varying current, Maxwell noticed an inconsistency in the Ampere's circuital law.



To find the magnetic field at a point such as P, in a region outside the parallel plate capacitor, consider a plane circular loop of radius r. A current i(t) passes through this surface.

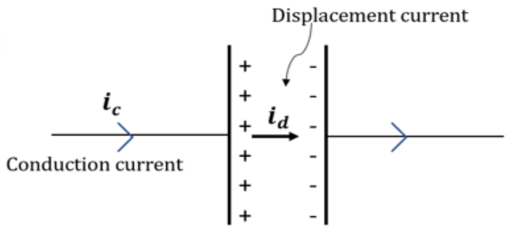
$$\oint B \cdot dl = \mu_0 i(t)$$



To find the magneticfield at the same point P, now consider a pot-shaped surface passing through the interior between the capacitor plates with the same rim as in first case. No current passes through this surface.

$$\oint B \cdot dl = 0 \qquad \text{(This is a contradiction)}$$

To remove this to inconsistency, Maxwell suggested the existence of an additional current , called, the displacement current .



Ampere-Maxwell law

According to Maxwell the source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field.

The total current i is the sum of the conduction current (i_c) and displacement current (i_d)

$$\mathbf{i = i_c + i_d}$$

$$i = i_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

Ampere's theorem become

$$\oint B \cdot dl = \mu_0 (i_C + i_d)$$

$$\oint B \cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

$$\boldsymbol{\oint B \cdot dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}}$$

This is known as Ampere-Maxwell law.

MAXWELL'S EQUATIONS

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q / \varepsilon_0$	(Gauss's Law for electricity)
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$	(Gauss's Law for magnetism)
3. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\Phi_B}{dt}$	(Faraday's Law)
4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	(Ampere – Maxwell Law)

Electromagnetic waves

Sources of Electromagnetic Waves

- A stationary charge produces only electrostatic fields.
- Charges in uniform motion (steady currents) can produce magnetic fields that, do not vary with time.
- An oscillating charge(accelerating charge) produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, as the electro magnetic wave propagates through the space.
  Thus an oscillating charge(accelerating charge) is the source of electromagnetic waves.

An electric charge oscillating harmonically with frequency ν, produces electromagnetic waves of the same frequency ν.

- The experimental demonstration of electromagnetic wave in the radio wave region was done by **Hertz** in1887.
- Seven years after Hertz, **Jagdish Chandra Bose**, succeeded in producing and observing electromagnetic waves of much shorter wavelength (25 mm to 5 mm).
- At around the same time**, Guglielmo Marconi** succeeded in transmitting electromagnetic waves over distances of many kilometres. Marconi's experiment marks the beginning of the field of communication using electromagnetic waves.

Nature of Electromagnetic Waves

1) In an e.m waves are transverse waves in which the electric and magnetic fields are perpendicular to each other, and also to the direction of propagation.

2) The speed of e.m.wave in vacuum is,

$$\mathbf{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

3)The speed of of electromagnetic waves in a material medium is

$$\mathbf{c = \frac{1}{\sqrt{\mu \varepsilon}}}$$

4) The electric and the magnetic fields in an electromagnetic wave are related as

$$\mathbf{\frac{E_0}{B_0} = c}$$

5) No material medium is required for the propagation of e.m.wave.

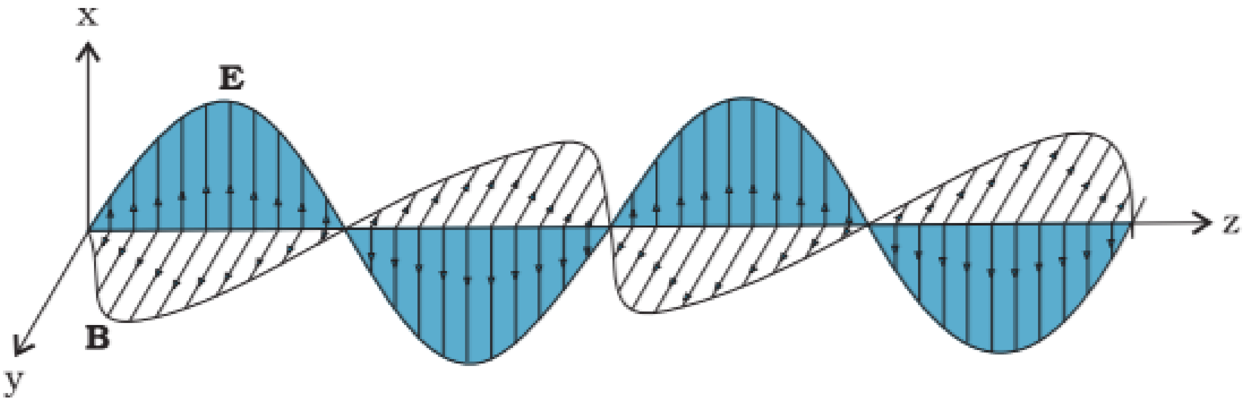
6) Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields.

7)Electromagnetic waves transport momentum as well. When these waves strike a surface, total momentum delivered to this surface is,

$$\mathbf{p = \frac{U}{c}}$$, where U is the energy

Expression for electric field and magnetic field

Consider an electromagnetic wave propagating along the z direction. Let the electric field E_x is along the x-axis and the magnetic field B_y is along the y-axis. Then



$$E_x = E_0 \sin(kz - \omega t)$$
$$B_y = B_0 \sin(kz - \omega t)$$

Here $\mathbf{k} = \frac{2\pi}{\lambda}$

k is the propagation constant

$$\omega = 2\pi v$$

ω is the angular frequency

$$\frac{\omega}{k} = \frac{2\pi v}{\frac{2\pi}{\lambda}} = v\lambda = c$$

Speed, $c = \frac{\omega}{k}$

Example

A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $E = 6.3\hat{j}$ V/m. What is B at this point?

$$\frac{E_0}{B_0} = c$$

$$B_0 = \frac{E_0}{c} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}\ \mathrm{T}$$

E is along y-direction and the wave propagates along x-axis.
Therefore, B should be in a direction perpendicular to both x- and y-axes.
i.e., B is along z-axis.

Example

The magnetic field in a plane electromagnetic wave is given by

$B_y = 2 \times 10^{-7} \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)$ T.

a) What is the wavelength and frequency of the wave?

b) Write an expression for the electric field.

(a) $$B_y = 2 \times 10^{-7} \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)$$

Comparing with general expression for magnetic field of an em wave travelling in x direction,

$$B_y = B_0 \sin(kx - \omega t)$$

$$k = 0.5\times10^3$$

$$k = \frac{2\pi}{\lambda} = 0.5\times10^3$$

$$\lambda = \frac{2\pi}{0.5\times10^3}$$

$$= 12.56 \times 10^{-3}\ \text{m}$$

$$\omega = 1.5\times10^{11}$$

$$\omega = 2\pi v = 1.5\times10^{11}$$

$$v = \frac{1.5\times10^{11}}{2\pi}$$

$$= 0.24 \text{ x} 10^{11}\ \text{Hz}$$

b) B is along y-direction and the wave propagates along x-axis. Therefore, E should be in a direction perpendicular to both x- and y-axes. **i.e., E is along z-axis.**

So expression for electric field is ,

$$E_z = E_0 \sin(k\,x - \omega t)$$

$$\frac{E_0}{B_0} = c$$

$$E_0 = B_0\ x\ c$$

$$= 2 \times 10^{-7} \text{ x } 3 \times 10^{8}$$

$$= 60\ \text{V/m}$$

$$\mathbf{E_z = 60\ \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)\ V/m}$$