Chapter 14. Statistics

Question-1

Find the mean deviation from the mean for the following data:
4, 7, 8, 9, 10, 12, 13, 17

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{80}{8} = 10$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 6 + 3 + 2 + 1 + 0 + 2 + 3 + 7 = 24$

M.D.$(\bar{x})$ = 24/8 = 3

Question-2

Find the mean deviation from the mean for the following data:
6.5, 5, 5.25, 5.5, 4.75, 4.5, 6.25, 7.75, 8.5

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{54}{9} = 6$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 0.5 + 1 + 0.75 + 0.5 + 1.25 + 1.5 + 0.25 + 1.75 + 2.5 = 10$

M.D.$(\bar{x})$ = 10/9 = 1.1

Question-3

Find the mean deviation from the mean for the following data:
38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{500}{10} = 50$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6 = 84$

M.D.$(\bar{x})$ = 84/10 = 8.4

Question-4

Find the mean deviation from the mean for the following data:
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{168}{12} = 14$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 1 + 3 + 2 + 0 + 3 + 1 + 4 + 2 + 3 + 4 + 2 + 3 = 28$

M.D.($\bar{x}$) = 28/12 = 2.33

Question-5

Find the mean deviation from the mean for the following data:
36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{500}{10} = 50$

$\sum_{i=1}^{8} |x_1 - \bar{x}| = 14 + 22 + 4 + 8 + 10 + 5 + 3 + 4 + 1 + 1 = 72$

M.D.($\bar{x}$) = 72/10 = 7.2

Question-12

Find the mean deviation from the median for the following data:
34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Solution:

No of observations n = 10

Arrangement in ascending order are as follows:

30, 34, 38, 40, 42, 44, 50, 51, 60, 66.

Median is 5th and 6th term i.e 42 and 44.

Therefore the median is (42 + 44)/2 = 43

$\sum |x_i - \text{Median}| = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 17 + 23$

Hence M.D (Median) = $|x_i - \text{Median}|/n$ = 87/10 = 8.7

Question-13

Find the mean deviation from the median for the following data:
22, 24, 30, 27, 29, 31, 25, 28, 41, 42

Solution:

No of observations n = 10

Arrangement in ascending order are as follows:

22, 24, 25, 27, 28, 29, 30, 31, 41, 42

Median is 5th and 6th term i.e 28 and 29.

Therefore the median is (28 + 29)/2 = 28.5

$\sum |x_i - \text{Median}| = 6.5 + 4.5 + 3.5 + 1.5 + 0.5 + 0.5 + 1.5 + 2.5 + 12.5 + 13.5$

Hence M.D (Median) = $|x_i - \text{Median}|/n = 47/10 = 4.7$

Question-14

Find the mean deviation from the median for the following data:
38, 70, 48, 34, 63, 42, 55, 44, 53, 47

Solution:

No of observations n = 10

Arrangement in ascending order are as follows:

34, 38, 42, 44, 47, 48, 55, 53, 63, 70,

Median is 5th and 6th term i.e 47 and 48.

Therefore the median is (47 + 48)/2 = 47.5

$\sum |x_i - \text{Median}| = 13.5 + 9.5 + 5.5 + 3.5 + 0.5 + 0.5 + 7.5 + 5.5 + 15.5 + 22.5$

Hence M.D (Median) = $|x_i - \text{Median}|/n = 84/10 = 8.4$

Question-17

Find the arithmetic mean of the series 1, 2, 2^2, ..., 2^{n-1}.

Solution:

$\sum x = 1 + 2 + 2^2 + \ldots\ldots + 2^{n-1}$

Sum are in G.P

$\therefore \sum x = \frac{1(2^n - 1)}{2 - 1} = 2^n - 1$

A.M = $\sum x/n = (2^n-1)/ n$

Question-19

Find the mean and variance for the following data:
6, 7, 10, 12, 13, 4, 8, 12

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9.$

The respective $(x_i - \bar{x})^2$ are $3^2, 2^2, 1^2, 3^2, 4^2, 5^2, 1^2, 3^2$.

$\sum (x_i - \bar{x})^2 = 9 + 4 + 1 + 9 + 16 + 25 + 1 + 9 = 74$

Hence variance (σ^2) = 74/8 = 9.25

Question-20

Find the mean and variance for the following data:

2, 4, 5, 6, 8, 17

Solution:

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{2+4+5+6+8+17}{6} = \frac{42}{6} = 7.$

The respective $(x_i - \bar{x})^2$ are $5^2, 3^2, 2^2, 1^2, 1^2, 10^2$.

$\sum (x_i - \bar{x})^2 = 25 + 9 + 4 + 1 + 1 + 100 = 140$

Hence variance $(\sigma^2) = 140/6 = 23.33$

Question-21

Find the mean for the following data:

First *n* natural numbers

Solution:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1+2+3........+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

Question-28

[Hint: First make the data continuous by making the classes as 32.5-36.5, 36.5-40.5,40.5-44.5,44.5-48.5, 48.5-52.5 and the proceed]

Solution:

Classes	x_i	$y_i = (x_i - 42.5)/4$	f_i	$f_i y_i$	$f_i y_i^2$
32.5-36.5	34.5	-2	15	-30	60
36.5-40.5	38.5	-1	17	-17	17
40.5-44.5	42.5	0	21	0	0
44.5-48.5	46.5	1	22	22	22
48.5-52.5	50.5	2	25	50	100
Total			100	25	199

Mean diameter of the circles = $\bar{x}$ = $\left[42.5+\frac{25}{100}\times 4\right]$ = 43.5

Variance (σ^2) = $[(4)^2/100][199 - 625/100]$ = 30.84

Hence the Standard Deviation is (σ) = $\sqrt{30.84}$ =5.55

Question-29

[Hint: Compare the variance of two groups. The group with greater variance is more variable]

Solution:

classes	x_i	$y_i = (x_i-45)/10$	Group A			Group B		
			f_i	$f_i y_i$	$f_i y_i^2$	f_i	$f_i y_i$	$f_i y_i^2$
10-20	15	-3	9	-27	81	18	-54	162
20-30	25	-2	17	-34	68	22	-44	88
30-40	35	-1	32	-32	32	40	-40	40
40-50	45	0	23	0	0	18	0	0
50-60	55	1	40	40	40	32	32	32
60-70	65	2	18	36	72	8	16	32
70-80	75	3	1	3	9	2	6	18
Total			140	-14	302	140	-84	372

Group A

Variance (σ^2) = $[(10)^2/140][302 - 196/140]$ = 214.7

Group B

Variance (σ^2) = $[(10)^2/140][372 - 7056/140]$ = 229.7

The variance group B is more than group A. Therefore group B has more variable.

Question-31

The mean and variance of 8 observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

Then mean = $\frac{6+7+10+12+12+13+x+y}{8} = 9$

$60 + x + y = 72$

$x + y = 12$ ....(i)

Variance = $\frac{(6-9)^2+(7-9)^2+(10-9)^2+(12-9)^2+(12-9)^2+(13-9)^2+(x-9)^2+(y-9)^2}{8} = 9.25$

$(-3)^2 + (-2)^2 + (1)^2 + (3)^2 + (3)^2 + (4)^2 + x^2 + y^2 - 18(x + y) + 2\times 9^2 = 9.25\times 8$

$x^2 + y^2 - 216 + 210 = 74$

$x^2 + y^2 = 80$ ....(ii)

But from (i)

$x^2 + y^2 = 144 - 2xy$ ....(iii)

$\therefore 144 - 2xy = 80$

$2xy = 64$ ....(iv)

Subtracting (iv) from (ii)

$x^2 + y^2 - 2xy = 80 - 64$

$(x - y)^2 = 16$

$x - y = \pm 4$ ....(v)

Hence solving (i) and (v)

$x = 8, y = 4$ and $x = 4, y = 8$

Therefore the remaining two observations are 4 and 8.

Question-32

The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

Solution:

Let the remaining two observations be x and y.

Then mean = $\frac{2+4+10+12+14+x+y}{7}=8$

$42 + x + y = 56$

$x + y = 14$ ....(i)

Variance = $\frac{(2-8)^2+(4-8)^2+(10-8)^2+(12-8)^2+(14-8)^2+(x-8)^2+(y-8)^2}{7} = 16$

$(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 16(x + y) + 2\times 8^2 = 16\times 7$

$x^2 + y^2 - 224 + 236 = 112$

$x^2 + y^2 = 100$ ....(ii)

But from (i)

$x^2 + y^2 = 196 - 2xy$ ....(iii)

$\therefore 196 - 2xy = 100$

$2xy = 96$ ....(iv)

Subtracting (iv) from (ii)

$x^2 + y^2 - 2xy = 100 - 96$

$(x - y)^2 = 4$

$x - y = \pm 2$ ....(v)

Hence solving (i) and (v)

x = 8, y = 6 and x = 6, y = 8

Therefore the remaining two observations are 8 and 6.

Question-33

The mean and variance of 6 observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:

Let the observations be x_1 , x_2, x_3, ..., x_{20} and $\bar{x}$ be their mean. Then

$$8 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$$

$$\text{or } \sum_{i=1}^{6}(x_i - \bar{x})^2 = 48$$

If each observation is multiplied by 3, the resulting observations are $3x_1$, $3x_2$, $3x_3$, ..., $3x_{20}$.

Their new mean $\bar{x} = \frac{3(x_1 + x_2 + x_3 + ... + x_n)}{n} = 3\bar{x} = 3\times 8 = 24$

and new variance $\frac{1}{6}\sum_{i=1}^{6}(3x_i - \bar{X})^2 = \frac{1}{6}\sum_{i=1}^{6}(3x_i - 3\bar{x})^2 = \frac{3}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2 = 3\times 48 = 144$

Therefore the new standard deviation is $\sqrt{144} = 12$

Question-34

Given that $\bar{x}$ is the mean and σ^2 is the variance of *n* observations x_1, x_2, x_3, ...x_n. Prove that the mean and variance of the observations ax_1, ax_2, ax_3, ...ax_n are a $\bar{x}$ and $a^2 \sigma^2$, respectively, ($a \neq 0$).

Solution:

Let the observations be x_1 , x_2, x_3, ..., x_n and $\bar{x}$ be their mean. Then $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

If each observation is multitplied by a, the resulting observations are ax_1, ax_2, ax_3,.....ax_n

Their new mean $\bar{x} = \frac{a(x_1 + x_2 + x_3 + ... + x_n)}{n} = a\bar{x}$

And new variance $\frac{1}{n}\sum_{i=1}^{n}(ax_i - \bar{X})^2 = \frac{1}{n}\sum_{i=1}^{n}(ax_i - a\bar{x})^2 = \frac{a}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = a\sigma^2$

Hence proved.

Question-35

The mean of 20 observations are found to be 10. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean in each of the following cases:
(i) If the wrong item is omitted.
(ii) If it is replaced by 12.

Solution:
Let the observations be x_1, x_2, x_3, ..., x_{20} and $\bar{x}$ be their mean. Then $\bar{x}$ = 10
2 = $\frac{1}{20}\sum_{i=1}^{20}(x_i-\bar{x})^2$ or $\sum_{i=1}^{20}(x_i-\bar{x})^2 = 40$

(i) Observation 8 is omitted.

New mean = $\bar{x} = \frac{20\times 10-8}{19}$ =10.11

(ii) Observation 8 is replaced by 12.

Difference = 12 – 8 = 4

New mean = $\bar{x} = \frac{20\times 10+4}{20}$ =10.2

Question-36

Prove that $(x_1-\bar{x})+(x_2-\bar{x})+\ldots\ldots(x_n-\bar{x})=0$ where $\bar{x}=\frac{x_1+x_2+\ldots..+x_n}{n}$

Solution:

$$(x_1-\bar{x})+(x_2-\bar{x})+\ldots\ldots..(x_n-\bar{x}) = x_1+x_2+x_3\ldots\ldots.x_n - n\frac{(x_1+x_2+x_3+\ldots\ldots..x_n)}{n} = 0.$$

Question-37

Prove the identity $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = \sum_{i=1}^{n} x^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}$.

Solution:

$\sum_{i=1}^{n}(x_i - \bar{x})^2$

$= \sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + n\bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\,n\bar{x} + n\bar{x}^2$ (Since $\sum_{i=1}^{n} x_i = n\bar{x}$)

$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - n(\sum_{i=1}^{n} \frac{x_i}{n})^2$

$= \sum_{i=1}^{n} x_i^2 - \frac{1}{n}(\sum_{i=1}^{n} x_i)^2$

Question-38

The mean of 9 items is 15. If one more item is added to this series, the mean becomes 16. Find the value of the 10^{th} item.

Solution:

Let the value of 9 items be x_1 , x , x_2 …… x_9

$15 = \frac{x_1 + x_2 + \ldots\ldots x_9}{9}$ ∴ $x_1+x_2+\ldots\ldots x_9 = 15 \times 9 = 135$

Let x_{10} be the 10^{th} item

AM of $x_1, x_2, \ldots\ldots x_9, x_{10} = 16$

$16 = \frac{x_1 + x_2 \ldots\ldots x_9 + x_{10}}{10}$ ∴ $x_1+x_2 \ldots\ldots\ldots\ldots x_9+x_{10} = 160$

$135 + x_{10} = 160$

$\Rightarrow x_{10} = 25$

Question-39

The average weight of a group of 25 items was calculated to be 78.4kg. It was later discovered that a weight was misread as 69kg instead of 96kg. Calculate correct average.

Solution:

No. of items = 25

Incorrect average = 78.4kg

Incorrect reading of weight of an item = 69kg

Correct reading of weight of an item = 96kg

Let the variable weight be denoted by 'x'

$\bar{x} = \frac{\sum x}{n}$

Incorrect $\bar{x} = \frac{\text{Incorrect} \sum x}{25}$

$78.4 = \frac{\text{Incorrect} \sum x}{25}$

Incorrect $\sum x = 78.4 \times 25 = 1960\text{kg}$

New correct $\sum x = \text{Incorrect} \sum x$ - incorrect weight of an item + correct weight of an item

Correct $\bar{x} = \frac{\text{correct} \sum x}{25} = \frac{1987}{25} = 79.48$ kg

Question-40

The mean of 9 items is 15. If one more item is added to this series, the mean becomes 16. Find the value of the 10th item.

Solution:

Let the value of 9 items be $x_1, x, x_2 \ldots\ldots x_9$

$15 = \frac{x_1 + x_2 + \ldots\ldots x_9}{9} \therefore x_1+x_2+\ldots\ldots x_9 = 15 \times 9 = 135$

Let x_{10} be the 10^{th} item

AM of $x_1, x_2, \ldots\ldots x_9, x_{10} = 16$

$16 = \frac{x_1 + x_2 \ldots\ldots x_9 + x_{10}}{10} \therefore x_1+x_2 \ldots\ldots\ldots\ldots x_9+x_{10} = 160$

$135 + x_{10} = 160$

$\Rightarrow x_{10} = 25$