CHAPTER 1

Numbers and Polynomials

SYNOPSIS

Factors

When a number is expressed as a product of two or more numbers, the latter numbers are called factors. Similarly, when an algebraic expression is as the product of two or more expressions each of these later quantities is called a factor of it. The determination of these is called resolution.

Example:

(i) $72 = 2^3 \times 2^2$
(ii) $7a^2 - 21ab = 7a(a - 3b)$

While resolving algebraic expression the following may be remembered $x^2 + (a + b)x + ab = (x + a)(x + b)$.

Multiples

If p divides q, then p is a <u>factor</u> of q; and q is a <u>multiple</u> of p.

Co-Primes

If two numbers are such that there is no common factor other than 1, then they are said to be relatively prime or co-prime to each other.

The two numbers individually may be prime or composite.

Example: 15 and 23; 13 and 29; 15 and 32 are co-primes.

Number of Factors of a given Number

If N is a composite number such that $N = a^p.b^q.c^r$ ...... where a, b, c are prime factors of N and p, q, r..... are positive integers, then

(i) the number of factors of N = (p + 1) (q + 1)(r + 1)... For example, $144 = 2^4 \times 3^2$. Hence 144 has(4 + 1) (2 + 1), i.e., 15 factors.
(ii) The number of ways in which N can be expressed as a product of two factors = $^1/_2\{(p + 1)(q + 1)(r + 1).....\}$

If p, q, r etc. are all even, then the product (p + 1) (q + 1)(r + 1)... becomes odd and the above rule will not be valid. If p, q, r,... are all even, it means that N is a perfect square.

So, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following two rules:

(a) product of two DIFFERENT factors(excluding $\sqrt{N} \times \sqrt{N}$) is $^1/_2\{(p + 1)(q + 1)(r + 1)... -1\}$ ways θ.
(b) product of two factors (including $\sqrt{N} \times \sqrt{N}$) is $^1/_2\{(p + 1)(q + 1)(r + 1)... +1\}$ ways.

- **LCM and HCF:**
  (i) If x is a factor of y, then x is the HCF of (x, y) and y is the LCM of (x, y).
  (ii) HCF of any two consecutive even natural numbers is 2 and their LCM is half of their product.

(iii) HCF of any two consecutive natural numbers or any two consecutive odd natural numbers is 1 and their LCM is their product.

(iv) For any two numbers (or polynomials), product of those two numbers (or polynomials) = Product of their LCM and HCF.

Factorial

Factorial is defined for any positive integer. It is denoted by ∠ or !. Thus "Factorial n" is written as n! or ∠n. n! is defined as the product of all the integers from 1 to n.

Thus n! = 1.2.3. ... $(n-1)^n$.

0! is defined to be equal to 1. Therefore 0! = 1 and 1! = 1.

- **Surds:** If a is a positive rational number, which is not the n^{th} power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds. Every surd is an irrational number (but every irrational number need not be a surd). So, the representation of monomial surd on a number line is same that of irrational numbers.

For example,

(i) $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.

(ii) π is an irrational number, but it is not a surd.

(iii) $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

Laws of Radicals

If a > 0, b > 0 and n is a positive rational number, then

1. $(\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$

2. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

4. $\sqrt[n]{a^p} = a^{p/n}$ and $\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}}$.

- **Rationalizing Factor (RF):** If the product of two surds is a rational number, then each of the two is a RF of the other.

  (i) $\left(\sqrt{a}+\sqrt{b}\right)$ is the rationalising factor of $\sqrt{a}-\sqrt{b}$ and vice versa, where a and b are rational. And also $a+\sqrt{b}$ is the rationalizing factor of $a-\sqrt{b}$.

  (ii) Two surds of the form $a+\sqrt{b}$ and $a-\sqrt{b}$, are called conjugate surds. The sum and product of conjugate surds are rational numbers.

- **Square root of a quadric surd:** Consider the real number $a+\sqrt{b}$, where a and b are rational numbers and $\sqrt{b}$ is a surd. Equate the square root of $a+\sqrt{b}$ to $\sqrt{x}+\sqrt{y}$, where x and y are rational numbers, i.e., $\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y}$

  Squaring both sides, $a+\sqrt{b} = x+y+2\sqrt{xy}$

  Equating the rational numbers on the two sides of the above equation we get a = x + y ……(1)

  and equating the irrational numbers, we get $\sqrt{b} = 2\sqrt{xy}$ ……(2)

  By solving (1) and (2) we get the values of x and y.

  Similarly, $\sqrt{a-\sqrt{b}} = \sqrt{x}-\sqrt{y}$

- **Square root of a trinomial quadratic surd:** Consider the real number $a+\sqrt{b}+\sqrt{c}+\sqrt{d}$, where a is a rational number and $\sqrt{b}, \sqrt{c}$ and $\sqrt{d}$ are surds. $\sqrt{a+\sqrt{b}+\sqrt{c}+\sqrt{d}} = \sqrt{x}+\sqrt{y}+\sqrt{z}$

  By squaring both sides, and comparing rational and irrational parts on either sides, we get, x + y + z = a, $x = \frac{1}{2}\sqrt{\frac{bd}{c}}$, $y = \frac{1}{2}\sqrt{\frac{bc}{d}}$ and $z = \frac{1}{2}\sqrt{\frac{cd}{b}}$

Remainder Theorem

When a rational integral function f(x) is divided by (x – a), the remainder is f (a).

For example, when $x^2 - 2x + 5$ is divided by x – 1, the remainder will be f (1), i.e., $1^2 - 2(1) + 5 = 4$

We can see that if f (x) is divided by (x + a), then the remainder will be f (–a).

For example, when $x^3 + x^2 - 5x - 4$ is divided by x +1, then the remainder will be f (–1),

i.e., $(-1)^3 (-1)^2 - 5(-1) - 4$. i.e., 1

If f(a) is zero, it means that the remainder is zero and hence, we can say that (x –a) is a factor of f (x).

- (x – a) is always a factor of $x^n - a^n$
- (x + a) is a factor of $x^n - a^n$ when 'n' is even
- (x+ a) is a factor of $(x^n + a^n)$ when n is odd.

Solved Examples

1. Find the number of factors of 324.

Solution: $324 = 18^2 = 3^2 \times 2 \times 3^2 \times 2 = 3^4 \times 2^2$

Number of factors = (4 + 1)(2 + 1) = 15.

2. Find the number of ways of expressing 216 as a product of two factors.

Solution: 216 is not a perfect square. The number of ways of expressing any non-perfect square as a product of two factors = $^1/_2$(number of its factors)

$216 = 2^3 \times 3^3$.

Number of its factors = (3 + 1)(3 + 1) = 16.

∴ The number of ways of writing 216 as a product of two factors = 8

3. Find the LCM of 216 and 240.

Solution: 216 = 24 × 9 and 240 = 24 × 10

L.C.M. (216, 240) = L.C.M. (24 × 9, 24 × 10)

= 24 L.C.M. (9, 10) = 90.

∴ LCM of 216, 240 = (24)(90) = 2160.

4. Find the HCF of 1159 and 1769.

Solution:

```
1159)1769(1
     1159
     ----
      610)1159(1
           610
           ---
           549)610(1
               549
               ---
                61)549(9
                   549
                   ---
                     0
```

HCF of 1159 and 1769 = 61

5. The LCM and HCF of two natural numbers are 480 and 12 respectively. If the numbers are in the ratio 5: 8, then find the numbers.

Solution: Let the natural numbers be 5x and 8x

HCF (5x, 8x) = x;

but given HCF = 12 ⇒ x = 12

The two numbers are

∴ 5x = 60 and 8x = 96

Alternately, LCM (5x, 8x) = 40x

40 x = 480 ⇒ x = 12

5x = 60 and 8x = 96.

6. Find the LCM of $\frac{3}{5}, \frac{4}{7}, \frac{6}{11}$

Solution: LCM of fractions

$$= \frac{\text{LCM of numerators}}{\text{HCF of denominators}} = \frac{\text{LCM of }(3,4,6)}{\text{HCF of }(5,7,11)}$$

$$= \frac{12}{1} = 12$$

7. Find the remainder when 2^{86} is divided by 7.

Solution: $\frac{2^{86}}{7} = \frac{2^{84}.2^2}{2^3-1} = \frac{\left(2^3\right)^{28}.4}{2^3-1}$

By remainder theorem, if $x = 2^3$ and $f(x) = 4x^{28}$

Required remainder = $f(1) = 4(1)^{28} = 4$.

8. Aloukya and Manoghna run in a circular track and they take 180 seconds and 150 seconds respectively to complete one revolution. If they start together at 9 am from the same point, how long it would take for them to meet again for the first time?

Solution: The required time taken is the LCM of 180 and 150.

$180 = 2^2 \times 3^2 \times 5^1$; $150 = 2 \times 3 \times 5^2$

LCM = $2^2 \times 3^2 \times 5^2$ = 900 sec = 15 minutes

They meet again for the first time at 9:15 a.m.

9. Compare the surds $A = \sqrt{8}+\sqrt{7}$ and $B = \sqrt{10}+\sqrt{5}$.

Solution: Since there is a positive sign, squaring both the surds, we get,

$$A^2 = \left(\sqrt{8}+\sqrt{7}\right)^2 = 8+7+2\sqrt{56} = 15+2\sqrt{56}$$

$$B^2 = \left(\sqrt{10}+\sqrt{5}\right)^2 = 10+5+2\sqrt{50} = 15+2\sqrt{50}$$

As = 56 > 50, $15+2\sqrt{56} > 15+2\sqrt{50}$

⇒ A > B i.e., $\sqrt{8} + \sqrt{7} > \sqrt{10} + \sqrt{5}$

10. If both a and b are rational numbers, then find the value of a and b in the following.

$$\frac{3+\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$$

Solution: (i) $\frac{3+\sqrt{5}}{3-\sqrt{5}}$

$3+\sqrt{5}$ is the rationalizing factor of $3-\sqrt{5}$.

$$\therefore \quad \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{\left(3+\sqrt{5}\right)^2}{(3)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{9+5+6\sqrt{5}}{9-5} = \frac{14+6\sqrt{5}}{4} = \frac{14}{4}+\frac{6}{4}\sqrt{5}$$

$$= \frac{7}{2}+\frac{3}{2}\sqrt{5} \;= a+b\sqrt{5} \text{ (given)}$$

$\therefore$ a = 7/2 and b = 3/2

11. Find the square root of $7+4\sqrt{3}$.

Solution: Let $\sqrt{7+4\sqrt{3}} = \sqrt{x}+\sqrt{y}$

Squaring both the sides, $7+4\sqrt{3} = x+y+2\sqrt{xy}$

$\Rightarrow$ x + y = 7 and $\sqrt{xy} = 2\sqrt{3} = \sqrt{12}$

$\Rightarrow$ xy = 12

By solving, we get x = 4 and y = 3

$\sqrt{x}+\sqrt{y} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

12. Find the HCF and the LCM of $3a^2b^3c^4$ and $9a^4b^3c^2$.

Solution: $3a^2b^3c^4 = (3a^2b^3c^2)c^2$ and
$9a^4b^3c^2 = (3a^2b^3c^2)3a^2$

The HCF of $3a^2b^3c^4$ and $9a^4b^3c^2$ is $3a^2b^3c^2$

The LCM of $3a^2b^3c^4$ and $9a^4b^3c^2$ is $9a^4b^3c^4$

13. Find the HCF and LCM of $(x-1)(x-2)^2(x+4)^3$ and $(x+1)(x-2)(x+4)^4$.

Solution: Clearly, the common factors of the expressions

$(x-1)(x-2)^2(x+4)^3$ and $(x+1)(x-2)(x+4)^4$ are $(x-2)$ and $(x+4)^3$.

$\therefore$ HCF = $(x-2)(x+4)^3$ and

LCM = $(x-2)^2(x-1)(x+1)(x+4)$.

14. Find the remainder when $f(x) = x^2 + 6x + 8$ is divided by 2x + 1.

Solution: Given polynomial is $f(x) = x^2 + 6x + 8$ and divisor is 2x + 1.

$$\therefore \quad \text{Remainder} = f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 6\left(-\frac{1}{2}\right) + 8$$

$$= \frac{1}{4} - 3 + 8 = 5\frac{1}{4}$$

15. The value of $ax^2 + bx + c$ is 5 when x = 0. The remainder is 6 when divided by x – 1 and 10 when divided by x + 1 then find value of 5a – 2b + 5c.

Solution: Let $f(x) = ax^2 + bx + c$

Given f(0) = c = 5 and $f(1) = a(1)^2 + b(1) + 5 = 6$

$\Rightarrow$ a + b = 1 ….. (1)

Also given f(–1) = 10, $a(-1)^2 + b(-1) + 5 = 10$

$\Rightarrow$ a – b = 5 ….. (2)

Solving (1) and (2) we have 2a = 6

$\Rightarrow$ a = 3 and b = –2

$\therefore$ Value of 5a – 2b + 5c = 5 (3) – 2(–2) + 5(5)
= 44.

PRACTICE EXERCISE 1 (A)

Directions for questions 1 to 50: Select the correct alternative from the given choices.

1. Find the smallest and the largest three-digit numbers which when divided by 22,33 and 55 leave a remainder of 5 in each case.

(1) 340, 980 (2) 335, 995
(3) 330, 990 (4) 325, 985

2. The HCF of two numbers is 8. The product of the two numbers is 1536. How many pairs of such numbers satisfy the above conditions?

(1) One (2) Two
(3) Four (4) None of these

3. Find the number of divisors of 3600 excluding one and itself.

(1) 44 (2) 45
(3) 47 (4) 43

4. In how many ways can 2304 be written as product of two different factors?

(1) 27 (2) 14
(3) 13 (4) 28

5. What values of n satisfy the following statements (given n is a natural number)?

(a) $2^{3n} - 1$ is divisible by 7
(1) Even values of n
(2) Odd values of n
(3) All values of n
(4) Cannot say

6. $5^n + 1$ is divisible by 6

(1) Even values of n (2) Odd values of n
(3) All values of n (4) Cannot say

7. Find the value of a if the expression $3x^2 + ax - 7$ is divisible by $(x - 1)$.

(1) –4 (2) 4
(3) 10 (4) –10

8. What is the remainder when 2^{63} is divided by 7?

(1) 2 (2) 4
(3) 1 (4) 5

9. What is the remainder when 5^{68} is divided by 8?

(1) 5 (2) 1
(3) 4 (4) 3

10. Find the highest power of 2 contained in 150!

(1) 148 (2) 146
(3) 147 (4) 145

11. The sum of the first N natural numbers is equal to x^2 where x is an integer less than 100. What are the values that N can take?

(1) 1, 9, 27
(2) 1, 7, 26
(3) 1, 8, 48
(4) 1, 8, 49

12. The LCM of two numbers is 1200. Which of the following cannot be their HCF?

(1) 600 (2) 500
(3) 200 (4) 400

13. Find the units digit of $(12)^{3^x} + (18)^{3^x}$ for all $x \in N$.

(1) 0 (2) 4
(3) 2 (4) 3

14. The LCM and the HCF of two numbers are 144 and 12 respectively. How many such pairs of numbers are possible?

(1) 2 (2) 3
(3) 4 (4) 5

15. P is the LCM of 2, 4, 6, 8, 10. Q is the LCM of 1, 3, 5, 7, 9. L is the LCM of P and Q. Find the relation between L and P.

(1) L = P
(2) L = 2P
(3) L = 21P
(4) L = 60P

16. Find the sum up to 91 terms of the following

$$\frac{1}{\sqrt{9}+\sqrt{10}} + \frac{1}{\sqrt{10}+\sqrt{11}} + \frac{1}{\sqrt{11}+\sqrt{12}} + \ldots\ldots .$$

(1) 90 (2) 10
(3) 7 (4) 1

17. Ashok had two vessels which contain 720 ml and 405 ml of milk respectively. Milk in each vessel was poured into glasses of equal capacity to their brim. Find the minimum number of glasses which can be filled with milk.

(1) 15 (2) 20
(3) 25 (4) 30

18. If m is a real number, then the simplified form of the expression $(m^{-16} \div m^{-8} \div m^{-6} \div m^{-4})$ is ______.

(1) 1 (2) m

(3) m^{-2} (4) m^2

19. If $a^m = b^m$, then ______ $(m > 0)$.

(1) $a = -b$

(2) $a = b$

(3) $2a + b = 0$

(4) $a^2 = b^2$

20. Simplify: $\left[\dfrac{(x+y)^{-1}\cdot(x-y)^{-m}}{(x+y)^{m+n}\cdot(x-y)^{1+n}}\right]\left(x^2-y^2\right)^{1+m+n}$

(where $x \neq y$ and $x + y \neq 0$)

(1) x^2 (2) x

(3) 0 (4) 1

21. If $ab = x^2$, $bc = y^2$ and $ca = z^2$, then find the value of $\left(\dfrac{abc}{xyz}\right)^{xyz}$, where xyz is an integer.

(1) 1 (2) −1

(3) ±1 (4) 0

22. Greatest among $\sqrt[6]{7}$, $\sqrt[4]{5}$, $\sqrt[5]{6}$, $\sqrt[6]{8}$ is

(1) $\sqrt[6]{7}$ (2) $\sqrt[4]{5}$

(3) $\sqrt[5]{6}$ (4) $\sqrt[6]{8}$

23. If $x = 5 + 2\sqrt{6}$ and $y = 5 - 2\sqrt{6}$, then find the value of $1/x^2 + 1/y^2$.

(1) 1 (2) 100

(3) 96 (4) 98

24. Simplify: $\sqrt[6]{15-2\sqrt{56}}\,\sqrt[3]{\sqrt{7}+2\sqrt{2}}$

(1) 8 (2) 7

(3) 2 (4) 1

25. If $p = 7 - 4\sqrt{3}$, then find the value of $\dfrac{p^2+1}{7p}$.

(1) 1 (2) 2

(3) 3 (4) 4

26. $(5^{1/3} + 5^{-1/3})(5^{2/3} - 1 + 5^{-2/3})$ = ______.

(1) 6 (2) 5

(3) 26/5 (4) 41/5

27. If $\sqrt{12+x\sqrt{2}} = 2\sqrt{2}-2$, then find x.

(1) 6 (2) 8

(3) −8 (4) −6

28. Simplify: $\sqrt{\sqrt[x]{2^{x}\ \sqrt[x^2]{3^{x^3}\ \sqrt[x^3]{6^{x^6}\ \sqrt[x^4]{9^{x^{10}}}}}}}$

(1) 2 (2) 3

(3) 12 (4) 18

29. If the HCF of the polynomials f(x) and g(x) is $4x - 6$, then f(x) and g(x) could be _____.

(1) $2, 2x - 3$ (2) $8x - 12, 2$

(3) $2(2x - 3)^2, 4(2x - 3)$ (4) $2(2x + 3), 4(2x + 3)$

30. The LCM of the polynomials $15a^2b(a^2 - b^2)$ and $40ab^2(a - b)$ is _____.

(1) $120ab(a^2 - b^2)$ (2) $120a^2b^2(a^2 - b^2)$

(3) $5a^2b^2(a^2 - b^2)$ (4) $120ab(a - b)$

31. Find the HCF of $(x^2 - 4)(x^2 - x - 2)$ and $(x^2 + 4x + 4)(x^2 - 3x + 2)$.

(1) $x^2 - 4$ (2) $(x + 2)^2$

(3) $(x - 2)^2$ (4) $x^2 - 1$

32. If $f(x) = (x - 2)(x^2 - x - a)$, $g(x) = (x + 2)(x^2 + x - b)$ and their HCF is $x^2 - 4$, then find $a - b$. (a and b are constants)

(1) 0 (2) 4

(3) 1 (4) 6

33. If the LCM of $f(x) = (x + 1)^5 (x + 2)^a$ and $g(x) = (x + 1)^b (x + 2)^a$ is $(x + 1)^a (x + 2)^b$, then find the minimum value of $a + b$.

(1) 5 (2) 8

(3) 10 (4) 12

34. $\dfrac{2x}{1+x^2+x^4} + \dfrac{1}{1+x+x^2} - \dfrac{1}{1-x+x^2}$. =

(1) 2x (2) x

(3) 1 (4) 0

35. If degree of each of f(x) and [f(x) + g(x)] is 18, then find the range of degree of g(x).

(1) 18 (2) ≥18

(3) ≤18 (4) can't say

36. Find the product of additive inverse and multiplicative inverse of $(x - 2)/(x^2 - 4)$.

(1) $x + 2$ (2) $x - 2$

(3) 1 (4) −1

37. If $f(x + 2) = x^2 + 7x - 13$, then find the remainder when f(x) is divided by $(x + 2)$.

(1) −25 (2) −12

(3) −23 (4) −11

38. If $ax^3 - 5x^2 + x + p$ is divisible by $x^2 - 3x + 2$, then find the values of a and p.

(1) a = 2, p = 2 (2) a = 2, p = 3
(3) a = 3, p = 1 (4) a = 1, p = 3

39. If the polynomials $f(x) = x^2 + 5x - p$ and $g(x) = x^2 - 2x + 6p$ have a common factor, then the common factor is _____.

(1) x + 2 (2) x
(3) x + 4 (4) Either (2) or (3)

40. Given f(x) is a cubic polynomial in x. If f(x) is divided by (x + 3), (x + 4), (x + 5) and (x + 6), then it leaves the remainders 0, 0, 4 and 6 respectively. Find the remainder when f(x) is divided by x + 7.

(1) 0 (2) 1
(3) 2 (4) 3

41. What is the largest number which divides 206, 368 and 449 and leaves the same remainder in each case?

(1) 32 (2) 48
(3) 81 (4) 96

42. $N = 161^3 - 77^3 - 84^3$, which of the following statements is not true?

(1) N is divisible by 4 and 23.
(2) N is divisible by 23 and 11.
(3) N is divisible by 4 and 7.
(4) N is divisible by 8 and 11.

43. Find the last digit of $1567^{143} \times 1239^{197} \times 2566^{1027}$.

(1) 2 (2) 3
(3) 4 (4) 6

44. If x is a composite number, which of the following is necessarily true?

(1) There is at most one factor of x (say a) such that $0 < a < \sqrt{x}$.
(2) There are at least two factors x (say a and b) such that $0 < a < \sqrt{x}$ and $0 < b < \sqrt{x}$.
(3) There are at least 4 factors of x.
(4) If, there are 3 factors not greater than $\sqrt{x}$, then there are 3 factors not less than $\sqrt{x}$.

45. Find the remainder when 1! + 2! + 3! + 4! + 5! + . . . + 49! is divided by 7.

(1) 0 (2) 1
(3) 5 (4) 6

46. A sweet shop sells laddus in boxes of different sizes. The laddus are priced at `10 per laddu upto 400 laddus. For every 10 additional laddus, the price of the entire lot goes down by 10 paise per laddu. Find the size of the box that would have the maximum cost.

(1) 600 (2) 500
(3) 700 (4) 800

47. When a natural number, N is divided by D, the remainder is 35. When 50N is divided by D, the remainder is 11. Find D.

(1) 1739 (2) 43
(3) 47 (4) Cannot be determined

48. If LCM (P, Q, R) = (P) (Q) (R), then HCF (P, R) =

(1) 1 (2) 2
(3) Q (4) Cannot say

49. How many numbers less than 2^{24} are co prime to it?

(1) 2^{12} (2) 2^{23}
(3) 2^{22} (4) None of these

50. A number when divided by 48 leaves a remainder of 31. Find the remainder if the same number is divided by 24.

(1) 5 (2) 7
(3) 9 (4) 11

PRACTICE EXERCISE 1 (B)

Directions for questions 1 to 50: Select the correct alternative from the given choices.

1. How many integers between 400 and 900 are exactly divisible by 2, 3 and 7?

(1) 10 (2) 11
(3) 12 (4) 13

2. Find the smallest and the largest four-digit numbers which when lessened by 12 are exactly divisible by 16, 24 and 40.

(1) 1208, 9848
(2) 1200, 9840
(3) 1212, 9852
(4) 1188, 9828

3. What is the remainder when 3^{21684} is divided by 5?

(1) 4 (2) 1
(3) 2 (4) 3

4. The product of two numbers whose HCF is 18 is 5832. How many pairs of numbers satisfy the above conditions?

(1) One (2) Two
(3) Three (4) None of these

5. Find the number of factors of 1728.

(1) 28 (2) 18
(3) 36 (4) 26

6. In how many ways can 2744 be resolved as a product of two factors?

(1) 16 (2) 14
(3) 6 (4) 8

7. For what values of n, is true (where n is a natural number)? the following statements are $5^n + 4^n$ is divisible by 9

(1) All values of n (2) Even values of n
(3) Odd values of n (4) Cannot say

8. $18^n - 2^{4n}$ is divisible by 34

(1) All values of n (2) Even values of n
(3) Odd values of n (4) No value

9. Find the highest power of 3 contained in 120!

(1) 57 (2) 60
(3) 59 (4) 58

10. A number when divided by a certain divisor leaves a remainder of 12. What is the divisor if a remainder of 9 is left when thrice the same number is divided by the same divisor?

(1) 18 (2) 24
(3) 27 (4) 30

11. If x and y are irrational numbers, then x + y – xy is

(1) a real number
(2) a complex number
(3) a rational number
(4) an irrational number

12. If $64 = x^y$, where $x > y$, $x \neq 4$ and $y \neq 1$, then x + y =

(1) 7 (2) 4
(3) 8 (4) 10

13. Two positive numbers have their HCF as 12 and their sum is 84. Find the number of pairs possible for the numbers.

(1) 4 (2) 3
(3) 2 (4) 1

14. If $X = 28 + (1 \times 2 \times 3 \times 4 \times \ldots \times 16 \times 28)$ and $Y = 17 + (1 \times 2 \times 3 \times \ldots \times 17)$, then X – Y is a

(1) Prime (2) Composite
(3) 1 (4) 0

15. There are 144 boys and 132 girls in a class. These students are arranged in rows for prayer in such a way that each row has either boys or girls, and every row has an equal number of students. Find the minimum number of rows in which all the students can be arranged.

(1) 12 (2) 20
(3) 23 (4) 24

16. If $a = \sqrt{11} + \sqrt{3}$, $b = \sqrt{12} + \sqrt{2}$ and $c = \sqrt{17} - \sqrt{3}$, then arrange them in descending order.

(1) a < b < c (2) a > b < c
(3) a = b = c (4) a > b > c

17. Find the remainder when the square of any prime number greater than 3 is divided by 6.

(1) 1 (2) 2
(3) 3 (4) 5

18. If x = 1/12, which is the greatest among x^{-1}, x^2 and 2/x?

(1) x^{-1} (2) x^2
(3) 2/x (4) All are equal

19. If $3^m \times 2^n = 18 \times 9 \times 27$, then $3^m/3^{2n}$ = ______.

(1) 243 (2) 192
(3) 125 (4) 81

20. If $\left(\frac{9}{4}\right)^{x+y} = \left(\frac{3}{2}\right)^{16}$ and $\left(\frac{49}{64}\right)^{x-2y} = \left(\frac{7}{8}\right)^{4}$, then xy =

(1) 12 (2) 14
(3) 15 (4) 16

21. If $x^5 + 1 = 7777$ and $y^4 - 1 = 9999$, then find the value of xy, where both x and y are positive integers.

(1) 70 (2) 60
(3) 80 (4) 40

22. If $x = 2 + 2^{\frac{1}{3}} + 2^{2/3}$, then prove that $x^3 - 6x^2 + 6x =$

(1) 0 (2) 2
(3) 1 (4) 6

23. Simplify: $\sqrt{7+2\sqrt{6}} - \sqrt{7-2\sqrt{6}}$

(1) $2\sqrt{6}$ (2) 1
(3) 2 (4) 14

24. If $x = \frac{1}{5+2\sqrt{6}}$, then find the value of $x^2 - 10x$ =.

(1) 0 (2) 1
(3) −1 (4) 5

25. The smallest rationalizing factor of $\sqrt[3]{63}$ is _____.

(1) $\sqrt[3]{37}$ (2) $\sqrt[3]{62}$
(3) $\sqrt[3]{147}$ (4) $\sqrt[3]{243}$

26. $\left[\frac{3}{\sqrt{19-2\sqrt{88}}} - \frac{8}{\sqrt{14+2\sqrt{33}}}\right].\left(\sqrt{8}-\sqrt{3}\right) =$

(1) $\sqrt{5}$ (2) $\sqrt{8}$
(3) −3 (4) 5

27. If $x^{3a} = y^{2b} = z^{4c} = xyz$, then $3ab + 4bc + 6ca$ =

(1) 3abc (2) 8abc
(3) 9abc (4) 12abc

28. The LCM of $x^2 - 1$, $x^2 + 1$ and $x^4 - 1$ is _____.

(1) $(x^2 + 1)(x^2 - 1)$ (2) $(x + 1)^2 (x - 1)^2$
(3) $(x + 1)^2 (x - 1)$ (4) None of these

29. The product of the HCF and the LCM of two polynomials is $(x^2 - 1)(x^4 - 1)$, then the product of the polynomials is _____.

(1) $(x^2 - 1)(x^2 + 1)$ (2) $(x^2 - 1)(x^2 + 1)^2$
(3) $(x^2 - 1)^2 (x^2 + 1)$ (4) None of these

30. If the LCM of $(x^2 + 3x)(x^2 + 3x + 2)$ and $(x^2 + 6x + 8)(x^2 + kx + 6)$ is $x(x + 1)(x + 2)^2 (x + 3)(x + 4)$, then find k.

(1) 2 (2) 3
(3) 5 (4) 8

31. If the LCM of f(x) and g(x) is $6x^2 + 13x + 6$ and their HCF is a linear polynomial, find the possible HCF of f(x) and g(x).

(1) 2x − 3 (2) 3x + 2
(3) 3x − 2 (4) 3x + 4

32. If (x + 6) is the HCF of $p(x) = x^2 - a$ and $q(x) = x^2 - bx + 6$, then $\frac{p(x)}{q(x)}$ in its lowest terms is _____.

(1) $\frac{x-6}{x-2}$ (2) $\frac{x+6}{x+1}$
(3) $\frac{x-6}{x-1}$ (4) $\frac{x-6}{x+1}$

33. If $f(x) = (x + 2)(x^2 + 8x + 15)$ and $g(x) = (x + 3)(x^2 + 9x + 20)$, then find the HCF of f(x) and g(x).

(1) x + 3 (2) $x^2 + 8x + 15$
(3) x + 4 (4) $x^2 + 9x + 20$

34. The LCM of $12x^3y^2$ and $18x^py^3$ is $36x^4y^3$. Find the number of integer values possible for p.

(1) 1 (2) 2
(3) 3 (4) 4

35. Simplify: $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}+\frac{b^2-(a-c)^2}{(a+b)^2-c^2}+\frac{c^2-(a-b)^2}{(b+c)^2-a^2}$.

(1) 1 (2) a − b − c
(3) a + b + c (4) 0

36. If x^5 is divided by $x^2 - 4x + 3$, then find its remainder.

(1) 121x + 120 (2) 121x −120
(3) 120x + 121 (4) 120x − 121

37. The remainder obtained when $2x^4 + 3x^2 - 2$ is divided by $x^2 + 2$ is _____.

(1) 0 (2) 12
(3) 14 (4) 16

38. Which of the following should be added to $9x^3 + 6x^2 + x + 2$ so that the sum is divisible by (3x + 1)?

(1) −4 (2) −3
(3) −2 (4) −1

39. Which of the following is/are factors of $x^3 + 3x^2 - x - 3$?

(a) x + 1 (b) x − 1
(c) x + 3

(1) Only (a) (2) Only (b)
(3) Both (a) and (b) (4) All (a), (b) and (c)

40. If (x − 2) and (x − 3) are two factors of $x^3 + ax + b$, then find the remainder when $x^3 + ax + b$ is divided by x − 5.

(1) 0 (2) 15
(3) 30 (4) 60

41. The sets S_x are defined to be (x, x + 1, x + 2, x + 3, x + 4) where x = 1, 2, 3,……80. How many of these sets contain 6 or its multiple?

(1) 65 (2) 66
(3) 59 (4) 60

42. N = (4711) (4713) (4715). Find the remainder when N is divided by 48.

(1) 19 (2) 21
(3) 17 (4) 23

43. What is the minimum number of identical square tiles required to completely cover a floor of dimensions 8 m 70 cm by 6 m 38 cm?

(1) 143 (2) 165
(3) 187 (4) 209

44. What is the last digit of $518^{163} + 142^{157}$?

(1) 2 (2) 4
(3) 6 (4) 8

45. Ravi distributed the chocolates with him equally between Rajesh and Suresh. He was left with a chocolate. Rajesh distributed his share equally among three of his friends and was also left with a chocolate. One of the three distributed his share equally among four of his friends and was left with no chocolate. Which of the following could be the number of chocolates that Rajesh received?

(1) 22 (2) 34
(3) 49 (4) 64

46. $(AB)^2$ = CCB where A, B and C are distinct single-digit natural numbers and 'AB' and 'CCB' are two-digit and three-digit natural numbers respectively. Find the number of possibilities for AB.

(1) 0 (2) 1
(3) 2 (4) 3

47. If N = 1223334444……. and is a 100-digit number, find the remainder when N is divided by 16.

(1) 15 (2) 13
(3) 11 (4) 9

48. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+......+\frac{1}{35^2-1}=$

(1) $\frac{4}{17}$ (2) $\frac{17}{72}$
(3) $\frac{19}{72}$ (4) $\frac{17}{36}$

49. If $N = 2^a \times 3^b \times 5^c$, how many numbers (in terms of N) are less than N and are co prime to it?

(1) $\frac{2}{15}N$ (2) $\frac{4}{15}N$
(3) $\frac{8}{15}N$ (4) $\frac{2}{5}N$

50. A number when divided by 18 leaves a remainder of 15. Which of the following could be the remainder when it is divided by 72?

(1) 33 (2) 51
(3) 15 (4) All the above

ANSWER KEYS

PRACTICE EXERCISE 1 (A)

1. 2	**2.** 2	**3.** 4	**4.** 3	**5.** 3	**6.** 2	**7.** 2	**8.** 3	**9.** 2	**10.** 2
11. 4	**12.** 2	**13.** 1	**14.** 1	**15.** 3	**16.** 3	**17.** 3	**18.** 4	**19.** 4	**20.** 4
21. 3	**22.** 2	**23.** 4	**24.** 4	**25.** 2	**26.** 3	**27.** 3	**28.** 4	**29.** 3	**30.** 2
31. 1	**32.** 1	**33.** 3	**34.** 4	**35.** 3	**36.** 4	**37.** 1	**38.** 1	**39.** 4	**40.** 1
41. 3	**42.** 4	**43.** 1	**44.** 4	**45.** 3	**46.** 3	**47.** 4	**48.** 1	**49.** 2	**50.** 2

PRACTICE EXERCISE 1 (B)

1. 3	**2.** 3	**3.** 2	**4.** 2	**5.** 1	**6.** 4	**7.** 3	**8.** 2	**9.** 4	**10.** 3
11. 1	**12.** 4	**13.** 2	**14.** 2	**15.** 3	**16.** 4	**17.** 1	**18.** 3	**19.** 1	**20.** 1
21. 2	**22.** 2	**23.** 1	**24.** 3	**25.** 3	**26.** 4	**27.** 4	**28.** 1	**29.** 3	**30.** 3
31. 2	**32.** 4	**33.** 2	**34.** 1	**35.** 1	**36.** 2	**37.** 1	**38.** 3	**39.** 3	**40.** 4
41. 2	**42.** 2	**43.** 2	**44.** 2	**45.** 3	**46.** 3	**47.** 4	**48.** 2	**49.** 2	**50.** 4