CHAPTER – 5
ARITHMETIC PROGRESSION

_n_th Term of an ARITHMETIC PROGRESSION (AP)

_n_th term a_n of the AP with first term a and common difference d is given by

$$a_n = a + (n - 1)\, d.$$

IMPORTANT QUESTIONS

Find the 15th term of the 21, 24, 27, . . .

Solution: Here, a = 21, d = 24 – 21 = 3

We know that $a_n = a + (n - 1)d$

So, $a_{15} = a + 14d = 21 + 14(3) = 21 + 42 = 63$

Which term of the AP : 3, 9, 15, 21, . . . , is 99?

Solution: Here, a = 3, d = 9 – 3 = 6

We know that $a_n = a + (n - 1)d$

Let $an = 99 \Rightarrow a + (n - 1)d = 99$

$\Rightarrow 3 + (n - 1)6 = 99 \Rightarrow (n - 1)6 = 99 - 3 = 96$

$\Rightarrow n - 1 = \frac{96}{6} = 16 \Rightarrow n = 16 + 1 = 17$

Hence, 17th term of the given AP is 99

Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution: We have $a_3 = a + (3 - 1)\, d = a + 2d = 5$ ..... (1)

and $a_7 = a + (7 - 1)\, d = a + 6d = 9$ ..... (2)

Solving the pair of linear equations (1) and (2), we get a = 3, d = 1

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Questions for practice

1. Find the 10th term of the AP : 2, 7, 12, . . .

2. Which term of the AP : 21, 18, 15, . . . is – 81?

3. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

4. How many two-digit numbers are divisible by 3?

5. How many three-digit numbers are divisible by 7?

6. How many multiples of 4 lie between 10 and 250?

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

10. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

12. The sum of 4th term and 8th term of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.

13. The sum of 5th term and 9th term of an AP is 72 and the sum of 7th and 12th terms is 97. Find the AP.

14. If the numbers n – 2, 4n – 1 and 5n + 2 are in AP, find the value of n.

15. Find the value of the middle most term (s) of the AP : –11, –7, –3,..., 49.

16. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.

17. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

18. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
19. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
20. The 26th, 11th and the last term of an AP are 0, 3 and –1/5 , respectively. Find the common difference and the number of terms.
21. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.
22. Determine k so that k2+ 4k + 8, 2k2 + 3k + 6, 3k2 + 4k + 4 are three consecutive terms of an AP.
23. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.
24. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.
25. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.
26. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.
27. Which term of the AP: 53, 48, 43,... is the first negative term?
28. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
29. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

***n*th Term from the end of an ARITHMETIC PROGRESSION (AP)**

Let the last term of an AP be '*l*' and the common difference of an AP is 'd' then the nth term from the end of an AP is given by

$$l_n = l - (n-1)\,d.$$

IMPORTANT QUESTIONS

Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Solution : Here, a = 10, d = 7 – 10 = – 3, l = – 62,

We know that nth term from the last is given by $l_n = l - (n-1)\,d$.

$\therefore l_{11} = l - 10d = -62 - 10(-3) = -62 + 30 = -32$

Questions for practice

1. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.
2. Find the 10th term from the last term of the AP : 4, 9 , 14, . . ., 254.
3. Find the 6th term from the end of the AP 17, 14, 11, …… (–40).
4. Find the 8th term from the end of the AP 7, 10, 13, …… 184.
5. Find the 10th term from the last term of the AP : 8, 10, 12, . . ., 126.
6. Find the 12th term from the end of the AP: –2, –4, –6,..., –100.

Sum of First *n* Terms of an ARITHMETIC PROGRESSION (AP)

The sum of the first *n* terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a = first term, d = common difference and n = number of terms.

or

$$S_n = \frac{n}{2}[a + l]$$

where *l* = last term

IMPORTANT QUESTIONS

Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

We know that $S = \frac{n}{2}[2a + (n-1)d]$

$\therefore S = \frac{22}{2}[16 + (22-1)\times(-5)]$ = 11(16 – 105) = 11(–89) = – 979

So, the sum of the first 22 terms of the AP is – 979.

Questions for practice

1. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
2. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?
3. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
4. Find the sum of first 24 terms of the list of numbers whose nth term is given by an = 3 + 2n
5. Find the sum of the first 40 positive integers divisible by 6.
6. Find the sum of the first 15 multiples of 8.
7. Find the sum of the odd numbers between 0 and 50.
8. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.
9. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
10. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
11. If $a_n = 3 - 4n$, show that a_1, a_2, a_3, ... form an AP. Also find S_{20} .
12. In an AP, if $S_n = n(4n + 1)$, find the AP.
13. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.
14. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$
15. Find the sum of first 17 terms of an AP whose 4th and 9th terms are –15 and –30 respectively.
16. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.
17. Find the sum of all the 11 terms of an AP whose middle most term is 30.
18. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.
19. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.
20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.
21. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP upto the term –77.
22. Find the sum of first seven numbers which are multiples of 2 as well as of 9.
23. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first 2n terms of another AP whose first term is – 30 and the common difference is 8. Find n.
24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.
25. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.
26. Find the sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.
27. Find the sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .
28. Find the sum of those integers from 1 to 500 which are multiples of 2 or 5.
29. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.
30. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.
31. Find the sum of the integers between 100 and 200 that are (i) divisible by 9 (ii) not divisible by 9

32. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.
33. Solve the equation : 1 + 4 + 7 + 10 +...+ x =287
34. Solve the equation – 4 + (–1) + 2 +...+ x = 437
35. Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

MCQ QUESTIONS (1 mark)

1. The 10th term of the AP: 5, 8, 11, 14, ... is
(a) 32 (b) 35 (c) 38 (d) 185

2. In an AP if a = –7.2, d = 3.6, an = 7.2, then n is
(a) 1 (b) 3 (c) 4 (d) 5

3. In an AP, if d = –4, n = 7, an = 4, then a is
(a) 6 (b) 7 (c) 20 (d) 28

4. In an AP, if a = 3.5, d = 0, n = 101, then an will be
(a) 0 (b) 3.5 (c) 103.5 (d) 104.5

5. The list of numbers – 10, – 6, – 2, 2,... is
(a) an AP with d = – 16 (b) an AP with d = 4
(c) an AP with d = – 4 (d) not an AP

6. The 11th term of the AP: –5, –5/2 , 0, 5/2 , ...is
(a) –20 (b) 20 (c) –30 (d) 30

7. The first four terms of an AP, whose first term is –2 and the common difference is –2, are
(a) – 2, 0, 2, 4 (b) – 2, 4, – 8, 16 (c) – 2, – 4, – 6, – 8 (d) – 2, – 4, – 8, –16

8. The 21st term of the AP whose first two terms are –3 and 4 is
(a) 17 (b) 137 (c) 143 (d) –143

9. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?
(a) 30 (b) 33 (c) 37 (d) 38

10. Which term of the AP: 21, 42, 63, 84,... is 210?
(a) 9th (b) 10th (c) 11th (d) 12th

11. If the common difference of an AP is 5, then what is a18 – a13 ?
(a) 5 (b) 20 (c) 25 (d) 30

12. What is the common difference of an AP in which a18 – a14 = 32?
(a) 8 (b) – 8 (c) – 4 (d) 4

13. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is
(a) –1 (b) – 8 (c) 7 (d) –9

14. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18^{th} term will be
(a) 7 (b) 11 (c) 18 (d) 0

15. The 4th term from the end of the AP: –11, –8, –5, ..., 49 is
(a) 37 (b) 40 (c) 43 (d) 58

16. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is (a) 0 (b) 5 (c) 6 (d) 15

17. The sum of first 16 terms of the AP: 10, 6, 2,... is
(a) –320 (b) 320 (c) –352 (d) –400

18. In an AP if a = 1, an = 20 and Sn = 399, then n is
(a) 19 (b) 21 (c) 38 (d) 42

19. The sum of first 100 multiples of 3 is
(a) 30300 (b) 15150 (c) 300 (d) none of these

20. The sum of first five multiples of 3 is
(a) 45 (b) 55 (c) 65 (d) 75