PART # 02

TRIGONOMETRY

EXERCISE # 01

SECTION-1 : (ONE OPTION CORRECT TYPE)

401. The difference between the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

(A) $\frac{2}{3}$ (B) $\frac{8}{7}$ (C) $\frac{9}{4}$ (D) $\frac{3}{8}$

402. If in a ΔABC $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(A) A.P. (B) H.P. (C) G.P. (D) A.G.P.

403. If $f(x) = \sin^{4n}x - \cos^{4n}x$ and $g(x) = \sin x + \cos x$, then general solution of $f(x) = \left[g\left(\frac{\pi}{10}\right)\right]$ is (where [.] is greatest integer less than equal to x)

(A) $2n\pi + \frac{\pi}{3}, n \in I$ (B) $n\pi + \frac{\pi}{2}, n \in I$ (C) $n\pi + \frac{\pi}{4}, n \in I$ (D) none of these

404. The maximum value of $(\sin\alpha_1)(\sin\alpha_2) \ldots (\sin\alpha_n)$ under the restrictions $0 \le \alpha_1, \alpha_2, \ldots \alpha_n \le \frac{\pi}{2}$ and $(\tan\alpha_1)(\tan\alpha_2) \ldots (\tan\alpha_n) = 1$ is

(A) $\frac{1}{2^n}$ (B) $\frac{1}{2n}$ (C) $\frac{1}{2^{n/2}}$ (D) 1

405. In a ΔABC, $(a + b + c)(b + c - a) = kbc$ if

(A) $k < 0$ (B) $k > 0$ (C) $0 < k < 4$ (D) $k > 4$

406. Least value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is

(A) $-\frac{\pi^3}{8}$ (B) $-\frac{\pi^3}{32}$ (C) $\frac{\pi^3}{32}$ (D) $\frac{\pi^3}{8}$

407. If r, r_0 be the in–radius and ex-radius of equilateral triangles having sides 2 and 3 respectively, then $r : r_0$ is equal to

(A) 2 : 3 (B) 1 : 3 (C) 1 : 9 (D) 2 : 9

408. In a Δ ABC, given that tan A : tan B : tan C = 3 : 4 : 5, then the value of sin A sin B sin C is

(A) $\frac{2}{\sqrt{5}}$ (B) $\frac{2\sqrt{5}}{7}$ (C) $\frac{2\sqrt{5}}{9}$ (D) $\frac{2}{3\sqrt{5}}$

409. The equation $\sin^{-1}x = |x - a|$ will have atleast one solution if

(A) $a \in [-1, 1]$ (B) $a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) $a \in \left[1-\frac{\pi}{2}, 1+\frac{\pi}{2}\right]$ (D) None of these

410. The solution set of x for which min(sinx, cosx) > min(–sinx, – cosx) where $x \in (0, 2\pi)$

(A) $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$ (B) $(0, \pi)$

(C) $\left(\frac{3\pi}{4}, 2\pi\right)$ (D) None of these

411. The value of $\tan(\sin^{-1} \cos \sin^{-1}x) \tan(\cos^{-1} \sin \cos^{-1}x)$ $\forall\, x \in \left(0, \frac{\pi}{2}\right)$ is

(A) 0 (B) 1

(C) –1 (D) none of these

412. If sin A = sin B and cos A = cos B, then

(A) $A = n\pi + (-1)^n B$ (B) $A = 2n\pi \pm B$

(C) $A = 2n\pi + B$ (D) none of these

413. If in a triangle ABC, tan A + tan B + tan C = 6 and tan A · tan B = 2, then the triangle is

(A) equilateral (B) obtuse angled

(C) acute angled (D) right angled isosceles

414. Inside a big circle exactly n small circles each of radius r can be drawn in such a way that each small circle touches the big circle and two small circles. If $n \ge 3$, then the radius of the bigger circle is

(A) $r\,\text{cosec}\left(\frac{\pi}{n}\right)$ (B) $r\left\{1 + \text{cosec}\left(\frac{\pi}{n}\right)\right\}$

(C) $r\left\{1 + \text{cosec}\left(\frac{2\pi}{n}\right)\right\}$ (D) $r\left\{1 + \text{cosec}\left(\frac{\pi}{2n}\right)\right\}$

415. In a right angled triangle ABC, if $\angle C = \frac{\pi}{2}$ and $\angle A = 2\angle B$, then $\frac{R}{r}$ is

(A) $\frac{\sqrt{3}+1}{2}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{2}{\sqrt{3}-1}$ (D) $\frac{2\sqrt{2}}{\sqrt{3}+1}$

416. If in a triangle $\sin^4 A + \sin^4 B + \sin^4 C = \sin^2 B \sin^2 C + 2 \sin^2 C \sin^2 A + 2 \sin^2 A \sin^2 B$, then angle A can be equal to

(A) 120° (B) 50°

(C) 30° (D) 45°

417. If the hypotenuse of the right angled triangle is twice the length of perpendicular drawn from opposite vertex to it, then the difference of two acute angles is

(A) 75 (B) 0

(C) 30 (D) 60

418. The area of the circle and area of a regular pentagon having perimeter equal to that of the circle are in the ratio

(A) $\tan\left(\frac{\pi}{5}\right) : \frac{\pi}{5}$ (B) $\cot\left(\frac{\pi}{5}\right) : \frac{\pi}{5}$ (C) $\sin\left(\frac{\pi}{5}\right) : \frac{\pi}{5}$ (D) $\cos\left(\frac{\pi}{5}\right) : \frac{\pi}{5}$

419. Let α, β, γ be the altitudes on the sides a, b, c respectively of a ΔABC. If α, β, γ are the roots of the equation $x^3 - 12x^2 + 44x - 48$, then the inradius of the ΔABC is :

(A) $\frac{11}{12}$ (B) $\frac{12}{11}$

(C) 5 (D) 3

420. The maximum value of the function f (x) = $(\sin^{-1}(\sin x))^2 - \sin^{-1}(\sin x)$ is

(A) $\frac{\pi}{4}[\pi + 2]$ (B) $\frac{\pi}{4}[\pi - 2]$

(C) $\frac{\pi}{2}[\pi + 2]$ (D) $\frac{\pi}{2}[\pi - 2]$

421. The value of $\tan^4\frac{\pi}{16} + 4\tan^3\frac{\pi}{16} - 6\tan^2\frac{\pi}{16} - 4\tan\frac{\pi}{16}$ is equal to

(A) 0 (B) 1

(C) –1 (D) 2

422. If $(\sin\theta, \cos\theta)$ and (3, 2) lie on the same side of the line x + y = 1, then θ lies between

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $(0, \pi)$

(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left(0, \frac{\pi}{4}\right)$

423. The number of possible real solutions of $\tan^{-1}(x^2 + x + 1) + \cos^{-1}(x^2 + 2x + 9) = \frac{3\pi}{2}$ is:

(A) 0 (B) 1

(C) 2 (D) 4

424. Sides of a ΔABC are in A.P. If a < min{b, c} and c > max{a, b}, then cosA is :

(A) $\frac{3c - 4b}{2b}$ (B) $\frac{3c - 4b}{2c}$

(C) $\frac{4c - 3b}{2a}$ (D) $\frac{4c - 3b}{2b}$

425. The number of ordered pairs (x, y) satisfying the system of equations given by
sinx + siny = sin(x + y) and |x| + |y| = 1 is

(A) 2 (B) 4

(C) 6 (D) none of these

Section-2 (MORE THAN ONE option correct type)

426. The equation $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x$ has a root for which

(A) sin2x = 1 (B) $\cos 2x = -\frac{1}{2}$ (C) sin2x = – 1 (D) $\cos x = \frac{1}{2}$

427. The side of ΔABC satisfy the equation $2a^2 + 4b^2 + c^2 = 4ab + 2ac$. Then -

(A) the triangle is isosceles (B) the triangle is obtuse

(C) $B = \cos^{-1}\frac{7}{8}$ (D) $A = \cos^{-1}\frac{1}{4}$

428. If $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)(1 + \tan^2 2y)(3 + \sin 3z) = 4$, then

(A) x may be a multiple of π (B) z can be a multiple of π

(C) y can be a multiple of $\frac{\pi}{2}$ (D) x cannot be an even multiple of π

429. If $\cos^{-1}\frac{x^2-1}{x^2+1} + \tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$, then x is equal to

(A) $\sqrt{3}$, when x > 1 (B) $2-\sqrt{3}$, when 0 < x < 1

(C) $2+\sqrt{3}$, when 0 < x < 1 (D) $-\frac{1}{\sqrt{3}}$, when x > 2

430. If inside a big circle exactly 24 small circles, each of radius 2, can be drawn in such a way that each small circle touches the big circle and also touch both its adjacent small circles, then radius of the big circle is

(A) $2\left(1+\operatorname{cosec}\frac{\pi}{24}\right)$ (B) $\left(\frac{1+\tan\frac{\pi}{24}}{\cos\frac{\pi}{24}}\right)$ (C) $2\left(1+\operatorname{cosec}\frac{\pi}{12}\right)$ (D) $\frac{2\left(\sin\frac{\pi}{48}+\cos\frac{\pi}{48}\right)^2}{\sin\frac{\pi}{24}}$

431. If $\tan^{-1}\left(x^2 + 3|x| - 4\right) + \cot^{-1}\left(4\pi + \sin^{-1}\sin 14\right) = \frac{\pi}{2}$, then the value of $\sin^{-1}\sin 2x$ is equal to

(A) $6 - 2\pi$ (B) $2\pi - 6$ (C) $\pi - 3$ (D) $3 - \pi$

432. Let f(x) = ab sinx + $b\sqrt{1-a^2}$ cosx + c, where |a| < 1, b > 0 then

(A) maximum value of f(x) is b if c = 0

(B) difference of maximum and minimum value of f(x) is 2b

(C) f(x) = c if x = $-\cos^{-1}a$ (D) f(x) = c if x = $\cos^{-1}a$

433. If in a triangle ABC, $A \le B \le C$ and $\sin A \le \sin B \le \sin C$, then the triangle may be

(A) equilateral (B) isosceles (C) obtuse angled (D) right angled

434. If $\cos\alpha = \frac{1}{2}\left(x + \frac{1}{x}\right)$ and $\cos\beta = \frac{1}{2}\left(y + \frac{1}{y}\right)$, (xy > 0) x, y, $\alpha, \beta \in R$ then

(A) $\sin(\alpha + \beta + \gamma) = \sin\gamma \ \forall \ \gamma \in R$ (B) $\cos\alpha \cos\beta = 1 \ \forall \ \alpha, \beta \in R$

(C) $(\cos\alpha + \cos\beta)^2 = 4 \ \forall \ \alpha, \beta \in R$ (D) $\sin(\alpha + \beta + \gamma) = \sin\alpha + \sin\beta + \sin\gamma \ \forall \ \alpha, \beta, \gamma \in R$

435. If $(a\cos\theta_1, a\sin\theta_1)$, $(a\cos\theta_2, a\sin\theta_2)$, $(a\cos\theta_3, a\sin\theta_3)$ represents the vertices of an equilateral triangle inscribed in a circle, then

(A) $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$ (B) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0$

(C) $\tan\theta_1 + \tan\theta_2 + \tan\theta_3 = 0$ (D) $\cot\theta_1 + \cot\theta_2 + \cot\theta_3 = 0$

436. $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left(\frac{1}{3}\tan\theta\right)$, then $\tan\theta$ is

(A) -2 (B) 1 (C) $\frac{2}{3}$ (D) 2

437. If $0 < \alpha, \beta < \pi$ and $\cos\alpha + \cos\beta - \cos(\alpha + \beta) = \frac{3}{2}$ then

(A) $\alpha = \frac{\pi}{3}$ (B) $\beta = \frac{\pi}{3}$ (C) $\alpha = \beta$ (D) $\alpha + \beta = \frac{\pi}{3}$

438. Let ABC be an isosceles triangle with base BC. If 'r' is the radius of the circle inscribed in ΔABC and 'ρ' be the radius of the circle described opposite to the angle A, then the product 'ρr' can be equal to

(A) $R^2 \sin^2 A$ (B) $R^2 \sin^2 2B$ (C) $\frac{1}{2}a^2$ (D) $\frac{a^2}{4}$

439. Which of the following functions have maximum value unity?

(A) $\sin^2 x - \cos^2 x$ (B) $\sqrt{\frac{6}{5}}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{3}}\cos x\right)$

(C) $\cos^6 x + \sin^6 x$ (D) $\cos^2 x + \sin^4 x$

440. Let $S_n = \tan^{-1}\frac{4}{7} + \tan^{-1}\frac{4}{19} + ... + \tan^{-1}\frac{4}{4n^2+3}$, then

(A) $S_n = \tan^{-1}\left(\frac{2n+5}{4n}\right)$ (B) $S_n = \cot^{-1}\left(\frac{2n+5}{4n}\right)$

(C) $S_\infty = \tan^{-1}2$ (D) $S_\infty = \cot^{-1}2$

441. If $\cos\alpha = \frac{1}{2}\left(x + \frac{1}{x}\right)$ and $\cos\beta = \frac{1}{2}\left(y + \frac{1}{y}\right)$, $(xy > 0)$ $x, y, \alpha, \beta \in R$ then

(A) $\sin(\alpha + \beta + \gamma) = \sin\gamma \ \forall\ \gamma \in R$ (B) $\cos\alpha \cos\beta = 1 \ \forall\ \alpha, \beta \in R$

(C) $(\cos\alpha + \cos\beta)^2 = 4 \ \forall\ \alpha, \beta \in R$ (D) $\sin(\alpha + \beta + \gamma) = \sin\alpha + \sin\beta + \sin\gamma \ \forall\ \alpha, \beta, \gamma \in R$

442. If [x] represents the greatest integer less than or equal to x, then which of the following statement is true

(A) sin[x] = cos[x] has no solution (B) sin[x] = tan[x] has infinitely many solutions

(C) sin[x] = cos[x] possess unique solution (D) sin[x] = tan[x] for no value of x

443. Triangles $A_1A_2A_3$ and $B_1B_2B_3$ have side lengths a_1, a_2, a_3 and b_1, b_2, b_3 respectively satisfying the relation $\sqrt{a_1+a_2+a_3}\sqrt{b_1+b_2+b_3} = \sqrt{a_1b_1} + \sqrt{a_2b_2} + \sqrt{a_3b_3}$, then which one of the following statements is/are true?

(A) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (B) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$

(C) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are similar (D) $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ are congruent

444. If $\theta_R \in [0, \pi]$ for $1 \le k \le 10$, then the maximum value of $\prod_{R=1}^{10}(1+\sin^2\theta_R)(1+\cos^2\theta_R)$ is -

(A) $\left(\frac{3}{2}\right)^{10}$ (B) $\left(\frac{9}{4}\right)^{10}$ (C) $\left(\frac{3}{2}\right)^{20}$ (D) $\left(\frac{9}{4}\right)^{5}$

445. If $0 \le \alpha, \beta \le \frac{\pi}{2}$ and $\cos\alpha + \cos\beta = 1$, then -

(A) $\alpha + \beta \ge \frac{\pi}{2}$ (B) $\cos(\alpha + \beta) \le 0$ (C) $\alpha + \beta \le \frac{\pi}{2}$ (D) $\cos(\alpha + \beta) \ge 0$

446. If $3 \sin \beta = \sin (2\alpha + \beta)$, then $\tan (\alpha + \beta) - 2 \tan \alpha$ is :
(A) independent of α (B) independent of β
(C) dependent of both α and β (D) independent of α but dependent of β

447. If $x = \sec \phi - \tan \phi$ & $y = \operatorname{cosec} \phi + \cot \phi$ then:
(A) $x = \frac{y+1}{y-1}$ (B) $y = \frac{1+x}{1-x}$
(C) $x = \frac{y-1}{y+1}$ (D) $xy + x - y + 1 = 0$

448. $(a + 2) \sin \alpha + (2a - 1) \cos \alpha = (2a + 1)$ if $\tan \alpha =$
(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{2a}{a^2+1}$ (D) $\frac{2a}{a^2-1}$

449. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:
(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n \in I$ (B) $x = n\pi + (-1)^n (\pi/6)$, $n \in I$
(C) $x = n\pi + (-1)^n (\pi/3)$, $n \in I$ (D) $x = n\pi - (-1)^n (\pi/6)$, $n \in I$

450. If the numerical value of $\tan (\cos^{-1} (4/5) + \tan^{-1} (2/3))$ is a/b then
(A) $a + b = 23$ (B) $a - b = 11$ (C) $3b = a + 1$ (D) $2a = 3b$

451. It is known that $\sin \beta = \frac{4}{5}$ & $0 < \beta < \pi$ then the value of $\frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos\frac{\pi}{6}}\cos(\alpha+\beta)}{\sin\alpha}$ is:
(A) independent of α for all β in $(0, \pi)$ (B) $\frac{5}{\sqrt{3}}$ for $\tan \beta > 0$
(C) $\frac{\sqrt{3}(7+24\cot\alpha)}{15}$ for $\tan \beta < 0$ (D) None

452. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha + \beta)\}$, then the length of the hypotenuse is:
(A) $2[1+\cos(\alpha - \beta)]$ (B) $2[1 - \cos(\alpha + \beta)]$ (C) $4 \cos^2 \frac{\alpha - \beta}{2}$ (D) $4\sin^2 \frac{\alpha + \beta}{2}$

453. If $\tan x = \frac{2b}{a-c}$, $(a \neq c)$
$y = a \cos^2 x + 2b \sin x \cos x + c \sin^2 x$
$z = a \sin^2 x - 2b \sin x \cos x + c \cos^2 x$, then
(A) $y = z$ (B) $y + z = a + c$ (C) $y - z = a - c$ (D) $y - z = (a - c)^2 + 4b^2$

454. $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$
(A) $2 \tan^n \frac{A-B}{2}$ (B) $2 \cot^n \frac{A-B}{2}$: n is even
(C) 0 : n is odd (D) none

455. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if
(A) $a \in (-1, 1)$ (B) $a \in \left(-1, -\frac{1}{2}\right)$ (C) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ (D) $a \in \left(\frac{1}{2}, 1\right)$

456. cos4x cos8x – cos5x cos9x = 0 if

(A) cos12x = cos 14 x (B) sin13 x = 0

(C) sinx = 0 (D) cosx = 0

457. In a ΔABC, following relations hold good. In which case(s) the triangle is a right angled triangle?

(A) $r_2 + r_3 = r_1 - r$ (B) $a^2 + b^2 + c^2 = 8R^2$

(C) $r_1 = s$ (D) $2R = r_1 - r$

458. In a triangle ABC, with usual notations the length of the bisector of angle A is :

(A) $\dfrac{2bc\cos\frac{A}{2}}{b+c}$ (B) $\dfrac{2bc\sin\frac{A}{2}}{b+c}$ (C) $\dfrac{abc\,\text{cosec}\,\frac{A}{2}}{2R(b+c)}$ (D) $\dfrac{2\Delta}{b+c}\cdot\text{cosec}\,\dfrac{A}{2}$

459. AD, BE and CF are the perpendiculars from the angular points of a Δ ABC upon the opposite sides, then :

(A) $\dfrac{\text{Perimeter of }\Delta DEF}{\text{Perimeter of }\Delta ABC} = \dfrac{r}{R}$ (B) Area of ΔDEF = 2 Δ cosA cosB cosC

(C) Area of ΔAEF = Δ $\cos^2 A$ (D) Circum radius of ΔDEF = $\dfrac{R}{2}$

460. In a triangle ABC, points D and E are taken on side BC such that BD = DE = EC. If angle ADE = angle AED = θ, then:

(A) $\tan\theta = 3\tan B$ (B) $3\tan\theta = \tan C$

(C) $\dfrac{6\tan\theta}{\tan^2\theta - 9} = \tan A$ (D) angle B = angle C

461. With usual notation, in a Δ ABC the value of $\Pi(r_1 - r)$ can be simplified as:

(A) $abc\,\Pi\tan\dfrac{A}{2}$ (B) $4rR^2$ (C) $\dfrac{(abc)^2}{R(a+b+c)^2}$ (D) $4Rr^2$

462. α, β and γ are three angles given by

$\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\dfrac{1}{\sqrt{2}} + \sin^{-1}\left(-\dfrac{1}{2}\right)$ and $\gamma = \cos^{-1}\dfrac{1}{3}$. Then

(A) α > β (B) β > γ (C) α < γ (D) α > γ

463. $\cos^{-1}x = \tan^{-1}x$ then

(A) $x^2 = \left(\dfrac{\sqrt{5}-1}{2}\right)$ (B) $x^2 = \left(\dfrac{\sqrt{5}+1}{2}\right)$

(C) $\sin(\cos^{-1}x) = \left(\dfrac{\sqrt{5}-1}{2}\right)$ (D) $\tan(\cos^{-1}x) = \left(\dfrac{\sqrt{5}-1}{2}\right)$

464. For the equation $2x = \tan(2\tan^{-1}a) + 2\tan(\tan^{-1}a + \tan^{-1}a^3)$, which of the following is invalid?

(A) $a^2x + 2a = x$ (B) $a^2 + 2ax + 1 = 0$

(C) $a \neq 0$ (D) $a \neq -1, 1$

465. The sum $\sum\limits_{n=1}^{\infty} \tan^{-1}\dfrac{4n}{n^4 - 2n^2 + 2}$ is equal to:

(A) $\tan^{-1}2 + \tan^{-1}3$ (B) $4\tan^{-1}1$ (C) $\pi/2$ (D) $\sec^{-1}(-\sqrt{2})$

SECTION - 3: (COMPREHENSION TYPE)

COMPREHENSION-1

Paragraph for Questions Nos. 466 to 468

The functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$, $\text{cosec}^{-1}x$ and $\sec^{-1}x$ are called inverse circular or inverse trigonometric functions which are defined as follows

$\sin^{-1}x$	$-1 \le x \le 1$	$\frac{\pi}{2} \le \sin^{-1} x \le \frac{3\pi}{2}$	
$\cos^{-1}x$	$-1 \le x \le 1$	$-\pi \le \cos^{-1} x \le 0$	
$\tan^{-1}x$	$x \in R$	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$	
$\text{cosec}^{-1}x$	$\|x\| \ge 1$	$\frac{\pi}{2} \le \text{cosec}^{-1}x \le \frac{3\pi}{2}$	$\ne \pi$
$\sec^{-1}x$	$\|x\| \ge 1$	$-\pi \le \sec^{-1} x \le 0$	$\ne -\frac{\pi}{2}$
$\cot^{-1}x$	$x \in R$	$0 < \cot^{-1} x < \pi$	

466. For $x \in [0, 1]$, $\sin^{-1} x$ is equal to

(A) $\cos^{-1}\sqrt{1-x^2} + \pi$ (B) $\cos^{-1}\sqrt{1-x^2} + \frac{\pi}{2}$ (C) $-\cos^{-1}\sqrt{1-x^2}$ (D) None of these

467. Number of solutions of $\tan^{-1}|x| - \cos^{-1} x = 0$ is/ are

(A) 2 (B) 1

(C) 0 (D) None of these

468. $\lim\limits_{x\to 0} \tan\left(\frac{\sin^{-1} x^4 + \sin^{-1} x^9}{4}\right)$

(A) Does not exist as L.H.L. and R.H.L. both are finite and unequal

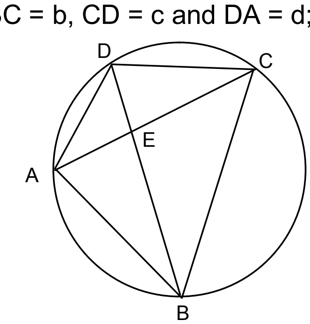
(B) Exist as L.H.L. = R.H.L.

(C) R.H.L. and L.H.L. are unequal (D) None of these

COMPREHENSION-2

Paragraph for Questions Nos. 469 to 471

ABCD be a cyclic quadrilateral and AB = a, BC = b, CD = c and DA = d; AC = x and BD = y,



469. If a = 2, b = 6, c = 4, d = 3 and y = 5, then the value of x will be

(A) 26 (B) $\frac{18}{5}$ (C) $\frac{8}{5}$ (D) $\frac{26}{5}$

470. If a, b, c and d have above values, then the value of $\angle B$ will be

(A) $\cos^{-1}\left(-\frac{15}{48}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$ (C) $\cos^{-1}\left(-\frac{1}{2}\right)$ (D) $\cos^{-1}\left(\frac{15}{48}\right)$

471. The minimum value of $\frac{(a^2+b^2+c^2)}{d^2}$ in any quadrilateral, where a, b, c and d are sides of quadrilateral, will be

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

COMPREHENSION-3

Paragraph for Questions Nos. 472 to 474

Consider the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=k,$ where x and k are real.

472. The values of x for which the equation is defined

(A) $x \neq n\pi$, $x \neq (2n-1)\frac{\pi}{2}$, $n \in I$

(B) $x \neq n\pi$, $x \neq (2n+1)\frac{\pi}{2}$, $n \in I$

(C) $x \neq n\pi$, $x \neq (4n+1)\frac{\pi}{2}$, $n \in I$

(D) none of these

473. The least value of 'k' for which the given equation has a solution in $\left(0, \frac{\pi}{2}\right)$ must be

(A) 3 (B) 6 (C) 9 (D) 5

474. If K = 10, then the number of solution in $\left(0, \frac{\pi}{2}\right)\cup\left(\frac{\pi}{2}, \pi\right)$ must be

(A) 0 (B) 1 (C) 2 (D) none of these

COMPREHENSION-4

Paragraph for Questions Nos. 475 to 477

ΔABC is inscribed in a circle and AL, BM and CN are diameters (2R) of circumcircle of ΔABC, then

475. The area of ΔBLC is

(A) $R^2 \sin A \sin B \sin C$

(B) $2R^2 \sin A \cos B \cos C$

(C) $2R^2 \sin A \sin B \sin C$

(D) $R^2 \sin A \sin B \cos C$

476. Area of ΔANB is

(A) $2R^2 \sin A \sin B \sin C$

(B) $2R^2 \sin A \sin B \cos C$

(C) $2R^2 \sin A \cos B \cos C$

(D) $2R^2 \sin C \cos A \cos B$

477. Area of ΔBLC + area of ΔCMA + area of ΔANB is equal to

(A) $\frac{abc}{R}$

(B) $\frac{abc}{2R}$

(C) $\frac{abc}{3R}$

(D) None of these

COMPREHENSION-5

Paragraph for Questions Nos. 478 to 480

Let Δ ABC be an equilateral triangle of sides of length a. On side AB produced, a point P is chosen such that PA = AB.

478. Inradius of ΔAPC is

(A) $\frac{a\sqrt{3}}{2}$ (B) $\frac{a}{2}$

(C) $\frac{a\sqrt{3}}{2\left(2+\sqrt{3}\right)}$ (D) None of these

479. Circumradius of ΔPBC is

(A) 2a (B) a

(C) $\frac{a}{2}$ (D) $\frac{a\sqrt{3}}{2}$

480. Let the excircle of ΔPBC w.r.t. side BC touch PC produced at E, then CE is equal to

(A) $\frac{3a+a\sqrt{3}}{2}$ (B) $\frac{3a-a\sqrt{3}}{2}$

(C) $a\sqrt{3}$ (D) None of these

COMPREHENSION-6

Paragraph for Questions Nos. 481 to 483

In a triangle ABC, the equation of the side BC is $2x - y = 3$ and its circumcentre and orthocentre are at (2, 4) and(1, 2) respectively.

481. Circumradius of triangle ABC is

(A) $\sqrt{\frac{61}{5}}$ (B) $\sqrt{\frac{51}{5}}$

(C) $\sqrt{\frac{41}{5}}$ (D) $\sqrt{\frac{43}{5}}$

482. The value of sin B sin C is equal to

(A) $\frac{9}{2\sqrt{61}}$ (B) $\frac{9}{4\sqrt{61}}$

(C) $\frac{9}{\sqrt{61}}$ (D) $\frac{9}{3\sqrt{61}}$

483. The distance of orthocentre to vertex A is equal to

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{6}{\sqrt{5}}$

(C) $\frac{3}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{5}}$

COMPREHENSION-7

Paragraph for Questions Nos. 484 to 486

Let S_1 be the set of all those solutions of the equation $(1 + a)\cos\theta\cos(2\theta - b) = (1 + a\cos2\theta)\cos(\theta - b)$ which are independent of a and b and S_2 be the set of all such solutions which are dependent on a and b. Then

484. The set S_1 and S_2 are

(A) $\{n\pi, n \in Z\}$ and $\frac{1}{2}\{n\pi + (-1)^n \sin^{-1}(a \sin b) + b; n \in Z\}$

(B) $\{n\frac{\pi}{2}, n \in Z\}$ and $\{n\pi + (-1)^n \sin^{-1}(a \sin b); n \in Z\}$

(C) $\{n\frac{\pi}{2}, n \in Z\}$ and $\{n\pi + (-1)^n \sin^{-1}(\frac{a}{2}\sin b); n \in Z\}$

(D) None of these

485. Conditions that should be imposed on a and b such that S_2 is non-empty

(A) $\left|\frac{a}{2}\sin b\right| < 1$ (B) $\left|\frac{a}{2}\sin b\right| \le 1$ (C) $|a \sin b| \le 1$ (D) None of these

486. All the permissible values of b if a = 0 and S_2 is a subset of $(0, \pi)$:

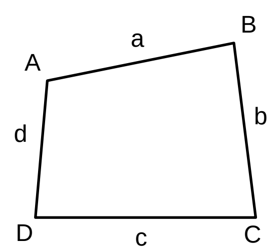
(A) $b \in (-n\pi, 2n\pi), n \in Z$ (B) $b \in (-n\pi, 2\pi - n\pi), n \in Z$

(C) $b \in (-n\pi, n\pi), n \in Z$ (D) None of these

COMPREHENSION-8

Paragraph for Questions Nos. 487 to 489

A quadrilateral ABCD is such that a circle can be inscribed in it and a circle can be circumscribed about it.



487. If $\frac{a}{b} = \frac{c}{d}$, then

(A) $\angle A = 90°$ (B) $\angle A = 90°$ (C) $\angle B = 90°$ (D) $\angle C = 90°$

488. $\tan^2\left(\frac{A}{2}\right)$ is

(A) $\frac{ab}{cd}$ (B) $\frac{bc}{ad}$

(C) $\frac{ac}{bd}$ (D) $\frac{bd}{ac}$

489. Let P_1 and P_2 be the points of contact of AB and AD respectively with the incircle of quadrilateral ABCD. Then $\cos A + \cos\angle P_1OP_2$ (where O is incentre of quadrilateral ABCD)

(A) 2cosB (B) 0

(C) 1 (D) can't be determined

COMPREHENSION-9

Paragraph for Questions Nos. 490 to 492

If $\theta = (2n + 1)\dfrac{\pi}{7}$ and n = 0, 1, 2, 3, 4, 5, 6, then

490. The value of $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7}$ is

(A) 4 (B) -3 (C) 3 (D) None of these

491. The value of $\sec^2\frac{\pi}{7}\sec^2\frac{3\pi}{7} + \sec^2\frac{3\pi}{7}\sec^2\frac{5\pi}{7} + \sec^2\frac{5\pi}{7}\sec^2\frac{\pi}{7}$ is

(A) -80 (B) 80 (C) 24 (D) -24

492. The value of $\tan^2\frac{\pi}{7}\tan^2\frac{3\pi}{7}\tan^2\frac{5\pi}{7}$ is

(A) 6 (B) 7 (C) 8 (D) None of these

COMPREHENSION-10

Paragraph for Questions Nos. 493 to 495

Consider $\left(1+\sin\theta+\sin^2\theta\right)^n = \sum_{r=0}^{2n} a_r (\sin\theta)^r$; $\theta \in R$.

493. $a_{n+1} + a_{n+2} + \ldots + a_{2n-1}$ equals

(A) $\dfrac{3^n}{2}$ (B) $\dfrac{3^n - a_n}{2}$

(C) $2(3^n - a_n)$ (D) $3^n - a_n$

494. $a_0^2 - a_1^2 + a_2^2 - \ldots a_{2n}^2$ is equal to

(A) a_n (B) a_n^2

(C) $2a_n^2$ (D) $\dfrac{a_n}{2}$

495. The value of $a_0 + 2a_1 + 3a_2 + \ldots + (2n + 1)a_{2n}$ is

(A) $n\,3^{n-1}$ (B) $n\,3^n$

(C) $(n + 1)3^n$ (D) None of these

SECTION - 4 (MATRIX MATCH Type)

496. Match the following:

List – I	List – II
(A) $\sin x \cos^3 x > \cos x \sin^3 x$, $0 \le x \le 2\pi$ is	(i) $\left[-\pi, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$
(B) $4\sin^2 x - 8\sin x + 3 \le 0$, $0 \le x \le 2\pi$, is	(ii) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$
(C) $\lvert\tan x\rvert \le 1$ and $x \in [-\pi, \pi]$ is	(iii) $\left(0, \frac{\pi}{4}\right)$
(D) $\cos x - \sin x \ge 1$ and $0 \le x \le 2\pi$	(iv) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

497. Match the following :

List – I	List – II
(A) The number of pairs (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ $\lvert x\rvert + \lvert y\rvert = 1$ is	(i) 3
(B) The number of values of x for which f (x) is valid $f(x) = \sqrt{\sec^{-1}\left(\frac{1-\lvert x\rvert}{2}\right)}$	(ii) 8
(C) If $x, y \in [0, 2\pi]$, then total number of ordered pairs (x, y) satisfying $\sin x \cos y = 1$	(iii) ∞
(D) $f(x) = \sin x - \cos x - kx + b$ decreases for all values of real values of x when $4\sqrt{2}k$ is always greater than	(iv) 6

498. Match the following

List – I	List – II
(A) If $y = \cos^{-1}(\cos x)$ then for $-\pi \le x \le 0$, value of y is	(i) $x - \pi$
(B) For $x \in (-\infty, -1] \cup (1, \infty)$ if $y = \sec(\sec^{-1} x)$, then value of y is equal t	(ii) $x + \pi$
(C) For $\frac{\pi}{2} < x < \frac{3\pi}{2}$ $y = \tan^{-1}(\tan x)$, then value of y is equal to	(iii) x
(D) For $-\frac{3\pi}{2} < x < -\frac{\pi}{2}$ if $y = \tan^{-1}(\tan x)$, then value of y is equal to	(iv) $-x$

499. Match the following :

List – I	List – II
(A) f(x) = $\int_0^{\sin x} t^2 dt$, then period of $f'(x)$ is	(i) $\frac{\pi}{14}$
(B) If area of ellipse $b^2x^2 + a^2y^2 = a^2b^2$ (a > b), enclosed by x-axis and the ordinates x = 0 and x = b be $\frac{1}{8}$th the area of entire ellipse, then $e\sqrt{1-e^2} + \sin^{-1}\sqrt{1-e^2}$ =	(ii) $\frac{\pi}{2}$
(C) Let f(x) = $\frac{\operatorname{cosec}^{-1}x + \cos^{-1}\left(\frac{1}{x}\right)}{\operatorname{cosec}x}$, then greatest value is	(iii) $\frac{\pi}{4}$
(D) $\cos^{-1}\left(\sin\left(\frac{46\pi}{7}\right)\right)$ is	(iv) 2π

500. Match the following :

List – I	List – II
(A) Period of $\tan\frac{\pi}{2}[x]$ (where [.] denotes the greatest integer function)	(i) 2π
(B) Period of $\sin\left(2\pi x + \frac{\pi}{3}\right) + 2\sin\left(3\pi x + \frac{\pi}{4}\right) + 3\sin 5\pi x$	(ii) $\frac{\pi}{2}$
(C) Period of $\sin^4 x + \cos^4 x$	(iii) 2
(D) Period of $1 + \sin^{10} x$	(iv) π

501. Match the following :

List – I	List – II
(A) The number of roots of equation 2 cosx – 2x + 1 = 0 in the interval $\left[\frac{\pi}{2}, \pi\right]$ is	(i) 2
(B) The number of solutions of $10[\ln x] + 10[2^x] = 31 + 10[\sin x]$ (where [.] denotes the greatest integer function) is	(ii) 3
(C) The number of solutions of $e^{-x^2} = \cos x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is	(iii) 0
(D) If tangents to the parabola $y^2 = 4x$ are normal to $x^2 = 4by$, $\lvert b\rvert < \frac{1}{\sqrt{k}}$, then the numerical quantity k should be	(iv) 8

502. Match the following:

List – I	List – II
(A) Fundamental period of $f(x) = \sec^2x - \tan^2x$ is	(i) no fundamental period
(B) Fundamental period of $f(x) = \sin^2x + \cos^2x$ is	(ii) π
(C) Fundamental period of $f(x) = \tan x.\cot x$ is	(iii) $\pi/2$
(D) Fundamental period of $f(x) = \text{cosec}^2x - \cot^2x + \{x\}$	(iv) non-periodic

503. Match the following:

List – I	List – II
(A) The value of 'a' for which the equation $4\ \text{cosec}^2\pi(a + x) + a^2 - 4a = 0$ has real solution is	(i) 1
(B) The number of solutions of equation $\tan^2x - \sec^{10}x + 1 = 0$ in (0, 10) is	(ii) 2
(C) $\sum_{n=1}^{\infty}\sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} = \frac{\pi}{a}$	(iii) 3

504. Match the list:

List – I	List – II
(A) In a ΔABC if area $\Delta = a^2 - (b - c)^2$, then 15tanA is	(i) $\frac{1}{3}$
(B) In a ΔABC $a = 6$, $b = 3$ and $\cos(A - B) = \frac{4}{5}$, then area of ΔABC is	(ii) 2
(C) If a, b, c, d are the sides of quadrilateral, then the minimum value of $\frac{a^2+b^2+c^2}{d^2}$ is	(iii) 3
(D) ΔPRQ is right angled triangle where P(3, 1), Q(6, 5) and R(x, y) and area of $\Delta PRQ = 7$, then number of such point R is	(iv) 8
	(v) 9

505. Match the following:

List – I	List – II
(A) $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\ldots+\infty$	(i) $\frac{\pi}{2}$
(B) $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)$	(ii) $\frac{\pi}{4}$
(C) $\sin^{-1}\left(\frac{4}{5}\right)+2\tan^{-1}\left(\frac{1}{3}\right)$	(iii) π
(D) $\cot^{-1}9+\text{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)$	(iv) $\frac{\pi}{3}$

506. Match the following with their minimum values, ($x \in R$)

(A) sinx + cosx — (i) –1

(B) sinx + |cosx| — (ii) $-\sqrt{2}$

(C) |sinx| + cosx — (iii) 1

(D) |sinx| + |cosx| — (iv) none

507. Match the following:

List I	**List – II**
(A) $\sin^{-1}x - \cos^{-1}x = 0$, then $\cos(5\cos^{-1}x + \sin^{-1}x)$ is equal to	(i) 3
(B) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right) = \frac{p}{q}$, (where p and q are coprime), then $3q - p$ is equal to	(ii) 2
(C) $\sin^{-1}x + 4\cos^{-1}x = 2\pi$, then x is equal to	(iii) 1
(D) In ΔABC, 2cosA sinC = sinB, then $\frac{2a}{c}$ is equal to	(iv) 0

508. Match the following:

List –I	List-II
(A) Number of solutions of the equation $\sin^{-1}x + \cos^{-1}x^2 = \frac{\pi}{2}$	(i) 1
(B) The number of ordered pairs (x, y) satisfying $\frac{\sin^{-1}x}{x} + \frac{\sin^{-1}y}{y} = 2$ is	(ii) 2
(C) Number of solutions of the equation cos(cosx) = sin(sinx) is	(iii) 0
(D) Number of solutions of the equation $\tan\left(x + \frac{\pi}{6}\right)$ = 2 tanx is	(iv) 3

509. Match the following

(A) Number of solutions of the equation $\sin^{-1}x + \cos^{-1}x^2 = \frac{\pi}{2}$ — (1) 1

(B) The number of ordered pairs (x, y) satisfying $\frac{\sin^{-1}x}{x} + \frac{\sin^{-1}y}{y} = 2$ is — (2) 2

(C) Number of solutions of the equation cos(cosx) = sin(sinx) is — (3) 0

(D) Number of solutions of the equation $\tan\left(x + \frac{\pi}{6}\right)$ = 2 tanx is — (4) 3

510. Match the following pair of curves with their angle of intersections :

	Column– I		**Column – II**
(A)	$x^2 + y^2 = 2\pi^2$ and $y = \sin^{-1}\left[x^2+\frac{1}{2}\right]+\cos^{-1}\left[x^2-\frac{1}{2}\right]$ (where [x] = greatest integer function)	(P)	$\frac{\pi}{4}$
(B)	$y^2 = 2x$ and $y = [\lvert\sin x\rvert + \lvert\cos x\rvert]$ where [x] = greatest integer function	(Q)	$\cot^{-1}(1/3)$
(C)	$x^2 = 4ay$, $y = \frac{8a^3}{x^2+4a^2}$	(R)	$\frac{3\pi}{4}$
(D)	$y^2 = \frac{2x}{\pi}$, $y = \sin x$	(S)	$\cot^{-1}(\pi)$

Section-5 : (INTEGER type)

511. The total number of positive integral solution of $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)+\cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right)=\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$ is ______________

512. If circum radius of ΔABC is 3 cm and its area is 6 cm^2 and DEF is triangle formed by foot of perpendicular drawn from A, B, C on sides BC, CA, AB respectively then perimeter of ΔDEF in cm is ________.

513. The greatest and least values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are I_{max} and I_{min} then $\frac{I_{max}}{I_{min}}$ is ___________

514. In a ΔABC, b cotB + c cotC = 2(r + R). If the base AC = 3 units and $\angle A = 60°$, BC is _____________

515. If in a ΔABC, a = 2, b = 3, c = 4, then the value of $a^3 \cos(B - C) + b^3 (C - A) + c^3 \cos(A - B)$ is ____________

516. In a right angled triangle ΔABC with C as a right angle, a perpendicular CD is drawn to AB. The radii of the circles inscribed into the triangles ACD and BCD are equal to 3 and 4 respectively. Then the radius of the circle inscribed into the ΔABC is _____________

517. In ΔABC, $\frac{\sum a\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{\sum \sin A} = nR$ where R is the radius of circumcircle, then n is equal to

518. In the quadrilateral, the length AC and BD are x and y respectively, AB = 5, BC = 7, CD = 6, AD = 8 and if angle between OD and OC is ω, where O is the point of intersection of two diagonals then, the value of 2xy cos ω is _______________

519. In an acute angled triangle the minimum value of secA secB secC(1 + secA)(1 + secB)(1 + secC) is——— .

520. P is any point and O being the origin. On the circle with OP as diameter two point Q and R are on same side of OP such that $\angle POQ = \angle QOR = \theta$. Let P, Q, R be z_1, z_2, z_3 such that $2\sqrt{3}z_2^2 = \left(2+\sqrt{3}\right)z_1 z_3$. Then the degree measure of θ is———— .

521. In a triangle ABC, side AB = 20, AC = 11, BC = 13, then the diameter of the semicircle inscribed in triangle ABC, whose diameter lies on AB and is touching AC and BC is _________

522. If $x^2 + y^2 \le 1$, then $\min\left\{\frac{kx^2}{y^2} + \frac{1}{k}\left(\frac{y+kx^2}{x^2}\right)\right\}$ (where k is positive) is ———————

523. The number of solutions that the equation sin(cos(sinx)) = cos(sin(cosx)) has in $\left[0, \frac{\pi}{2}\right]$ is _________.

524. If $\sin^{-1}x \in \left(0, \frac{\pi}{2}\right)$, then the value of $\tan\left[\frac{\cos^{-1}\left(\sin\left(\cos^{-1}x\right)\right) + \sin^{-1}\left(\cos\left(\sin^{-1}x\right)\right)}{2}\right]$ is ________

525. If $\sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = k$, when θ lies in third quadrant, then k is equal to

526. The smallest positive integral value of p for which the equation tan (p sin x) = cot (p cos x) in x has a solution in $[0, 2\pi]$ is :

527. Let $A_1, A_2, \ldots\ldots, A_n$ be the vertices of an n–sided regular polygon such that; $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.

Find the value of n.

528. Find the value of $\operatorname{cosec}\frac{4\pi}{15} + \operatorname{cosec}\frac{8\pi}{15} + \operatorname{cosec}\frac{16\pi}{15} + \operatorname{cosec}\frac{32\pi}{15}$

529. In any ΔABC, then minimum value of $\frac{r_1 r_2 r_3}{r^3}$ is equal to :

530. The radii r_1, r_2, r_3 of escribed circles of a triangle ABC are in harmonic progression. If its area is 24 sq. cm and its perimeter is 24 cm, find the sum of squares of lengths of its sides.

CO-ORDINATE GEOMETRY

EXERCISE # 01

SECTION-1 : (ONE OPTION CORRECT TYPE)

531. The co-ordinates of the point on the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$ are

(A) $(-2, 4)$ (B) $(2, -4)$ (C) $(2, 4)$ (D) none of these

532. The values of k for which the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 4\lambda x + 8 = 0$ have two common tangents is

(A) $\left(-\frac{9}{4}, \frac{9}{4}\right)$ (B) $\left(-\infty, -\frac{9}{4}\right) \cup \left(\frac{9}{4}, \infty\right)$

(C) $\left(-\infty, -\frac{9}{4}\right] \cup \left[\frac{9}{4}, \infty\right)$ (D) None of these

533. The chords of the hyperbola $x^2 - y^2 = a^2$ touches parabola $y^2 = 4ax$, then the locus of their mid-point is

(A) $y^2 (a - x) = x^3$ (B) $x^2 (a + x) = y^3$ (C) $y^2 (x - a) = x^3$ (D) $y^2 (x + a) = x^3$

534. Consider a triangle ABC, where B and C are $(-a, 0)$ and $(a, 0)$ respectively. A be any point (h, k). P, Q, R divides the sides of this triangle in same ratio. Then centroid of ΔPQR is

(A) $(0, 0)$ (B) $\left(\frac{h}{3}, 0\right)$ (C) $\left(0, \frac{k}{3}\right)$ (D) $\left(\frac{h}{3}, \frac{k}{3}\right)$

535. If a focal chord of the parabola $y^2 = 4ax$ be at a distance d from the vertex, then its length is equal to

(A) $\frac{a^2}{d^2}$ (B) $\frac{d^2}{a}$ (C) $\frac{4a^3}{d^2}$ (D) $\frac{2a^2}{d}$

536. An ellipse slides between two perpendicular straight lines, then the locus of its centre is

(A) circle (B) parabola (C) ellipse (D) hyperbola

537. If the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ cuts the parabola again at $Q(aT^2, 2aT)$, then

(A) $-2 \le T \le 2$ (B) $T \in (-\infty, -8) \cup (8, \infty)$

(C) $T^2 < 8$ (D) $T^2 \ge 8$

538. If the tangents at P and Q on a parabola meet in R and S be its focus. If p = SP, q = SR and r = SQ, the roots of the equation $px^2 + 2qx + r = 0$ are

(A) rational (B) real and equal (C) imaginary (D) real and unequal

539. Equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(A) $3y = 9x + 2$ (B) $y = 2x + 1$

(C) $2y = x + 8$ (D) $y = x + 2$

540. The tangent drawn from (α, β) to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = c^2$, then the locus of (α, β) is

(A) an ellipse (B) a circle (C) a parabola (D) none of these

541. Consider a point P(at^2, 2at) on the parabola $y^2 = 4ax$. A focal chord PS (S being focus) is drawn to meet parabola again at Q. From Q, a normal is drawn to meet parabola again at R. From R, a tangent is drawn to the parabola to meet focal chord PSQ (extended) at T. The area of ΔQRT is

(A) $8a^2\left(t^2+\frac{1}{t^2}\right)^3$ (B) $a^2\left(t+\frac{1}{t}\right)^4$ (C) $\frac{8a^2}{3}\left(t+\frac{1}{t}\right)^3$ (D) None of these

542. Two pair of straight lines have the equations $y^2 + xy - 20 = 0$ and $ax^2 + 2hxy + by^2 = 0$. One line will be common among them if

(A) a = 8(h – 2b)or a = –5(2h + 5b) (B) a = 4(h – 2b) or a = –3(2h + 5b)
(C) a = 8(h – 2b) or a = 5(2h + 5b) (D) None of these

543. Let PQ, RS are the tangents at the extremities of a diameters PR of a circle of radius r such that PS, RQ intersect at a point X on the circumference of the circle, then 2r equals

(A) $\frac{PQ+RS}{2}$ (B) $\sqrt{PQ\cdot RS}$ (C) $\frac{2PQ\cdot RS}{PQ+RS}$ (D) $\sqrt{\frac{PQ^2+RS^2}{2}}$

544. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that OPQ is an equilateral triangle. O being the centre of the hyperbola, then the eccentricity e of the hyperbola satisfies

(A) $1 < e < \frac{2}{\sqrt{3}}$ (B) $e = \frac{2}{\sqrt{3}}$ (C) $e = \frac{\sqrt{3}}{2}$ (D) $e > \frac{2}{\sqrt{3}}$

545. An equilateral triangle is inscribed in the circle $x^2 + y^2 = a^2$ with vertex (a, 0) the equation of the side opposite to the vertex is

(A) 2x – a = 0 (B) x + a = 0
(C) 2x + a = 0 (D) 3x – 2a = 0

546. If the normal at any point P on an ellipse meets major and minor axis at G and G′ and OF be the perpendicular drawn from centre O to this normal then PF. PG must be equal to

(A) b^2 (B) a^2
(C) ab (D) None of these

547. The diameter of the smallest circle which touches the line y = 3x – 3 and passes through a point on the parabola $y = x^2 + 7x + 2$

(A) $\frac{1}{2\sqrt{5}}$ (B) $\frac{1}{\sqrt{10}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{10}$

548. If four distinct points of the curve $y = 2x^4 + 7x^3 + 3x - 5$ are collinear, then the A.M. of the x-co-ordinate of the four points is

(A) $-\frac{7}{8}$ (B) $\frac{3}{4}$ (C) $\frac{7}{8}$ (D) $-\frac{3}{4}$

549. Given a fixed circle C and a line L through the centre O of C. Take a variable point P on L and let K be the circle centre P through O. Let T be the point where a common tangent to C and K meets K. The locus of T is

(A) a circle (B) a parabola
(C) a pair of straight lines (D) None of these

550. If an ellipse slides between two perpendicular straight lines, then the point of intersection of these two lines lies on

(A) auxiliary circle of ellipse
(B) director circle of ellipse
(C) a line passes through centre of ellipse and perpendicular to axis of ellipse
(D) none of these

551. If a variable straight line x cosα + y sinα = p which is a chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (b > a) subtends a right angle at the centre of the hyperbola then it always touches a fixed circle whose radius is

(A) $\frac{a^2b^2}{\sqrt{b^2-a^2}}$ (B) $\frac{ab}{\sqrt{a^2-b^2}}$ (C) $\frac{ab}{\sqrt{b^2-a^2}}$ (D) $\frac{ab}{b^2-a^2}$

552. Equation of the circle of which the points (1, 2) and (2, 3) are the ends of a chord of segment containing an angle 45^0 is

(A) $x^2 + y^2 - 4x - 4y + 7 = 0$
(B) $x^2 + y^2 - 4x - 4y - 14 = 0$
(C) $x^2 + y^2 + 4x + 4y + 7 = 0$
(D) none of these

553. The locus of the mid point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

(A) $x = -a$ (B) $x = \frac{a}{2}$ (C) $x = -\frac{a}{2}$ (D) $x = 0$

554. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(A) $r < 2$ (B) $r = 2$ (C) $r > 2$ (D) $2 < r < 8$

555. A line $3x - 4y + 4 = 0$ is tangent to a circle whose radius is 3/4. If another straight line $3x - 4y + \lambda = 0$ is also tangent to same circle, then value of λ is

(A) $\frac{3}{4}$ (B) $-\frac{7}{2}$ (C) $-\frac{4}{3}$ (D) $-\frac{2}{7}$

SECTION–2 : (MORE THAN ONE OPTION CORRECT TYPE)

556. If the circle $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other, then α is

(A) $-\frac{4}{3}$ (B) 0 (C) 1 (D) $\frac{4}{3}$

557. If the ellipse $\frac{x^2}{4}+y^2 = 1$ meets the ellipse $x^2+\frac{y^2}{a^2}=1$ in four distinct points and $a = b^2 - 5b + 7$ then b can take values

(A) $(-\infty, 0)$
(B) [4, 5]
(C) [2, 3]
(D) $(0, \infty)$

558. The straight line $x+y=0, 3x+y-4=0$ and $x+3y-4=0$ form a triangle which is

(A) isosceles
(B) right -angled
(C) obtuse - angled
(D) equilateral

559. The point P(α, α + 1) will lie inside the ΔABC whose vertices are A(0, 3), B(–2, 0) and C(6, 1) if

(A) $\alpha = -1$ (B) $\alpha = -\frac{1}{2}$ (C) $\alpha = \frac{1}{2}$ (D) $-\frac{6}{7} < \alpha < \frac{3}{2}$

560. Tangents are drawn from any point with eccentric angle θ on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ to the circle $x^2 + y^2 = 16$. If (x_1, y_1) is midpoint of chord of contact, then θ is equal to

(A) $\sec^{-1}\frac{4x_1}{\left(x_1^2 + y_1^2\right)}$ (B) $\sec^{-1}\frac{16x_1^2}{\left(x_1^2 + y_1^2\right)}$

(C) $\tan^{-1}\frac{16y_1}{3\left(x_1^2 + y_1^2\right)}$ (D) $\tan^{-1}\frac{4y_1^2}{\left(x_1^2 + y_1^2\right)}$

561. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola, then a point of intersection of the circle and the parabola is

(A) $\left(\frac{p}{2}, p\right)$ (B) $\left(\frac{p}{2}, -p\right)$ (C) $\left(\frac{p}{4}, p\right)$ (D) $\left(\frac{p}{4}, -p\right)$

562. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$ where c is

(A) 1 (B) 2 (C) 3 (D) 6

563. The points one line x = 2 from which tangents drawn to circle $x^2 + y^2 = 16$ are at right angles is (are)

(A) $\left(2, 2\sqrt{7}\right)$ (B) $\left(2, 2\sqrt{5}\right)$

(C) $\left(2, -2\sqrt{7}\right)$ (D) $\left(2, -2\sqrt{5}\right)$

564. The line(s) tangents to the curve $y = x^2 - x$

(A) $x - y = 0$ (B) $x + y = 0$ (C) $x - y = 1$ (D) $x + y = 1$

565. The area of a triangle formed by tangent at any point on the curve and co-ordinate axes is constant, then the curve may be

(A) a straight line (B) a circle

(C) a parabola (D) a hyperbola

566. The equation of a circle is $S_1 \equiv x^2 + y^2 = 1$. The orthogonal tangents to S_1 meet at another circle S_2 and orthogonal tangents to S_2 meet at the third circle S_3, then

(A) radius of S_2 and S_3 are in the ratio $1 : \sqrt{2}$ (B) radius of S_2 and S_3 are in the ratio 1 : 2

(C) the circles S_1, S_2 and S_3 are concentric (D) None of these

567. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 8y + \lambda = 0$ and the pair of radii at the point of contact of these tangent to the circle is $2\sqrt{6}$ sq. units, then the value of λ must be

(A) $4\sqrt{6}$ (B) 24 (C) 1 (D) $12\sqrt{6}$

568. The locus of the point of intersection of the tangents at the extremities of the chord of the circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ passes through the point

(A) $\left(\frac{a}{2}, 0\right)$ (B) $\left(0, \frac{a}{2}\right)$

(C) (0, a) (D) (a, 0)

569. The equation of the tangent to the hyperbola $3x^2 - y^2 = 3$ parallel to the line $y = 2x + 4$ is

(A) $y = 2x + 3$ (B) $y = 2x + 1$

(C) $y = 2x - 1$ (D) $y = 2x + 2$

570. From a point P, the chord of contact to the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = (a + b)$ …(1)

touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ …(2) then the locus of the point P is

(A) director circle of (1) (B) auxillary circle of (2)

(C) $x^2 + y^2 = (a + b)^2$ (D) $x^2 + y^2 = a^2 + b^2$

571. The point (1, – 3) is inside the circle S, $S \equiv x^2 + y^2 - 8x + 4y + k = 0$. What are the possible values of k if the circle S neither touches the axes nor cuts them?

(A) 5 (B) 6

(C) 7 (D) 8

572. AB and CD are two equal and parallel chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Tangents to the ellipse at A and B intersect at P and at C and D at Q. The line PQ

(A) passes through the origin (B) is bisected at the origin

(C) cannot pass through the origin (D) is not bisected at the origin

573. If the equation $ax^2 - 6xy + y^2 + bx + cy + d = 0$ represents pair of lines whose slopes are m and m^2, then value of a is / are

(A) $a = -8$ (B) $a = 8$

(C) $a = 27$ (D) $a = -27$

574. The tangent to the hyperbola, $x^2 - 3y^2 = 3$ at the point $(\sqrt{3}, 0)$ when associated with two asymptotes constitutes :

(A) isosceles triangle

(B) an equilateral triangle

(C) a triangles whose area is $\sqrt{3}$ sq. units

(D) a right isosceles triangle.

575. Variable chords of the parabola $y^2 = 4ax$ subtend a right angle at the vertex. Then:

(A) locus of the feet of the perpendiculars from the vertex on these chords is a circle

(B) locus of the middle points of the chords is a parabola

(C) variable chords passes through a fixed point on the axis of the parabola

(D) none of these

SECTION - 3: (COMPREHENSION TYPE)

COMPREHENSION-1

Paragraph for Questions Nos. 576 to 578

The straight line(s) which passes through a given point (a, b) and make a given angle α with the given straight line y = mx + c where m = tan θ [(0 < θ < 90°) and $\alpha \neq 0, 90°$], then

576. How many such lines are possible

(A) one (B) two (C) infinite (D) None of these

577. If β is the angle made by the line L with positive direction of x-axis, then tan β is equal to

(A) $\frac{\tan\alpha + m}{1 - m\tan\alpha}$ (B) $\frac{\tan\alpha + m}{\tan\alpha - m}$ (C) $\frac{m - \tan\alpha}{m + \tan\alpha}$ (D) $\frac{m - \tan\alpha}{1 - m\tan\alpha}$

578. The equation of line L is

(A) $(y - b) = \frac{m + \tan\alpha}{1 + m\tan\alpha}(x - a)$

(B) $(y - b) = \frac{\tan\alpha + m}{\tan\alpha - m}(x - a)$

(C) $(y - b) = \frac{m - \tan\alpha}{m + \tan\alpha}(x - a)$

(D) $(y - b) = \frac{m - \tan\alpha}{1 + m\tan\alpha}(x - a)$

COMPREHENSION-2

Paragraph for Questions Nos. 579 to 581

A boy moving along the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at each and every point of it he is drawing a tangent and finding the area of triangle formed by it with co-ordinate axes at point P, Q, R and S starting from positive axis anticlockwise. He found that area of triangle is $\geq$ m and is m at P, Q, R and S.

579. The co-ordinate of point Q are

(A) $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$

(B) $\left(-\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$

(C) $\left(\frac{a}{\sqrt{2}}, -\frac{b}{\sqrt{2}}\right)$

(D) $\left(-\frac{a}{\sqrt{2}}, -\frac{b}{\sqrt{2}}\right)$

580. The area of Δ formed by P, Q and S is

(A) ab

(B) πab

(C) $\sqrt{ab}$

(D) $\pi\sqrt{ab}$

581. Equation of normal to the ellipse at S is

(A) $ax + by = \frac{b^2}{\sqrt{2}}$

(B) $ax + by = \frac{a^2 - b^2}{\sqrt{2}}$

(C) $bx + ay = \frac{a^2}{2}$

(D) $bx + ay = \frac{b^2 - a^2}{\sqrt{2}}$

COMPREHENSION-3

Paragraph for Questions Nos. 582 to 584

Let an ellipse having major axis and minor axis parallel to x-axis and y-axis respectively. Its two foci S and S′ are (2, 1), (4, 1) and a line $x + y = 9$ is a tangent to this ellipse at point P.

582. Eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{12}}$ (B) $\frac{1}{\sqrt{13}}$

(C) $\frac{1}{2}$ (D) None of these

583. Length of major axis

(A) $\sqrt{13}$ (B) $2\sqrt{11}$

(C) $\sqrt{52}$ (D) $2\sqrt{12}$

584. The latus rectum of ellipse

(A) $\frac{12}{13}$ (B) $\frac{12}{\sqrt{13}}$

(C) $\frac{24}{\sqrt{13}}$ (D) $\frac{25}{\sqrt{13}}$

COMPREHENSION-4

Paragraph for Questions Nos. 585 to 587

Perpendiculars are drawn from the focus S of the parabola $y = ax^2 + bx + c$ upon the tangents to the parabola at the points A(– 1, 0) and B(1, 2) meeting them at the point $C\left(-\frac{1}{4}, \frac{9}{4}\right)$ and $D\left(\frac{3}{4}, \frac{9}{4}\right)$ respectively.

585. The co-ordinates of the focus are

(A) $\left(\frac{1}{2}, \frac{9}{4}\right)$ (B) $\left(\frac{1}{2}, 2\right)$ (C) $\left(0, \frac{9}{4}\right)$ (D) (– 1, 2)

586. The normals at A and B intersect at a point P. The foot of the third normal through the point P is

(A) $\left(\frac{1}{2}, \frac{9}{4}\right)$ (B) $\left(-\frac{1}{2}, \frac{5}{4}\right)$ (C) (0, 2) (D) $\left(\frac{3}{2}, \frac{5}{4}\right)$

587. Area of the region bounded by the parabola and the x– axis is

(A) $\frac{5}{4}$ (B) 5

(C) $\frac{5}{2}$ (D) $\frac{9}{2}$

COMPREHENSION-5

Paragraph for Questions Nos. 588 to 590

Consider an ellipse $\frac{x^2}{4}+y^2=\alpha$; (α is parameter > 0) and a parabola $y^2 = 8x$. If a common tangent to the ellipse and the parabola meets the co-ordinate axes at A and B respectively, then

588. Locus of mid point of AB is
(A) $y^2 = -2x$
(B) $y^2 = -x$
(C) $y^2 = -\frac{x}{2}$
(D) $\frac{x^2}{4}+\frac{y^2}{2}=1$

589. If the eccentric angle of a point on the ellipse where the common tangent meets it is $\left(\frac{2\pi}{3}\right)$, then α is equal to
(A) 4 (B) 5
(C) 26 (D) 36

590. If two of the three normals drawn from the point (h, 0) on the ellipse to the parabola $y^2 = 8x$ are perpendicular, then
(A) h = 2 (B) h = 3
(C) h = 4 (D) h = 6

COMPREHENSION-6

Paragraph for Questions Nos. 591 to 593

Two straight lines rotate about two fixed points (– a, 0) and (a, 0). If they start from their position of coincidence such that one rotates at the rate double that of the other, then

591. The point (– a, 0) always lies
(A) inside the curve (B) Outside the curve
(C) on the curve (D) None of these

592. Locus of the curve is
(A) circle (B) straight line
(C) parabola (D) ellipse

593. Distance of the point (a, 0) from the variable point on the curve is
(A) 0 (B) 2a
(C) 3a (D) 4a

COMPREHENSION-7

Paragraph for Questions Nos. 594 to 596

Consider a point P on a parabola such that 2 of the normals drawn from it to the parabola are at right angles on parabola, then

594. If the equation of parabola is $y^2 = 8x$, then locus of P is

(A) $x^2 = 4(y - 6)$ (B) $y^2 = 2(x - 6)$

(C) $y^2 = 8(x - 6)$ (D) $2x^2 = (y - 6)$

595. The ratio of latus rectum of given parabola and that of made by locus of point P is

(A) 4 : 1 (B) 2 : 1

(C) 16 : 1 (D) 1 : 1

596. If $P \equiv (x_1, y_1)$, the slope of third normal is

(A) $\frac{y_1}{8}$ (B) $\frac{y_1}{2}$

(C) $-\frac{y_1}{8}$ (D) $-\frac{y_1}{2}$

COMPREHENSION-8

Paragraph for Questions Nos. 597 to 599

Read the following writeup carefully:

Observe the following facts for a parabola.

(i) Axis of the parabola is the only line which can be the perpendicular bisector of the two chords of the parabola.

(ii) If AB and CD are two parallel chords of the parabola and the normals at A and B intersect at P and the normals at C and D intersect at Q, then PQ is a normal to the parabola.

597. The vertex of the parabola passing through (0, 1), (– 1, 3), (3, 3) and (2, 1) is

(A) $\left(1, \frac{1}{3}\right)$ (B) $\left(\frac{1}{3}, 1\right)$

(C) (1, 3) (D) (3, 1)

598. The directrix of the parabola is

(A) $y - \frac{1}{24} = 0$ (B) $y + \frac{1}{24} = 0$

(C) $y + \frac{1}{12} = 0$ (D) $y - \frac{1}{12} = 0$

599. For the parabola $y^2 = 4x$, AB and CD are any two parallel chords having slope 1. C_1 is a circle passing through O, A and B and C_2 is a circle passing through O, C and D. C_1 and C_2 intersect at

(A) (4, – 4) (B) (– 4, 4)

(C) (4, 4) (D) (– 4, – 4)

COMPREHENSION-9

Paragraph for Questions Nos. 600 to 602

To the circle $x^2 + y^2 = 4$ two tangents are drawn from P(–4, 0), which touches the circle at T_1 and T_2 and a rhombus $PT_1\,P'T_2$ is completed.

600. Circum centre of the triangle PT_1T_2 is at

(A) (–2, 0) (B) (2, 0)

(C) $\left(\frac{\sqrt{3}}{2}, 0\right)$ (D) None of these

601. Ratio of the area of triangle PT_1P' to that the $P'T_1T_2$ is

(A) 2 : 1 (B) 1 : 2

(C) $\sqrt{3}:2$ (D) None of these

602. If P is taken to be at (h, 0) such that P′ lies on the circle, the area of the rhombus is

(A) $6\sqrt{3}$ (B) $2\sqrt{3}$

(C) $3\sqrt{3}$ (D) None of these

COMPREHENSION-10

Paragraph for Questions Nos. 603 to 605

A circle C whose radius is 1 unit, touches the x–axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is a right angled triangle at A and its perimeter is 8 units.

603. The length of QP is

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$

(C) $\frac{5}{3}$ (D) None of these.

604. Equation of circle C is

(A) $\{x-(2+\sqrt{3})\}^2+(y-1)^2=1$ (B) $\{x-(\sqrt{3}+\sqrt{2})\}^2+(y-1)^2=1$

(C) $(x-\sqrt{3})^2+(y-2)^2=1$ (D) None of these.

605. Equation of tangent OT is

(A) $x-\sqrt{3}y=0$ (B) $x-\sqrt{2}y=0$

(C) $y-\sqrt{3}x=0$ (D) None of these

COMPREHENSION-11

Paragraph for Questions Nos. 606 to 608

If a circle with centre C(α, β) intersects a rectangular hyperbola with centre L (h, k) at four points P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3) and S(x_4, y_4), then the mean of the four points P, Q, R, S is the mean of the points C and L. In other words, the mid-point of CL coincides with the mean point of P, Q, R, S. Analytically

$$\frac{x_1+x_2+x_3+x_4}{4}=\frac{\alpha+h}{2};\ \frac{y_1+y_2+y_3+y_4}{4}=\frac{\beta+k}{2}.$$

606. Five points are selected on a circle of radius a. The centres of the rectangular hyperbolas, each passing through four of these points lie on a circle of radius

(A) a (B) 2a (C) $\frac{a}{\sqrt{2}}$ (D) $\frac{a}{2}$

607. A, B, C and D are the points of intersection of a circle and a rectangular hyperbola which have different centres. If AB passes through the centre of the hyperbola, then CD passes through

(A) centre of the hyperbola (B) centre of the circle

(C) mid-point of the centres of the circle and hyperbola

(D) none of these

608. A circle with fixed centre (3h, 3k) and of variable radius cuts the rectangular hyperbola $x^2 - y^2 = 9a^2$ at the points A, B, C , D. The locus of the centroid of the triangle ABC is given by

(A) $(x - 2h)^2 - (y - 2k)^2 = a^2$ (B) $(x - h)^2 - (y - k)^2 = a^2$

(C) $\frac{x^2}{h^2}-\frac{y^2}{k^2}=a^2$ (D) $\frac{x^2}{h^2}+\frac{y^2}{k^2}=a^2$

COMPREHENSION-12

Paragraph for Questions Nos. 609 to 611

The vertices of a ΔABC lies on a rectangular hyperbola such that the orthocentre of the triangle is (3, 2) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axis. If the two perpendicular tangents of the hyperbola intersect at the point (1, 1).

609. The equation of the asymptotes is

(A) $xy - 1 = x - y$ (B) $xy + 1 = x + y$

(C) $2xy = x + y$ (D) None of these

610. Equation of the rectangular hyperbola is

(A) $xy = 2x + y - 2$

(B) $2xy = x + 2y + 5$

(C) $xy = x + y + 1$

(D) None of these

611. Number of real normals that can drawn from the point (1, 1) to the rectangular hyperbola is

(A) 4 (B) 0

(C) 3 (D) 2

COMPREHENSION-13

Paragraph for Questions Nos. 612 to 614

Let ABCD be a parallelogram whose diagonals equations are AC: $x + 2y = 3$; BD: $2x + y = 3$. If length of diagonal AC = 4 units and area of ABCD = 8 sq. units.

612. The length of other diagonal BD is

(A) $\frac{10}{3}$ (B) 2 (C) $\frac{20}{3}$ (D) None of these

613. The length of side AB is equal to

(A) $\frac{2\sqrt{58}}{3}$ (B) $\frac{4\sqrt{58}}{9}$ (C) $\frac{3\sqrt{58}}{9}$ (D) $\frac{4\sqrt{58}}{9}$

614. The length of BC is equal to

(A) $\frac{2\sqrt{10}}{3}$ (B) $\frac{4\sqrt{10}}{3}$

(C) $\frac{8\sqrt{10}}{3}$ (D) None of these

COMPREHENSION-14

Paragraph for Questions Nos. 615 to 617

A coplanar beam of light emerging from a point source have equation $\lambda x - y + 2(1 + \lambda) = 0$, $\lambda \in R$. the rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x - y + 2(1 - \mu) = 0$, $\mu \in R$. Further it is found that the foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$.

615. The eccentricity of the ellipse is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

616. The area of the largest triangle that an incident ray and the corresponding reflected ray can enclose with the axis of the ellipse is equal to

(A) $4\sqrt{5}$ (B) $2\sqrt{5}$

(C) $\sqrt{5}$ (D) None of these

617. Total distance travelled by an incident ray and the corresponding reflected ray is the least if the point of incidence coincides with

(A) an end of the minor axis (B) an end of the major axis

(C) an end of the latus rectum (D) None of these

COMPREHENSION-15

Paragraph for Questions Nos. 618 to 620

A circular arc of radius 2 units subtend an angle of x radians at the centre O such that $x \in \left(0, \frac{\pi}{2}\right)$. Tangents at the end points P and Q of the arc intersect at R. Let f(x) be the area of triangle PQR and let $\phi(x)$ be the area of region enclosed by the chord PQ and the arc PQ.

618. Length of OR is given by

(A) $\tan\frac{x}{2}$ (B) $2\tan\frac{x}{2}$ (C) $2\sec\frac{x}{2}$ (D) $2\cot\frac{x}{2}$

619. $\phi(x)$ will be given by

(A) $x - \sin x$ (B) $2(x - \sin x)$ (C) $\frac{x}{2} - \tan\frac{x}{2}$ (D) $\tan\frac{x}{2}\sec\frac{x}{2}$

620. $\lim_{x\to 0}\frac{\phi(x)}{f(x)}$ is equal to

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

SECTION - 4 (MATRIX MATCH Type)

621. Match the following:

List – I	List – II
(A) The length of latus rectum of parabola $2y^2 + 3y - 4x - 3 = 0$ is	(i) 3/4
(B) The length of tangent from point (0, – 1) to the circle $\frac{(x-3)^2}{5^2} + \frac{(y-4)^2}{5^2} = 1$ is	(ii) 2
(C) The length of shortest focal chord of parabola $y^2 = 4x - 3$ is	(iii) 3
(D) Straight line with slope m represents the locus of middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to y = 2x, then m is	(iv) 4

622. Match the following:

List – I	List – II
(A) The length of common chord of the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ and circle $(x-1)^2 + (y-2)^2 = 1$ is	(i) 0
(B) $\operatorname{cosec}^2 A \cdot \cot^2 A - \sec^2 A \cdot \tan^2 A - (\cot^2 A - \tan^2 A)(\sec^2 A \operatorname{cosec}^2 A - 1)$ is	(ii) 1
(C) If $a \ne p, b \ne q, c \ne r$ $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-1}$ is equal to	(iii) 2
(D) The circle $x^2 + y^2 = 4x - 7y + 12 = 0$ cuts an intercept on y-axis of length	(iv) 3

623. Match the following:

List – I	List – I
(A) The value of 'a' for which the image of point (a, a, –1) w.r.t. the line mirror $3x + y = 6a$ is is the point $(a^2 + 1, a)$ is	(i) 3
(B) The normal chord at a point 't' on the parabola $16y^2 = x$ subtends right angle at the vertex. Then t^2 is equal to	(ii) 2
(C) If e and e′ be the eccentricity of a hyperbola and its conjugate. Then $\frac{1}{e^2} + \frac{1}{e'^2} =$	(iii) 5
(D) The shortest distance between parabola $y^2 = 4x$ and circle $x^2 + y^2 + 6x - 12y + 20 = 0$ is $4\sqrt{2} - A$. The A is	(iv) 1

624. Match the following:

List – I	List – II
(A) Let $\alpha = \lim\limits_{m\to\infty}\lim\limits_{n\to\infty}\cos^{2m}\angle n\pi x$, where x is rational, $\beta = \lim\limits_{m\to\infty}\lim\limits_{n\to\infty}\cos^{2m}\angle n\pi x$, where x is irrational, then the area of the triangle having vertices (α, β), (– 2, 1) and (2, 1) is	(i) 56
(B) If the circumference of the circle $x^2 + y^2 + 8x + 8y - b = 0$ is bisected by the circle $x^2 + y^2 - 2x + 4y + a = 0$, then $\|a + b\| =$	(ii) 2
(C) Given a circle of radius 3, tangents are drawn from points A and B lying on one of its diameters which meet at a point P lying on another diameter perpendicular to the other diameter. The minimum area of triangle PAB is	(iii) 90
(D) If the radius of the circle $(x - 1)^2 + (y - 2)^2 = 1$ and $(x - 7)^2 + (y - 10)^2 = 4$ are increasing uniformly w.r.t. time as 0.3 and 0.4 unit/sec, if they touch each other internally after t sec. then t is equal to	(iv) 18

625. Match the following:

ABC be a triangle with a = 3, b = 4, c = 5

List– I	List–II
(A) Distance between circumcentre and orthocentre	(i) 1/3
(B) Distance between centroid and circumcentre	(ii) 5/2
(C) Distance between centroid and incentre	(iii) 5/6
(D) Distance between centroid and orthocentre	(iv) 5/3

626. Match the following curve with their corresponding orthogonal trajectory

List – I	List – II
(A) $ay^2 = x^3$	(i) $y = kx$
(B) $y = ax^2$	(ii) $y^{2/3} - x^{2/3} = c$
(C) $x^{2/3} + y^{2/3} = a^{2/3}$	(iii) $y^{2/3} + x^{2/3} = c$
(D) $x^2 + y^2 = a^2$	(iv) $2y^2 + x^2 = c$
	(v) $3y^2 + 2x^2 = c$

627. Match the following

List – I		List – II	
(A)	For a rectangular hyperbola $xy = c^2$ (c is purely imaginary) if a point on it is (a, α) where 'a' is a positive number, α can take value	(i)	3
(B)	If sides of a triangle are in A.P. and $(a - b + c)s = kb^2$ (where s is the semi perimeter) then k is equal to	(ii)	$\frac{1-\sqrt{3}}{2}$
(C)	Radius of the circle having centre (3, 4) and touching the circle $x^2 + y^2 = 4$ can be	(iii)	17
(D)	Maximum distance of any point on the circle $(x-7)^2 + \left(y-2\sqrt{30}\right)^2 = 16$ from the centre of the ellipse $25x^2 + 16y^2 = 400$ is	(iv)	$\frac{3}{2}$

628. Match the following

(A) The number of rational points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is — (1) ∞

(B) Number of integral points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is — (2) 4

(C) Number of rational points on the curve $\frac{x^2}{3}+y^2=1$ is — (3) 0

(D) Number of integral points on the curve $\frac{x^2}{3}+y^2=1$ is — (4) 2

629. Match the following:

List – I	List – II
(A) The triangle PQR is inscribed in the circle $x^2 + y^2 = 169$. If Q and R have coordinates (5, 12) and (–12, 5) respectively find $\angle QPR$	(i) $\frac{\pi}{4}$
(B) What is the angle between the line joining origin to the point of intersection of the line $4x + 3y = 24$ with circle $(x - 3)^2 + (y - 4)^2 = 25$	(ii) $\frac{\pi}{2}$
(C) Two parallel tangents drawn to given circle are cut by a third tangent. The angle subtended by the third tangent at the centre is	(iii) π
(D) For a circle if a chord is drawn along the point of contact of tangents drawn from a point P. If the chord subtends an angel $\frac{\pi}{2}$ then find the angle at P	(iv) $\frac{\pi}{6}$

630. Match the following:

List – I	List – II
(A) The normal at an end of a latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through an end of the minor axis if e^4 is equal to	(i) $\frac{1}{9}$
(B) PQ is a double ordinate of a parabola $y^2 = 4ax$. If the locus of its points of trisection is another parabola length of whose latus rectum is k times the length of the latus rectum of the given parabola then k is equal to	(ii) $\frac{1}{a}$
(C) If e and e′ are the distances of the extremities of any focal chord from the focus f of the parabola $y^2 = 4ax$, then $\frac{1}{e}+\frac{1}{e'}$ is equal to	(iii) 1

(D) If e and e′ be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2}+\frac{1}{e'^2}$ is equal to	(iv) $1 - e^2$

631. Match the following:

List – I		List – II
(A)	The equation of tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which cuts off equal intercepts on axes is $x - y = a$ where a equal to	(i) $\sqrt{2}$
(B)	The normal $y = mx - 2am - am^3$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex if m equals to	(ii) $\sqrt{3}$
(C)	The equation of the common tangent to parabola $y^2 = 4x$ and $x^2 = 4y$ is $x + y + \frac{k}{\sqrt{3}} = 0$, then k is equal to	(iii) $\sqrt{8}$
(D)	An equation of common tangent to parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is $4x - 2y + \frac{k}{\sqrt{2}} = 0$, then k is equal to	(iv) $\sqrt{41}$

632. The parabola $y^2 = 4ax$ has a chord AB joining points $A\left(at_1^2, 2at_1\right)$ and $B\left(at_2^2, 2at_2\right)$. Then match the following

List – I	List – II
(A) AB is a normal chord	(i) $t_2 = -t_1 - \frac{1}{2}$
(B) AB is a focal chord	(ii) $t_2 = -\frac{4}{t_1}$
(C) AB subtends 90° at point (0, 0)	(iii) $t_2 = -\frac{1}{t_1}$
(D) AB is inclined at 45° to the axis of parabola	(iv) $t_2 = -t_1 - \frac{2}{t_1}$

Section-5 : (INTEGER type)

633. A point P moves in such a way that $\frac{AP}{PB} = 3$ where A(1, 2) and B(3, 5), then maximum distance of P from A is ___________

634. The line L has intercepts 1 and 1/2 on the co-ordinate axes. When keeping the origin fixed, the co-ordinate axes are rotated through a fixed angle, if the same line has intercepts p and q on the rotates axes. Then $\frac{1}{p^2}+\frac{1}{q^2}$ is ______________

635. If PSQ is the focal chord of the parabola $y^2 = 8x$ such that SP = 6, then the length SQ is _______________

636. If the ellipse $x^2 + 81y^2 - 81 = 0$ and the circle $x^2 + y^2 - 9 = 0$ intersect an angle θ in first quadrant, then the value of $(-3 \tan \theta)$ is ____________

637. Let (x_i, y_i) where i = 1, 2, 3, 4 are the integral solutions of equation $2x^2y^2 + y^2 - 6x^2 - 12 = 0$. The area of quadrilateral whose vertices are (x_i, y_i), i = 1, 2, 3, 4 is _______

638. Square of diameter of the circle having tangent at (1, 1) as $x + y - 2 = 0$ and passing through (2, 2) is ________

639. If P be a point on ellipse $4x^2 + y^2 = 8$ with eccentric angle $\frac{\pi}{4}$. Tangent and normal at P intersects the axes at A, B, A′ and B′ respectively, then the ratio of area of $\Delta APA'$ and area of $\Delta BPB'$ is ___________ .

640. P is the positive extremity of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and A is the positive major vertex and B is the positive minor vertex then the 10 times of the area bounded by BPA and chords BP and AP is ___________

641. The minimum of the distances from the point (0, 1) to the points of intersection of the lines $(3\cos\theta + 4\sin\theta)x + (2\cos\theta + 2\sin\theta)y - (5\cos\theta + 6\sin\theta) = 0$, where different values of θ gives different lines, is ____________

642. A line passes through the point P (2, 3) and makes an angle θ with positive direction of x-axis. If it meets the lines represented by $x^2 - 2xy - y^2 = 0$ at the points A and B. If $PA \cdot PB = 17$, then the value of θ in degrees is ________________

643. Six points (x_i, y_i), i = 1, 2, …, 6 are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocentre of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed points (h, k), then h + k is ________

644. The square of the length of the intercept on the normal at the point P(18, 12) of the parabola $y^2 = 8x$ made by the circle on the line joining the focus and P as diameter, is ___________

645. The angle between the straight lines $x\cos\alpha + y\sin\alpha = p$ and $ax + by + p = 0$ is $\frac{\pi}{4}$. They meet the straight line $x\sin\alpha - y\cos\alpha = 0$ in the same point, then the value of $a^2 + b^2$ is ___________

646. Area of the rectangle formed by a asymptotes of the hyperbola $xy - 3y - 2x = 0$ and co–ordinate axes is

647. The tangent at the point A(12, 6) to a parabola intersects its directrix at the point B(– 1, 2). The focus of the parabola lies on x-axis. The number of such parabolas is

648. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 - 2x - 4 = 0$ at the point A and B. If the circle $x^2 + y^2 - 4x - k = 0$ passes through A and B, then the value of k is

649. If G is the centroid of ΔABC with vertices A(a, 0), B(– a, 0) and C(b, c) then $\frac{AB^2 + BC^2 + CA^2}{GA^2 + GB^2 + GC^2} =$

650. A man running round a race course notes that the sum of the distance of two flag posts from him is always 10 meter and distance between flag posts is 8 m. The area of the path, he encloses (in square meters) is $k\pi$. What is the value of k?