Mind Map-2

INVERSE TRIGONOMETRIC FUNCTIONS

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Inverse Function	Domain	Principal Value Branch
$y=\sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y=\text{cosec}^{-1}x$	$R-(-1, 1)$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$y=\sec^{-1}x$	$R-(-1, 1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$y=\tan^{-1}x$	R	$\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
$y=\cot^{-1}x$	R	$[0, \pi]$

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Properties Of Inverse Trigonometric Functions

Property-1

(i) $\sin^{-1}(\sin\theta)=\theta$, if $\frac{-\pi}{2}\le\theta\le\frac{\pi}{2}$

(ii) $\cos^{-1}(\cos\theta)=\theta$, if $0\le\theta\le\pi$

(iii) $\tan^{-1}(\tan\theta)=\theta$, if $\frac{-\pi}{2}<\theta<\frac{\pi}{2}$

(iv) $\cot^{-1}(\cot\theta)=\theta$, if $0<\theta<\pi$

(v) $\sec^{-1}(\sec\theta)=\theta$, if $0\le\theta<\frac{\pi}{2}$ or $\frac{\pi}{2}<\theta\le\pi$

(vi) $\text{cosec}^{-1}(\text{cosec}\,\theta)=\theta$, if $-\frac{\pi}{2}\le\theta<0$ or $0<\theta\le\frac{\pi}{2}$

Property-2

(i) $\sin(\sin^{-1}x)=x$, if $-1\le x\le1$

(ii) $\cos(\cos^{-1}x)=x$, if $-1\le x\le1$

(iii) $\tan(\tan^{-1}x)=x$, if $-\infty<x<\infty$

(iv) $\cot(\cot^{-1}x)=x$, if $-\infty<x<\infty$

(v) $\sec(\sec^{-1}x)=x$, if $-\infty<x\le-1$ or $1\le x<\infty$

(vi) $\text{cosec}(\text{cosec}^{-1}x)=x$, if $-\infty<x\le-1$ or $1\le x<\infty$

Property-3

(i) $\sin^{-1}(-x)=-\sin^{-1}x$, if $-1\le x\le1$

(ii) $\cos^{-1}(-x)=\pi-\cos^{-1}x$, if $-1\le x\le1$

(iii) $\tan^{-1}(-x)=-\tan^{-1}x$, if $-\infty<x<\infty$

(iv) $\cot^{-1}(-x)=\pi-\cot^{-1}x$, if $-\infty<x<\infty$

(v) $\sec^{-1}(-x)=\pi-\sec^{-1}x$, if $-\infty<x\le-1$ or $1\le x<\infty$

(vi) $\text{cosec}^{-1}(-x)=-\text{cosec}^{-1}x$, if $-\infty<x\le-1$ or $1\le x<\infty$

Property-4

(i) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $x\in[-1,1]$

(ii) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $x\in R$

(iii) $\sec^{-1}x+\text{cosec}^{-1}x=\frac{\pi}{2}$, $x\in(-\infty,-1]\cup[1,\infty)$

Property-5

(i) $\sin^{-1}x=\text{cosec}^{-1}\left(\frac{1}{x}\right)$, $-1\le x\le1$

(ii) $\text{cosec}^{-1}x=\sin^{-1}\left(\frac{1}{x}\right)$, $x\in R-(-1,1)$

(iii) $\cos^{-1}x=\sec^{-1}\left(\frac{1}{x}\right)$, $-1\le x\le1$

(iv) $\sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right)$, $x\in R-(-1,1)$

(v) $\tan^{-1}x=\cot^{-1}\left(\frac{1}{x}\right)$, $x\in R$

(vi) $\cot^{-1}x=\tan^{-1}\left(\frac{1}{x}\right)$, $x\in R$

Property-6

(i) $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy<1$

(ii) $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy>-1$

(iii) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z$
$=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$,
if $x>0, y>0, z>0$
and $(xy+yz+zx)<1$

Property-7

(i) $\sin^{-1}x+\sin^{-1}y$
$=\sin^{-1}\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\}$,
if $-1\le x, y\le1$
and $x^2+y^2\le1$
or if $xy<0$ and $x^2+y^2>1$

(ii) $\sin^{-1}x-\sin^{-1}y$
$=\sin^{-1}\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\}$,
if $-1\le x, y\le1$
and $x^2+y^2\le1$
or if $xy>0$
and $x^2+y^2>1$

Property-8

(i) $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\}$,
if $-1\le x, y\le1$ and $x+y\ge0$

(ii) $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\}$,
if $-1\le x, y\le1$ and $x\le y$

Property-9

(i) $2\sin^{-1}x=\sin^{-1}(2x\sqrt{1-x^2})$, if $\frac{-1}{\sqrt2}\le x\le\frac{1}{\sqrt2}$

(ii) $3\sin^{-1}x=\sin^{-1}(3x-4x^3)$, if $\frac{-1}{2}\le x\le\frac{1}{2}$

Property-10

(i) $2\cos^{-1}x=\cos^{-1}(2x^2-1)$, if $0\le x\le1$

(ii) $3\cos^{-1}x=\cos^{-1}(4x^3-3x)$, if $\frac{1}{2}\le x\le1$

Property-11

(i) $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, if $-1<x\le1$

(ii) $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, if $\frac{-1}{\sqrt3}<x<\frac{1}{\sqrt3}$

Property-12

(i) $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $-1\le x\le1$

(ii) $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, if $0\le x<\infty$

Property-13

(i) $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$
$=\cot^{-1}\frac{\sqrt{1-x^2}}{x}=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)=\text{cosec}^{-1}\left(\frac{1}{x}\right)$

(ii) $\cos^{-1}x=\sin^{-1}\sqrt{1-x^2}=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$
$=\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sec^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

(iii) $\tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\cot^{-1}\left(\frac{1}{x}\right)$
$=\sec^{-1}\sqrt{1+x^2}=\text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$