OPEN CHANNEL FLOW

open channel flow (Basics / Intro) :-

- free surface flow → constant pressure → atm. pressure
  ($1\,atm$ Pressure $= 101.3\,kPa$)

OCF driving force → gravity force component
'or'
weight component of liquid along slope.

if slope θ = zero → no OCF.

Rigid Boundry channel	mobile Boundry channel
Boundry → not deformable	deformable
no major scouring, erosion, deposition	scouring, erosion, deposition
degree of freedom – 1 (only depth of flow)	4 → depth of flow, Bed width, slope, layout of channel

nonuniform flow ($dy/dx \neq 0$)

- Gradually varied flow (GVF)
  - friction consider
  - curvature of streamlines → nil
- Rapid Varied flow (RVF)
  - friction neglect
  - curvature more
- Spatially varied flow (SVF)
  - Steady
    - SVF with Increasing discharge
      - Ex (flow in side channel Spillway)

    - SVF with decreasing discharge
      - Ex.
        - flow over side weir → to divert excess stormwater from urban drainage system
        - Bottom rack → Provided at bottom to divert Part of flow.
  - unsteady
    - Ex. Surface runoff due to rainfall

flow

- Steady flow
  - uniform $\left(\frac{dy}{dx} = 0\right)$
    - Ex. (I) canal flow
    - (II) flow in Prismatic channel
  - nonuniform $\left(\frac{dy}{dx} \neq 0\right)$
    - GVF — Ex. Back water curve due to obstruction
    - RVF — Ex. Jump
- unsteady flow
  - GVUF — gradually varied Unsteady flow — Ex. flood flow in rivers
  - RVUF — rapid varied unsteady flow — Ex. Surge

note: unsteady uniform → not possible

$$\boxed{F_r = \frac{V}{\sqrt{\frac{gD\cos\theta}{\alpha}}}}$$

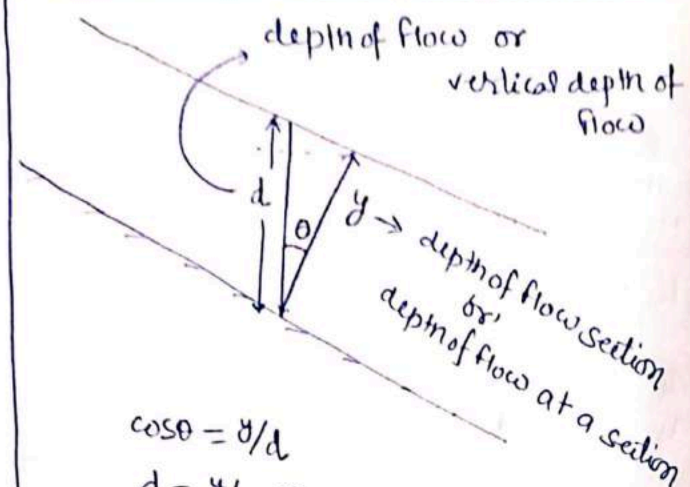
θ → slope

α → K.E energy correction factor

$D \rightarrow A/T$ — Hydraulic depth; A → area of flow; T → top width

- $F_r < 1$
  - Subcritical flow or streaming flow or Tranquil flow
  - $y > y_c$, $V_c > V$
- $F_r > 1$
  - supercritical flow 'or' Torrential flow 'or' Shooting flow 'or' Rapid flow
  - $y < y_c$, $V > V_c$
- $F_r = 1$
  - critical flow
  - $y = y_c$, $V = V_c$

note:



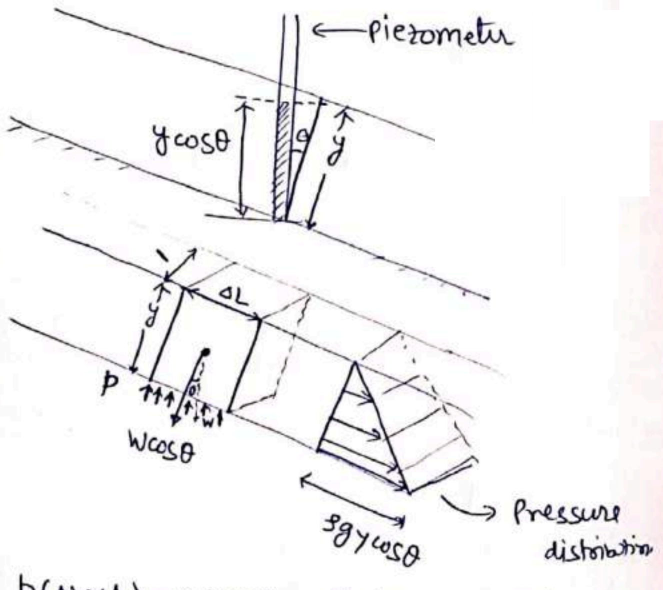
$\cos\theta = y/d$

$d = y/\cos\theta$

θ small $d \simeq y$

Pressure distribution in OCF :-

θ → large Ex. spillways, chutes.



$p(\Delta L \times 1) = W\cos\theta = \rho g(\Delta L \times 1 \times y\cos\theta)$

$p = \rho g y\cos\theta$

$p/\rho g = y\cos\theta$ = piezometric head.

HGL in OCF :-

$HGL = p/\rho g + z = y\cos\theta + z$

• HGL will lie below free surface $\{\cos\theta < 1\}$

But if $\theta = 0$ $\cos\theta = 1$

then HGL will touch free surface.

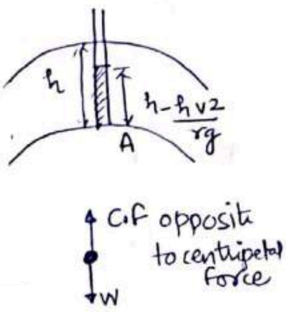
(or if nothing is mentioned in question)

Pressure distribution over curve boundary
'or'
Pressure distribution in curvilinear flow.

$\boxed{\text{convex}}$ | $\boxed{\text{concave}}$

Convex

Ex. spillway crest



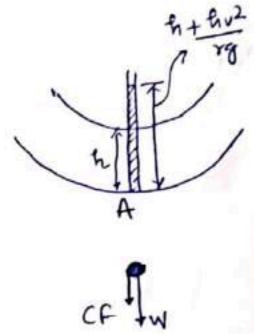
$p_A = \rho g h - \rho\left(\frac{v^2}{r}\right)h$

$\boxed{\text{Pressure head} = \frac{p_A}{\rho g} = h - \frac{hv^2}{rg}}$

⇓

$\boxed{\text{hence Pressure head correction factor} < 1}$

Concave



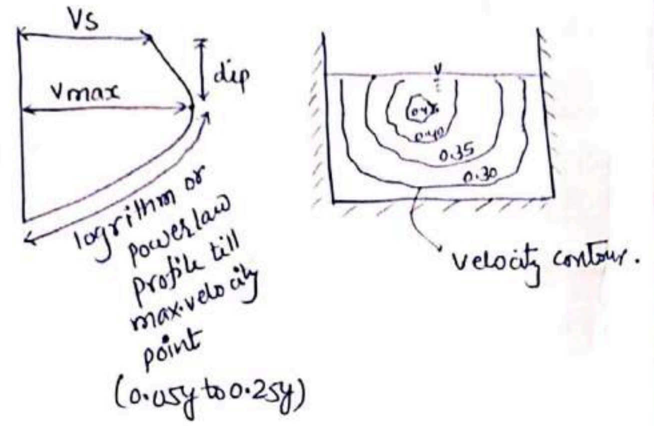
Pressure head

$p_A = \boxed{h + \frac{hv^2}{rg}}$

⇓

$\boxed{\text{Pressure head correction factor} > 1}$

velocity distribution in OCF :



1. Velocity is zero at solid Boundry (∵ resistance → maximum) and velocity gradually increases with distance from boundry.

2. dip of max. velocity point is due to secondry current.

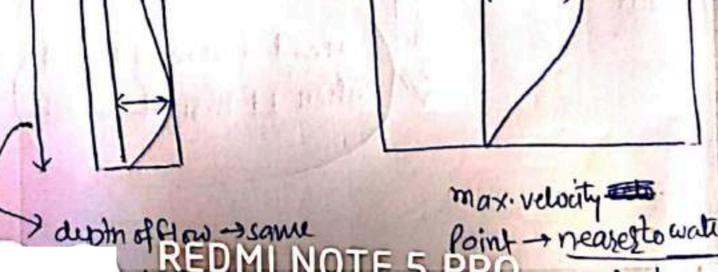
{ transverse component of velocity gives rise to secondry current }

- secondry current = f^n (Aspect ratio)

$$\boxed{\text{Aspect ratio} = \frac{\text{Depth}(D)}{\text{width}(B)}}$$

note:- Field observation (depth of flow → same)

- narrow channel B↓
- wide channel B↑



depth of flow → same

max. velocity point → nearest to water surface

(Prismatic channel (no Bend))

avg. velocity :-

$$\boxed{V_{avg} = \frac{\int u\,dA}{\int A}}$$

$$\boxed{V_{avg} = K V_s}$$

K depend on channel section (0.80 – 0.95) — Based on field observation

- Sallow: $\boxed{V_{avg} = V_{0.6y}}$ { dist 'y' from free surface }
- deep channel: $\boxed{V_{avg} = \frac{V_{0.2y} + V_{0.8y}}{2}}$ (2 point velocity formula more accurate)

correction factor — used to compensate velocity variation.

- α : Kinetic energy correction factor 'or' Coriolis cofficient

$$\boxed{\alpha = \frac{\int U^3 dA}{V^3 A}}$$

use :- velocity head calculation

Put $\frac{\alpha V^2}{2g}$

- β : momentum correction factor / Bousinesq coff.

$$\boxed{\beta = \frac{\int U^2 dA}{V^2 A}}$$

use :- Calculation of momentum per unit time $\beta \rho V^2 A$

note:- $\boxed{\text{nonuniformity} \uparrow \Rightarrow \alpha, \beta \uparrow}$

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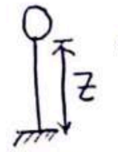
Potential energy — Pressure energy — Kinetic energy

Head (height of fluid column) 'or' energy per unit weight of fluid w.r.t. datum

Datum head, or elevation head 'or' potential head.	Pressure head / static head	Kinetic head or velocity head or dynamic head
(Z)	$((P/\rho g) = y\cos\theta)$	$(V^2/2g)$



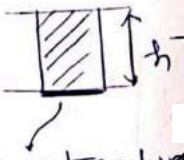
- work done to lift this mass at Z

→ Potential energy stored in fluid mass

= Force × displacement
$\cancel{(f)} \rightarrow W \quad (Z)$

∴ Potential energy per unit weight of fluid mass $= \frac{WZ}{W} = Z$

- head of fluid when fluid is static.



$h \rightarrow$ static fluid

- water column will exert pressure on it.

- Pressure energy F/A $= \frac{\text{wt. of fluid column}}{\text{area}}$

$= \frac{(\rho g h)A}{A}$

$= \rho g h$

∴ Pressure energy per unit weight $= \frac{\rho g h}{\rho g} = h$

- head of flowing fluid due to its velocity.

$KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}\left(\frac{W}{g}\right)(v^2)$

$$\boxed{\therefore \frac{KE}{W} = \frac{v^2}{2g}}$$

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note:-

imp **

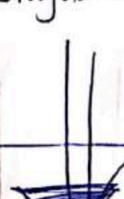
① Piezometric head = datum head (Z) + Pressure head $(y\cos\theta)$

to know this → Install piezometer at that point

if datum ⇒ reference then rise in piezometer will give you pressure head.

imp

② Stagnation head = Pressure head $(P/\rho g)$ + velocity head $(V^2/2g)$



Head of flowing fluid when it brought to rest.

equation used in OCF → continuity eqⁿ / energy " / momentum "

① continuity eqⁿ (mass conservation) :-

assumption : ⊛ fluid Incompressible $\boxed{d\rho/ds = 0}$
* 1D flow

• Unsteady flow $\boxed{\frac{dQ}{dx} + T\frac{dy}{dt} = 0}$

• Steady Flow $\left(\frac{dy}{dt} = 0\right)$ $\boxed{\frac{dQ}{dx} = 0}$

$$\left\{\begin{array}{l} Q_2 = Q_1 \\ A_1V_1 = A_2V_2 \end{array}\right\}$$

note:- if spatially varied flow (SVF)
then $\frac{dQ}{dx} + T\frac{dy}{dt} = \pm q'$

② Energy eqⁿ (energy conservation) :-

assumption :- * flow steady * 1D flow

$\boxed{\text{total head} = z + y\cos\theta + \alpha V^2/2g}$

∴ $\boxed{H_1 = H_2 + h_L}$ → headloss

frictionloss | form or eddy Loss

• energy loss inform of heat due to change in velocity < magnitude / direction
like in Bed.

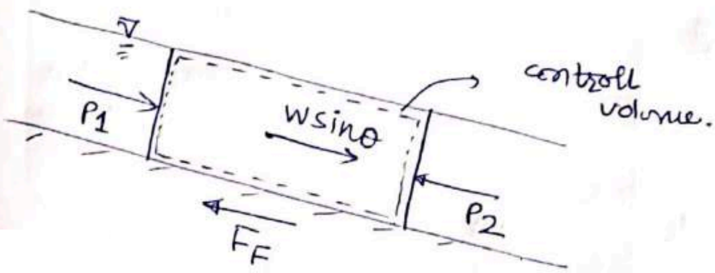
note:-
$\theta = 0$ horizontal channel
$h_f = 0$ frictionless channel
$h_e = 0$ Prismatic channel (no Bend)

③ momentum eqⁿ (momentum conservation) :-

{Based on Lineal momentum eqⁿ 'or' newton's 2nd Law}

for steady flow :-

Sum of all external force on fluid mass in the direction of flow = Rate of change of lineal momentum in direction of flow



$$P_1 - P_2 - W\sin\theta - F_f = M_2 - M_1$$

$P_1 = \gamma A_1\bar{z}_1\cos\theta$, $P_2 = \gamma A_2\bar{z}_2\cos\theta$, $M_2 = \beta\rho QV_2$, $M_1 = \beta\rho QV_1$ (momentum per unit time)

** Imp note :- controll volume such that required force becomes external to controll volume.

specialcase :- $\theta = 0$ (Horizontal) + $F_f = 0$ (frictionless)

$\gamma A\bar{z}\cos\theta \to 1$

$$\left\{\frac{P_1 + M_1}{\rho g} = \frac{P_2 + M_2}{\rho g} = \text{specific force } (P_s)\right\}$$

imp.

$$\boxed{P_s = A\bar{z} + \frac{Q^2}{Ag}} \longrightarrow \left\{\begin{array}{l}\text{when} \\ \theta = 0 \\ f_f = 0\end{array}\right\}$$

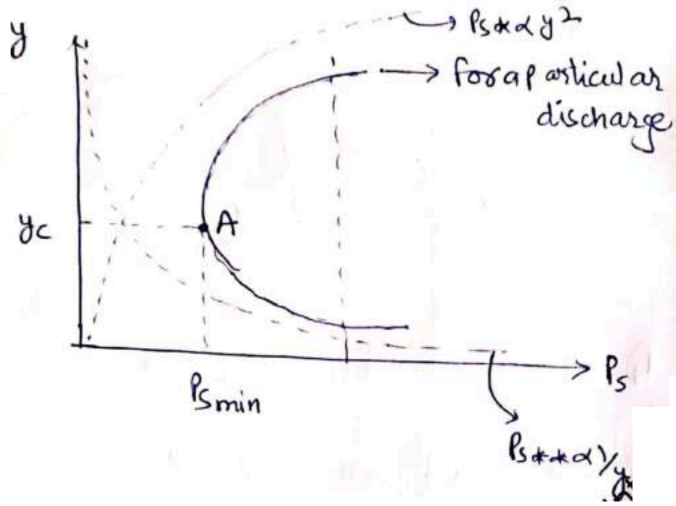
$$\boxed{P_s = A\bar{z} + \frac{Q^2}{Ag}} \quad (\theta = 0,\ F_f = 0)$$

$A\bar{z} \to P_{s*}$, $\frac{Q^2}{Ag} \to P_{s**}$

for rectangular channel.

$A\bar{z} = (By)(y/2) \Rightarrow P_{s*} \propto y^2$

$\frac{Q^2}{Ag} = \frac{Q^2}{(By)g} \Rightarrow P_{s**} \propto 1/y$



at critical depth (for particular discharge)

$\frac{dP_s}{dy} = 0$ (slope at A = 0)

& $\frac{d^2P_s}{dy^2} > 0$

V.Imp

- conjugate depth ⇒ at which specific force is same.
- Alternate depth ⇒ at which specific energy is same.

momentum eqn for unsteady flow :-

Sum of all external force =

Rate of change in lineal momentum

+ time rate of Increase of momentum in that direction in controll volume.

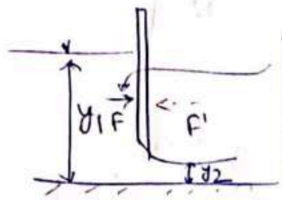
note:- for Rapid varied flow / curvilineal flow →

Streamlines has curvature so pressure force must be corrected for curvature effect of streamlines.

But in RVF (Jump) we assume that Before & after flow is uniform

$P = \gamma A\bar{z}\cos\theta$ only

force by water on sluice gate :-



$$\boxed{F = \frac{1}{2}\gamma\frac{(y_1 - y_2)^3}{y_1 + y_2}}$$

kN/m

kN per unit width of gate

(I) $P_1 - P_2 - F' = M_2 - M_1$

(II) assume loss = 0 $\quad y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + h_L^{\,0}$

if F_1, F_2 → froude no. at Alternate depth y_1, y_2

rectangular channel

Imp

$$\left(\frac{F_2}{F_1}\right)^{2/3} = \frac{2 + F_2^2}{2 + F_1^2}$$

Triangular channel

$$\left(\frac{F_2}{F_1}\right)^{2/5} = \frac{4 + F_2^2}{4 + F_1^2}$$

Uniform flow :- Properties

1) $\boxed{\frac{dy}{dx} = 0}$ $\boxed{\frac{dv}{dx} = 0}$ $\boxed{\frac{dQ}{dx} = 0}$

$\frac{dv}{dx} = 0$ → velocity profile is constant at any section.

$\frac{dy}{dx} = 0$ → this constant depth is known as normal depth (y_n) 'or' uniform flow depth.

$\frac{dQ}{dx} = 0$ → Spatially varied flow can never be uniform

2) $\boxed{\text{specific energy (E)} \longrightarrow \text{constant}}$

$$E = y\cos\theta + \cancel{\alpha}\frac{V^2}{2g} \quad (\alpha = 1)$$

3) $\boxed{S_o = S_w = S_e}$

$S_o \rightarrow$ Bed slope

$S_w \rightarrow$ Slope of water surface or HGL slope

$S_e \rightarrow$ slope of TEL

4) frictional force (f_f) balance by wt. component in direction of flow {gravity component}

$\boxed{f_f = W\sin\theta}$

proof $P_1 - P_2 - f_f - W\sin\theta = M_2 - M_1$

{as per linear momentum theorem

in uniform flow $P_1 = P_2$

$M_1 = M_2$

$\{ \therefore f_f = W\sin\theta \}$

note :- Uniform means → steady uniform

∴ unsteady uniform → not Possible

avg. Boundary shear stress on wetted perimeter under uniform flow condition.

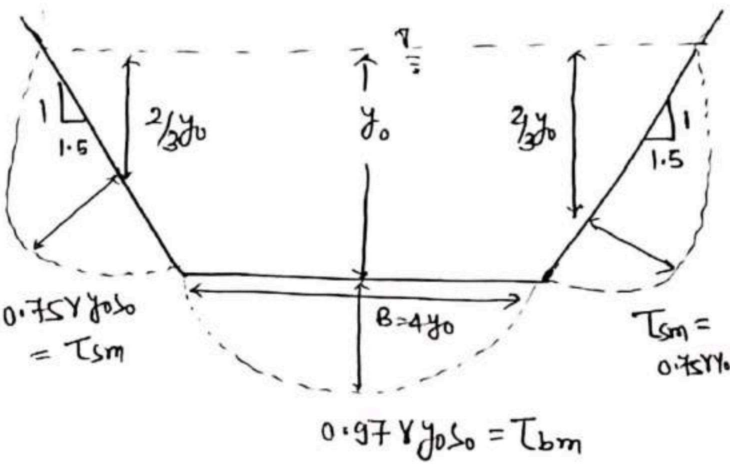
$\boxed{\tau_o = \gamma R S_o}$

Proof : $f_f = W\sin\theta$

$\tau_o P L = \gamma A L \cancel{\tan\theta}^{S_o}$ { for small θ, $\sin\theta = \tan\theta$

$\tau_o = \gamma R S_o$

actual shear stress distribution in trapezoidal channel $[B = 4y_o,\ Z = 1.5]$



Non uniform distribution of shear stress

- due to turbulence in flow
- due to presence of secondry current.

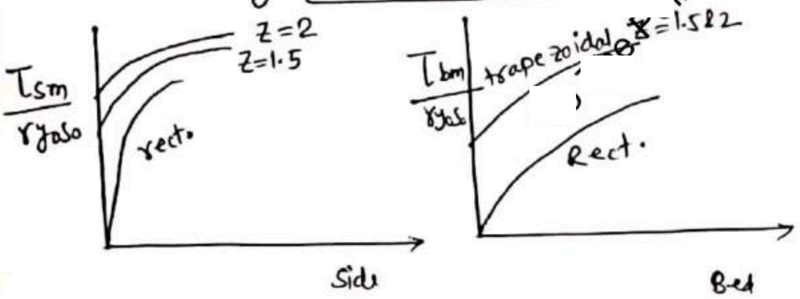
note:-

(I) avg. Boundry shear stress on bottom $= \gamma R S_0$

(Apply by water in uniform flow condition)

(II) Avg. Boundry shear stress on side $= 0.75 \gamma R S_0$

note:- Lane derived the shear stress distribution curve for trapezoidal, rectangular channel using $\boxed{\text{membrane analogy}}$



note:- OCF ⟶ turbulent flow (most of the time)

∴ avg. Boundry shear stress $(\tau_0) \propto$ dynamic pressure $\left(\rho v^2/2\right)$

$$\therefore \tau_0 = K \frac{\rho v^2}{2} = \gamma R S_0$$

$$\boxed{\therefore V = C\sqrt{R S_0} = \sqrt{\frac{2\gamma}{K\rho}}\sqrt{R S_0}} \longrightarrow \text{chezy's formula}$$

$C = \sqrt{\frac{2\gamma}{K\rho}} \Rightarrow$ depend on nature of surface & nature of flow.

$$\boxed{C = \sqrt{\frac{8g}{f}} \quad \text{or} \quad f = \frac{8g}{C^2}}$$

$$\boxed{C = \frac{1}{n} R^{1/6}} \qquad \left\{ \because V = \frac{1}{n} R^{2/3} S^{1/2} = C R^{1/2} S^{1/2} \right.$$

$$\boxed{C = \frac{\frac{1}{n} + \left(23 + \frac{0.00155}{S_0}\right)}{1 + \left(23 + \frac{0.00155}{S_0}\right)\frac{n}{\sqrt{R}}}} \longrightarrow \text{kutter formula}$$

$$\boxed{V = \frac{1}{n} R^{2/3} S^{1/2}} \longrightarrow \text{manning eq}^n$$

note:- Imp

$V = \frac{1}{n} R^{2/3} S_0^{1/2} \longrightarrow y_n$ uniform flow depth or normal depth

$V = \frac{1}{n} R^{2/3} S_f^{1/2} \longrightarrow y$ actual depth of flow

Stricklar formula :-

$$\boxed{n = \frac{d^{1/6}}{24}} \qquad d \to \text{meter}$$

valid for rigid Boundry channel}

canal design

lined canal design (IS:4745–1968)

Imp

criteria → most efficient or economical section

unlined canal design (IS:7112–1973)

Alluvial formation — like sand-silt

- Kenedy's Theory
- Lacy's Theory

Non Alluvial formation — like clay → less susseptible to erosion

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Hydraulically efficient channel

- most economical channel section → in which cost of construction is minimum
- most efficient channel section → in which max. discharge should pass for given A, n, s

Imp

note:- out of all section for given area of flow, semi circular section has minimum wetted perimeter.

note:-

Cost

- excavation cost — fixed — based on c/s area, length
- lining cost — major cost in lined channel → Based on perimeter → hence wetted perimeter should be minimum. ***

note:-

$$Q = A\cdot\frac{1}{n}\left(\frac{A}{P}\right)^{2/3}S^{1/2}$$

$$Q \propto \frac{1}{P^{2/3}}$$

P↓ Q↑

hence wetted perimeter should be min

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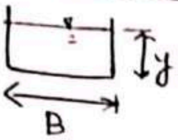
condition for Hydraulically efficient channel ⟶

① Rectangular channel :-

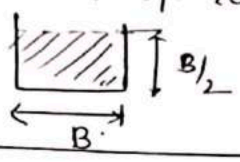


$$\boxed{y = B/2}$$

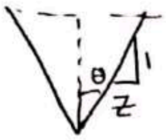
$$\boxed{R = y/2}$$

$$\left(\frac{dP}{dy} = 0\right)$$

channel ⟶ half square of side B



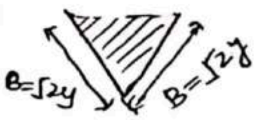
② Triangular channel :- $(dP/d\theta = 0)$



$$\boxed{\theta = 45^\circ \text{ or } Z = 1}$$

$$\boxed{R = y/2\sqrt{2}}$$

channel — half square of side $(B = \sqrt{2}y)$ with its diagonal horizontal



③ Trapezoidal channel :-

Sideslope (Z) ⟶ fix

$$\left(\frac{dP}{dy} = 0\right) \left\{\begin{matrix} A, Z \\ \text{constant} \end{matrix}\right\}$$

(T) topwidth / 2 = sideslope length (L)

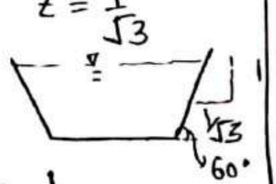
$$\frac{B + 2yZ}{2} = y\sqrt{1+Z^2}$$

$$\boxed{R = y/2}$$

Sideslope (Z) ⟶ vary

$$\left(\frac{dP}{dZ} = 0\right) \left\{\begin{matrix} A, y \\ \text{constant} \end{matrix}\right\}$$

$$Z = \frac{1}{\sqrt{3}}$$

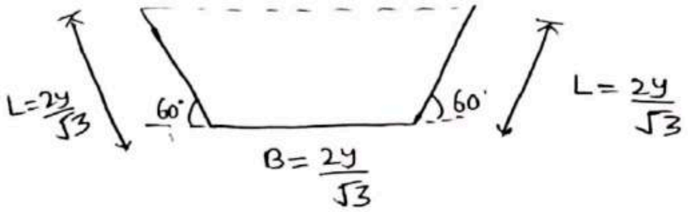


Best side slope = 60° to horizontal 'or 30' to vertical

note:- if $Z = 1/\sqrt{3}$

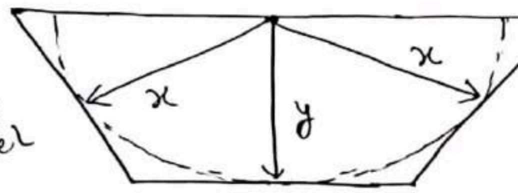
then $$\frac{B + 2y(1/\sqrt{3})}{2} = y\sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\boxed{B = \frac{2y}{\sqrt{3}}} \quad \& \quad \boxed{L = \frac{2y}{\sqrt{3}}}$$



Best trapezoidal channel ⟶ Half of regular hexagon with $B = 2y/\sqrt{3}$

note:-



Best Trapezoidal channel ⟶ $\boxed{x = y}$ (normal flow depth)

Imp

• conclusion ⟶ a circle of radius (y) should be Inscribed in trapezoidal section means circle will touch all 3 sides of trapezoidal section.

(4) circular section :-

(a) manning formula best for OCF where Raynold's no. is large but in circular section sometime free surface is absent hence prefer chezy's formula.

(b) $\boxed{Q = AV}$

→ Rectangular, Triangular section

if A → constant then Qmax condition = Vmax condition

hence we maximise Q then automatically Vmax condition achieved.

→ circular section

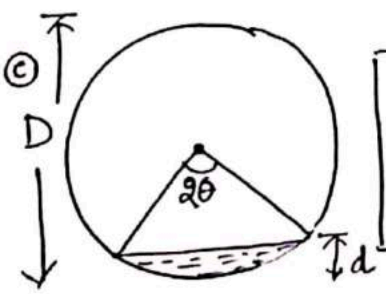
these Qmax condition is not same as Vmax. condition

→ Qmax condition

∵ sometimes our aim to pass max. discharge through circular section.

→ Vmax condition

∵ Sometime Vmax is needed to avoid siltation.

(c) 

$$A = R^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$P = R(2\theta)$$

As per chezy's eqn

→ Qmax condition

$$Q = A \times C\sqrt{A/P}\sqrt{S}$$

∵ $C, S \to$ constant

$A, P \to f^n(\theta)$

hence for Qmax

$$\frac{d}{d\theta}(A^3/P) = 0 \quad *$$

$$\boxed{\frac{d}{D} = 0.95}$$

$$2\theta = 308°$$

→ Vmax condition

$$V = C\sqrt{A/P}\sqrt{S}$$

for Vmax

$$\frac{d}{d\theta}(A/P) = 0$$

$$\boxed{\frac{d}{D} = 0.81} \; *$$

$$2\theta = 257°27' = 258°$$

Imp

note:-

chezy
- Qmax $\frac{d}{D} = 0.95$ → +5% more than full discharge
- Vmax $d/D = 0.81$ → +12.5% than full velocity

note :-

mannings
- Qmax $\frac{d}{D} = 0.938$ → +7% than full discharge
- Vmax ⇒ $d/D = 0.81$ → +14% than full velocity.

Vyas 3/3/2020

Specific energy :-

① total energy head

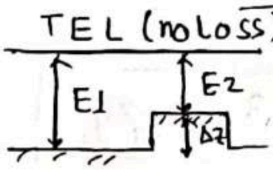
$$\boxed{H = Z + y\cos\theta + \frac{\alpha V^2}{2g}}$$

when $Z = 0$

then $H = \boxed{E = y\cos\theta + \frac{\alpha V^2}{2g}}$

Backmeteff eqn

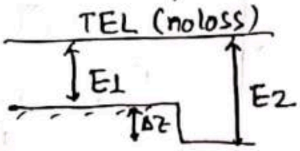
② total energy head always decrease in direction of flow due to Loss (eddy loss, friction loss) But specific energy may increase or decrease (depends on ~~wther~~ It is hump case or drop case) assume



$E_1 = E_2 + \Delta Z$

$$\boxed{\therefore E_2 = E_1 - \Delta Z}$$

In hump case, Specific energy decreases.



$$\boxed{E_2 = E_1 + \Delta Z}$$

In drop case, Specific energy Increases.

③ In uniform flow :-

$\because y, V \longrightarrow$ constant

$\therefore E \longrightarrow$ constant

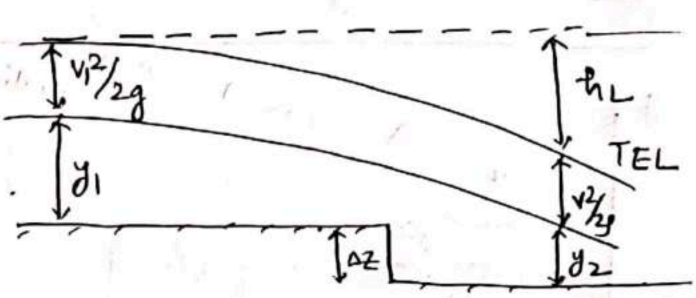
$(E_1 = E_2)$

④ for horizontal & frictionless channel :-

$$\boxed{E_1 = E_2}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

⑤



$$\Delta Z + \left(y_1 + \frac{V_1^2}{2g}\right) = \left(y_2 + \frac{V_2^2}{2g}\right) + h_L$$

$$\Delta Z + E_1 = E_2 + h_L$$

specific energy diagram : y vs E

$\downarrow$

$\boxed{\text{cubic parabola}}$ *

$\because\ E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$

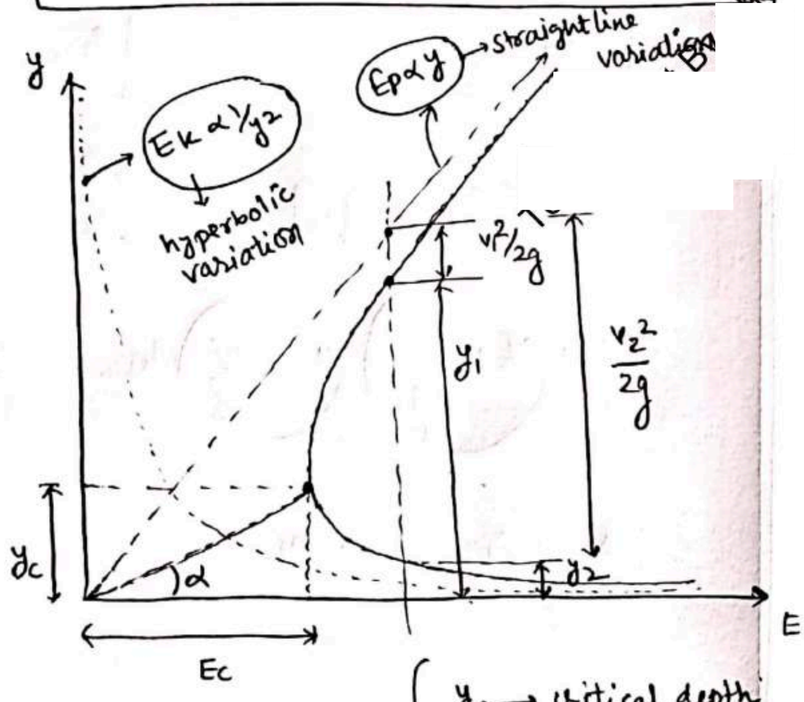
$\frac{Q^2}{2gA^2} \to f(y)$

$E = E_p + E_k$

$E_p \to$ Potential head or datum head

$E_k \to$ velocity head or kinetic head.

$\therefore\ E - E_p = \frac{V^2}{2g}$



$y_c \to$ critical depth

$E_c \to$ critical specific energy

$\boxed{\tan\alpha = \frac{y_c}{E_c}}$

$\because$ for Rectangular channel

$y_c = 2/3\,E_c$

$\alpha = 33.690°$

Imp

note:- flow at/near critical state is unstable because minor change in specific energy leads to major change in depth.

for given specific energy

y_1, $v_1^2/2g$ — Subcritical flow

y_2, $\frac{v_2^2}{2g}$ — Supercritical flow

Alternate depths :- has same specific energy (E)

for rectangular channel :-

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

$$V_1 = \frac{Q^2}{B^2 y_1^2} = \frac{q^2}{y_1^2} \qquad V_2 = \frac{q^2}{y_2^2}$$

$$\therefore\ \boxed{\frac{q^2}{g} = (y_c)^3 = \frac{2y_1^2 y_2^2}{y_1 + y_2}}$$

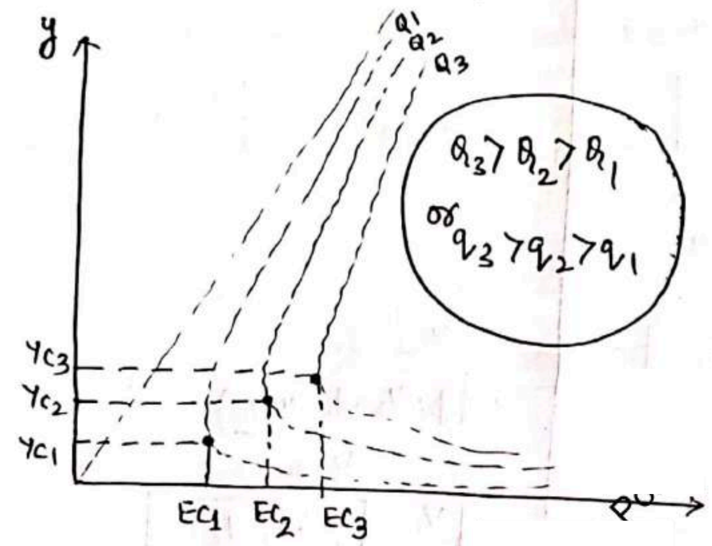
relation in Alternate depths.

$$\therefore\ \boxed{E = \frac{y_1^2 + y_2^2 + y_1 y_2}{y_1 + y_2}}$$

effect of discharge over specific energy diagram :-

∵ for a given channel discharge may vary.

take example of rectangular channel for easy understanding.



∵ for rectangular channel

$$(y_c)^3 = q^2/g$$

$$\text{if } q\uparrow \Rightarrow y_c\uparrow \Rightarrow E_c\uparrow \quad \{E_c = 1.5y_c\}$$

hence

$\boxed{\text{conclusion} \rightarrow Q\uparrow \Rightarrow \text{curve shifted right upward.}}$

conclusion :- E constant, $Q\uparrow$ then difference between alternate depth decreases.

critical flow properties :-

(I) $Fr = 1 = \dfrac{V}{\sqrt{gD\frac{\cos\theta}{\alpha}}} = \dfrac{V}{\sqrt{g\frac{A}{T}\frac{\cos\theta}{\alpha}}}$

$\theta \rightarrow$ Bed slop

$\alpha \rightarrow$ K.E correction factor

(II) velocity head$\left(\frac{V^2}{2g}\right)$ = $\dfrac{\text{hydraulic depth}(D)}{2}$

(III) $Q^2/g = A^3/T$

(IV) for given discharge, $\left.\begin{matrix}E\\P_s\end{matrix}\right\}$ min

E → specific energy

P_s → specific force

$\frac{dE}{dy} = 0$ & $\frac{d^2E}{dy^2} > 0$, $\frac{dP_s}{dy} = 0$ $\frac{d^2P_s}{dy^2} > 0$

(V) for given specific energy, Q → max.

$dq/dy = 0$ $\quad d^2q/dy^2 < 0$

(VI) for given specific force, Q → max.

$dQ/dy = 0$ $\quad d^2Q/dy^2 < 0$

(VII) critical flow unstable (∵ minor change in specific energy cause major change in depth)

(VIII) if flow changes from subcritical to supercritical (viceversa) then flow has to go through critical condition.

Critical depth, critical specific energy, critical velocity Relation :-

channel	Critical depth (y_c)	critical specific energy (E_c)	critical velocity (V_c)
rectangular y_c	$\boxed{(y_c)^3 = q^2/g}$	$\boxed{E_c = 1.5 y_c}$ $E_c = \underset{E_p}{y_c} + \underset{E_K}{y_c/2}$	$\frac{V_c}{\sqrt{g y_c}} = 1$ $\quad \boxed{V_c = \sqrt{g y_c}}$
Triangular θ, 1, Z	$\boxed{y_c = \left(\frac{2Q^2}{g Z^2}\right)^{1/5}}$	$\boxed{E_c = 1.25 y_c}$ $E_c = \underset{E_p}{y_c} + \underset{E_K}{y_c/4}$	$\frac{V_c}{\sqrt{g\left(\frac{y_c}{2}\right)}} = 1$ $\quad \boxed{V_c = \sqrt{0.5 g y_c}}$
Parabolic $T = K\sqrt{y_c}$, y_c $A = 2/3\,(y_c)(K\sqrt{y_c})$	$\boxed{y_c = \left(\frac{27 Q^2}{8 g K^2}\right)^{1/4}}$	$\boxed{E_c = 1.33 y_c}$ $E_c = \underset{E_p}{y_c} + \underset{E_K}{y_c/3}$	$\frac{V_c}{\sqrt{g\left(\frac{2/3 \times y_c \times K\sqrt{y_c}}{K\sqrt{y_c}}\right)}} = 1$ $\boxed{V_c = \sqrt{0.66 g y_c}}$

Trapezoidal ⟶ no direct solution ⟶ use hit & trial method

discharge diagram :

y vs q or Q

- for simplicity we take rectangular channel.

$$E = y + \frac{q^2}{2 g y^2} \qquad \boxed{q = \sqrt{2g(E-y)y^2}}$$

for discharge to be max. (at given E)

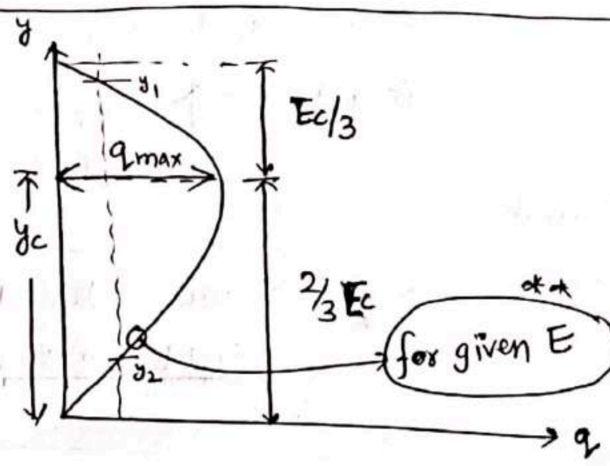
$dq/dy = 0$ & $d^2q/dy^2 < 0$

or

$\frac{d}{dy}(q^2) = 0$ ⟶ for more easier calculation

$\boxed{y = 0 \;;\; y = 2/3\,E}$

$y = 0$ ⟶ no meaning



note :- for given E, discharge (q) can pass in 2 ways

Subcritical flow $y_1 (> y_c)$

Super critical flow $y_2 (< y_c)$

effect of specific energy over discharge diagram :-

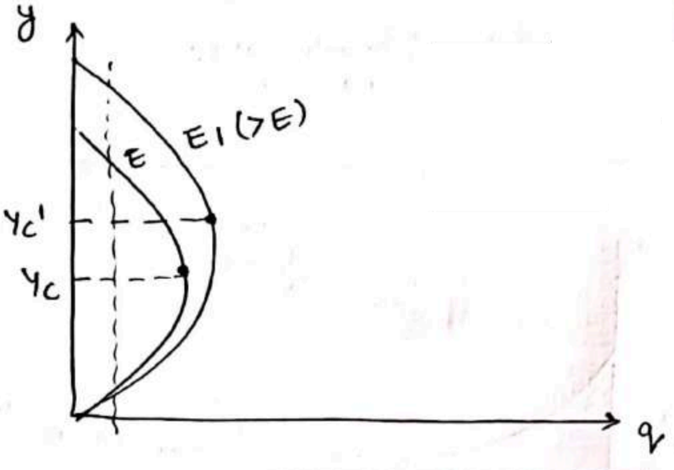
$$\boxed{q = \sqrt{2g(E-y)\,y^2}}$$

(i) $E\uparrow \Rightarrow q\uparrow$

$\therefore$ max. q point will shift in right.

(ii) $q\uparrow \Rightarrow y_c\uparrow$ hence max q point will shift upward.

$\therefore$ net $\longrightarrow$ upward right shifting



note:- In subcritical flow, same discharge with increased depth can be pass (flow behaviour same)

note:- In supercritical flow, same discharge with decreased depth can be pass without changing flow behaviour.

Transition

- Vertical contraction
  - Ex. hump construction, drop construction
- Horizontal contraction
  - Ex. increase or decrease width of channel
  - Use:- Bridge construction
  - we narrowdown the channel width then flow analysis need to be done.

Channel transition Application (discharge measuring device)

- Venturi flume
- Standing wave flume or Critical wave flume
- Parshall Flume

Analysis of Transition assumption :-

(I) Horizontal + frictionless channel

{means TEL parallel to channel bed ∵ no loss exist in this case.}

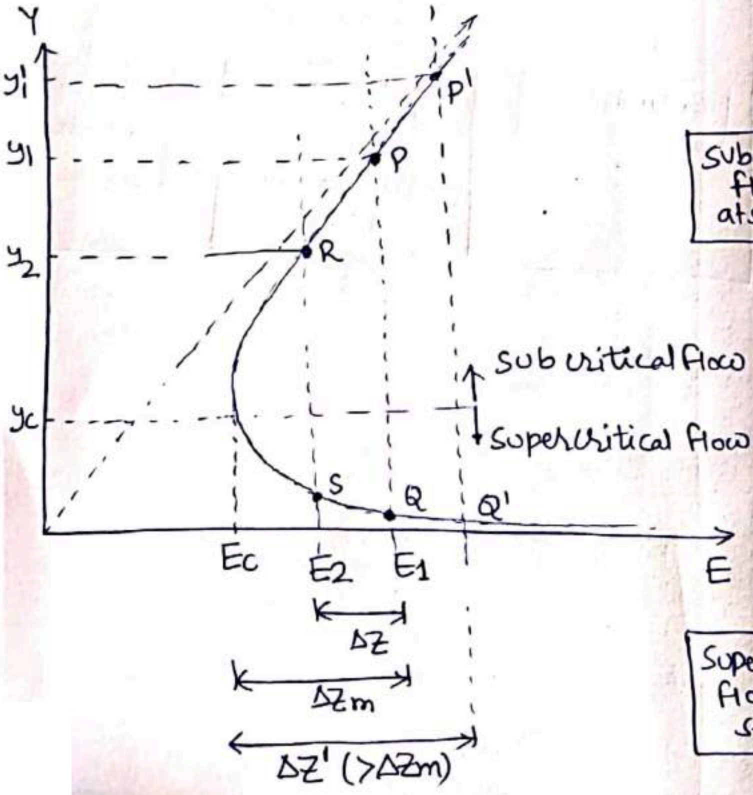
(II) rectangular channel → to simplyfy the analysis

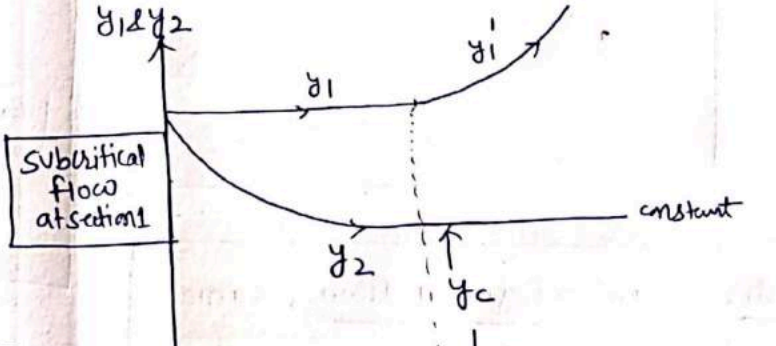
cases

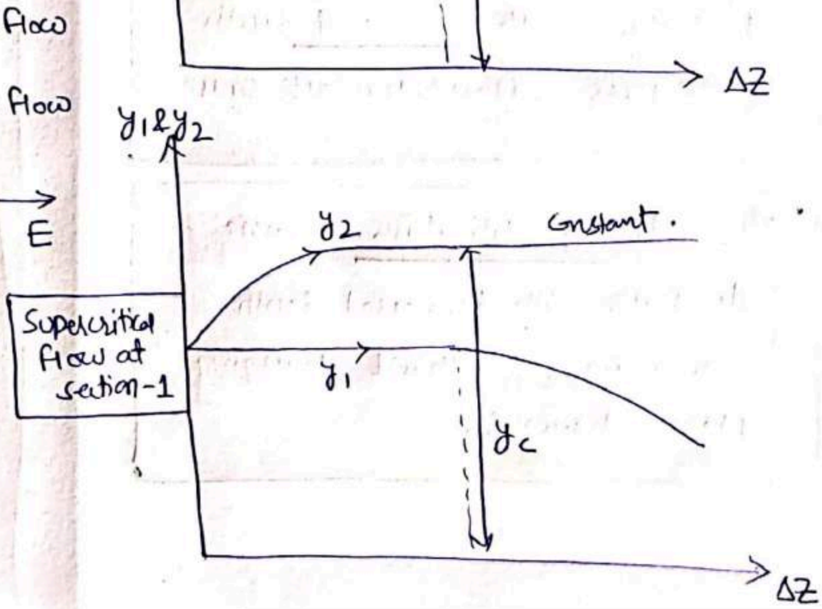
Hump construction / drop construction	width contraction
Q → constant	E → constant
∴ use → specific energy diagram	∴ use → discharge diagram

(I) Hump construction

- Subcritical flow at section 1 :- free water surface dropdown at hump (P to R)
- supercritical flow at section 1 :- free water surface rise (Q to S)

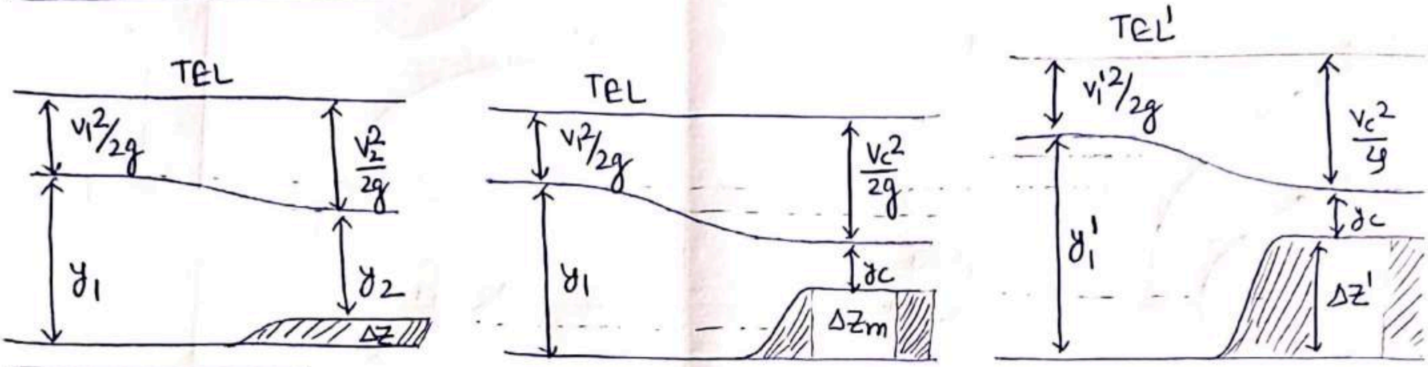






Subcritical flow at section-1

Subcritical flow Analysis :-



$\boxed{(E_1 = E_2 + \Delta Z)}$ $\boxed{(E_1 = E_c + \Delta Z_m)}$ $\boxed{E_1' = E_c + \Delta Z' \ (> \Delta Z_m)}$

$y_c \rightarrow$ same

---

v. Imp

- minimum height of hump for critical flow over hump
  'or'
  max. height of hump for which upstream flow does not affected

v. Imp **

$$\boxed{\Delta Z_m = E_1 - E_c}$$

Special case :- if $\Delta Z' > \Delta Z_m \Rightarrow$ no flow possible / chocking condition

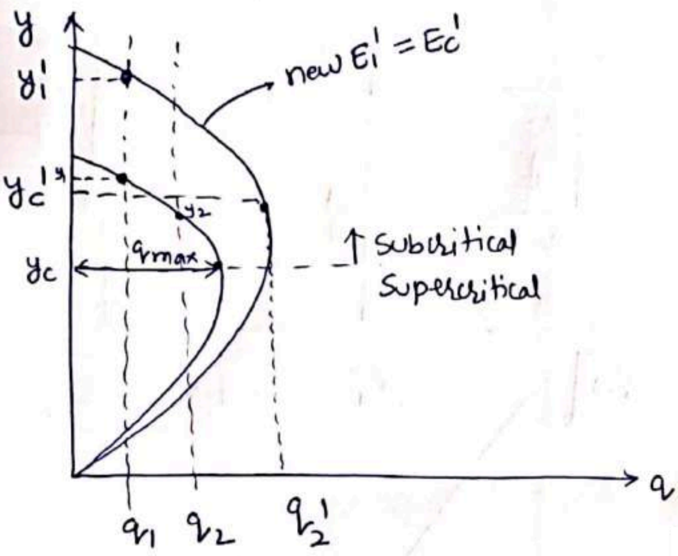
means $\rightarrow$ Same discharge can not pass with given specific energy at section-1

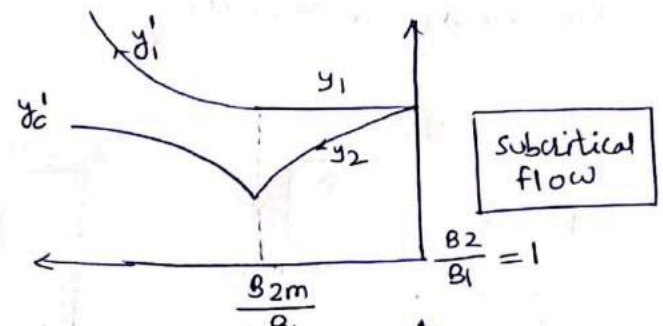
So for flow to be possible $\longrightarrow$ $\boxed{E_1' > E_1}$

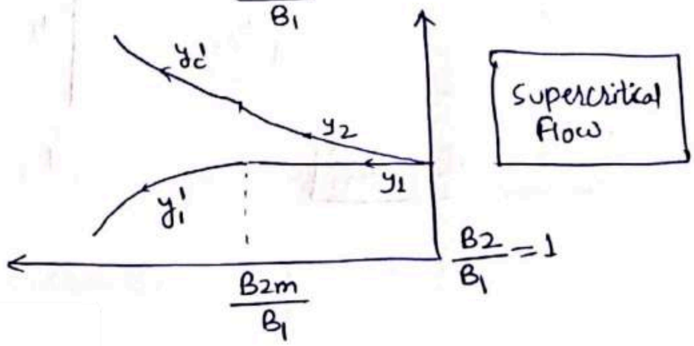
in subcritical flow depth to be increased hence flooding at upstream (or section 1) now E_1' sufficient to cause critical flow over hump.

(II) Width contraction :-

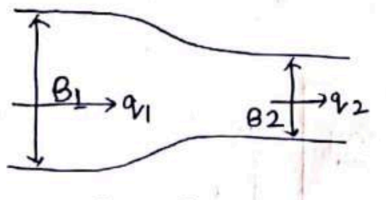
(Analysis for subcritical flow)



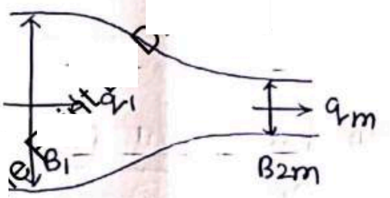




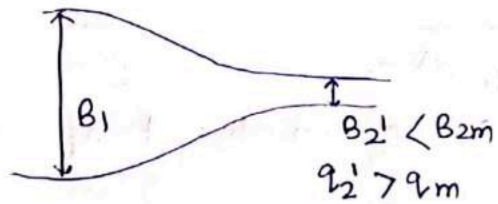
(Plan)



$B_1 > B_2$
$q_1 < q_2$

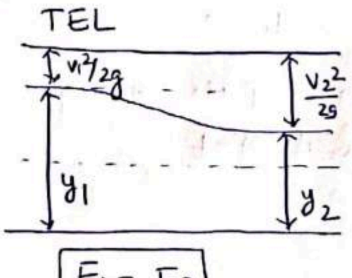


$B_1 > B_{2m}$
$q_m > q_1$ ($q_m \to q_{max}$)

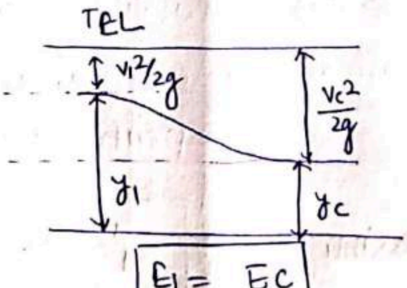


$B_2' < B_{2m}$
$q_2' > q_m$

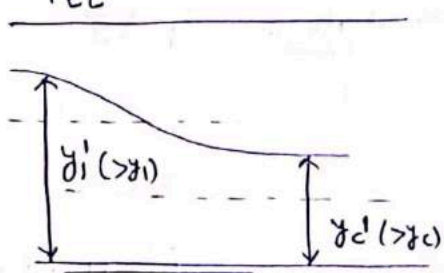
subcritical



$\boxed{E_1 = E_2}$



$\boxed{E_1 = E_c}$



$\boxed{E_1' = E_c'}$

$$\left\{ \begin{array}{l} E_c' = 1.5 y_c' \\ \quad y_c' = \left(\frac{q_2'^2}{g}\right)^{1/3} \\ q_2' > q_{max} \\ \therefore y_c' > y_c \end{array} \right.$$

if $\boxed{B_2' < B_{2m}}$ → flow not possible

$q_2' > q_{2m}$

hence to pass this discharge there will be rise in waterlevel at section 1 $\{y_1' > y_1\}$ ∵ It is a case of subcritical flow.

∴ $E_1' = E_c' \rightarrow 1.5y_c' \rightarrow \left(\frac{q_2'^2}{g}\right)^{1/3}$

$y_1' + \frac{Q^2}{2gB^2y_1'^2}$

to get y_1' ?

Imp

* maximum contracted width (B_{2m})

ⓐ U/S water level does not affect

$E_1 = E_{cm}$ or $E_{2m} = \frac{3}{2}y_c$

$y_c = \left(\frac{q_{2m}^2}{g}\right)^{1/3}$, $q_{2m} = Q/B_{2m}$

∴

$$\boxed{B_{2m} = \sqrt{\frac{27Q^2}{8gE_1^3}}}$$ **

if beyond this $B < B_{2m}$

flooding (in subcritical flow)

waterlevel decrease at section (1) (in supercritical flow)

3/3/2020

GVF (Gradually varied flow) :- Properties

① type of non uniform flow

$$\left\{ \frac{dy}{dx} \neq 0 \Rightarrow \frac{dy}{dx} << 1 \right\}$$

(II) component of gravity force in the direction of flow $\neq$ resistance force (mainly due to Boundry friction)

∴ Balance disturb ⟶ velocity change

{∴ depth changes from section to section. (GVF)}

(III) Curvature $\frac{1}{R} \longrightarrow 0$ (very less)

{∵ depth changes very gradually.}

(IV) Loss due to Boundry friction only mainly.

{∵ friction play imp. role in GVF}

(V) $S_0 \neq S_w \neq S_f$

S_0 ⟶ Bedslope

S_w ⟶ water surface slope

S_f ⟶ TEL slope.

GVF analysis assumption :-

① Pressure distribution ⟶ hydrostatic

[hence curvature —— less

streamlines more or less straight and parallel]

∴ no centrifugal force, no normal acceleration in GVF we assume.

(II) Resistance to flow given by chezy or manning equation.

[Put $S_0 \longrightarrow S_f$ to get actual depth of flow] in GVF $S_0 \neq S_f$

(III) $\theta \rightarrow$ small

$HGL = y\cancel{\cos\theta}^{1} + Z$ ∴ HGL lie at free surface.

(IV) $\alpha \longrightarrow 1$ variation in velocity ⟶ neglect

(V) Prismatic channel [Area, slope constant throughout]

(VI) Resistance cofficient C, n does not vary with depth {otherwise analysis will be very difficult}

(VII) no lateral Inflow or outflow

(VIII) no air entrainment

{otherwise 'n' will change}

GVF theory :-

Total energy Head at any section

$$\boxed{H = Z + y\cos\theta + \frac{\alpha V^2}{2g}}$$

here $y \rightarrow f(x)$ $\therefore H \rightarrow f(x)$

$\frac{dZ}{dx} \rightarrow -S_0$, $\frac{dH}{dx} \rightarrow$ wrt horizontal

$\frac{dH}{dx} \rightarrow -S_f$

(-)ve sign $\because$ decreases

$\frac{dy}{dx} \rightarrow$ wrt channel bottom

$$\boxed{\text{differential equation of GVF } \frac{dE}{dx} = S_0 - S_f}$$

dynamic eqn of GVF :-

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 T}{gA^3}} = \frac{S_0 - S_f}{1 - \frac{\alpha V^2}{gD}} = \frac{S_0 - S_f}{1 - \alpha F_r^2}}$$

$\frac{dy}{dx} = f^n \{ S_0, S_f, F_r \}$

dynamic eqn for wide rectangular channel

$$\frac{dy}{dx} = \frac{S_0\left[1 - \left(\frac{y_n}{y}\right)^{10/3}\right]}{1 - \left(\frac{y_c}{y}\right)^3} \rightarrow \text{as per manning formula}$$

$$\frac{dy}{dx} = \frac{S_0\left[1 - \left(\frac{y_n}{y}\right)^3\right]}{\left[1 - \left(\frac{y_c}{y}\right)^3\right]} \rightarrow \text{as per chezy eqn}$$

dynamic eqn in terms of discharge :-

$Q_n = K\sqrt{S_0}$

$Q = K\sqrt{S_f}$

$\therefore S_f/S_0 = \left(Q/Q_n\right)^2$

$\frac{Q_c^2 T}{gA^3} = 1$

$\frac{Q^2 T}{gA^3} = F_r^2$

$$\therefore \boxed{\frac{dy}{dx} = S_0\left[\frac{1 - \left(\frac{Q}{Q_n}\right)^2}{1 - \left(\frac{Q}{Q_c}\right)^2}\right]}$$

dynamic eqn in terms of conveyance 'Z'

$Q = K_0\sqrt{S_0} = K\sqrt{S_f}$ $\quad S_f/S_0 = (K_0/K)^2$

$Z^2 = A^3/T$ $\{\because Z = A\sqrt{D} = A\sqrt{A/T}\}$

$Z_c^2 = \frac{A_c^3}{T_c} = \frac{Q^2}{g}$

$\therefore \frac{Q^2 T}{gA^3} = \left(Z_c/Z\right)^2$

$$\boxed{\frac{dy}{dx} = S_0\left[\frac{1 - \left(\frac{K_0}{K}\right)^2}{1 - \left(\frac{Z_c}{Z}\right)^2}\right]}$$

classification of flowprofile

$\left\{ \begin{array}{l} 1 \rightarrow \text{Top} \\ 2 - \text{middle} \\ 3 - \text{bottom(lowest most)} \end{array} \right.$

Based on channel category

(mild, steep, critical, horizontal, adverse)

Based on flow Zone | region

(1, 2, 3)

Bedslope

- Sustainable slope: $S_0 > 0$
  - mild
  - Steep
  - Critical
- Unsustainable slope: $S_0 \leq 0$
  - Horizontal
  - adverse

---

total flow profiles $(12) = \{15 - 3\}$, where $3 = C_2, A_1, H_1$

- Subcritical: 6
- supercritical: 6

note:- $C_2, A_1, H_1 \longrightarrow$ don't exist

Points to draw GVF profile :-

need to remember $\dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1 - Fr^2}$

for wide rect. $\dfrac{dy}{dx} = S_0 \dfrac{\left(1 - \left(\frac{y_n}{y}\right)^{10/3}\right)}{1 - \left(\frac{y_c}{y}\right)^3}$

(I) region 1 & 3 $\boxed{\dfrac{dy}{dx} > 0}$

$\therefore$ Back water / rising curve

(II) region 2 $\dfrac{dy}{dx} < 0$ $\boxed{\text{drawdown curve}}$

(III) $y \rightarrow y_n$ profile will Approach NDL asymptotically.

$$\left\{ \frac{dy}{dx} = S_0 \frac{\left(1 - \left(\frac{y_n}{y}\right)^{10/3}\right)}{1 - \left(\frac{y_c}{y}\right)^3} \quad \text{when } y \rightarrow y_n \quad \frac{dy}{dx} \rightarrow 0 \right\}$$

(4) $y \rightarrow y_c$ Profile will approach CDL vertically means at very steep slope.

$\left\{ \because \dfrac{dy}{dx} \rightarrow \infty \right.$ {very high curvature}

$\because$ GVF don't exist in such region hence shown by dashlines - - -

(5) $y \rightarrow \infty$ {larger water depth}

water surface profile will be horizontal

$\because \dfrac{dy}{dx} = S_0$ {horizontal Asymptotic}

(6) $y \rightarrow 0$ $\therefore \dfrac{dy}{dx} \rightarrow \infty$ (high curvature)

$\therefore$ y approach to Bedslope normally or perpendicular.

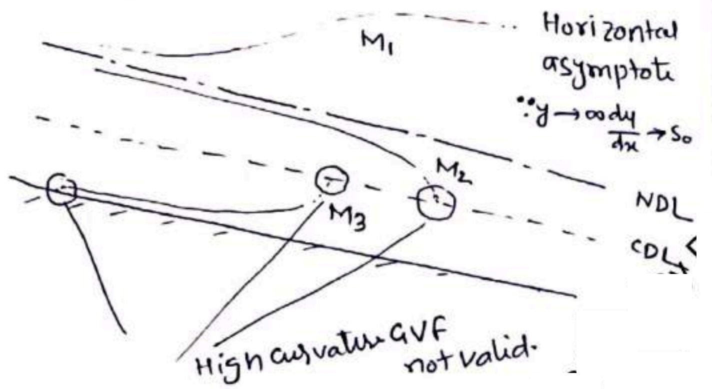
note:-

NDL — · — · —

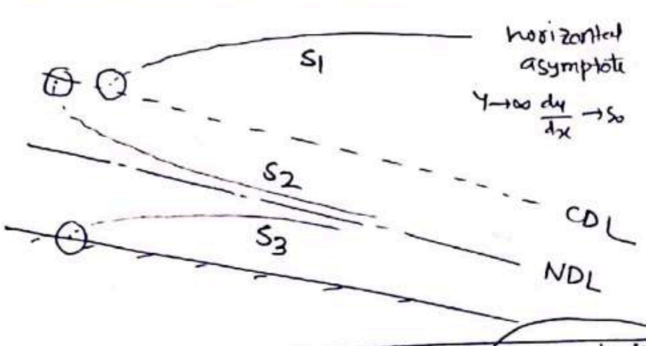
CDL - - - - - -

∴ less stable { minor change in energy leads to major change in depth.

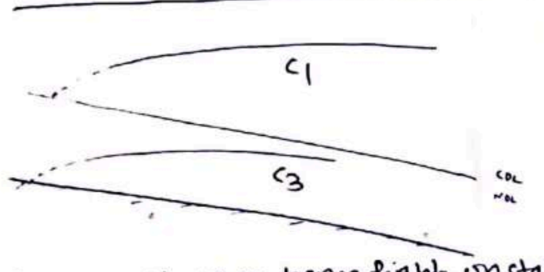
mild slope profiles :- (m_1/m_2/m_3)



steep slope profiles (S_1/S_2/S_3) :-



critical slope profiles C_1/~~C_2~~/C_3 (C_2 does not exist) : only rising curve

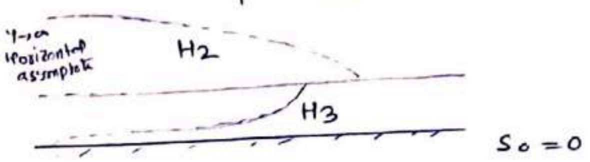


$C_1, C_3 \to$ very rare hence highly unstable

note:- So where we have to reduce height of water this can not be provided.

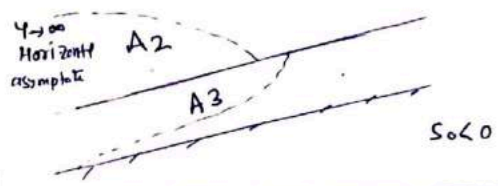
Horizontal slope profile (H_2 H_3)

$H_1 \longrightarrow$ does not exist.



adverse slope profile (A_2, A_3)

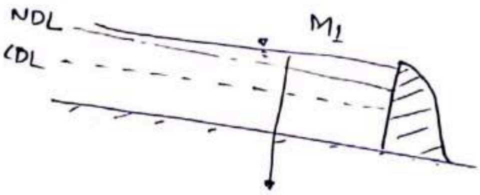
$A_1 \to$ does not exist.



Practical case :-

① **M_1 profile :-** $\longrightarrow$ most common profile

when subcritical flow ($y > y_c$) obstructed by dam/weir extends several kms before joining normal depth of flow.



Back water curve
'or'
rising curve
$\frac{dy}{dx} > 0$

note:- water always wants to flow at normal depth.

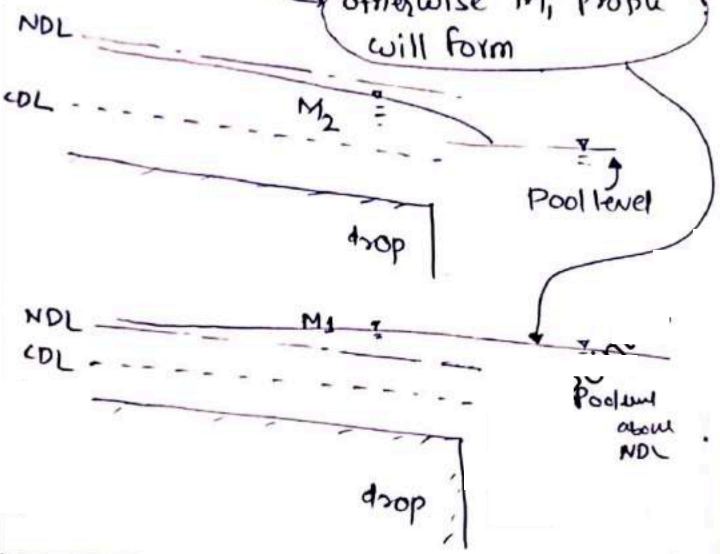
M_2 Profile :-

- when subcritical flow ($y > y_c$) experience a sudden drop in Channel Bed

Ex. Canal outlet into pool

{ Pool level below NDL at end of channel profile }

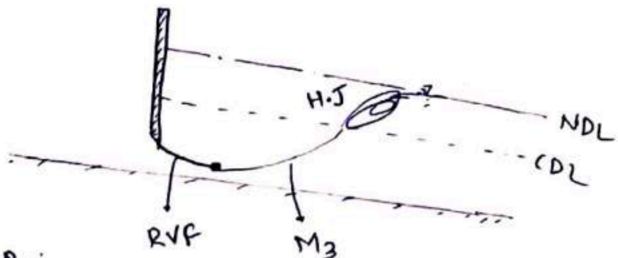
** otherwise M_1 Profile will form



M_3 Profile :-

- when supercritical flow ($y < y_c$) enters mild slope channel

Ex. Flow leading from spillway / sluice gate to mild slope.



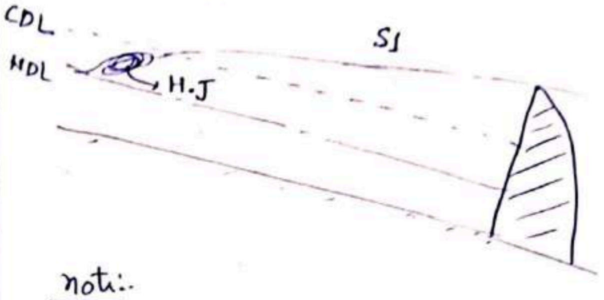
Jump

note: ① Beginning of M_3 curve is usually followed by small stretch of RVF and d/s of M_3 is terminated by hydraulic Jump.

② M_3 is relatively short in comparison to M_1, M_2 due to supercritical flow.

S_1 Profile :-

- when supercritical flow ($y < y_c$) over Steep slope terminated by deep pool created by dam/weir.



note:-

① during S_1 curve, flow changes from supercritical to subcritical through a formation of H.J

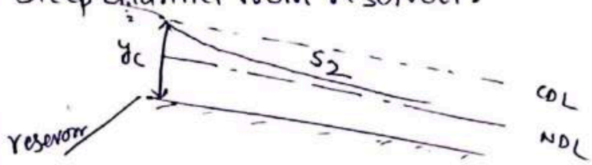
hence S_1 curve is preceded by H.J.

most :- S_1 के पहले Jump
M_3, A_3, H_3 के बाद Jump

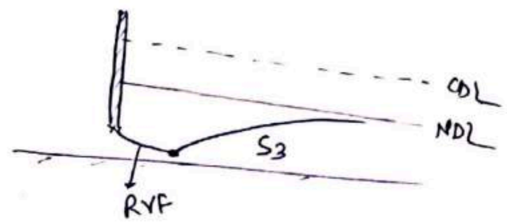
most :- flow over weir is always critical ($y = y_c$)

∵ weir treated as case of large hump $\Delta Z > \Delta Z_{max}$

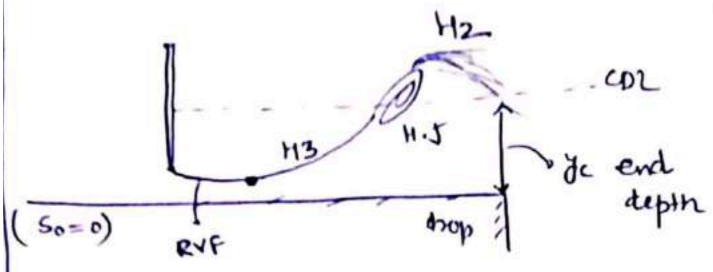
S_2 profile :- forms at entrance region of Steep channel from reservoir.



S_3 Profile :- when free flow from sluice gate enters steep slope.

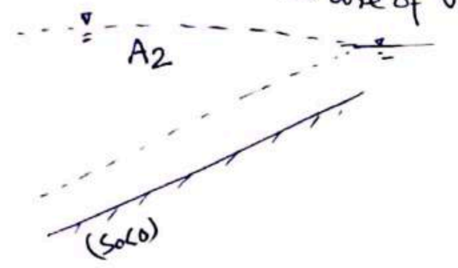


Type H profiles :- → (rare & stable)



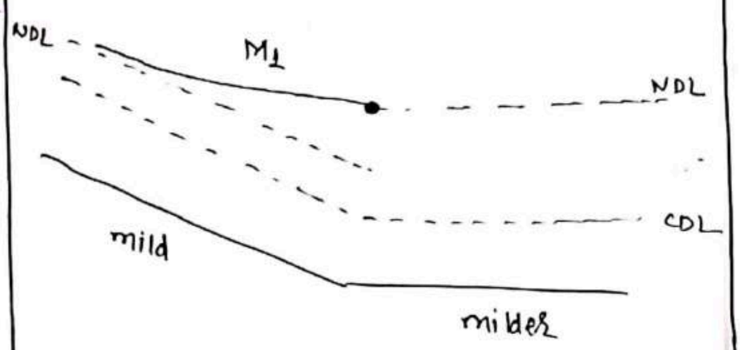
note:- S_1 के पहले जम्प M_3, A_3, H_3 के बाद Jump

Type A Profiles :- → rarest of rate
→ are of very short length.

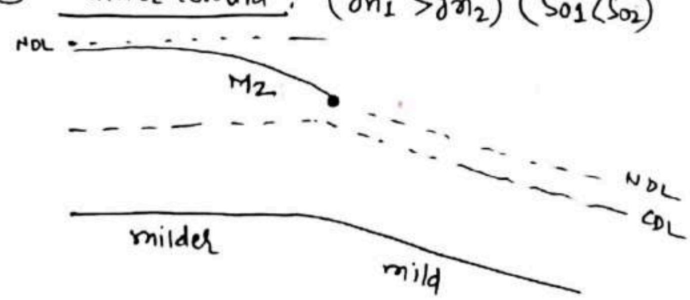


cases where slopes meets :-

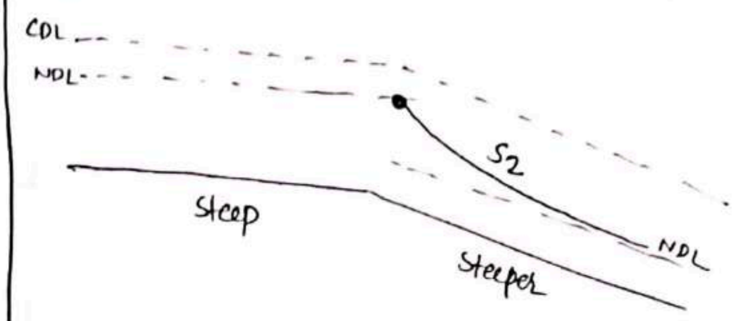
① mild to milder ($y_{n1} < y_{n2}$)
$S_{01} > S_{02}$



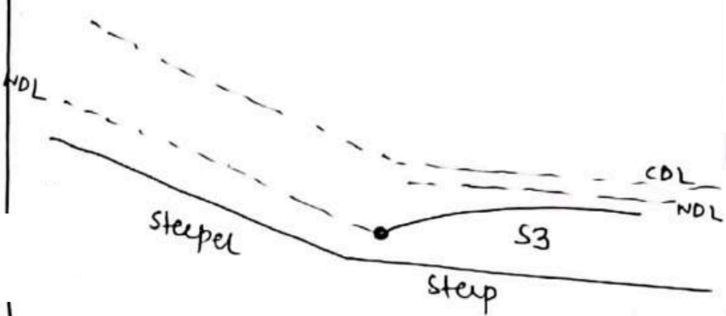
② milder to mild : ($y_{n1} > y_{n2}$) ($S_{01} < S_{02}$)



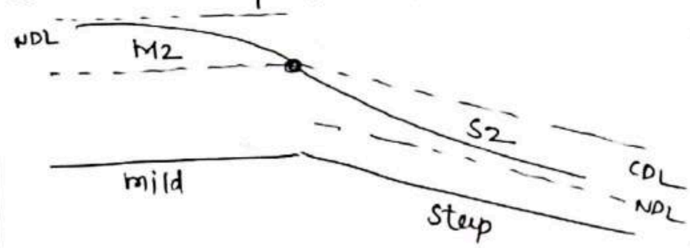
③ Steep to steeper :-



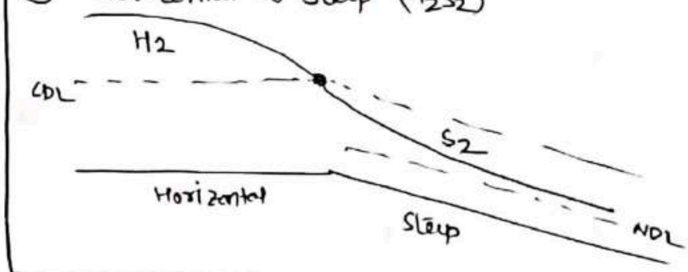
④ steeper to steep



⑤ mild to steep ($M_2 S_2$)



⑥ Horizontal to steep ($H_2 S_2$)

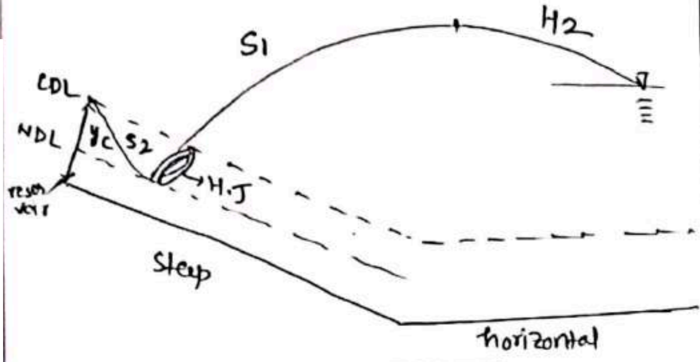


⑦ same adverse to steep- $A_2 S_2$

⑧ (SH) **Steep to horizontal** :- 2 case possible based on post depth or depth on horizontal slope.

Case-1 :-

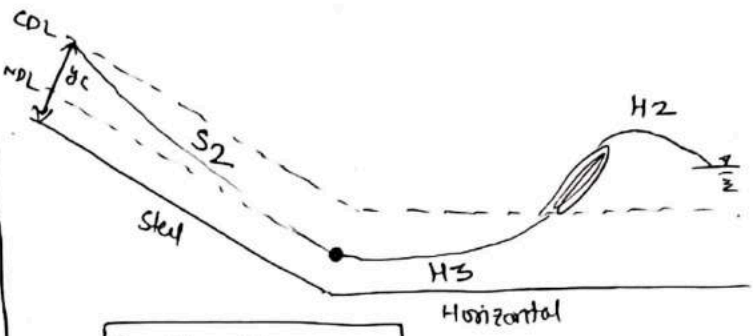
Hydraulic Jump forms in steep slope before S_1



$\boxed{S_2 - \text{Jump} - S_1 - H_2}$

Case-2 :-

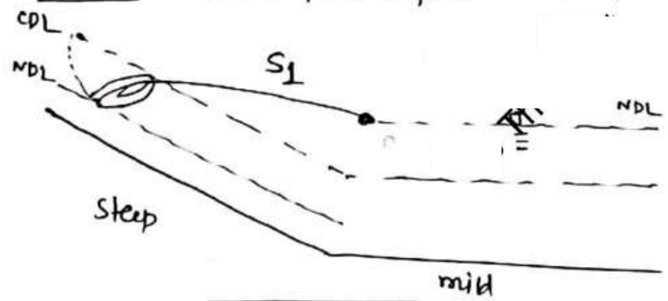
Hydraulic Jump forms in horizontal slope after S_3



$\boxed{S_2 - H_3 - \text{Jump} - H_2}$

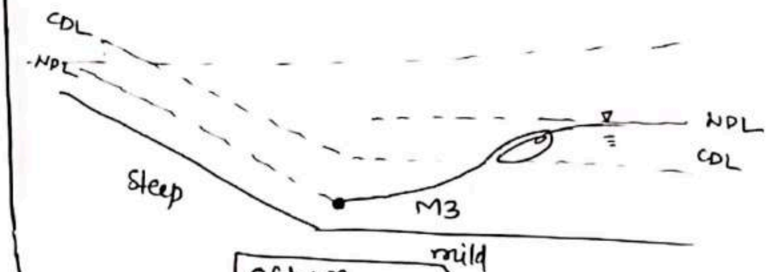
⑨ (SM) **steep to mild** : 2 case possible based on post depth (~~sequent~~ post depth requirement)

Case 1 :- when post depth → more



$\boxed{\text{Before } S_1\text{-Jump}}$

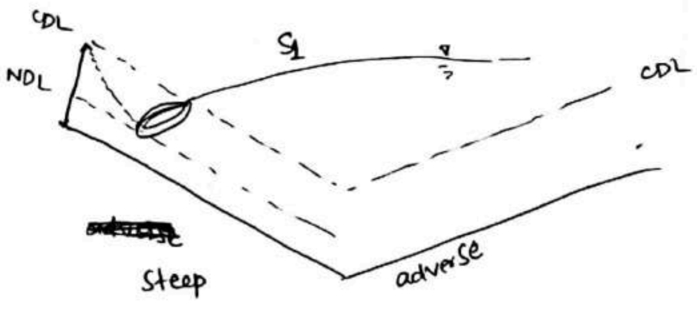
Case-2 when post depth → less



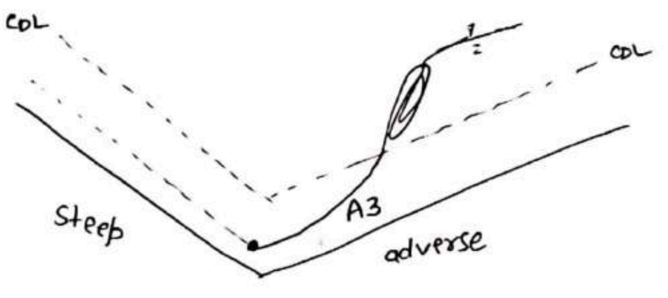
$\boxed{\text{after } M_3\text{-Jump}}$

⑩ (SA) **steep to adverse** :-

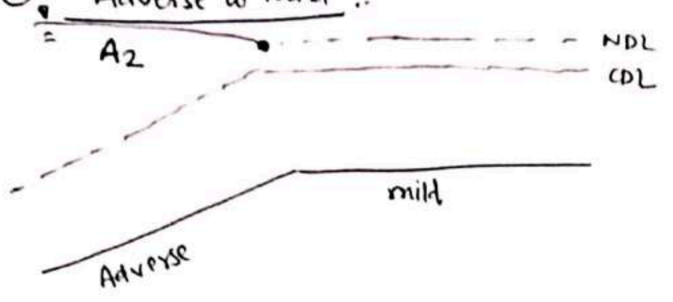
case-1 :- Before S_1 → Hydraulic jump



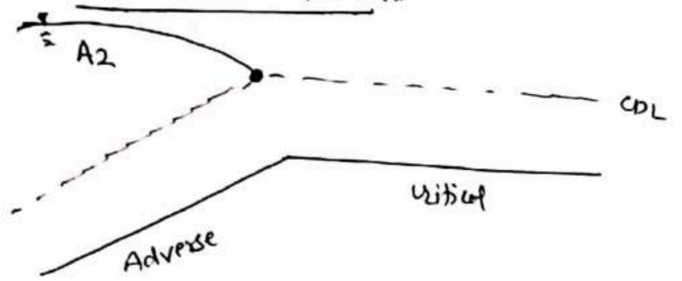
Case-2 after A_3 → Hydraulic jump



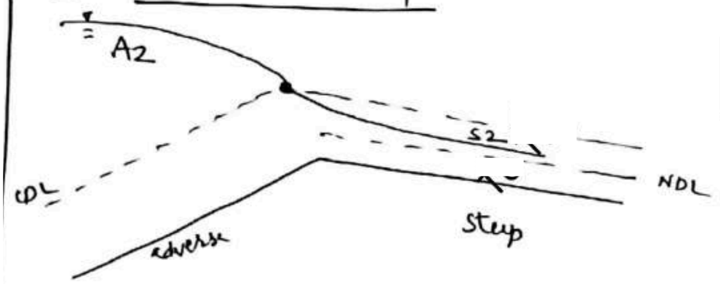
(11) Adverse to mild :-



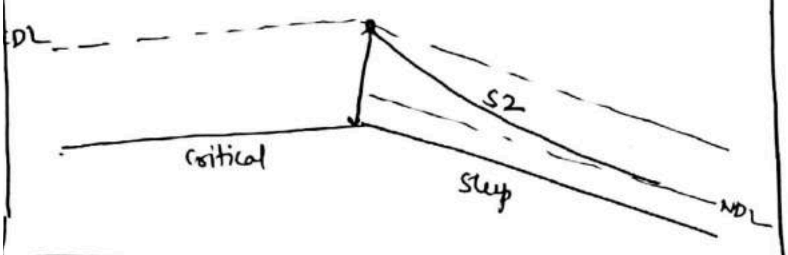
(12) Adverse to critical :-



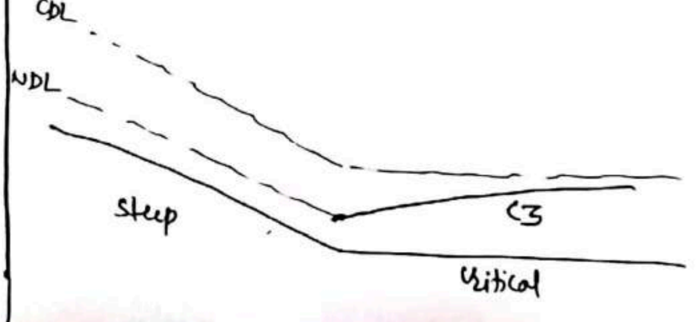
(13) Adverse to steep



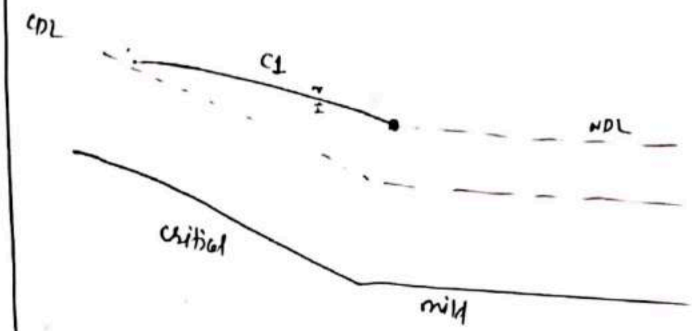
(14) critical to steep :-



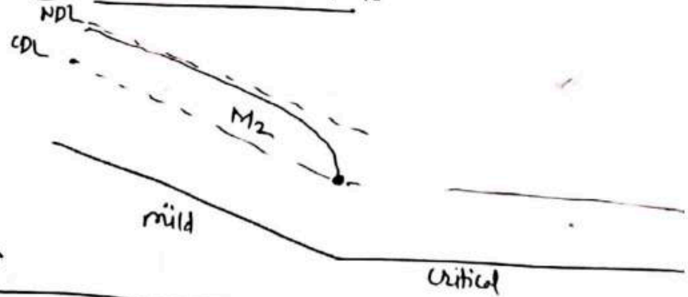
(15) steep to critical



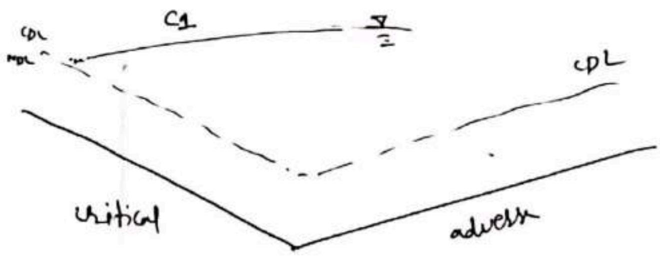
(16) criticaltomild :-



(17) mild to critical :-

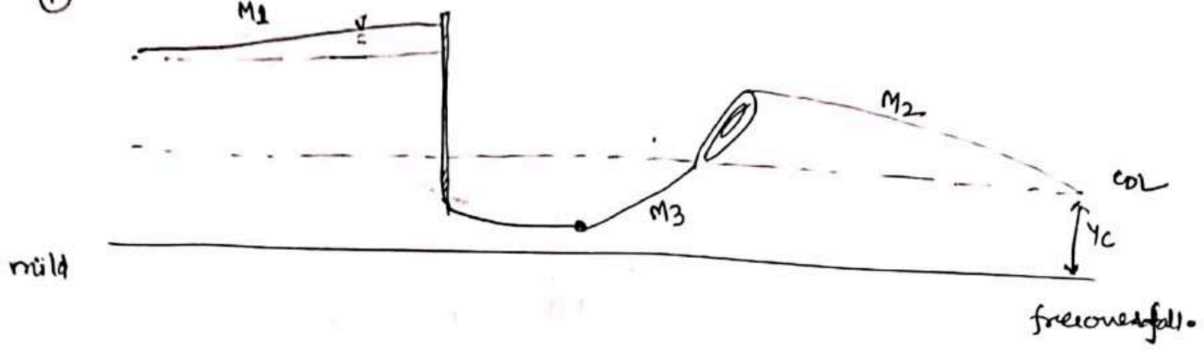


(18) critical to adverse :-

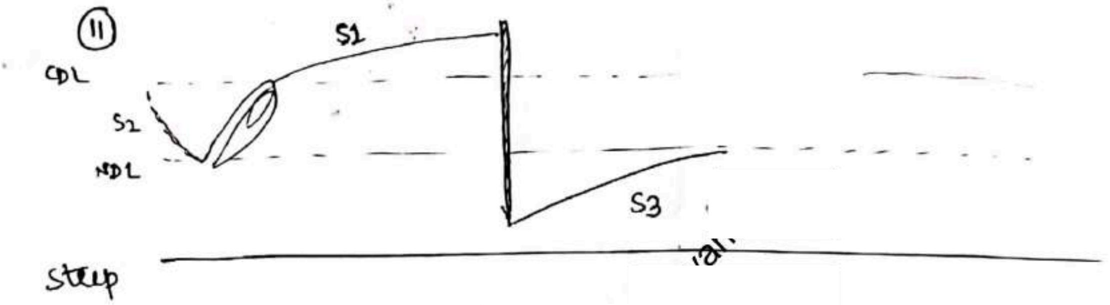


Real life problem :-

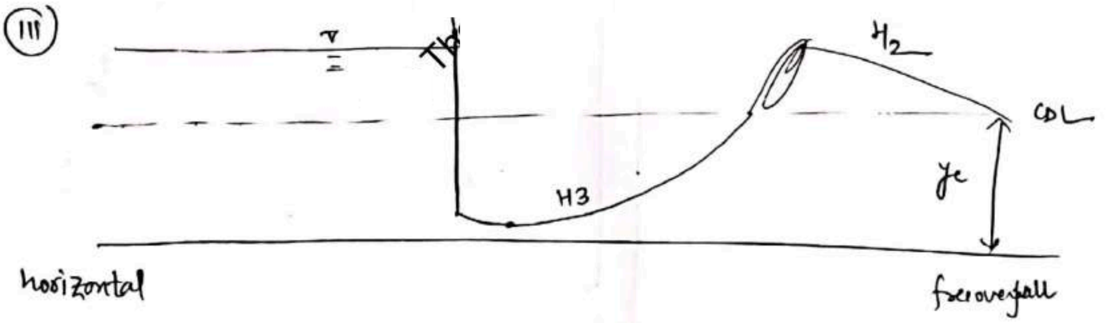
(I)



(II)



(III)



GVF Profile computation

Uses →

(I) to find effect of hydraulic structure on flow pattern in channel.

(II) Inundation of land due to dam/weir construction.

(III) estimation of flood zones

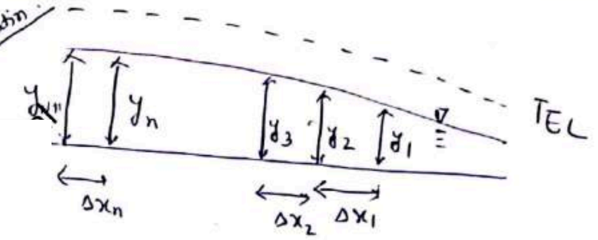
note:: control section

- Subcritical flow (in downstream) → hence GVF calculation must proceed in U/S direction.
- Supercritical flow (in upstream)

GVF computation method

- direct Integration method 'or' Chow's method (for wide rectangular channel)
- numerical methods → mostly used
  - direct Step method → artificial channel 'or' Prismatic channel (simplest method)
  - Standard Step method → natural channel → Prismatic + nonprismatic
- Graphical methods → natural channel → Practically obsolete

Derivation



$$\Delta x = \Delta x_1 + \Delta x_2 - - - \Delta x_n$$

$$\because \frac{dE}{dx} = S_0 - S_f$$

'or' $\frac{\Delta E}{\Delta x} = S_0 - \bar{S_f}$ [infinite form]

$$\therefore \boxed{\Delta x = \frac{\Delta E}{S_0 - \bar{S_f}}} \quad \Delta E \rightarrow E_2 - E_1$$

$$\bar{S_f} = \left[\underbrace{\frac{S_{f_1}+S_{f_2}}{2} \quad \sqrt{S_{f_1} S_{f_2}} \quad \frac{2 S_{f_1} S_{f_2}}{S_{f_1}+S_{f_2}}}_{\text{depends on question}}\right]$$

But if nothing is given use $\bar{S_f} = \frac{S_{f_1}+S_{f_2}}{2}$

direct step method — for Artificial or prismatic channel

- entire length of channel is divided into short reaches so that flow profiles can be assumed as straight & curvature can be neglected for short reaches.

note:: for wide rectangular channel

$$Q = \frac{1}{n}(A_{avg})(R_{avg})^{2/3}\sqrt{\bar{S_f}}$$

$A_{avg} \rightarrow B y_{avg}$, $R_{avg} \rightarrow y_{avg}$

$$\boxed{\bar{S_f} = \frac{n^2 q^2}{(y_{avg})^{10/3}}}$$

Rapid varied Steady Flow :- Hydraulic Jump :- $\left(\frac{dy}{dx} \to 1\right)$

(RVSF)

'or' shock wave
'or' standing wave

RVF :- (I) sudden change in depth over short length hence friction neglected.

(II) $\frac{dy}{dx} \to 1$ streamlines $\longrightarrow$ high curvature $\to$ turbulence $\to$ High energy loss

∴ pressure distribution $\longrightarrow$ non hydrostatic.

location where Hydraulic Jump forms :-

1- d/s of sluicegate

2- toe of spillway

3- Steep to mild channel
(refer GVF profiles)

Hydraulic Jump Applications :-

1- energy dissipation < sluicegate, spillway

2- to avoid erosion in stilling Basin

3- to mix chemical (use turbulence)

4- aeration in water treatment plant

5- reduce uplift pressure { 'or' increase weight of apron.

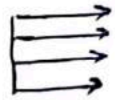
6- raise water level on d/s of metering flume
∵ higher water level needed for Irrigation purpose.

7. Desalination of sea water.

Assumption in analysis of Hydraulic Jump

(I) Before & after of Jump
Flow depth $\longrightarrow$ uniform

∴ velocity distribution $\longrightarrow$ uniform



(II) Pressure distribution $\longrightarrow$ Hydrostatic
(curvature of streamlines neglected)

(III) Length of Jump small hence
frictional resistance $\longrightarrow$ neglected
(F_f) (0)

(IV) slope of channel $\longrightarrow$ small
$(\theta = 0)$

(V) air entrainment effect $\longrightarrow$ neglected
(wind effect)

equation used in HJ analysis :-

- continuity eqn
- momentum eqn

Imp

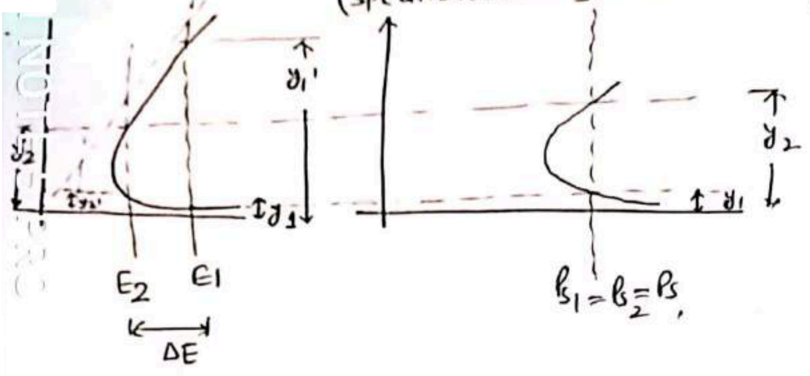
note :- ∵ energy loss $\longrightarrow$ not known Initially

hence energy eqn not used

Analysis of Jump in horizontal, frictionless, rectangular channel :-

($\theta = 0$ $f_f = 0$ rectangular channel)

(specific force diagram)



(specific energy diagram)

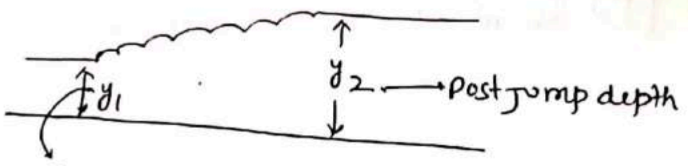
Conclusion based on above 2 diagrams :-

Alternate depth (y_1') corresponding to E_1 > y_2 Post Jump depth > y_2' Alternate depth (E_2) > Pre depth y_1

explanation :- $y_1 \xrightarrow{\text{wants}} y_1'$ but due to eddy / turbulence

$y_1 \longrightarrow y_2$

Supercritical depth → subcritical depth.



$$P_1 - P_2 = M_2 - M_1$$

$$\boxed{\frac{P_1 + M_1}{\rho g} = \frac{P_2 + M_2}{\rho g}}$$

→ valid for all type channel (rectangular, Triangular, trapezoidal)

$$\boxed{P_s = A\bar{Z} + \frac{Q^2}{Ag}}$$ specific force

H.J formula's for rectangular type channel

$$\boxed{y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g}}$$

$(q = Q/B)$

$y_c = \left(\frac{q^2}{g}\right)^{1/3}$

$E_c = 1.5 y_c$

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_1^2}\right]}$$

$$\boxed{\frac{y_1}{y_2} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_2^2}\right]}$$

y_1 → Prejump depth

y_2 → Post Jump depth

y_1, y_2 → sequent depth / conjugate depth {for which specific force is constant)

$$\boxed{F_1^2 = \frac{(F_2)^2}{\left(\frac{y_1}{y_2}\right)^3}}$$

$$\boxed{F_2^2 = \frac{F_1^2}{\left(\frac{y_2}{y_1}\right)^3}}$$

$$\boxed{F_r^2 = \left(\frac{y_c}{y}\right)^3}$$

$$\boxed{F_1^2 = \left(\frac{y_c}{y_1}\right)^3}$$

$$\boxed{F_2^2 = \left(\frac{y_c}{y_2}\right)^3}$$

$$\boxed{\Delta E = E_1 - E_2 = \frac{(y_2 - y_1)^3}{4 y_1 y_2}}$$ (energy loss)

$$\boxed{\text{Power } P = \rho g Q \Delta E}$$ (1 HP = 746 watt)

Relative energy loss :- $= \frac{\Delta E}{E_1}$

How to solve : $\frac{\Delta E / y_1}{E_1 / y_1}$

$\boxed{\Delta E / y_1}$

$$\left[\frac{(y_2 - y_1)^3}{4 y_1 y_2}\right] \times \frac{1}{y_1}$$

$$\frac{y_1^3 \left(\frac{y_2}{y_1} - 1\right)^3}{4 y_1^2 y_2}$$

$$\boxed{\frac{(y_2/y_1 - 1)^3}{4 (y_2/y_1)}}$$

$\boxed{E_1 / y_1}$

$$\frac{y_1 + \frac{q^2}{2 g y_1^2}}{y_1}$$

$$\boxed{1 + \frac{F_1^2}{2}}$$

both are f^n of F_1

other Terms :

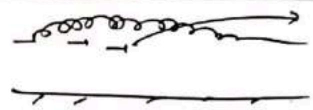
(I) Jump efficiency $\boxed{\eta = \frac{E_2}{E_1} = \frac{E_1 - \Delta E}{E_1} = 1 - \text{relative energy loss}}$ $\boxed{\text{if } \Delta E = 0 \text{ then } \eta = 100\%}$

(II) Jump height $= \boxed{y_2 - y_1}$

(III) relative (मतलब E_1 के respect में)

energy loss	height of Jump	Initial depth	sequent/post Jump depth
$\left(\frac{\Delta E}{E_1}\right)$	$\frac{y_2 - y_1}{E_1}$	$\frac{y_1}{E_1}$	$\frac{y_2}{E_1}$

Types of Jump :

Initial froude no. F_1	Jump name	$\Delta E * 100/E_1$ relative energy loss	Properties / characteristics
1 – 1.7	Undular	≃ 0	• Undulation of surface. 
1.7 – 2.5	weak	18 %	 • small roller forms • d/s water surface smooth
2.5 – 4.5	Oscillating	18 – 45 %	 oscillation • tailwater depth fluctuates • max. damage to earthen bank
4.5 – 9	steady [Best Jump]*	45 – 70 %	• well established Jump • tail water does not fluctuates • energy loss sufficient
> 9	strong or choppy	> 70 %	water surface is rough & choppy continuous for a long distance. लहरें भरा

Hydraulic jump in triangular channel :-

- Side wall not vertical
- lateral expansion of stream in addition to increase in depth.

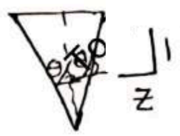
note:- for same F_1, $\frac{y_2}{y_1}$ is less {if compare with rectangular channel}

Proof :-

Pre — Post

$$A_1\bar{z}_1 + \frac{Q^2}{A g} = A_2\bar{z}_2 + \frac{Q^2}{A_2 g}$$

$(y_1^2 z)$ $(\gamma_1/3)$ $(y_2^2 z)$



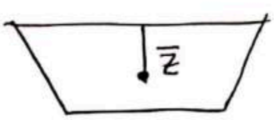
after solving :- **

$$\boxed{\frac{F_1^2}{2} = \frac{Q^2}{g y_1^5} = \left(\frac{r^3-1}{r^2-1}\right)\left(\frac{r^2}{3}\right)}\quad r = \frac{y_2}{y_1}$$

→ triangle with vertex angle = 90°

---

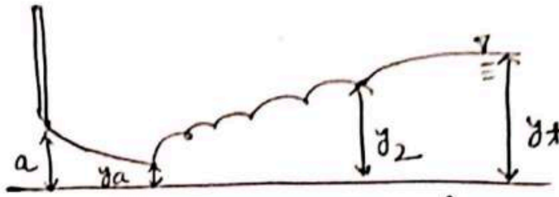
note:- HJ in Trapezoidal channel



$P_{s_1} = P_{s_2}$ — solve hit & Trial.

$$A_1\bar{z}_1 + \frac{Q^2}{A_1 g} = A_2\bar{z}_2 + \frac{Q^2}{A_2 g}$$

H.J classification based on tail water depth (y_t)

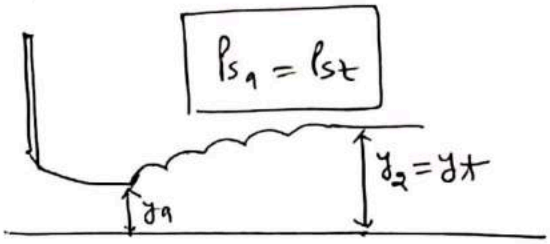


$\boxed{C_c = y_a/a}$ assumption $\begin{cases}\theta = 0 \\ FF = 0\end{cases}$

① free Jump :- $\boxed{\text{if } y_2 = y_t}$

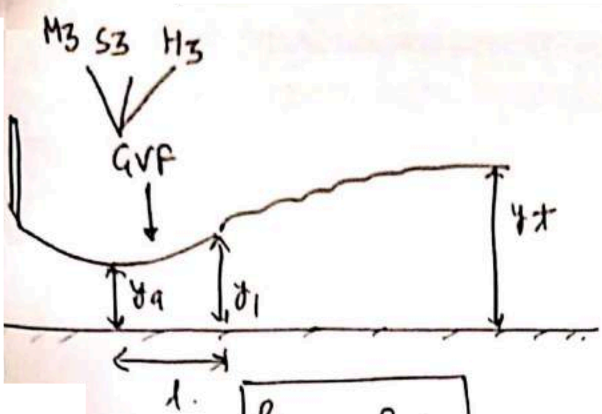
→ forms at vena contracta.

$\boxed{P_{s_a} = P_{st}}$



② free repelled Jump :- $\boxed{\text{if } y_2 > y_t}$

- due to less tail water depth (y_t) Formation of Jump is delayed.
- Jump is repelled at d/s of vena contracta at some distance 'L' and at some depth y_1 ($y_1 > y_a$) and now Jump will end at y_t.



$$\boxed{F_{s1} = F_{st}}$$

$$\boxed{\frac{y_1}{y_t} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 F_t^2}\right]}$$

F_t → known

now

$y_1 \longrightarrow$ will be known

now 'l' can be find out by

$$\boxed{l = \Delta x = \frac{E_2 - E_1}{S_0 - \bar{S}_f}}$$

note:-

$$\boxed{\text{submerge factor } (S) = \frac{y_t - y_2}{y_2} = \frac{y_t}{y_2} - 1}$$

$$\boxed{\Delta E \propto \frac{1}{S}} \longrightarrow \text{if submerge factor is less, there will be more energy loss.}$$

ΔE → (energy loss)

③ Submerged Jump (Drowned Jump) :-

- $\boxed{\text{if } y_t > y_2 \text{ (tail water is more)}}$
- Forms at vena contracta, But energy loss is less.

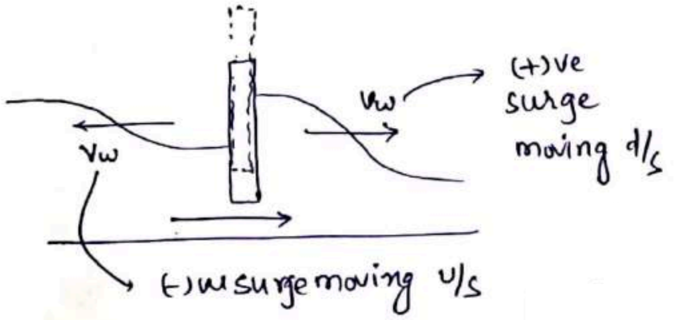
1/3/2020

Rapid varied unsteady flow : surge

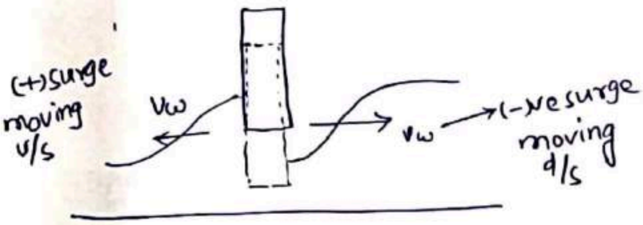
Surge { sudden change in depth / discharge or both }

- Positive — depth ↑
- negative — depth ↓

Case-1 :- sudden gate open



Case-2 sudden gate close

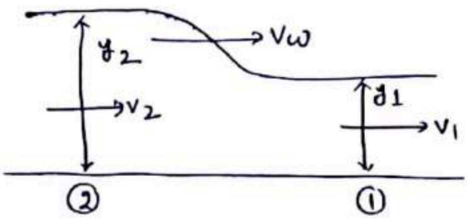


surge analysis.

(I) ∵ Rapid varied flow ∴ friction role neglected.

(II) ∵ unsteady flow hence equivalent steady flow formed by considering flow wrt. surge / wave itself.

Analysis of (+)ve surge moving d/s :-



(v_1, v_2, v_w ⟶ with respect to ground)

v_1, y_1 ⟶ known in normal flow condition

note:-

$\vec{V}_{stream/wave} = \vec{V}_{stream/ground} + \vec{V}_{ground/wave}$

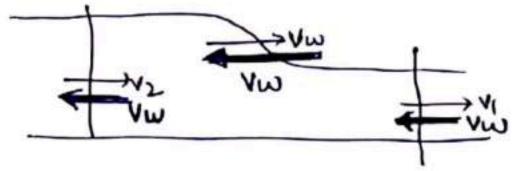
'or'

$\vec{V}_{stream/wave} = \vec{V}_{stream/grd} - \underbrace{\vec{V}_{wave/grd}}$

∴ net velocity $\vec{V}_{stream/wave}$ at 1 ⇒ $V_1 - V_w$

at 2 ⇒ $V_2 - V_w$

'or' apply Vw opposite to make steady state



① Apply continuity eqn (∵ now steady state)

$$\boxed{Q = y_1(v_1 - V_w) = y_2(v_2 - V_w)}$$

② Apply momentum eqⁿ :-

assumption
(I) $\theta = 0$ (Bed slope-horizontal)
(II) Rectangular section
(III) friction neglected ($F_f = 0$)
(IV) hydrostatic pressure distribution

$\therefore$ first section (P_2) – last section (P_1) = last(M_1) – first(M_2)

$$\frac{1}{2}\rho g y_2^2 - \frac{1}{2}\rho g y_1^2 = \rho Q(v_1 - v_w) - \rho Q(v_2 - v_w)$$

put $Q = y_1(v_1 - v_w)$ $\because$ v_1, y_1 known inflow condition normal

$\therefore$

$$\boxed{\frac{(v_w - v_1)^2}{g y_1} = \frac{y_2}{2y_1}\left(\frac{y_2}{y_1} + 1\right)}$$ ** (RVUF)

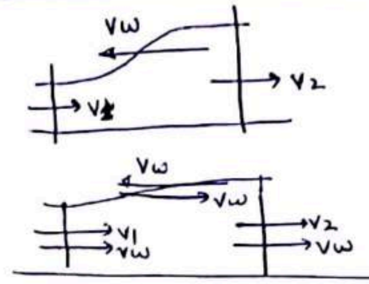
{ $v_1, v_2, v_w, y_1, y_2 \longrightarrow$ ⑤ variable
3 will be given 2 will be found out by continuity & momentum eqⁿ }

Special case :- Rapid varied steady flow → Hydraulic Jump

$\therefore v_w = 0$

hence $\boxed{y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g}}$

Analysis of (+)ve surge moving u/s :-



**

$$\boxed{\frac{(v_w + v_1)^2}{g y_1} = \frac{y_2}{2y_1}\left(\frac{y_2}{y_1} + 1\right)}$$

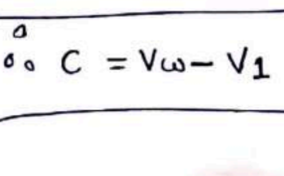
Celerity :- velocity of wave with respect to fluid media (water)

$$\boxed{\vec{V}_{wave/water} = \text{celerity}}$$

$$\vec{V}_{wave/water} = \vec{V}_{wave/grd} + \vec{V}_{grd/water}$$

$$\boxed{C = V_w - \vec{V}_{water/grd}}$$

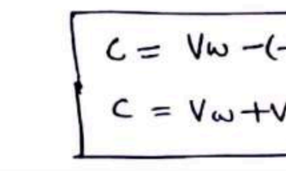
(+) surge moving d/s



reference direction same as Vw direction

$$\boxed{\therefore C = V_w - V_1}$$

(+)ve surge moving u/s



reference direction same as Vw direction

$$\boxed{\begin{aligned} C &= V_w - (-V_1) \\ C &= V_w + V_1 \end{aligned}}$$

hence

$$\frac{c^2}{gy_1} = \frac{y_2}{2y_1}\left(\frac{y_2}{y_1}+1\right)$$

$$\boxed{\therefore \quad c = \sqrt{\frac{g y_2}{2 y_1}(y_1+y_2)}}$$

if surge height $= y_2 - y_1$

if assume $y_2 \simeq y_1 \simeq y$

$$\boxed{\therefore \quad c = \sqrt{gy}}$$

Imp. Lagrange eqn of celerity for rectangular channel when surge height is very small.

Conclusion :- Speed of small amplitude wave is proportional to square root of fluid depth and independent of wave amplitude.

General equation of 'c' (small height wave) or surge height of negligible thickness

$$\boxed{c = \sqrt{gD}}$$

$D \rightarrow A/T$

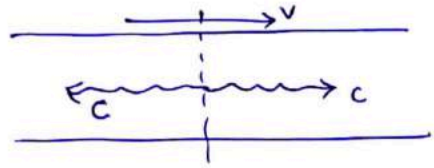
wave propogation :-

$$Fr = \frac{V}{\sqrt{gD}}$$

rectangular $\boxed{Fr = \frac{V}{\sqrt{gy}} = \frac{V}{C}}$ — when surge height negligible

- Critical flow: $\boxed{V = C}$ $\because Fr = 1$
- Super critical flow: $\boxed{V > C}$ $\because Fr > 1$
- Subcritical flow: $\boxed{V < C}$ $\because Fr < 1$

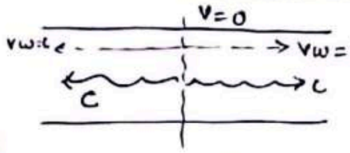
$$\text{also} \quad \vec{V}_{wave/grd} = \vec{V}_{\frac{wave}{water}} + \vec{V}_{\frac{water}{grd}}$$



$\therefore$ velocity of wave in d/s direction $= C + V$

———— u/s ———— $\Rightarrow C - V$

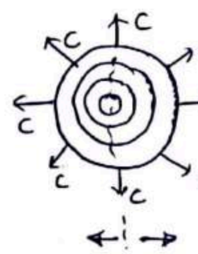
Case-1 :- In still water (flow velocity $V=0$)



∴ velocity of wave in d/s direction $= c+v = c$

—— u/s $= c-v = c$

Conclusion :-



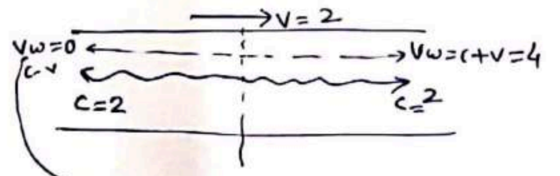
wave propogation in u/s & d/s equally

disturbance on both side equally

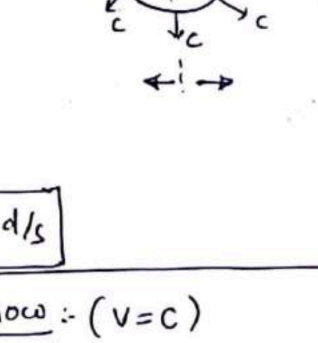
flow affected in u/s & d/s

Case-2 :- In critical flow :- ($V=C$)

assume $V = C = 2$ unit



conclusion :-



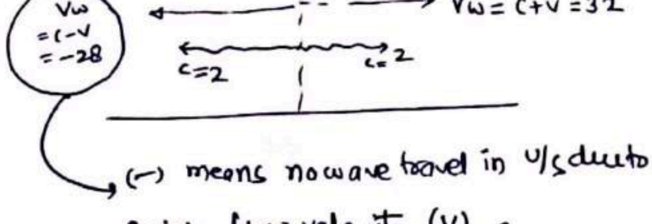
no wave will travel in u/s at all

no disturbance in u/s at all

(or) flow in d/s will be affected only

Case 3 :- In supercritical flow ($V>C$)

assume $V = 30$ $C = 2$ unit

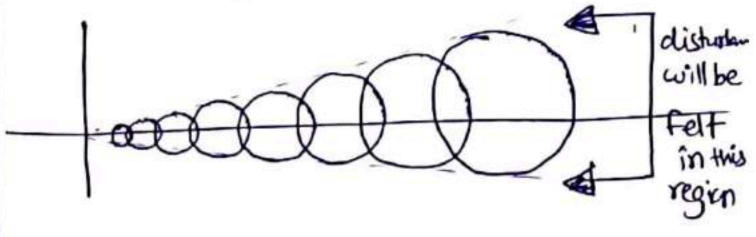


(−) means no wave travel in u/s due to heigh flow velocity (V) .

Conclusion :- (I) at high velocity (supercritical flow)

Small disturbance can not travel upstream here disturbance only in d/s side .

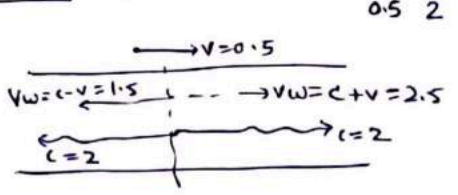
(II) ripples so formed will get progressively wash away to d/s end due to high velocity



note :- controll section :-

as disturbance does not travel in u/s hence flow of u/s doesnot know what is happening in d/s of that location hence to change the flow condition in supercritical flow , flow-condition must be changed at on upstream location hence Upstream controll . (in supercritical flow)

Case 4 :- Subcritical flow ($V<C$) $fr = 0.25 < 1$
0.5 2 (very low velocity)



Conclusion :- disturbance on both side but more in d/s than u/s .

controll section :- d/s