RACES AND GAMES

Race : A contest of speed is called a *race*.

Race-course : The ground or path on which contests are arranged is called a *race-course*.

Dead - heat Race :

If all the persons contesting a race reach the goal exactly at the same time, then the race is called *dead-heat race*.

Now, suppose *A* and *B* are two participants in a race. If, before the start of the race, *A* is at the starting point and *B* is ahead of *A* by 25 metres, then *A* is said to give *B* a start of 25 metres. To cover a race of 100 metres in this case, *A* will cover a distance of 100 metres and *B* will cover 100 – 25 = 75 metres only.

Note : In the above case, we may say that "*A* has given a lead of 25 metres to *B*" .

Games : If we say that it is a game of 100, then the person among the participants who scores 100 points first is the winner. If, when *A* scores 100 while *B* scores only 80 points, then we say that "*A* can give 20 points to *B*" or, "*A* can give *B* 20 points" in a game of 100.

Example : *A* is $1\frac{2}{3}$ times as fast as *B*. If *A* gives *B* a start of 60 metres, how long should the racecourse be so that both of them reach at the same time?

Solution : *A*'s speed : *B*'s speed = $1\frac{2}{3} : 1$

$= 5/3 : 1$

$= 5 : 3$

We may say that *A* gains 5 – 3 = 2 m in a race of 5 metres.

Therefore, he will gain 60 m in a race of $\frac{5}{2} \times 60$

= 150 m

Quicker Maths (Direct formula) :

$$\text{Distance of race-course} = \text{lead} \left(\frac{1}{1 - \frac{\text{B's speed}}{\text{A's speed}}} \right)$$

$$= 60 \left(\frac{1}{1 - 3/5} \right) = 60 \left(\frac{5}{5-3} \right) = 150 \text{ m.}$$

Example : In a 100 m race, *A* runs at 5 km/hr. *A* gives *B* a start of 8 metres and still beats him by 8 seconds. Find the speed of *B*.

Solution : Time taken by *A* to cover 100 m.

$$= 100 \div (5 \times 5/18) = \frac{100 \times 18}{25} = 72 \text{ seconds}$$

So, *B* covers (100 – 8) or 92 m in (72 + 8) or 80 seconds.

$$\text{So, speed of } B = \frac{92}{80} \times \frac{18}{5} = 4.14 \text{ km/hr}$$

Quicker Math (Direct Formula) :

$$\text{B's speed} = \frac{100\text{m} - 8\text{m}}{\text{A's time to cover 100m} + 8 \text{ sec}}$$

$$= \frac{92}{72+8} = \frac{92}{80} \text{ m/s} = 4.14 \text{ km/hr.}$$

Example : In a game of billiards, *A* can give *B* 12 points in 60 and *A* can give *C* 10 in 90. How many can *C* give *B* in a game of 70?

Solution : *A* : *B* = 60 : 48 = 90 : 72

A : *C* = 90 : 80 = 90 : 80

So, *C* : *B* = 80 : 72 = 80 (70/80) : 72 (70/80)

= 70 : 63

So, *C* gives *B* 7 points in the game of 70 points.

EXERCISE

1. *A* can run 100 m in 20 seconds and *B* in 25 seconds. *A* will beat *B* by :
(*a*) 10 m (*b*) 20 m
(*c*) 25 m (*d*) 12 m
(*e*) None of these

2. At a game of billiards, *A* can give *B* 15 points in 60 and *A* can give *C* 20 in 60. How many can *B* give *C* in a game of 90?
(*a*) 10 points (*b*) 20 points
(*c*) 30 points (*d*) 40 points
(*e*) None of these

3. In a 500 m race, the ratio of speeds of two runners *A* and *B* is 3 : 4. *A* has a start of 140 m. Then *A* wins by :
(*a*) 30 m (*b*) 34 m
(*c*) 20 m (*d*) 10 m
(*e*) None of these

4. In a 100 m race, *A* runs at 6 km/hr. If *A* gives *B* a start of 4 m and still beats him by 12 seconds, what is the speed of *B*?
(*a*) 4 km/hr (*b*) 4.5 km/hr
(*c*) 4.8 km/hr (*d*) 5 km/hr
(*e*) None of these

5. Two men *A* and *B* run a 4 km race on a course. 250 m round. If their rates be 5 : 4, how often does the winner pass the other :
(*a*) 1 time (*b*) 4 times
(*c*) 2 times (*d*) 5 times
(*e*) None of these

6. *A* runs 1.75 times as fast as *B*. If A gives *B* a start of 60 m, how far must the winning post be in order that *A* and *B* reach at the same time?
(*a*) 105 m (*b*) 80 m
(*c*) 140 m (*d*) 45 m
(*e*) 50 m

7. At a game of billiards, *A* can give *B* 10 points in 60 and he can give *C* 15 in 60. How many can *B* give *C* in a game of 90?
(*a*) 10 points (*b*) 9 points
(*c*) 8 points (*d*) 7 points
(*e*) None of these

8. In a 1000 m race *A* can give *B* 100 m and *C* 280 m. In the same race, *B* can give *C* :
(*a*) 180 m (*b*) 200 m
(*c*) 270 m (*d*) 90 m
(*e*) None of these

9. *A* and *B* run a km and *A* wins by 1 minute. *A* and *C* run a km and *A* wins by 375 m. *B* and *C* run a km and *B* wins by 30 seconds. Find the time taken by A and B ro run a km race.
(*a*) 150, 210 sec (*b*) 200, 160 sec
(*c*) 300, 150 sec (*d*) 150, 200 sec
(*e*) None of these

10. In a race of 300 m *A* beats *B* by 15 m or 5 seconds. *A*'s time over the course is :
(*a*) 100 sec (*b*) 95 sec
(*c*) 105 sec (*d*) 90 sec
(*e*) None of these

EXPLANATORY ANSWERS

1. (*b*): Distance covered by *B* in 5 seconds

$$= \frac{100}{25} \times 5 = 20 \text{ m}$$

So, *A* beats *B* by 20 m.

2. (*a*) : $A : B : C = 60 : 45 : 40$

$$\text{So, } B : C = \frac{45}{40} = \frac{45 \times 2}{40 \times 2} = \frac{90}{80}$$

So, *B* gives *C* 10 points in a game of 90.

3. (*c*) : To reach the winning points
A covers 500 – 140 =360 m.
So, *B* covers 360 (4/3) = 480 m when *A* reaches the winning point.
So, *A* reaches the winning point while *B* remains 20 m behind.
So, *A* wins by 20 m.

4. (*c*) : *B*'s speed

$$= \frac{100 \text{ m} - 4 \text{ m}}{A \text{ s time to cover } 100 \text{ m} + 12 \text{ second}}$$

A's time to cover 100 m

$$= 100 \div (6 \times 5/18)$$

$$= \frac{100 \times 18}{6 \times 5} = 60 \text{ sec}$$

So, B's speed $= \frac{96}{60+12} = \frac{4}{3}$ m/s

$$= \frac{4}{3} \times \frac{18}{5} = 4.8 \text{ km/hr.}$$

5. (*a*) : A's rate : B's rate = 5 : 4

$\Rightarrow$ When A makes 5 rounds, B makes 4 rounds

$\Rightarrow$ When A covers $\frac{5 \times 250}{1000} = \frac{5}{4}$ km.

B covers $\frac{4 \times 250}{1000} = 1$ km

$\Rightarrow$ A passes B each time, when A makes 5 rounds.

$\Rightarrow$ In covering 5/4 km, A passes B 1 time.

6. (*c*) : $60 \times \left(\frac{1}{1-4/7}\right)$

$= 60 \times \frac{7}{3} = 140$ m

7. (*b*) : $A : B = 60 : 50$ $\qquad A : C = 60 : 45$

So, $B : C = 50 : 45$

$= 50\,(90/50) : 45\,(90/50) = 90 : 81$

Hence, B gives C 9 points in a game of 90.

8. (*b*) : $A : B : C = 100 : 900 : 720$

So, $B : C = \frac{900}{720} = \frac{900 \times \frac{1000}{900}}{720 \times \frac{1000}{900}}$

$= \frac{1000}{800} = 1000 : 800$

So, B can give C 200 m.

9. (*a*) : A beats B by 60 seconds, B beats C by 30 seconds.

So, A beats C by 90 seconds or 375 m (given)

So, C covers 375 m in $\frac{90}{375} \times 1000$

$= 240$ seconds

Thus, time taken by A to cover 1 km

$= 240 - 90 = 150$ sec.

and time taken by B to cover 1 km

$= 240 - 30 = 210$ sec.

10. (*b*) : 15 m is covered by B in 5 seconds

So, 300 m is covered by B in $\frac{5}{15} \times 300$

$= 100$ seconds

Hence, A's time over the course $= 100 - 5$

$= 95$ sec.