Three Dimensional Geometry

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BASIC CONCEPTS

1. Distance between two given points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

2. Direction ratios of a line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are $x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1$.
3. Angle between two lines, whose direction ratios are a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}.\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

   (*i*) If lines are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

   (*ii*) If lines are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

4. Vector equation of a straight line passing through a fixed point with the position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a parameter and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
5. Cartesian equation (symmetrical form) of the straight line passing through a fixed point (x_1, y_1, z_1) having the direction ratios a, b, c is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.
6. The parametric equations of the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are

   $x = x_1 + a\lambda,\ y = y_1 + b\lambda,\ z = z_1 + c\lambda$, where λ is a parameter.

7. The coordinates of any point on the line $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ are

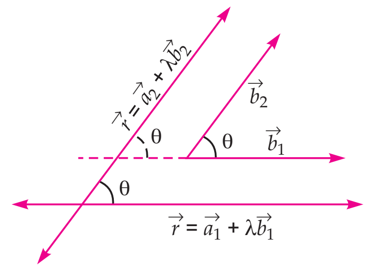
   $(x_1 + a\lambda, y_1 + b\lambda, z_1 + c\lambda)$, where $\lambda \in R$.

8. Equation of straight line passing through the point (x_1, y_1, z_1) having direction cosines l, m, n is $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$.
9. Vector equation of two straight lines passing through two given points with position vector $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, where λ is a parameter.
10. Cartesian equation of a straight line passing through two given points $A\ (x_1, y_1, z_1)$ and $B\ (x_2, y_2, z_2)$ is given by $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$.

11. Angle between two straight lines: Angle between two straight lines whose vector equations are $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$, is equal to the angle between $\vec{b}_1$ and $\vec{b}_2$, because $\vec{b}_1$ and $\vec{b}_2$ are parallel vector to the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ respectively.

If θ is angle between both lines, then

$$\cos\theta = \left|\frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1| . |\vec{b}_2|}\right|$$



(*i*) If $\vec{b}_1 . \vec{b}_2 = 0$, then $\cos\theta = 0°$

$\Rightarrow \quad \theta = 90° \quad \Rightarrow \quad \vec{b}_1 \perp \vec{b}_2$

$\Rightarrow$ Both lines are perpendicular to each other.

(*ii*) If $\vec{b}_1 = \lambda\vec{b}_2$, then

$$\Rightarrow \quad \cos\theta = \frac{\lambda\vec{b}_2 . \vec{b}_2}{\lambda|\vec{b}_2| . |\vec{b}_2|} = \frac{\lambda|\vec{b}_2|^2}{\lambda|\vec{b}_2|^2}$$

$\Rightarrow \quad \cos\theta = 1 \quad \Rightarrow \quad \theta = 0° \quad \Rightarrow \quad \vec{b}_1 \parallel \vec{b}_2$

$\Rightarrow$ Both lines are parallel to each other.

12. Shortest distance between two lines: Let l_1 and l_2 be two skew lines given by $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ respectively, where $\vec{a}_1$ and $\vec{a}_2$ are position vectors of points on l_1 and l_2 then shortest distance between two given points is given by

$$\frac{(\vec{a}_2 - \vec{a}_1) . (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

Note: If two lines are intersecting, then shortest distance between them is zero, *i.e.*,

$$(\vec{a}_2 - \vec{a}_1) . (\vec{b}_1 \times \vec{b}_2) = 0$$

13. Shortest distance between two parallel lines: Let l_1 and l_2 be two parallel lines given by $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}$ respectively. Then shortest distance between them is $\left|\frac{\vec{b} . (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$.

14. Shortest distance between two skew lines in cartesian form: Let $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ are two skew lines, then shortest distance between them is given by $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$.

Note: If $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$. Then lines are intersecting.

15. Equation of a plane passing through given point, whose position vector is $\vec{a}$ and perpendicular to a given vector $\vec{n}$, is $\vec{r} . \vec{n} = \vec{a} . \vec{n}$ or $(\vec{r} - \vec{a}) . \vec{n} = 0$ or $(\vec{r} - \vec{a}) . \hat{n} = 0$ when $\hat{n} = \frac{\vec{n}}{|\vec{n}|}$.

16. Cartesian equation of plane passing through a given point (x_1, y_1, z_1) and perpendicular to the normal, whose direction ratios are a, b, c respectively is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

17. Equation of a plane in normal form:

(*i*) When a unit vector $(\hat{n})$ perpendicular (normal) to the plane is given and its perpendicular distance d from the origin is also given, then the equation of plane is $\vec{r} \cdot \hat{n} = d$.

(*ii*) If unit vector $\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$ where l, m, n are direction cosines and p, the perpendicular distance from origin to normal are given, then equation of the plane is $lx + my + nz = p$.

18. Angle between line and plane: Angle between line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r} \cdot \vec{n} = d$ is complementary to the angle between line and the normal to the plane.

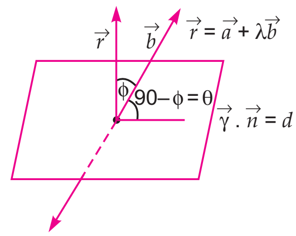
Let θ be the angle between line and plane and ϕ be the angle between line and normal of plane.

Now, $\cos\phi = \left|\dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}| \cdot |\vec{n}|}\right|$

$\because$ $\theta = 90 - \phi$ or $\phi = 90 - \theta$

$\therefore$ $\cos(90 - \theta) = \left|\dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}| \cdot |\vec{n}|}\right|$

$\Rightarrow$ $\sin\theta = \left|\dfrac{\vec{b} \cdot \vec{n}}{|\vec{b}| \cdot |\vec{n}|}\right|$



(*i*) If $\vec{b} = \lambda\vec{n}$, then $\sin\theta = \left|\dfrac{\lambda\,\vec{n} \cdot \vec{n}}{\lambda\,\vec{n} \cdot \vec{n}}\right| = 1 \quad \Rightarrow \quad \theta = 90^\circ$

$\Rightarrow$ Line is perpendicular to the plane.

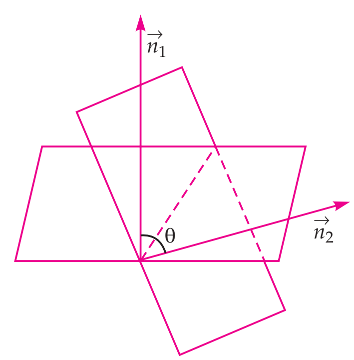
(*ii*) If $\vec{b} \cdot \vec{n} = 0 \Rightarrow \sin\theta = 0 \quad \Rightarrow \quad \theta = 0^\circ$

$\Rightarrow$ Line is parallel to the plane.

19. Angle between two planes: The angle between two planes is defined as the angle between their normals.

Case I. If θ be the angle between normals $\vec{n_1}$ and $\vec{n_2}$ of the planes $\vec{r} \cdot \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$ respectively, then

$$\cos\theta = \left|\frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}| \cdot |\vec{n_2}|}\right|$$



Note: (*i*) If $\vec{n_1} \cdot \vec{n_2} = 0$ then given planes are perpendicular.

(*ii*) If $\vec{n_1} = \lambda\vec{n_2}$, then both planes are parallel.

(*iii*) The angle between two planes is always taken as acute angle.

Case II. If $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be two planes and θ is the angle between them, where a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of normals to the planes, then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

(*i*) If $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$, then planes are perpendicular to each other.

(*ii*) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then given planes are parallel to each other.

Determination of plane under given conditions:

20. An equation of first degree in x, y, z of the form $ax + by + cz + d = 0$ where at least one of a, b, c is non-zero real number, *i.e.*, $a^2 + b^2 + c^2 \neq 0$ represents a plane.

21. Equation of plane in intercept form: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where a, b, c are intercepts made by the plane on the x-axis, y-axis and z-axis respectively.

22. Equation of plane passing through three given points:

Case I. Vector equation of the plane passing through three given points having position vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by

$$[(\vec{r} - \vec{a}) \quad (\vec{b} - \vec{a}) \quad (\vec{c} - \vec{a})] = 0$$

or $$(\vec{r} - \vec{a}).(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = 0$$

Case II. Cartesian equation of the plane passing through points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

23. Condition for coplanarity of two lines:

Case I. When lines are in vector form:

(*i*) Let $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$ be two lines then these lines are coplanar if

$$[\vec{a_2} - \vec{a_1}\ \vec{b_1}\ \vec{b_2}] = 0 \ \ i.e \left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 0$$

(*ii*) Equation of plane containing these two lines is

$$(\vec{r} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0 \qquad \text{or} \qquad (\vec{r} - \vec{a_2}).(\vec{b_1} \times \vec{b_2}) = 0 \ .$$

Case II. When lines are in cartesian form:

(*i*) Let $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ be the two lines

then these lines are coplanar iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(*ii*) Equation of plane containing these lines is

$$\frac{x - x_1}{b_1 c_2 - b_2 c_1} = \frac{y - y_1}{a_2 c_1 - a_1 c_2} = \frac{z - z_1}{a_1 b_2 - a_2 b_1} \quad \text{or} \quad \frac{x - x_2}{b_1 c_2 - b_2 c_1} = \frac{y - y_2}{a_2 c_1 - a_1 c_2} = \frac{z - z_2}{a_1 b_2 - a_2 b_1}$$

(*iii*) The length of perpendicular from a point having position vector $\vec{a}$ to the plane

$\vec{r}.\hat{n} = d$ is given by $|d - \vec{a}.\hat{n}|$.

(*iv*) The length of perpendicular from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is given by

$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Selected NCERT Questions

1. If a line makes angles 90°, 135°, 45° with the x, y and z axis respectively, find its direction cosines.

Sol. Since the line makes angle 90°, 135°, 45° with the x, y and z axis respectively

then $\alpha = 90°$, $\beta = 135°$ and $\gamma = 45°$

$l = \cos 90° = 0$, $m = \cos 135° = \cos(180 - 45)° = -\cos 45° = -\frac{1}{\sqrt{2}}$ and $n = \cos 45° = \frac{1}{\sqrt{2}}$

Thus, direction cosines of the line are 0, $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

2. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

Sol. Let $A(1, -1, 2)$ and $B(3, 4, -2)$ be given points.

Direction ratios of AB are

$$(3 - 1), \{(4 - (-1)\}, (-2 - 2) \text{ i.e., } 2, 5, -4.$$

Let $C(0, 3, 2)$ and $D(3, 5, 6)$ be given points.

Direction ratios of CD are

$$(3 - 0), (5 - 3), (6 - 2) \text{ i.e., } 3, 2, 4.$$

We know that two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\therefore\ 2 \times 3 + 5 \times 2 + (-4) \times 4 = 6 + 10 - 16 = 0$, which is true.

It will shows that lines AB and CD are perpendicular.

3. Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

Sol. The equation of given line is

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}.$$

The direction ratios of the given line are 3, 5, 6. Since the required line is parallel to given line, so, the direction ratios of required line are proportional *i.e.*, 3, 5, 6.

Now the equation of the line passing through point (– 2, 4, –5) and having direction ratios 3, 5, 6 is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

which is equation of required line.

4. Find the angle between the following pair of lines:

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

Sol. Here the equation of given lines are

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \text{ and } \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

$\therefore$ Direction ratios of two lines are 2, 2, 1 and 4, 1, 8.

Let θ be the angle between two given lines then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2\times 4 + 2\times 1 + 1\times 8}{\sqrt{(2)^2 + (2)^2 + (1)^2}.\ \sqrt{(4)^2 + (1)^2 + (8)^2}.} = \frac{8 + 2 + 8}{\sqrt{4 + 4 + 1}\ \sqrt{16 + 1 + 64}}$$

$$\therefore \quad \cos\theta = \frac{2}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\frac{2}{3}.$$

5. Find the value of p so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{5z-10}{11} \quad \text{and} \quad \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$ **[*CBSE Delhi 2009*]**

are perpendicular to each other.

Sol. The given lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{5z-10}{11} \text{ and } \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \text{ are rearranged to get}$$

$$\frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-2}{11/5} \qquad ...(i)$$

$$\frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5} \qquad ...(ii)$$

Direction ratios of lines are

$$-3, \frac{2p}{7}, \frac{11}{5} \quad \text{and} \quad \frac{-3p}{7}, 1, -5$$

As the lines are perpendicular, we get

$$\therefore \quad -3\left(\frac{-3p}{7}\right) + \frac{2p}{7}\times 1 + \frac{11}{5}(-5) = 0$$

$$\Rightarrow \quad \frac{9p}{7} + \frac{2p}{7} - 11 = 0 \quad \Rightarrow \quad \frac{11}{7}p = 11 \quad \Rightarrow \quad p = 7$$

6. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$ **[*CBSE (F) 2011*]**

Sol. Given lines are

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \qquad \dots (i)$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}) \qquad \dots (ii)$$

Comparing the equation (*i*) and (*ii*) with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, we get

$$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k} \qquad \vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b_1} = \hat{i} - \hat{j} + \hat{k} \qquad \vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Now, $\vec{a_2} - \vec{a_1} = (2i - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 2\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = (-2-1)\hat{i} - (2-2)\hat{j} + (1+2)\hat{k} = -3\hat{i} + 3\hat{k}$$

$$\therefore \quad \left|\vec{b_1}\times\vec{b_2}\right| = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$$

$$\therefore \text{ Shortest distance} = \left|\frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right| = \left|\frac{(\hat{i}-3\hat{j}-2\hat{k}).(-3\hat{i}+0\hat{j}+3\hat{k})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

$$= \left|\frac{-3-0-6}{3\sqrt{2}}\right| = \frac{9}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{9\sqrt{2}}{3\times 2} = \frac{3\sqrt{2}}{2}$$

7. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the plane is $3\hat{i}+5\hat{j}-6\hat{k}$.

Sol. Normal vector of the plane is

$$\vec{n} = 3\hat{i}+5\hat{j}-6\hat{k}$$

$$\therefore \quad |\vec{n}| = \sqrt{(3)^2+(5)^2+(-6)^2} = \sqrt{9+25+36} = \sqrt{70}$$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{1}{\sqrt{70}}(3\hat{i}+5\hat{j}-6\hat{k}) = \frac{3}{\sqrt{70}}\hat{i}+\frac{5}{\sqrt{70}}\hat{j}-\frac{6}{\sqrt{70}}\hat{k}.$$

The required equation of plane is $\vec{r}.\hat{n} = 7$.

$$\therefore \quad \vec{r}.\left(\frac{3}{\sqrt{70}}\hat{i}+\frac{5}{\sqrt{70}}\hat{j}-\frac{6}{\sqrt{70}}\hat{k}\right) = 7$$

8. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k}=7)$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and through the point (2, 1, 3).

Sol. Let the equation of plane passing through the intersection of two planes be.

$$\vec{r}.\left[(2\hat{i}+2\hat{j}-3\hat{k})+\lambda(2\hat{i}+5\hat{j}+3\hat{k})\right] = 7+9\lambda$$

$$\vec{r}.\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right] = 7+9\lambda \qquad ...(i)$$

$$(x\hat{i}+y\hat{j}+z\hat{k}).\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right] = 7+9\lambda$$

$$(2+2\lambda)x+(2+5\lambda)y+(-3+3\lambda)z = 7+9\lambda$$

$\because$ It contains point (2, 1, 3).

$$\therefore \quad (2+2\lambda)\times 2 + (2+5\lambda)\times 1+(-3+3\lambda)\times 3 = 7+9\lambda$$

$$4+4\lambda+2+5\lambda-9+9\lambda = 7+9\lambda \Rightarrow 18\lambda-3 = 7+9\lambda \Rightarrow 18\lambda-9\lambda = 7+3$$

$$9\lambda = 10 \Rightarrow \lambda = \frac{10}{9}$$

Put in equation (*i*), we get

$$\vec{r}.\left(\frac{38}{9}\hat{i}+\frac{68}{9}\hat{j}+\frac{3}{9}\hat{k}\right) = 17$$

$\vec{r}.(38\hat{i}+68\hat{j}+3\hat{k}) = 153$ is the required vector equation in plane.

9. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$.

Sol. Let the equation of the plane passing through the intersection of two planes be

(equation of (*i*) plane) + λ (equation of (*ii*) plane) = $d_1+\lambda\, d_2$

$$(x+y+z)+\lambda(2x+3y+4z) = 1+5\lambda \qquad ...(i)$$

$$x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda) = 1+5\lambda$$

$a_1 = (1+2\lambda)$, $b_1 = (1+3\lambda)$, $c_1 = (1+4\lambda)$

This plane is perpendicular to the plane $x-y+z=0$.

$a_2 = 1$, $b_2 = -1$, $c_2 = 1$

As plane is perpendicular to another plane then, $a_1a_2+b_1b_2+c_1c_2 = 0$

$$(1+2\lambda)\times 1+(1+3\lambda)\times(-1)+(1+4\lambda)\times 1 = 0$$

$$1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0 \quad \Rightarrow \quad -\lambda + 4\lambda = -1 \quad \Rightarrow \quad 3\lambda = -1$$

$$\lambda = -\frac{1}{3}$$

Put value of λ in equation (*i*), we get

$$(x + y + z) - \frac{1}{3}(2x + 3y + 4z) = 1 + 5\left(\frac{-1}{3}\right)$$

$$\frac{3x + 3y + 3z - (2x + 3y + 4z)}{3} = \frac{3-5}{3}$$

$x - z = -2 \Rightarrow x - z + 2 = 0$ is the required equation of the plane.

10. Find the coordinates of the point where the line through (3, – 4, – 5) and (2, – 3, 1) crosses the plane $2x + y + z = 7$.

Sol. The equation of given plane is $2x + y + z = 7$ ...(*i*)

Equation of the line passing through points (3, – 4, – 5) and (2, –3, 1) is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} \quad \Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \text{ (say)}$$

$$\therefore \quad \frac{x-3}{-1} = \lambda \quad \Rightarrow \quad x - 3 = -\lambda \quad \Rightarrow \quad x = 3 - \lambda$$

$$\frac{y+4}{1} = \lambda \quad \Rightarrow \quad y + 4 = \lambda \quad \Rightarrow \quad y = -4 + \lambda$$

$$\frac{z+5}{6} = \lambda \quad \Rightarrow \quad z + 5 = 6\lambda \quad \Rightarrow \quad z = -5 + 6\lambda$$

Putting value of x, y and z in (*i*), we have

$$2(3 - \lambda) + (-4 + \lambda) + (-5 + 6\lambda) = 7$$

$$\Rightarrow \quad 6 - 2\lambda - 4 + \lambda - 5 + 6\lambda = 7$$

$$\Rightarrow \quad 5\lambda = 7 + 3 \quad \Rightarrow \quad \lambda = 2$$

$$\therefore \quad x = 3 - 2 = 1,\ y = -4 + 2 = -2 \text{ and } z = -5 + 6 \times 2 = -5 + 12 = 7$$

Thus, coordinates of required point are (1, – 2, 7).

11. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

Sol. Here the equations of given planes are

$$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1 \text{ and } \vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

The equation of a plane passing through the intersection of the given planes is

$$\left[\vec{r}.(\hat{i} + \hat{j} + \hat{k}) - 1\right] + \lambda\left[\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4\right] = 0$$

$$\Rightarrow \quad \vec{r}.\left[(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}\right] + 4\lambda - 1 = 0 \quad ...(i)$$

Since the above plane is parallel to x-axis *i.e.*, $\hat{i} + 0\hat{j} + 0\hat{k}$.

$$\therefore \quad \left[(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}\right].(\hat{i} + 0\hat{j} + 0\hat{k}) = 0 \quad \Rightarrow \quad 2\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$$

Putting value of λ in (*i*), we have

$$\vec{r}.\left[\left(2 \times \left(-\frac{1}{2}\right) + 1\right)\hat{i} + \left(3 \times \left(-\frac{1}{2}\right) + 1\right)\hat{j} + \left(1 + \frac{1}{2}\right)\hat{k}\right] + 4 \times \left(-\frac{1}{2}\right) - 1 = 0$$

$$\Rightarrow \quad \vec{r}.\left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right) - 3 = 0 \quad \Rightarrow \quad \vec{r}.(-\hat{j} + 3\hat{k}) - 6 = 0$$

Putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we have

$$(x\hat{i} + y\hat{j} + z\hat{k})(-\hat{j} + 3\hat{k}) - 6 = 0 \quad \Rightarrow \quad -y + 3z - 6 = 0 \quad \Rightarrow \quad y - 3z + 6 = 0$$

which is required equation of the plane.

12. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$. ***[CBSE Delhi 2011, 2013]***

Sol. The given planes are

$$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0 \qquad ...(i)$$

and $$\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0 \qquad ...(ii)$$

Therefore, a plane which contains the line of intersection of the planes (*i*) and (*ii*) is

$$\Rightarrow \quad \vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4+\lambda\{\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5\}=0$$

$$\Rightarrow \quad \vec{r}\cdot[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}]-4+5\lambda=0 \qquad ...(iii)$$

Now, the plane (*iii*) is perpendicular to the plane

$$\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0 \qquad ...(iv)$$

Therefore from (*iii*) and (*iv*), we get

$$(1+2\lambda).\,5+(2+\lambda).\,3+(3-\lambda).\,(-6)=0 \qquad [\because \vec{n}_1\cdot\vec{n}_2=0]$$

$$\Rightarrow \quad 5+10\lambda+6+3\lambda-18+6\lambda=0$$

$$\Rightarrow \quad 19\lambda-7=0 \quad \Rightarrow \lambda=\frac{7}{19}$$

Now, putting the value of λ in (*iii*), we get

$$\vec{r}\cdot\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{k}\right]-4+5\times\frac{7}{19}=0$$

$$\vec{r}\cdot\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]+\frac{35-76}{19}=0$$

$\Rightarrow \quad \vec{r}\cdot(33\hat{i}+45\hat{j}+50\hat{k})-41=0$, which is the required equation.

13. Find the distance of the point (–1, –5, –10), from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$. ***[CBSE Delhi 2014; (AI) 2011; (F) 2014]***

Sol. Given line and plane are

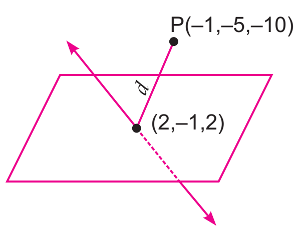
$$\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k}) \qquad ...(i)$$

$$\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5 \qquad ...(ii)$$

For intersection point, we solve equations (*i*) and (*ii*) by putting the value of $\vec{r}$ from (*i*) in (*ii*).

$$[(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})]\cdot(\hat{i}-\hat{j}+\hat{k})=5$$

$$\Rightarrow \quad (2+1+2)+\lambda\,(3-4+2)=5 \quad \Rightarrow \quad 5+\lambda=5 \quad \Rightarrow \quad \lambda=0$$



Hence, position vector of intersecting point is $2\hat{i}-\hat{j}+2\hat{k}$

i.e., coordinates of intersection of line and plane is (2, –1, 2).

Hence, required distance

$$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}=\sqrt{9+16+144}$$

$$=\sqrt{169}=13 \text{ units}$$

14. Find the vector equation of the line passing through (1, 2, – 4) and perpendicular to the two lines:

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

Sol. Equation of any line through the point (1, 2, – 4) is

$$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c} \quad \ldots(i)$$

where a, b and c are direction ratios of line (i).

Now the line (i) is perpendicular to the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

having direction ratios 3, – 16, 7 and 3, 8, – 5 respectively.

$$\therefore \quad 3a-16b+7c=0 \quad \ldots(ii)$$

$$3a+8b-5c=0 \quad \ldots(iii)$$

Solving (ii) and (iii) by cross-multiplication method, we have

$$\frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48} \Rightarrow \frac{a}{24}=\frac{b}{36}=\frac{c}{72}$$

$$\Rightarrow \quad \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$$

Let $\quad \frac{a}{2}=\frac{b}{3}=\frac{c}{6}=\lambda \quad \Rightarrow \quad a=2\lambda, b=3\lambda \text{ and } c=6\lambda$

The equation of required line which passes through point (1, 2, –4) and parallel to vector $2\hat{i}+3\hat{j}+6\hat{k}$ is $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$.

15. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

Sol. Let the equation of plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \ldots(i)$

$\therefore$ Length of perpendicular from origin to plane (i) is

$$\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}=\frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

It is given that

$$p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow \sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}=\frac{1}{p} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}. \text{ (on squaring both sides)}$$

Multiple Choice Questions [1 mark]

Choose and write the correct option in the following questions.

1. The co-ordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is [*CBSE* 2020, (65/2/1)]

(a) (2, 3, 4) (b) (– 2, – 3, – 4) (c) (0, –3, 0) (d) (2, 0, 4)

2. The two planes $x-2y+4z=10$ and $18x+17y+kz=50$ are perpendicular, if k is equal to [*CBSE* 2020, (65/4/1)]

(a) –4 (b) 4 (c) 2 (d) – 2

3. **Distance of the point (α, β, γ) from y-axis is**

(*a*) β (*b*) $|\beta|$ (*c*) $|\beta|+|\gamma|$ (*d*) $\sqrt{\alpha^2+\gamma^2}$

4. **If the direction cosines of a line are k, k, k then** [*NCERT Exemplar*]

(*a*) $k > 0$ (*b*) $0 < k < 1$ (*c*) $k = 1$ (*d*) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

5. **The distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)=1$ from the origin is**

(*a*) 1 (*b*) 7 (*c*) $\frac{1}{7}$ (*d*) none of these

6. **The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is** [*NCERT Exemplar*]

(*a*) $\frac{10}{6\sqrt{5}}$ (*b*) $\frac{4}{5\sqrt{2}}$ (*c*) $\frac{2\sqrt{3}}{5}$ (*d*) $\frac{\sqrt{2}}{10}$

7. **The reflection of the point (α, β, γ) in the xy-plane is** [*NCERT Exemplar*]

(*a*) $(\alpha, \beta, 0)$ (*b*) $(0, 0, \gamma)$ (*c*) $(-\alpha, -\beta, \gamma)$ (*d*) $(\alpha, \beta, -\gamma)$

8. ***P* is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of *P* is 5, then its y co-ordinate is** [*NCERT Exemplar*]

(*a*) 2 (*b*) 1 (*c*) –1 (*d*) –2

9. **If α, β, γ are the angles that a line makes with the positive direction of x, y, z axis, respectively, then the direction cosines of the line are** [*NCERT Exemplar*]

(*a*) $\sin\alpha, \sin\beta, \sin\gamma$ (*b*) $\cos\alpha, \cos\beta, \cos\gamma$ (*c*) $\tan\alpha, \tan\beta, \tan\gamma$ (*d*) $\cos^2\alpha, \cos^2\beta, \cos^2\gamma$

10. **The distance of a point *P* (*a*, *b*, *c*) from x-axis is**

(*a*) $\sqrt{a^2+c^2}$ (*b*) $\sqrt{a^2+b^2}$ (*c*) $\sqrt{b^2+c^2}$ (*d*) b^2+c^2

11. **The equations of x-axis in space are** [*NCERT Exemplar*]

(*a*) $x = 0, y = 0$ (*b*) $x = 0, z = 0$ (*c*) $x = 0$ (*d*) $y = 0, z = 0$

12. **A line makes equal angles with co-ordinate axis. Direction cosines of this line are** [*NCERT Exemplar*]

(*a*) $\pm(1, 1, 1)$ (*b*) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (*c*) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (*d*) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

13. ***P* is the point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of *P* is 5, then its y co-ordinate is**

(*a*) 2 (*b*) 1 (*c*) –1 (*d*) –2

14. **The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is**

(*a*) $\frac{10}{6\sqrt{5}}$ (*b*) $\frac{4}{5\sqrt{2}}$ (*c*) $\frac{2\sqrt{3}}{5}$ (*d*) $\frac{\sqrt{2}}{10}$

15. **The area of the quadrilateral *ABCD* where *A* (0, 4, 1), *B* (2, 3, –1), *C* (4, 5, 0) and *D* (2, 6, 2) is equal to**

(*a*) 9 sq units (*b*) 18 sq units (*c*) 27 sq units (*d*) 81 sq units

16. **The intercepts made by the plane $2x-3y+5z+4=0$ on the coordinate axes are**

(*a*) $-2, \frac{4}{3}$ and $-\frac{4}{5}$ (*b*) $-2, -\frac{4}{3}$ and $\frac{4}{5}$ (*c*) $\frac{4}{3}, -\frac{4}{3}$ and $\frac{7}{3}$ (*d*) $-2, -\frac{4}{3}$ and $-\frac{4}{5}$

17. **The shortest distance between the lines given by**

$\vec{r} = (8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ **and** $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$ **is**

(*a*) 7 units (*b*) 2 units (*c*) 14 units (*d*) 3 units

18. **The image of the point (1, 6, 3) in the line** $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ **is**

(*a*) (2, 0, 5) (*b*) (1, 3, 4) (*c*) (1, 0, 7) (*d*) (– 3, – 2, 0)

19. **The coordinates of the point where the line through (3, – 4, – 5) and (2, –3, 1) crosses the plane passing through three points (2, 2, 1), (3, 0, 1) and (4, – 1, 0) are**

(*a*) (0, – 2, 7) (*b*) (3, – 2, 5) (*c*) (1, – 2, – 7) (*d*) (1, – 2, 7)

20. **The co-ordinates of the foot of perpendicular drawn from point** ***A*****(1, 8, 4) to the line joining the points** ***B*****(0, –1, 3) and** ***C*****(2, –3, –1) are**

(*a*) $\left(\frac{-7}{3}, \frac{2}{3}, \frac{11}{3}\right)$ (*b*) $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ (*c*) $\left(\frac{4}{3}, \frac{2}{3}, \frac{11}{3}\right)$ (*d*) None of these

Answers

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*d*)	**5.** (*a*)	**6.** (*d*)
7. (*d*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)	**11.** (*d*)	**12.** (*b*)
13. (*a*)	**14.** (*d*)	**15.** (*a*)	**16.** (*a*)	**17.** (*c*)	**18.** (*c*)
19. (*d*)	**20.** (*b*)				

Solutions of Selected Multiple Choice Questions

1. The x and z co-ordinates on y-axis are 0.

$\therefore$ Required point is (0, –3, 0) on y-axis.

2. We have angle between two given plants is given by

$$\cos\frac{\pi}{2} = \frac{1\times 18 + (-2)\times 17 + 4\times k}{\sqrt{(1)^2+(-2)^2+(4)^2}\,.\sqrt{(18)^2+(17)^2\,(k)^2}}$$

$\Rightarrow\ 0 = 18 - 34 + 4k \qquad \Rightarrow \qquad 4k = 16$

$\Rightarrow \qquad k = 4$

4. Since, direction cosines of a line are k, k and k.

$\therefore \qquad l = k,\ m = k$ and $n = k$

We know that, $l^2 + m^2 + n^2 = 1 \quad \Rightarrow \quad k^2 + k^2 + k^2 = 1 \quad \Rightarrow \quad k^2 = \frac{1}{3}$

$\therefore \qquad k = \pm\frac{1}{\sqrt{3}}$

6. We have, the equation of line as

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

Now, the line passes through point (2, 3, 4) and having direction ratios (3, 4, 5).

Since, the line passes through point (2, 3, 4) and parallel to the vector $(3\hat{i} + 4\hat{j} + 5\hat{k})$.

$\therefore \qquad \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Also, the cartesian form of the given plane is $2x - 2y + z = 5$.

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k})(2\hat{i} - 2\hat{j} + \hat{k}) = 5$

$\therefore \qquad \vec{n} = (2\hat{i} - 2\hat{j} + \hat{k})$

We know that, $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|.|\vec{n}|} = \dfrac{|(3\hat{i}+4\hat{j}+5\hat{k}).(2\hat{i}-2\hat{j}+\hat{k})|}{\sqrt{3^2+4^2+5^2}.\sqrt{4+4+1}}$

$$= \frac{|6-8+5|}{\sqrt{50}\,.\,3} = \frac{3}{15\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\sin\theta = \frac{\sqrt{2}}{10}$$

7. In xy-plane, the reflection of the point (α, β, γ) is $(\alpha, \beta, -\gamma)$.
8. Let P divides the line segment in the ratio of $\lambda : 1$, x-coordinate of the point P may be expressed as $x = \dfrac{6\lambda+3}{\lambda+1}$ giving $\dfrac{6\lambda+3}{\lambda+1} = 5$ so that $\lambda = 2$. Thus y-coordinate of P is $\dfrac{2\lambda+2}{\lambda+1} = 2$.
10. The required distance is the distance of $P(a, b, c)$ from $Q(a, 0, 0)$, which is $\sqrt{b^2+c^2}$.
11. On x-axis the y-co-ordinate and z-co-ordinate are zero.
12. Let the line makes angle α with each of the axis.

    Then, its direction cosines are $\cos\alpha$, $\cos\alpha$, $\cos\alpha$.

    Since $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$. Therefore, $\cos\alpha = \pm\dfrac{1}{\sqrt{3}}$

Fill in the Blanks [1 mark]

1. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ is ____________ units. [CBSE 2020 (65/2/1)]
2. If $p(1, 0, -3)$ is the foot of the perpendicular from the origin to the plane, then the Cartesian equation of the plane is ____________. [CBSE 2020 (65/2/1)]
3. Vector equation of the line $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$ is ____________.
4. If a line makes an angle of $\dfrac{\pi}{4}$ with each of y and z–axis, then the angle which it makes with x – axis is ____________. [CBSE 2020 (65/3/1)]
5. The cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k}) = 2$ is ____________.

Answers

1. 2   2. $x - 3z - 10 = 0$ 3. $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

4. $\dfrac{\pi}{2}$ 5. $x + y - z = 2$

Solutions of Selected Fill in the Blanks

1. Given parallel planes are

   $2x + y - 2z - 6 = 0$ ...(*i*)

   and $4x + 2y - 4z = 0$

   $\Rightarrow 2x + y - 2z = 0$ ...(*ii*)

   Required distance between planes (*i*) and (*ii*) is given by

   $$D = \left|\frac{-6-0}{\sqrt{(2)^2+(1)^2+(-2)^2}}\right| = \frac{6}{3} = 2 \text{ units}$$

2. Direction ratios of the normal to the plane are given by $1-0, 0-0, -3-0$

   $\Rightarrow 1, 0, -3$

   $\therefore$ Equation of the plane be

$$a(x-1)+b(y-0)+c(z-(-3))=0$$

$$\Rightarrow 1.\ (x-1)+0.(y-0)+(-3)(z+3)=0$$

$$\Rightarrow x-1+0-3z-9=0$$

$$\Rightarrow x-3z-10=0$$

4. Let the line makes angle $\propto$ with x–axis.

$\therefore \quad \cos^2\propto+\cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}=1 \qquad (\because l^2+m^2+n^2=1)$

$$\Rightarrow \quad \cos^2\alpha+\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2=1 \qquad \Rightarrow \qquad \cos^2\alpha+\frac{1}{2}+\frac{1}{2}=1$$

$$\Rightarrow \quad \cos^2\alpha=0 \qquad \Rightarrow \qquad \cos\alpha=0$$

$$\Rightarrow \quad \alpha=\frac{\pi}{2}$$

5. We have, $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}-\hat{k})=2$

$\Rightarrow x+y-z=2$

Very Short Answer Questions [1 mark]

1. If a line has direction ratios 2, –1, –2, then what are its direction cosines? *[CBSE Delhi 2012]*

Sol. Here direction ratios of line are 2, –1, –2

$\therefore$ Direction cosines of line are $\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$

i.e., $\quad \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

[**Note:** If a, b, c are the direction ratios of a line, the direction cosines are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$]

2. Find the co-ordinate of the point, where the line $\frac{x+2}{1}=\frac{y-5}{3}=\frac{z+1}{5}$ cuts the yz-plane. **[*CBSE* 2019 (65/5/3)]**

Sol. Let the required point be (α, β, γ) where given line cuts yz-plane.

$\therefore \quad \frac{\alpha+2}{1}=\frac{\beta-5}{3}=\frac{\gamma+1}{5}=k(\text{say})$

If $\frac{\alpha+2}{1}=k \quad \Rightarrow \quad \alpha=-2+k, \qquad \beta=5+3k, \gamma=-1+5k$

Since this point lies in yz-plane.

$\therefore \quad \alpha=0 \quad \Rightarrow \quad -2+k=0 \quad \Rightarrow \quad k=2$

So, $\alpha=0, \beta=11, \gamma=9$

$\therefore$ Required point is (0, 11, 9) where given line cuts yz-plane.

3. Write the direction cosine of a line equally inclined to the three coordinate axes. *[CBSE (AI) 2009, (F) 2011]*

Sol. Any line equally inclined to coordinate axes will have direction cosines l, l, l

$\therefore \qquad l^2+l^2+l^2=1$

$$3l^2 = 1 \quad \Rightarrow \quad l = \pm\frac{1}{\sqrt{3}}$$

$\therefore$ Direction cosines are $+\frac{1}{\sqrt{3}}, +\frac{1}{\sqrt{3}}, +\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

4. Write the distance of the following plane from the origin. **[*CBSE (AI) 2010*]**

$$\mathbf{2x - y + 2z + 1 = 0}$$

Sol. We have given plane

$$2x - y + 2z + 1 = 0$$

Distance from origin $= \left|\frac{(2\times0)-(1\times0)+(2\times0)+1}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right| = \left|\frac{1}{\sqrt{4+1+4}}\right| = \frac{1}{3}$

5. Find the acute angle between the planes **[*CBSE* 2019 (65/4/1)]**

$$\vec{r}\,.(\hat{i} - 2\hat{j} - 2\hat{k}) = 1 \text{ and } \vec{r}\,.(3\hat{i} - 6\hat{j} + 2\hat{k}) = 0.$$

Sol. We have, $\vec{n_1} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{n_2} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Let θ be the angle between the normals to the planes drawns from some common point.

We have, $\cos\theta = \left|\frac{\vec{n_1}\,.\,\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right| = \left|\frac{3+12-4}{\sqrt{9}\,\sqrt{49}}\right| = \left|\frac{11}{3\times7}\right| = \frac{11}{21}$

$\therefore$ $\theta = \cos^{-1}\left(\frac{11}{21}\right)$.

6. Write the direction cosines of a line parallel to *z*-axis. **[*CBSE (F) 2012*]**

Sol. The angle made by a line parallel to z-axis with x, y and z-axis are 90°, 90° and 0° respectively.

$\therefore$ The direction cosines of the line are cos 90°, cos 90°, cos 0° *i.e.*, 0, 0, 1.

7. Write the cartesian equation of a plane, bisecting the line segment joining the points *A*(2, 3, 5) and *B*(4, 5, 7) at right angles. **[*CBSE (F) 2013*]**

Sol. One point of required plane = mid point of given line segment.

$$= \left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{5+7}{2}\right) = (3, 4, 6)$$

Also dr's of normal to the plane = (4 – 2), (5 – 3), (7 – 5) = (2, 2, 2)

Therefore, required equation of plane is

$$2(x-3) + 2(y-4) + 2(z-6) = 0$$

$$2x + 2y + 2z = 26 \text{ or } x + y + z = 13$$

8. Write the vector equation of the plane, passing through the point (*a*, *b*, *c*) and parallel to the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$. **[*CBSE Delhi 2014*]**

Sol. Since, the required plane is parallel to plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$.

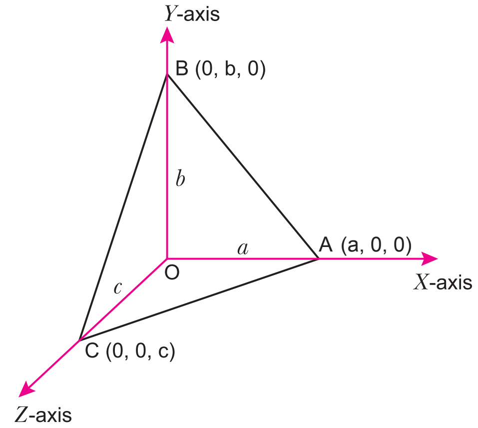
$\therefore$ Normal of required plane is normal of given plane.

$\Rightarrow$ Normal of required plane $= \hat{i} + \hat{j} + \hat{k}$

$\therefore$ Required vector equation of plane

$$\{\vec{r} - (a\hat{i} + b\hat{j} + c\hat{k})\}.(\hat{i} + \hat{j} + \hat{k}) = 0$$

9. Find the sum of the intercepts cut off by the plane $2x + y - z = 5$, on the coordinate axes. [CBSE (F) 2015]



Sol. Let a, b, c be the intercepts cut off by the plane

$2x + y - z = 5 \ldots (i)$ on x, y and z-axis respectively.

$\Rightarrow$ $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$ satisfy the equation (i)

Hence, $2a + 0 - 0 = 5 \quad \Rightarrow \quad a = \dfrac{5}{2}$

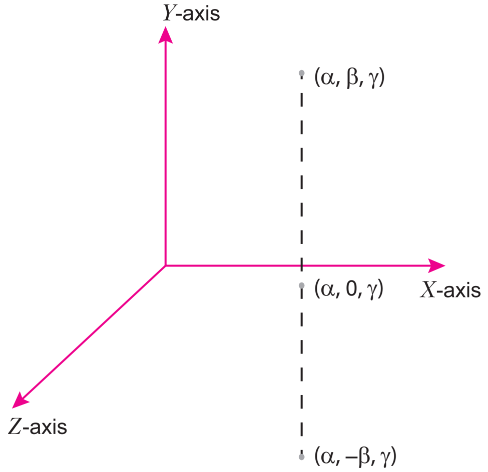
and $2 \times 0 + b - 0 = 5 \quad \Rightarrow \quad b = 5$

and $2 \times 0 + 0 - c = 5 \quad \Rightarrow \quad c = -5$

$\therefore \quad a + b + c = \dfrac{5}{2} + 5 - 5 = \dfrac{5}{2}$

10. Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane. [CBSE East 2016]

Sol. The reflection of the point (α, β, γ) in the XZ plane is $(\alpha, -\beta, \gamma)$.



11. Find the distance between the planes $\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0$ and $\vec{r}.(6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$ [CBSE South 2016]

Sol. Given two planes are

$$\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) - 4 = 0 \quad \text{and} \quad \vec{r}.(6\hat{i} - 9\hat{j} + 18\hat{k}) + 30 = 0$$

Given planes may be written in cartesian form as

$$2x - 3y + 6z - 4 = 0 \qquad \ldots (i)$$

$$6x - 9y + 18z + 30 = 0 \qquad \ldots (ii)$$

Let $P(x_1, y_1, z_1)$ be a point on plane (i)

$\therefore \quad 2x_1 - 3y_1 + 6z_1 - 4 = 0$

$\Rightarrow \quad 2x_1 - 3y_1 + 6z_1 = 4 \qquad \ldots (iii)$

The length of the perpendicular from $P(x_1, y_1, z_1)$ to plane (ii)

$$= \left| \frac{6x_1 - 9y_1 + 18z_1 + 30}{\sqrt{6^2 + (-9)^2 + 18^2}} \right| = \left| \frac{3(2x_1 - 3y_1 + 6z_1) + 30}{\sqrt{36 + 81 + 324}} \right|$$

$$= \left| \frac{3 \times 4 + 30}{\sqrt{441}} \right| = \left| \frac{42}{21} \right| = 2 \qquad \text{[Using } (iii)\text{]}$$

12. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes. **[*CBSE (F) 2016*]**

Sol. Obviously, a vector equally inclined to co-ordinate axes is given by $\hat{i} + \hat{j} + \hat{k}$

$\therefore$ Unit vector equally inclined to co-ordinate axes $= \dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}} = \dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

Therefore, required equation of plane is

$$\vec{r} . \left\{ \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) \right\} = 5\sqrt{3} \qquad \Rightarrow \qquad \vec{r} . (\hat{i} + \hat{j} + \hat{k}) = 15 \text{ or } x + y + z = 15$$

13. If a line makes angles 90° and 60° respectively with the positive directions of *x* and *y* axes, find the angle which it makes with the positive direction of *z*-axis. **[*CBSE Delhi 2017*]**

Sol. Let the angle made by line with positive direction of z-axis be θ then,

We know that

$$\cos^2 90° + \cos^2 60° + \cos^2 \theta = 1$$

$$\Rightarrow \quad 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1 \qquad \Rightarrow \qquad \frac{1}{4} + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \frac{1}{4} \qquad \Rightarrow \qquad \cos^2\theta = \frac{3}{4}$$

$$\Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 60° \text{ or } \frac{\pi}{3} \text{ if } \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \theta = 150° \text{ or } \frac{5\pi}{6} \text{ if } \cos\theta = -\frac{\sqrt{3}}{2}$$

14. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$. **[*CBSE (AI) 2017*]**

Sol. Let $P(x_1, y_1, z_1)$ be any point on plane $2x - y + 2z = 5$.

$$\Rightarrow \quad 2x_1 - y_1 + 2z_1 = 5$$

Now distance of point $P(x_1, y_1, z_1)$ from plane $5x - 2.5y + 5z = 20$ is given by

$$d = \left| \frac{5x_1 - 2.5y_1 + 5z_1 - 20}{\sqrt{5^2 + (2.5)^2 + (5)^2}} \right| = \left| \frac{2.5(2x_1 - y_1 + 2z_1 - 8)}{\sqrt{25 + 6.25 + 25}} \right| = \left| \frac{2.5(5 - 8)}{\sqrt{56.25}} \right|$$

$$= \frac{7.5}{7.5} = 1 \text{ unit}$$

15. Find the equation of a plane that cuts the coordinates axes at (a, 0, 0), (0, b, 0) and (0, 0, c). **[*NCERT Exemplar*]**

Sol. The equation of such plane is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

16. Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 3$. **[*CBSE* 2019 (65/5/3)]**

Sol. We have equation of line

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$$

$$\therefore \quad \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$$

Equation of plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=3$

$\vec{n}=\hat{i}+\hat{j}+\hat{k}$

Let θ be the required angle

$$\therefore \quad \cos\theta = \left|\frac{\vec{b}\,.\,\vec{n}}{|\vec{b}||\vec{n}|}\right| = \left|\frac{3-1+2}{\sqrt{9+1+4}\,\sqrt{1+1+1}}\right| = \left|\frac{4}{\sqrt{14}\,\sqrt{3}}\right| = \frac{4}{\sqrt{42}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{4}{\sqrt{42}}\right).$$

Short Answer Questions-I [2 marks]

1. Find the points of intersection of the line $\hat{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\hat{r}.(\hat{i}-\hat{j}+\hat{k})=5$ [*CBSE 2020, (65/2/1)*]

Sol. Given line be

$$\hat{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$$

Its Cartesian form is

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda\ (let)$$

$\therefore$ points on this line be $(3\lambda+2, 4\lambda-1, 2\lambda+2)$

i.e., $x=3\lambda+2,\ y=4\lambda-1,\ z=2\lambda+2$ ...(*i*)

and plane be $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$

$\Rightarrow \quad x-y+z=5$

$\Rightarrow \quad (3\lambda+2)-(4\lambda-1)+2\lambda+2=5$ (from (*i*)

$\Rightarrow \quad \lambda+5=5 \quad \Rightarrow \quad \lambda=0$

Putting $\lambda=0$ in (*i*), we get the co-ordinates of the point, $x=2, y=-1, z=2$

$\therefore$ Point of intersection be (2, –1, 2).

2. If the *x*-coordinate of a point *P* on the join of *Q*(2, 2, 1) and *R*(5, 1, –2) is 4, then find its *z*-coordinate. [*NCERT Exemplar*]

Sol. Let *P* divides *QR* in the ratio λ: 1

Then coordinates of *P* are $\left(\frac{5\lambda+2}{\lambda+1}, \frac{\lambda+2}{\lambda+1}, \frac{-2\lambda+1}{\lambda+1}\right)$

It is given that *x*-coordinate of *P* is 4.

$$\therefore \quad \frac{5\lambda+2}{\lambda+1}=4 \quad \Rightarrow \quad 5\lambda+2=4\lambda+4 \quad \Rightarrow \quad \lambda=2$$

So, *z* -coordinate of $P=\frac{-2\lambda+1}{\lambda+1}=\frac{-4+1}{2+1}=-1.$

3. Show that the points $(\hat{i}-\hat{j}+3\hat{k})$ and $3(\hat{i}+\hat{j}+\hat{k})$ are equidistant from the plane $\vec{r}.(5\hat{i}+2\hat{j}-7\hat{k})+9=0$ and lies on opposite side of it. [*NCERT Exemplar*]

Sol. To show that these given points $(\hat{i}-\hat{j}+3\hat{k})$ and $3(\hat{i}+\hat{j}+\hat{k})$ are equidistant from the plane $\vec{r}.(5\hat{i}+2\hat{j}-7\hat{k})+9=0,$ we first find out the mid-point of the points which is $2\hat{i}+\hat{j}+3\hat{k}$.

On substituting $\vec{r}$ by the mid-point in plane, we get

$$\text{LHS} = (2\hat{i}+\hat{j}+3\hat{k})\cdot(5\hat{i}+2\hat{j}-7\hat{k})+9 = 10+2-21+9=0=\text{RHS}$$

Hence, the two points lie on opposite sides of the plane are equidistant from the plane.

4. If the plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α, then prove that the equation of the plane in its new position is $ax + by \pm \left(\sqrt{a^{*} + b^{*}}\tan\alpha\right) z = 0$. **[*NCERT Exemplar*]**

Sol. Given, planes are $\quad ax + by = 0 \qquad$ ...(*i*)

and $\quad z = 0 \qquad$ ...(*ii*)

Therefore, the equation of any plane passing through the line of intersection of planes (*i*) and (*ii*) may be taken as $ax + by + k = 0$. ...(*iii*)

Then, direction cosines of a normal to the plane (*iii*) are

$\dfrac{a}{\sqrt{a^2+b^2+k^2}}, \dfrac{b}{\sqrt{a^2+b^2+k^2}}, \dfrac{c}{\sqrt{a^2+b^2+k^2}}$ and direction cosines of the normal to the plane (*i*) are

$\dfrac{a}{\sqrt{a^2+b^2}}, \dfrac{b}{\sqrt{a^2+b^2}}, 0.$

Since, the angle between the planes (*i*) and (*ii*) is α,

$$\therefore \qquad \cos\alpha = \frac{a\cdot a + b\cdot b + k\cdot 0}{\sqrt{a^2+b^2+k^2}\sqrt{a^2+b^2}} = \sqrt{\frac{a^2+b^2}{a^2+b^2+k^2}}$$

$$\Rightarrow \qquad k^2\cos^2\alpha = a^2(1-\cos^2\alpha) + b^2(1-\cos^2\alpha)$$

$$\Rightarrow \qquad k^2 = \frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}$$

$$\Rightarrow \qquad k = \pm\sqrt{a^2+b^2}\tan\alpha$$

On putting this value in plane (*iii*), we get the equation of the plane as

$$ax + by + z\sqrt{a^2+b^2}\tan\alpha = 0$$

5. Find the co-ordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX - plane. **[*CBSE 2020, (65/3/1)*]**

Sol. We have,

Equation of the line passing through $(-1, 1, -8)$ and $(5, -2, 10)$ be

$$\frac{x-(-1)}{5+1} = \frac{y-1}{-2-1} = \frac{z-(-8)}{10+8}$$

$$\Rightarrow \quad \frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} \qquad ..(i)$$

Now, for the co-ordinates of the point where the line (*i*) crosses the ZX–plane, put $y = 0$ in (*i*), we get

$$\frac{x+1}{6} = \frac{0-1}{-3} = \frac{z+8}{18}$$

$$\Rightarrow \quad \frac{x+1}{6} = \frac{1}{3} \text{ and } \frac{1}{3} = \frac{z+8}{18}$$

$$\Rightarrow \quad x = 1 \text{ and } z = -2$$

$\therefore$ Co-ordinates of the required point be $(1, 0, -2)$

Short Answer Questions-II **[3 marks]**

1. Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$ **[CBSE (F) 2014]**

Sol. Comparing the given equations with equations $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$.

We get $\vec{a_1} = \hat{i} + \hat{j}, \vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}, \ \vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore, $\vec{a_2} - \vec{a_1} = (\hat{i} - \hat{k})$ and

$$\vec{b_1} \times \vec{b_2} = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{9 + 1 + 49} = \sqrt{59}$$

Hence, the shortest distance between the given lines is given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| = \left| \frac{3 - 0 + 7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}} \text{ units.}$$

2. Find the distance between the lines l_1 and l_2 given by

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}); \quad l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$ **[CBSE (F) 2014]**

Sol. Given lines are

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

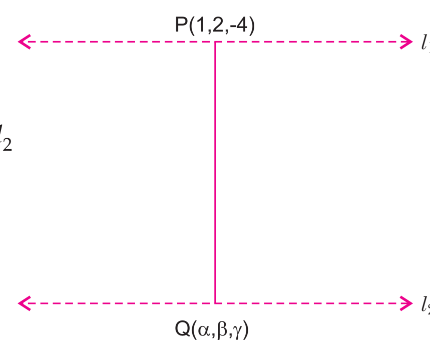
After observation, we get $l_1 || l_2$

Therefore, it is sufficient to find the perpendicular distance of a point of line l_1 to line l_2.

The coordinate of a point of l_1 is $P(1, 2, -4)$

Also the cartesian form of line l_2 is

$$\frac{x - 3}{4} = \frac{y - 3}{6} = \frac{z + 5}{12} \qquad \ldots (i)$$



Let $Q(\alpha, \beta, \gamma)$ be foot of perpendicular drawn from P to line l_2

$\because \quad Q(\alpha, \beta, \gamma)$ lie on line l_2

$\therefore \quad \dfrac{\alpha - 3}{4} = \dfrac{\beta - 3}{6} = \dfrac{\gamma + 5}{12} = \lambda$ (say)

$\Rightarrow \quad \alpha = 4\lambda + 3, \beta = 6\lambda + 3, \gamma = 12\lambda - 5$

Again, $\because \vec{PQ}$ is perpendicular to line l_2.

$\Rightarrow \quad \vec{PQ} \cdot \vec{b} = 0$, where $\vec{b}$ is parallel vector of l_2

$\Rightarrow \quad (\alpha - 1).4 + (\beta - 2).6 + (\gamma + 4).12 = 0 \qquad [\because \vec{PQ} = (\alpha - 1)\hat{i} + (\beta - 2)\hat{j} + (\gamma + 4)\hat{k}]$

$\Rightarrow \quad 4\alpha - 4 + 6\beta - 12 + 12\gamma + 48 = 0 \quad \Rightarrow \quad 4\alpha + 6\beta + 12\gamma + 32 = 0$

$\Rightarrow \quad 4(4\lambda + 3) + 6(6\lambda + 3) + 12(12\lambda - 5) + 32 = 0 \quad \Rightarrow \quad 16\lambda + 12 + 36\lambda + 18 + 144\lambda - 60 + 32 = 0$

$\Rightarrow \quad 196\lambda + 2 = 0 \qquad \Rightarrow \qquad \lambda = \dfrac{-2}{196} = \dfrac{-1}{98}$

Coordinate of $Q \equiv \left(4\times\left(-\frac{1}{98}\right)+3, 6\times\left(-\frac{1}{98}\right)+3, 12\times\left(-\frac{1}{98}\right)-5\right)$

$$\equiv\left(-\frac{2}{49}+3, -\frac{3}{49}+3, -\frac{6}{49}-5\right)\equiv\left(\frac{145}{49}, \frac{144}{49}, -\frac{251}{49}\right)$$

Therefore required perpendicular distance is

$$\sqrt{\left(\frac{145}{49}-1\right)^2+\left(\frac{144}{49}-2\right)^2+\left(\frac{-251}{49}+4\right)^2}=\sqrt{\left(\frac{96}{49}\right)^2+\left(\frac{46}{49}\right)^2+\left(\frac{-55}{49}\right)^2}$$

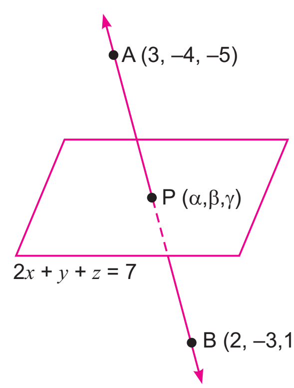
$$=\sqrt{\frac{96^2+46^2+55^2}{49^2}}=\sqrt{\frac{9216+2116+3025}{49^2}}$$

$$=\frac{\sqrt{14357}}{49}=\frac{7\sqrt{293}}{49}=\frac{\sqrt{293}}{7}\text{ units}$$

3. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1) crosses the plane $2x + y + z = 7$. **[*CBSE (AI) 2012*]**

Sol. The equation of line passing through the points (3, – 4, – 5) and (2, – 3, 1) is

$$\frac{x-3}{2-3}=\frac{y+4}{-3+4}=\frac{z+5}{1+5}\Rightarrow\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}\qquad ...(i)$$



Let the line (*i*) crosses at point $P(\alpha, \beta, \gamma)$ to the plane $2x + y + z = 7$ ...(*ii*)

$\because$ P lies on line (*i*), therefore (α, β, γ) satisfy equation (*i*)

$\therefore$ $\frac{\alpha-3}{-1}=\frac{\beta+4}{1}=\frac{\gamma+5}{6}=\lambda$ (say)

$\alpha = -\lambda + 3;\ \beta = \lambda - 4$ and $\gamma = 6\lambda - 5$

Also $P(\alpha, \beta, \gamma)$ lie on plane (*ii*)

$\therefore$ $2\alpha + \beta + \gamma = 7 \Rightarrow 2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) = 7$

$\Rightarrow$ $-2\lambda + 6 + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow 5\lambda = 10 \Rightarrow \lambda = 2$

Hence, the coordinate of required point P is $(-2 + 3, 2 - 4, 6 \times 2 - 5)$ *i.e.*, $(1, -2, 7)$

4. A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$. Obtain its equation in vector and cartesian form.

[*CBSE (AI) 2014*]

Sol. Let $\vec{b}$ be parallel vector of required line.

$\Rightarrow$ $\vec{b}$ is perpendicular to both given line.

$\Rightarrow$ $\vec{b}=(2\hat{i}-2\hat{j}+\hat{k})\times(\hat{i}+2\hat{j}+2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -2 & 1\\ 1 & 2 & 2\end{vmatrix}=(-4-2)\hat{i}-(4-1)\hat{j}+(4+2)\hat{k}=-6\hat{i}-3\hat{j}+6\hat{k}.$$

Hence, the equation of line in vector form is

$$\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(-6\hat{i}-3\hat{j}+6\hat{k})\qquad \vec{r}=(2\hat{i}-\hat{j}+3\hat{k})-3\lambda(2\hat{i}+\hat{j}-2\hat{k})$$

$$\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\mu(2\hat{i}+\hat{j}-2\hat{k})\qquad [\mu=-3\lambda]$$

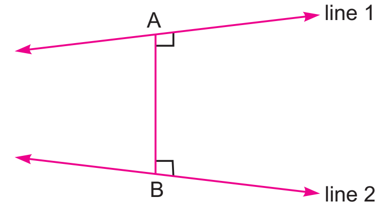
Equation in cartesian form is

$$\frac{x-2}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$$

5. Find the shortest distance between the following lines :

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \quad \text{and} \quad \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$ **[*CBSE Delhi 2008, (F) 2013, 2014*]**

Sol. Let $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}=\lambda$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}=k$



Now, let's take a point on first line as

$A\ (\lambda+3, -2\lambda+5, \lambda+7)$ and let

$B\ (7k-1, -6k-1, k-1)$ be point on the second line

The direction ratio of the line AB

$7k-\lambda-4, -6k+2\lambda-6, k-\lambda-8$

Now, as AB is the shortest distance between line 1 and line 2 so,

$$(7k-\lambda-4)\times 1+(-6k+2\lambda-6)\times(-2)+(k-\lambda-8)\times 1=0 \qquad ...(i)$$

and $$(7k-\lambda-4)\times 7+(-6k+2\lambda-6)\times(-6)+(k-\lambda-8)\times 1=0 \qquad ...(ii)$$

Solving equation (*i*) and (*ii*), we get

$\lambda=0$ and $k=0$

$\therefore$ $A\equiv(3,5,7)$ and $B\equiv(-1,-1,-1)$

Hence, $AB=\sqrt{(3+1)^2+(5+1)^2+(7+1)^2}=\sqrt{16+36+64}=\sqrt{116}$ units $=2\sqrt{29}$ units

6. Find the equation of planes passing through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ and are at a unit distance from the origin. **[*CBSE* 2019 (65/5/3)]**

Sol. We are given planes:

$$\vec{r}.(2\hat{i}+6\hat{j})+12=0 \qquad ...(i)$$

$$\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0 \qquad ...(ii)$$

So equation of the required plane can be written as:

$$(\vec{r}.(2\hat{i}+6\hat{j})+12)+\lambda(\vec{r}.(3\hat{i}-\hat{j}+4\hat{k}))=0$$

$$\Rightarrow \quad \vec{r}.\{(2+3\lambda)\hat{i}+(6-\lambda)\hat{j}+4\lambda\hat{k}\}+12=0 \qquad ...(iii)$$

In cartesian form

$$(2+3\lambda)x+(6-\lambda)y+4\lambda z+12=0 \qquad ...(iv)$$

Since direction ratios of the normal to the plane are $(2+3\lambda),(6-\lambda),4\lambda;$ the direction cosines of it are:

$$\frac{2+3\lambda}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}},\frac{6-\lambda}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}},\frac{4\lambda}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}}$$

So the distance of the plane from the origin is $\frac{12}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}}$

We are given that distance from origin is unity

$$\therefore \quad \frac{12}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}}=1$$

$$\Rightarrow \quad \frac{144}{4+9\lambda^2+12\lambda+36-12\lambda+\lambda^2+16\lambda^2}=1 \quad \Rightarrow \quad 144=26\lambda^2+40 \qquad \text{(Squaring both sides)}$$

$\Rightarrow \quad 26\lambda^2 = 104 \quad \Rightarrow \quad \lambda^2 = \frac{104}{26} = 4$

$\Rightarrow \quad \lambda = \pm 2$

$\therefore$ Required equation of the plane is $8x + 4y + 8z + 12 = 0$.

In vector form

$$\vec{r}.(8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0$$

7. Find the vector equation of the plane determined by the points *A*(3, –1, 2), *B*(5, 2, 4) and *C*(–1, –1, 6). Hence, find the distance of the plane, thus obtained from the origin.

[*CBSE 2019 (65/4/2)*]

Sol. Required equation of plane is given by:

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} 0 \Rightarrow (x-3)\,12 - (y+1)\,16 + (z-2)\,12 = 0$$

$\Rightarrow \quad 12x - 16y + 12z - 36 - 16 - 24 = 0 \quad \Rightarrow \quad 12x - 16y + 12z - 76 = 0 \quad \Rightarrow \quad 3x - 4y + 3z - 19 = 0$

Vector form:

$$\vec{r}.(3\hat{i} - 4\hat{j} + 3\hat{k}) = 19. \qquad ...(i)$$

Distance of plane (*i*) from origin

$$\vec{r}.\frac{3i - 4j + 3k}{\sqrt{9+16+9}} = \frac{19}{\sqrt{34}} \qquad [\because \vec{r}.\hat{n} = d]$$

Therefore distance of plane from origin is $\frac{19}{\sqrt{34}}$ units.

8. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$. [*CBSE (AI) 2014*]

Sol. Let the cartesian equation of the line passing through (2, 1, 3) be

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \qquad \ldots(i)$$

Since, line (*i*) is perpendicular to given line

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \qquad \ldots(ii)$$

and $$\frac{x}{-3} = \frac{y}{2} = \frac{z}{5} \qquad \ldots(iii)$$

$\therefore \quad a + 2b + 3c = 0 \qquad \ldots(iv)$

$-3a + 2b + 5c = 0 \qquad \ldots(v)$

From equation (*iv*) and (*v*), we get

$$\frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6} \quad \Rightarrow \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = \lambda \qquad \text{(say)}$$

$\Rightarrow \quad a = 4\lambda, b = -14\lambda, c = 8\lambda$

Putting the value of *a*, *b* and *c* in (*i*), we get

$$\frac{x-2}{4\lambda} = \frac{y-1}{-14\lambda} = \frac{z-3}{8\lambda} \quad \Rightarrow \quad \frac{x-2}{4} = \frac{y-1}{-14} = \frac{z-3}{8}$$

$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$, which is the cartesian form.

The vector form is $\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$.

9. Find the distance of the point P (6, 5, 9) from the plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6). **[*CBSE (AI) 2010; (F) 2012; Delhi 2013; Ajmer 2015*]**

Sol. Plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6) is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 5-3 & 2+1 & 4-2 \\ -1-3 & -1+1 & 6-2 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} x-3 & y+1 & x-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$$(x-3)\begin{vmatrix} 3 & 2 \\ 0 & 4 \end{vmatrix} - (y+1)\begin{vmatrix} 2 & 2 \\ -4 & 4 \end{vmatrix} + (z-2)\begin{vmatrix} 2 & 3 \\ -4 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad 12x - 36 - 16y - 16 + 12z - 24 = 0 \quad \Rightarrow \quad 3x - 4y + 3z - 19 = 0$

Distance of this plane from point P (6, 5, 9) is

$$\left|\frac{(3\times 6) - (4\times 5) + (3\times 9) - 19}{\sqrt{(3)^2 + (4)^2 + (3)^2}}\right| = \left|\frac{18 - 20 + 27 - 19}{\sqrt{9 + 16 + 9}}\right| = \frac{6}{\sqrt{34}} \text{ units.}$$

10. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection. **[*CBSE Delhi 2014*]**

Sol. Given lines are

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \quad \ldots(i)$$

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} \quad \ldots(ii)$$

Let two lines (*i*) and (*ii*) intersect at a point $P(\alpha, \beta, \gamma)$.

$\Rightarrow \quad (\alpha, \beta, \gamma)$ satisfy line (*i*)

$\Rightarrow \quad \frac{\alpha+1}{3} = \frac{\beta+3}{5} = \frac{\gamma+5}{7} = \lambda \quad$ (say)

$\Rightarrow \quad \alpha = 3\lambda - 1, \quad \beta = 5\lambda - 3, \quad \gamma = 7\lambda - 5 \quad , \quad \ldots(iii)$

Again (α, β, γ) also lie on (*ii*), we get

$$\frac{\alpha-2}{1} = \frac{\beta-4}{3} = \frac{\gamma-6}{5} \quad \Rightarrow \quad \frac{3\lambda-1-2}{1} = \frac{5\lambda-3-4}{3} = \frac{7\lambda-5-6}{5}$$

$$\Rightarrow \quad \underset{\text{I}}{\frac{3\lambda-3}{1}} = \underset{\text{II}}{\frac{5\lambda-7}{3}} = \underset{\text{III}}{\frac{7\lambda-11}{5}}$$

From I and II

$$\frac{3\lambda-3}{1} = \frac{5\lambda-7}{3}$$

$\Rightarrow \quad 9\lambda - 9 = 5\lambda - 7$

$\Rightarrow \quad 4\lambda = 2$

$\Rightarrow \quad \lambda = \frac{1}{2}$

From II and III

$$\frac{5\lambda-7}{3} = \frac{7\lambda-11}{5}$$

$\Rightarrow \quad 25\lambda - 35 = 21\lambda - 33$

$\Rightarrow \quad 4\lambda = 2$

$\Rightarrow \quad \lambda = \frac{1}{2}$

Since, the value of λ in both the cases is same

$\Rightarrow \quad$ Both lines intersect each other at a point.

$\therefore \quad$ Intersecting point $= (\alpha, \beta, \gamma) = \left(\frac{3}{2} - 1, \frac{5}{2} - 3, \frac{7}{2} - 5\right) \quad$ [From (*iii*)]

$$= \left(\frac{1}{2}, -\frac{1}{2}, \frac{-3}{2}\right)$$

11. Find the vector and cartesian equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.
[*CBSE Delhi 2012, 2017*]

OR

Find the equation of a line passing through the point (1, 2, –4) and perpendicular to two lines $\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

[*CBSE Allahabad 2015; Delhi 2016*]

Sol. Let the cartesian equation of line passing through (1, 2, – 4) be

$$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c} \qquad ...(i)$$

Given lines are

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \qquad ...(ii)$$

$$\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5} \qquad ...(iii)$$

Obviously parallel vectors $\vec{b}_1, \vec{b}_2$ and $\vec{b}_3$ of (*i*), (*ii*) and (*iii*) respectively are given as

$$\vec{b}_1=a\hat{i}+b\hat{j}+c\hat{k};\ \vec{b}_2=3\hat{i}-16\hat{j}+7\hat{k};\ \vec{b}_3=3\hat{i}+8\hat{j}-5\hat{k}$$

According to question

$$(i)\perp(ii) \quad\Rightarrow\quad \vec{b}_1\perp\vec{b}_2 \quad\Rightarrow\quad \vec{b}_1.\vec{b}_2=0$$

$$(i)\perp(iii) \quad\Rightarrow\quad \vec{b}_1\perp\vec{b}_3 \quad\Rightarrow\quad \vec{b}_1.\vec{b}_3=0$$

Hence, $3a-16b+7c=0$ ...(*iv*)

and $3a+8b-5c=0$ ...(*v*)

From equation (*iv*) and (*v*), we get

$$\frac{a}{80-56}=\frac{b}{21+15}=\frac{c}{24+48}$$

$$\Rightarrow \quad \frac{a}{24}=\frac{b}{36}=\frac{c}{72} \quad\Rightarrow\quad \frac{a}{2}=\frac{b}{3}=\frac{c}{6}=\lambda \qquad \text{(say)}$$

$$\Rightarrow \quad a=2\lambda,\ b=3\lambda,\ c=6\lambda$$

Putting the value of *a, b, c* in (*i*), we get the required cartesian equation of line as

$$\frac{x-1}{2\lambda}=\frac{y-2}{3\lambda}=\frac{z+4}{6\lambda} \quad\Rightarrow\quad \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$$

Hence, vector equation is

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$

12. A line passing through the point A with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$. Find the length of the perpendicular drawn on this line from a point *P* with position vector $\vec{r}_1=\hat{i}+2\hat{j}+3\hat{k}$. **[*CBSE Panchkula 2015*]**

Sol. The equation of line passing through the point *A* and parallel to $\vec{b}$ is given in cartesian form as

$$\frac{x-4}{2}=\frac{y-2}{3}=\frac{z-2}{6} \qquad ...(i)$$

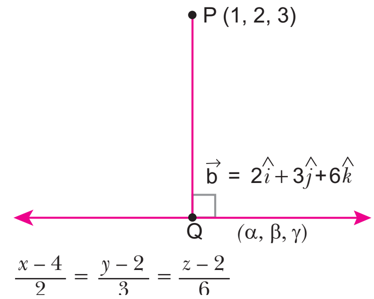
Let $Q(\alpha,\beta,\gamma)$ be foot of perpendicular drawn from point *P* to the line (*i*).

Co-ordinate or point $P\equiv(1,2,3)$ $\quad [\because P.V.$ of P is $\hat{i}+2\hat{j}+3\hat{k}]$

Since, Q lie on line (i)

$$\frac{\alpha-4}{2}=\frac{\beta-2}{3}=\frac{\gamma-2}{6}=\lambda \quad \Rightarrow \quad \alpha=2\lambda+4, \beta=3\lambda+2, \gamma=6\lambda+2$$

Now, $\overrightarrow{PQ}=(\alpha-1)\hat{i}+(\beta-2)\hat{j}+(\gamma-3)\hat{k}$



Obviously, $\overrightarrow{PQ} \perp \vec{b}$ $\quad \therefore \quad \overrightarrow{PQ}\cdot\vec{b}=0$

$\Rightarrow \quad 2(\alpha-1)+3(\beta-2)+6(\gamma-3)=0$

$\Rightarrow \quad 2\alpha-2+3\beta-6+6\gamma-18=0 \quad \Rightarrow \quad 2\alpha+3\beta+6\gamma-26=0$

Putting the value of $\alpha, \beta, \gamma;$ we get

$2(2\lambda+4)+3(3\lambda+2)+6(6\lambda+2)-26=0$

$\Rightarrow \quad 4\lambda+8+9\lambda+6+36\lambda+12-26=0$

$\Rightarrow \quad 49\lambda=0 \quad \Rightarrow \quad \lambda=0$

Hence, the co-ordinate of $Q \equiv (4, 2, 2)$

$\therefore$ Length of perpendicular $PQ=\sqrt{(4-1)^2+(2-2)^2+(2-3)^2}$

$=\sqrt{9+0+1}=\sqrt{10}$ units.

13. Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x-y+z-5=0$. Also find the angle between the line and the plane. **[*CBSE Delhi 2013*]**

Sol. Let the given line

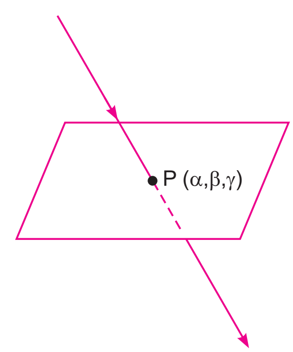
$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2} \qquad \ldots(i)$$

intersect at point $P(\alpha, \beta, \gamma)$ to the plane $x-y+z-5=0$ ...(ii)

$\because \quad P(\alpha, \beta, \gamma)$ lie on line (i)

$\therefore \quad \frac{\alpha-2}{3}=\frac{\beta+1}{4}=\frac{\gamma-2}{2}=\lambda$ (say)

$\alpha=3\lambda+2; \beta=4\lambda-1; \gamma=2\lambda+2$



Also, $P(\alpha, \beta, \gamma)$ lies on plane (ii)

$\therefore \quad (3\lambda+2)-(4\lambda-1)+(2\lambda+2)-5=0$

$\Rightarrow \quad 3\lambda+2-4\lambda+1+2\lambda+2-5=0 \quad \Rightarrow \quad \lambda=0$

$\therefore \quad \alpha=2, \beta=-1, \gamma=2$

Hence, co-ordinate of required point $=(2,-1,2)$

Now, find angle between line (i) and plane (ii)

If θ be the required angle, then

$$\sin\theta=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|=\left|\frac{1}{\sqrt{9+16+4}\cdot\sqrt{1^2+(-1)^2+1^2}}\right|=\left|\frac{1}{\sqrt{29}\cdot\sqrt{3}}\right| \qquad \left[\begin{array}{l}\because \vec{b}=3\hat{i}+4\hat{j}+2\hat{k} \\ \vec{n}=\hat{i}-\hat{j}+\hat{k} \\ \therefore \vec{b}\cdot\vec{n}=3-4+2=1\end{array}\right]$$

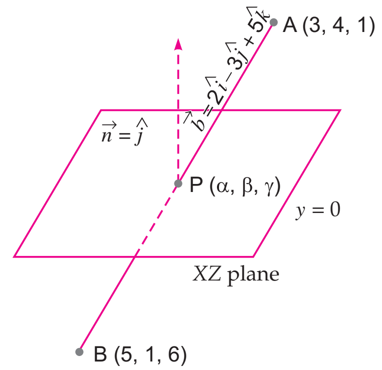
$$\sin\theta=\frac{1}{\sqrt{87}} \quad \Rightarrow \quad \theta=\sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$$

14. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane. **[*CBSE (Central) 2016*]**

Sol. Let $P(\alpha, \beta, \gamma)$ be the point at which the given line crosses the XZ plane.

Now the equation of given line AB is

$$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} \quad ...(i)$$



Since $P(\alpha, \beta, \gamma)$ lies on line (i)

$$\therefore \quad \frac{\alpha-3}{2} = \frac{\beta-4}{-3} = \frac{\gamma-1}{5} = \lambda \text{ (say)}$$

$\Rightarrow \quad \alpha = 2\lambda + 3; \beta = -3\lambda + 4$ and $\gamma = 5\lambda + 1$

Also $P(\alpha, \beta, \gamma)$ lie on XZ plane, *i.e.*, $y = 0$ $(0x + 1y + 0z = 0)$

$$0\alpha + 1.\beta + 0.\gamma = 0$$

$\Rightarrow \quad \beta = 0 \quad \Rightarrow \quad -3\lambda + 4 = 0 \quad \Rightarrow \quad \lambda = \frac{4}{3}$

Hence, the co-ordinates of required point P is

$$\alpha = 2\times\frac{4}{3} + 3 = \frac{8}{3} + 3 = \frac{17}{3};\ \beta = -3\times\frac{4}{3} + 4 = 0;\ \gamma = 5\times\frac{4}{3} + 1 = \frac{23}{3}$$

$\therefore$ Co-ordinate of required point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

Let θ be the angle made by line AB with XZ plane.

$$\therefore \quad \sin\theta = \left|\frac{\vec{n}.\vec{b}}{|\vec{b}||\vec{n}|}\right|$$

Here $\quad \vec{n} = \hat{j}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

$$|\vec{n}| = 1 \text{ and } |\vec{b}| = \sqrt{4+9+25} = \sqrt{38}$$

$$\Rightarrow \quad \sin\theta = \left|\frac{\hat{j}.(2\hat{i} - 3\hat{j} + 5\hat{k})}{1.\sqrt{38}}\right| = \left|\frac{-3}{\sqrt{38}}\right|$$

$$\Rightarrow \quad \sin\theta = \frac{3}{\sqrt{38}} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$

15. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$. **[*CBSE Delhi 2017*]**

Sol. Given line is

$$5x - 25 = 14 - 7y = 35z$$

$$\Rightarrow \quad \frac{x-5}{\frac{1}{5}} = \frac{2-y}{\frac{1}{7}} = \frac{z-0}{\frac{1}{35}} \quad \Rightarrow \quad \frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z-0}{\frac{1}{35}}$$

$$= \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-0}{1} \quad \ldots (i)$$

Hence, parallel vector of given line *i.e.*, $\vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$

Since required line is parallel to given line (i)

$\Rightarrow \quad \vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$ will also be parallel vector of required line which passes through $A(1, 2, -1)$.

Therefore, required vector equation of line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$

16. Find the co-ordinates of the point where the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from origin. **[*CBSE (South) 2016*]**

Sol. We know that the equation of plane is

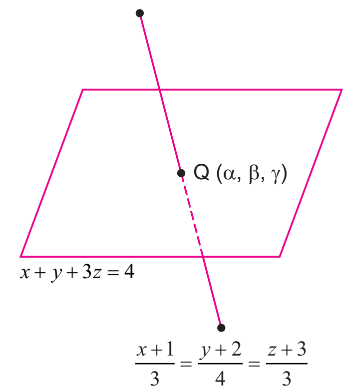
$\vec{r}.\hat{n} = d$; where $\hat{n}$ is normal unit vector and d is perpendicular distance from origin.

Here, $\hat{n} = \dfrac{\hat{i}+\hat{j}+3\hat{k}}{\sqrt{1^2+1^2+3^2}} = \dfrac{1}{\sqrt{11}}(\hat{i}+\hat{j}+3\hat{k})$ and $d = \dfrac{4}{\sqrt{11}}$

$\therefore$ Equation of plane

$$\vec{r}.\frac{1}{\sqrt{11}}(\hat{i}+\hat{j}+3\hat{k}) = \frac{4}{\sqrt{11}} \quad \Rightarrow \quad \vec{r}.(\hat{i}+\hat{j}+3\hat{k}) = 4$$

$\Rightarrow$ $$x + y + 3z = 4 \qquad ...(i)$$



Equation of given line

$$\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$$

Its cartesian form is

$$\frac{x+1}{3} = \frac{y+2}{4} = \frac{z+3}{3} \qquad ...(ii)$$

Let Q (α, β, γ) be the point of intersection of (i) & (ii)

$\because$ Q lies on (ii)

$\therefore$ $$\frac{\alpha+1}{3} = \frac{\beta+2}{4} = \frac{\gamma+3}{3} = \lambda$$

$\Rightarrow$ $\alpha = 3\lambda - 1, \beta = 4\lambda - 2, \gamma = 3\lambda - 3$

Also, Q lies on (i)

$\therefore$ $\alpha + \beta + 3\gamma = 4 \Rightarrow 3\lambda - 1 + 4\lambda - 2 + 9\lambda - 9 = 4 \Rightarrow 16\lambda = 16 \Rightarrow \lambda = 1$

$\therefore$ $\alpha = 2, \beta = 2, \gamma = 0$

$\therefore$ Required point of intersection $\equiv (2, 2, 0)$

17. A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2} = \dfrac{1}{p^2}$.

[*CBSE (AI) 2017*]

Sol. Let the given variable plane meets X, Y and Z axes at $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$.

Therefore the equation of given plane is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \ldots (i)$$

Let (α, β, γ) be the coordinates of the centroid of triangle ABC. Then

$$\alpha = \frac{a+0+0}{3} = \frac{a}{3} \Rightarrow a = 3\alpha; \quad \beta = \frac{0+b+0}{3} = \frac{b}{3} \Rightarrow b = 3\beta$$

$$\gamma = \frac{0+0+c}{3} = \frac{c}{3} \Rightarrow c = 3\gamma$$

$\because$ $3p$ is the distance from origin to the plane (i)

$$\Rightarrow \quad 3p = \frac{0.\frac{1}{a} + 0.\frac{1}{b} + 0.\frac{1}{c} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \quad \Rightarrow \quad \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} = -\frac{1}{3p}$$

Squaring both sides, we have

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \Rightarrow \frac{1}{9\alpha^2} + \frac{1}{9\beta^2} + \frac{1}{9\gamma^2} = \frac{1}{9p^2} \text{ [Putting value of } a = 3\alpha, b = 3\beta, c = 3\gamma\text{]}$$

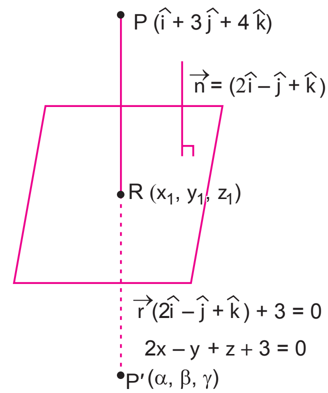
$$\Rightarrow \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{1}{p^2}$$

Therefore, locus of (α, β, γ) is $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2} = \dfrac{1}{p^2}$ Hence proved.

18. Find the image P' of the point P having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})+3=0$. Hence find the length of PP'. **[*CBSE (F) 2013, 2017*]**

Sol. Let given point be P (1, 3, 4) and the equation of given plane in cartesian form be

$$2x-y+z+3=0 \qquad ...(i)$$



Let R (x_1, y_1, z_1) be the foot of perpendicular and P' (α, β, γ) be the image of P

Since, R (x_1, y_1, z_1) lie on plane (i)

$$2x_1-y_1+z_1+3=0 \qquad ...(ii)$$

Also, normal vector $\vec{n}$ of plane (i) is $\vec{n}=2\hat{i}-\hat{j}+\hat{k}$

and $\quad \overrightarrow{PR}=(x_1-1)\hat{i}+(y_1-3)\hat{j}+(z_1-4)\hat{k}$

$\therefore \quad \overrightarrow{PR} \,||\, \vec{n}$

$$\Rightarrow \quad \frac{x_1-1}{2}=\frac{y_1-3}{-1}=\frac{z_1-4}{1}=\lambda$$

$$\Rightarrow \quad x_1=2\lambda+1, y_1=-\lambda+3, z_1=\lambda+4$$

Putting x_1, y_1, z_1 in (ii), we get

$\Rightarrow \quad 2(2\lambda+1)-(-\lambda+3)+(\lambda+4)+3=0$

$\Rightarrow \quad 4\lambda+2+\lambda-3+\lambda+4+3=0$

$\Rightarrow \quad 6\lambda+6=0 \quad \Rightarrow \quad \lambda=-1$

$\therefore \quad R\equiv(x_1, y_1, z_1)\equiv(-1, 4, 3)$

Since R is the mid point of PP'

$\therefore \quad -1=\dfrac{\alpha+1}{2} \quad \Rightarrow \quad \alpha=-3$

$4=\dfrac{\beta+3}{2} \quad \Rightarrow \quad \beta=5$

$3=\dfrac{\gamma+4}{2} \quad \Rightarrow \quad \gamma=2$

Hence, image $P'=(-3, 5, 2)$

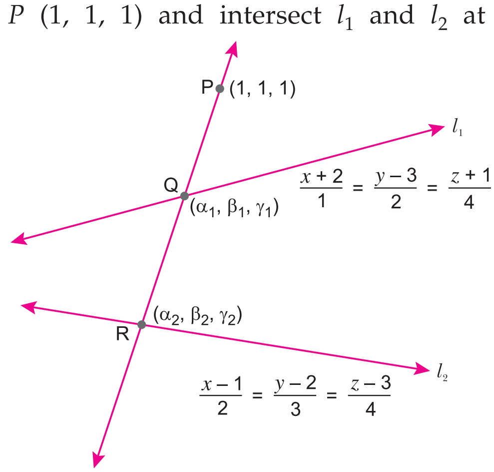
$$PP'=\sqrt{(-3-1)^2+(5-3)^2+(2-4)^2}=\sqrt{16+4+4}=\sqrt{24}=2\sqrt{6} \text{ units}$$

19. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1). **[*CBSE Sample Paper 2018*]**

Sol. Let l_1, l_2 be given lines as

$$l_1: \frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}; \qquad l_2: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

Let l be the required line, which passes through P (1, 1, 1) and intersect l_1 and l_2 at Q $(\alpha_1, \beta_1, \gamma_1)$ and R $(\alpha_2, \beta_2, \gamma_2)$ respectively.



Now, Q $(\alpha_1, \beta_1, \gamma_1)$ lie on line l_1

$$\therefore \quad \frac{\alpha_1+2}{1}=\frac{\beta_1-3}{2}=\frac{\gamma_1+1}{4}=\lambda \text{ (say)}$$

$$\alpha_1=\lambda-2, \quad \beta_1=2\lambda+3, \quad \gamma_1=4\lambda-1$$

Similarly, R $(\alpha_2, \beta_2, \gamma_2)$ lie on line l_2

$$\frac{\alpha_2-1}{2}=\frac{\beta_2-2}{3}=\frac{\gamma_2-3}{4}=\mu \text{ (say)}$$

$$\Rightarrow \quad \alpha_2=2\mu+1, \quad \beta_2=3\mu+2, \quad \gamma_2=4\mu+3$$

$$\therefore \quad \overrightarrow{PQ}=(\alpha_1-1)\hat{i}+(\beta_1-1)\hat{j}+(\gamma_1-1)\hat{k}$$

$$= (\lambda - 3)\hat{i} + (2\lambda + 2)\hat{j} + (4\lambda - 2)\hat{k}$$

Similarly, $\overrightarrow{PR} = 2\mu\hat{i} + (3\mu + 1)\hat{j} + (4\mu + 2)\hat{k}$

$\because \quad \overrightarrow{PQ} \parallel \overrightarrow{PR} \quad \Rightarrow \quad \dfrac{\lambda - 3}{2\mu} = \dfrac{2\lambda + 2}{3\mu + 1} = \dfrac{4\lambda - 2}{4\mu + 2} = M$ (say)

Now, $\dfrac{\lambda - 3}{2\mu} = M \Rightarrow \lambda - 3 = 2M\mu \Rightarrow M\mu = \dfrac{\lambda - 3}{2}$

Also, $\dfrac{2\lambda + 2}{3\mu + 1} = M \Rightarrow 2\lambda + 2 = 3M\mu + M$

$$\Rightarrow 2\lambda + 2 = \frac{3\lambda - 9}{2} + M \qquad \Rightarrow 2\lambda + 2 - \frac{3\lambda - 9}{2} = M$$

$$\Rightarrow \frac{4\lambda + 4 - 3\lambda + 9}{2} = M \qquad \Rightarrow \frac{\lambda + 13}{2} = M$$

Also, $\dfrac{4\lambda - 2}{4\mu + 2} = M \Rightarrow 4\lambda - 2 = 4M\mu + 2M$

$$\Rightarrow 4\lambda - 2 = \frac{4\lambda - 12}{2} + 2M \qquad \Rightarrow \frac{8\lambda - 4 - 4\lambda + 12}{2} = 2M$$

$$\Rightarrow \frac{4\lambda + 8}{2} = 2M \qquad \Rightarrow \frac{4(\lambda + 2)}{2} = 2M$$

$$\Rightarrow \lambda + 2 = M \qquad \Rightarrow \lambda + 2 = \frac{\lambda + 13}{2}$$

$$\Rightarrow 2\lambda + 4 = \lambda + 13 \Rightarrow \lambda = 9 \quad \Rightarrow \quad M = 11 \Rightarrow \mu = \frac{3}{11}$$

$\therefore \qquad \overrightarrow{PQ} = 6\hat{i} + 20\hat{j} + 34\hat{k}$

Hence, equation of required line is $\dfrac{x-1}{6} = \dfrac{y-1}{20} = \dfrac{z-1}{34} \Rightarrow \dfrac{x-1}{3} = \dfrac{y-1}{10} = \dfrac{z-1}{17}$

Long Answer Questions [5 marks]

1. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations $\dfrac{x+2}{1} = \dfrac{z-3}{2} = \dfrac{z+1}{4}$ and $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines . [*CBSE 2020 (65/5/1)*]

Sol. Let the cartesian equation of the required line be

$$\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c} \qquad ...(i)$$

where, a, b, c are direction ratios and given lines are

$$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4} \qquad ...(ii)$$

and $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \qquad ...(iii)$$

Since the line (*i*) is perpendicular to both the lines (*ii*) and (*iii*)

$\therefore \qquad a \times 1 + b \times 2 + c \times 4 = 0 \qquad \Rightarrow \qquad a + 2b + 4c = 0$

Also, $\qquad a \times 2 + b \times 3 + c \times 4 = 0 \qquad \Rightarrow \qquad 2a + 3b + 4c = 0$

On solving these two equations, we get

$$\frac{a}{8-12} = \frac{-b}{4-8} = \frac{c}{3-4} \qquad \Rightarrow \qquad \frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$$

$\therefore$ Direction ratios of the required line be – 4, 4, –1

$\therefore$ Vector and cartesian equation of required line be

$$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(-4\hat{i} + 4\hat{j} - \hat{k})$$

and, $\dfrac{x-1}{-4} = \dfrac{y-1}{4} = \dfrac{z-1}{-1}$ respectively.

Let θ be the angle between given lines

$$\therefore \quad \cos\theta = \left|\frac{1\times 2 + 2\times 3 + 4\times 4}{\sqrt{1+4+16}\times\sqrt{4+9+16}}\right| = \frac{24}{\sqrt{609}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$$

2. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point *P* (5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also find the image of *P* in this line. **[*CBSE (AI) 2012*]**

Sol. Given line is

$$\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$$

It can be written in cartesian form as

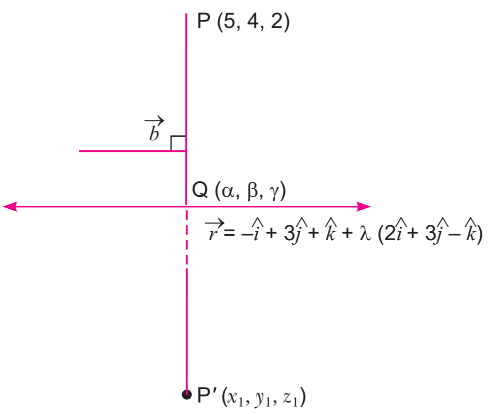
$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} \qquad \ldots(i)$$

Let $Q(\alpha, \beta, \gamma)$ be the foot of perpendicular drawn from $P(5, 4, 2)$ to the line (*i*) and $P'(x_1, y_1, z_1)$ be the image of P on the line (*i*)

$\because$ $Q(\alpha, \beta, \gamma)$ lie on line (*i*)

$$\therefore \quad \frac{\alpha+1}{2} = \frac{\beta-3}{3} = \frac{\gamma-1}{-1} = \lambda \text{ (say)}$$

$$\Rightarrow \quad \alpha = 2\lambda - 1;\ \beta = 3\lambda + 3 \text{ and } \gamma = -\lambda + 1 \qquad \ldots(ii)$$



Now, $\overrightarrow{PQ} = (\alpha - 5)\hat{i} + (\beta - 4)\hat{j} + (\gamma - 2)\hat{k}$

Parallel vector of line (*i*) $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.

Obviously $\overrightarrow{PQ} \perp \vec{b} \quad \Rightarrow \quad \overrightarrow{PQ}\cdot\vec{b} = 0$

$$2(\alpha - 5) + 3(\beta - 4) + (-1)(\gamma - 2) = 0$$

$$\Rightarrow \quad 2\alpha - 10 + 3\beta - 12 - \gamma + 2 = 0$$

$$\Rightarrow \quad 2\alpha + 3\beta - \gamma - 20 = 0$$

$$\Rightarrow \quad 2(2\lambda - 1) + 3(3\lambda + 3) - (-\lambda + 1) - 20 = 0$$

[Putting value of α, β, γ from (*ii*)]

$$\Rightarrow \quad 4\lambda - 2 + 9\lambda + 9 + \lambda - 1 - 20 = 0$$

$$\Rightarrow \quad 14\lambda - 14 = 0 \ \Rightarrow \lambda = 1$$

Hence the coordinates of foot of perpendicular Q are $(2\times 1 - 1,\ 3\times 1 + 3, -1 + 1)$, *i.e.*, (1, 6, 0)

$$\therefore \quad \text{Length of perpendicular} = \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$$

$$= \sqrt{16+4+4} = \sqrt{24} = 2\sqrt{6} \text{ units.}$$

Also, since Q is mid-point of PP'

$$\therefore \quad 1 = \frac{x_1 + 5}{2} \quad \Rightarrow \quad x_1 = -3$$

$$6 = \frac{y_1 + 4}{2} \quad \Rightarrow \quad y_1 = 8 \qquad 0 = \frac{z_1 + 2}{2} \quad \Rightarrow \quad z_1 = -2$$

Therefore required image is (–3, 8, –2).

3. Find the equation of the plane that contains the point (1, – 1, 2) and is perpendicular to both the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$. Hence find the distance of point $P(-2, 5, 5)$ from the plane obtained above. **[*CBSE (F) 2014*]**

Sol. Equation of plane containing the point (1, –1, 2) is given by

$$a(x-1) + b(y+1) + c(z-2) = 0 \qquad ...(i)$$

$\because$ (i) is perpendicular to plane $2x + 3y - 2z = 5$

$$\therefore \quad 2a + 3b - 2c = 0 \qquad ...(ii)$$

Also, (i) is perpendicular to plane $x + 2y - 3z = 8$

$$a + 2b - 3c = 0 \qquad ...(iii)$$

From (ii) and (iii), we get

$$\frac{a}{-9+4} = \frac{b}{-2+6} = \frac{c}{4-3}$$

$$\Rightarrow \quad \frac{a}{-5} = \frac{b}{4} = \frac{c}{1} = \lambda \text{ (say)} \quad \Rightarrow \quad a = -5\lambda, b = 4\lambda, c = \lambda$$

Putting these values in (i), we get

$$-5\lambda(x-1) + 4\lambda(y+1) + \lambda(z-2) = 0$$

$$\Rightarrow \quad -5(x-1) + 4(y+1) + (z-2) = 0$$

$$\Rightarrow \quad -5x + 5 + 4y + 4 + z - 2 = 0 \quad \Rightarrow \quad -5x + 4y + z + 7 = 0$$

$$\Rightarrow \quad 5x - 4y - z - 7 = 0 \qquad ...(iv) \text{ is the required equation of plane.}$$

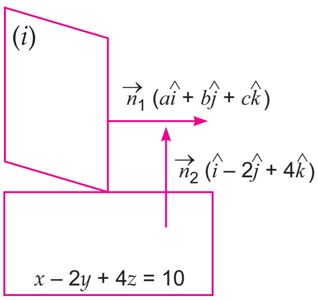
Again, if d be the distance of point P (–2, 5, 5) to plane (iv), then

$$d = \left|\frac{5\times(-2) + (-4)\times 5 + (-1)\times 5 - 7}{\sqrt{5^2 + (-4)^2 + (-1)^2}}\right| = \left|\frac{-10 - 20 - 5 - 7}{\sqrt{25+16+1}}\right| = \frac{42}{\sqrt{42}} = \sqrt{42} \text{ units}$$

4. Find the vector equation of the plane passing through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$. **[*CBSE (AI) 2013, (F) 2012, 2013*]**

Sol. Let the equation of plane through (2, 1, –1) be

$$a(x-2) + b(y-1) + c(z+1) = 0 \qquad ...(i)$$



$\because$ (i) passes through (–1, 3, 4)

$$\therefore \quad a(-1-2) + b(3-1) + c(4+1) = 0$$

$$\Rightarrow \quad -3a + 2b + 5c = 0 \qquad ...(ii)$$

Also plane (i) is perpendicular to plane $x - 2y + 4z = 10$

$$\Rightarrow \quad \vec{n}_1 \perp \vec{n}_2 \Rightarrow \vec{n}_1 . \vec{n}_2 = 0$$

$$\therefore \quad 1a - 2b + 4c = 0 \qquad ...(iii)$$

From (ii) and (iii), we get

$$\frac{a}{8+10} = \frac{b}{5+12} = \frac{c}{6-2} \quad \Rightarrow \quad \frac{a}{18} = \frac{b}{17} = \frac{c}{4} = \lambda \text{ (say)}$$

$$\Rightarrow \quad a = 18\lambda, b = 17\lambda, c = 4\lambda$$

Putting the value of a, b, c in (i), we get

$$18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) = 0$$

$$\Rightarrow \quad 18x - 36 + 17y - 17 + 4z + 4 = 0 \quad \Rightarrow \quad 18x + 17y + 4z = 49$$

$\therefore$ Required vector equation of plane is

$$\vec{r}\,.\,(18\hat{i} + 17\hat{j} + 4\hat{k}) = 49 \qquad ...(iv)$$

Obviously plane (*iv*) contains the line

$$\vec{r} = (-\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda\,(3\hat{i} - 2\hat{j} - 5\hat{k}) \qquad ...(v)$$

Since, point $(-\hat{i} + 3\hat{j} + 4\hat{k})$ satisfy equation (*iv*) and vector $(18\hat{i} + 17\hat{j} + 4\hat{k})$ is perpendicular to, $(3\hat{i} - 2\hat{j} - 5\hat{k})$, as $(-\hat{i} + 3\hat{j} + 4\hat{k})\,.\,(18\hat{i} + 17\hat{j} + 4\hat{k}) = -18 + 51 + 16 = 49$

and $(18\hat{i} + 17\hat{j} + 4\hat{k})\,.\,(3\hat{i} - 2\hat{j} - 5\hat{k}) = 54 - 34 - 20 = 0$

Therefore, (*iv*) contains line (*v*).

5. Find the vector and Cartesian equations of a plane containing the two lines.
$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda\,(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu\,(3\hat{i} - 2\hat{j} + 5\hat{k})$ [*CBSE Delhi 2013*]

Sol. Given lines are

$$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda\,(\hat{i} + 2\hat{j} + 5\hat{k}) \qquad ...(i)$$

$$\vec{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu\,(3\hat{i} - 2\hat{j} + 5\hat{k}) \qquad ...(ii)$$

Here $\vec{a}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$; $\quad \vec{a}_2 = 3\hat{i} + 3\hat{j} + 2\hat{k}$

$\vec{b}_1 = \hat{i} + 2\hat{j} + 5\hat{k}$; $\quad \vec{b}_2 = 3\hat{i} - 2\hat{j} + 5\hat{k}$

Now, $$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= (10 + 10)\hat{i} - (5 - 15)\hat{j} + (-2 - 6)\hat{k} = 20\hat{i} + 10\hat{j} - 8\hat{k}$$

Hence, vector equation of required plane is

$$(\vec{r} - \vec{a}_1)\,.\,(\vec{b}_1 \times \vec{b}_2) = 0 \quad \Rightarrow \quad \vec{r}\,.\,(\vec{b}_1 \times \vec{b}_2) = \vec{a}_1\,.\,(\vec{b}_1 \times \vec{b}_2)$$

$$\Rightarrow \quad \vec{r}\,.\,(20\hat{i} + 10\hat{j} - 8\hat{k}) = (2\hat{i} + \hat{j} - 3\hat{k})\,.\,(20\hat{i} + 10\hat{j} - 8\hat{k})$$

$$\Rightarrow \quad \vec{r}\,.\,(20\hat{i} + 10\hat{j} - 8\hat{k}) = 40 + 10 + 24$$

$$\Rightarrow \quad \vec{r}\,.\,(20\hat{i} + 10\hat{j} - 8\hat{k}) = 74 \quad \Rightarrow \quad \vec{r}\,.\,(20\hat{i} + 10\hat{j} - 8\hat{k}) = 74 \quad \Rightarrow \quad \vec{r}\,.\,(10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$$

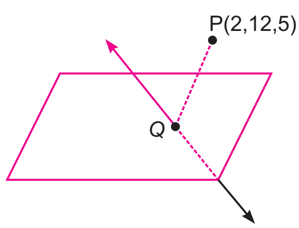
Therefore, Cartesian equation is $10x + 5y - 4z = 37$

6. Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\,(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}\,.\,(\hat{i} - 2\hat{j} + \hat{k}) = 0$. [*CBSE (AI) 2014*]

Sol. Given line and plane are

$$\vec{r} = (2\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda\,(3\hat{i} + 4\hat{j} + 2\hat{k}) \qquad ...(i)$$

and $$\vec{r}\,.\,(\hat{i} - 2\hat{j} + \hat{k}) = 0 \qquad ...(ii)$$



For intersection point Q, we solve equations (*i*) and (*ii*) by putting the value of $\vec{r}$ from (*i*) in (*ii*)

$$[(2\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda\,(3\hat{i} + 4\hat{j} + 2\hat{k})]\,.\,(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow \quad [(2 + 3\lambda)\hat{i} - (4 - 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}]\,.\,(\hat{i} - 2\hat{j} + \hat{k}) = 0 \quad \Rightarrow \quad (2+3\lambda) + 2\,(4 - 4\lambda) + (2 + 2\lambda) = 0$$

$$\Rightarrow \quad 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$$

$\Rightarrow\ 12 - 3\lambda = 0$

$\Rightarrow\quad \lambda = 4$

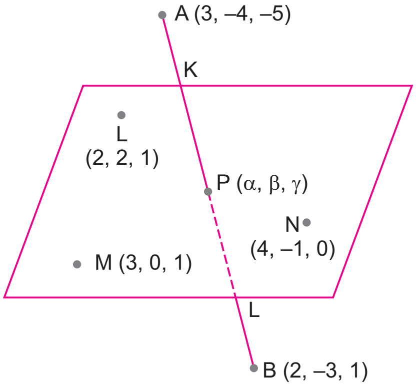
Hence, position vector of intersecting point is $14\hat{i} + 12\hat{i} + 10\hat{k}$.

Co-ordinate of intersecting point, $Q \equiv (14, 12, 10)$

Required distance $= \sqrt{(14-2)^2 + (12 - 12)^2 + (10 - 5)^2} = \sqrt{144 + 25} = \sqrt{169}$ units

$= 13$ units.

7. Find the coordinate of the point P where the line through $A(3, -4, -5)$ and B (2, – 3, 1) crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB. ***[CBSE Delhi 2016]***



Sol. Let the coordinate of P be (α, β, γ).

Equation of plane passing through $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$ is given by

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ 3-2 & 0-2 & 1-1 \\ 4-2 & -1-2 & 0-1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

$\Rightarrow\quad (x-2)(2-0) - (y-2)(-1-0) + (z-1)(-3+4) = 0$

$\Rightarrow\quad 2(x-2) + (y-2) + z - 1 = 0$

$\Rightarrow\quad 2x - 4 + y - 2 + z - 1 = 0$

$\Rightarrow\quad 2x + y + z - 7 = 0 \qquad ...(i)$

Now, the equation of line passing through $A(3, -4, -5)$ and $B(2, -3, 1)$ is given by

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} \quad \Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \qquad ...(ii)$$

$\because\quad P(\alpha, \beta, \gamma)$ lie on line AB

$\Rightarrow\quad \dfrac{\alpha-3}{-1} = \dfrac{\beta+4}{1} = \dfrac{\gamma+5}{6} = \lambda$ (say) $\Rightarrow\quad \alpha = -\lambda + 3, \beta = \lambda - 4, \gamma = 6\lambda - 5$

Also $\quad P(\alpha, \beta, \gamma)$ lie on plane (i)

$\Rightarrow\quad 2\alpha + \beta + \gamma - 7 = 0 \quad \Rightarrow \quad 2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 0$

$\Rightarrow\quad -2\lambda + 6 + \lambda - 4 + 6\lambda - 5 - 7 = 0 \quad \Rightarrow \quad 5\lambda - 10 = 0 \quad \Rightarrow \quad \lambda = 2$

$\therefore\quad \alpha = 1, \beta = -2, \gamma = 7$

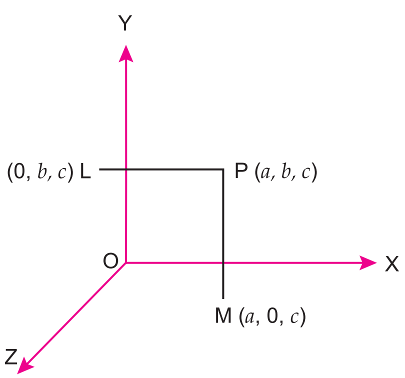
$\therefore\quad$ Co-ordinate of $P \equiv (1, -2, 7)$

Let P divides AB in the ratio $K : 1$.

$\therefore\quad 1 = \dfrac{K \times 2 + 1 \times 3}{K+1} \quad \Rightarrow \quad K + 1 = 2K + 3 \quad \Rightarrow \quad K = -2$

$\Rightarrow\quad P$ divides AB externally in the ratio 2 : 1.

8. From the point $P(a, b, c)$, perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM. ***[CBSE Bhubaneshwar 2015]***



Sol. Obviously, the coordinates of O, L and M are $(0, 0, 0)$, $(0, b, c)$ and $(a, 0, c)$.

Therefore, the equation of required plane is given by

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 0-0 & b-0 & c-0 \\ a-0 & 0-0 & c-0 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$

$\Rightarrow \quad x(bc - 0) - y(0 - ac) + z(0 - ab) = 0$

$\Rightarrow \quad bcx + acy - abz = 0$

9. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin. ***[CBSE (AI) 2014; (F) 2017]***

Sol. The equation of a plane passing through the intersection of the given planes is

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\Rightarrow \quad (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0 \qquad \ldots (i)$$

Since, (i) is perpendicular to $x - y + z = 0$

$$\Rightarrow \quad (1 + 2\lambda)1 + (1 + 3\lambda)(-1) + (1 + 4\lambda)1 = 0$$

$$\Rightarrow \quad 1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0 \quad \Rightarrow \quad 3\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{3}$$

Putting the value of λ in (i), we get

$$\left(1-\frac{2}{3}\right)x + (1-1)y + \left(1-\frac{4}{3}\right)z - \left(1-\frac{5}{3}\right) = 0 \qquad \Rightarrow \qquad \frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

$\Rightarrow \quad x - z + 2 = 0,$ it is required plane.

Let d be the distance of this plane from origin.

$$\therefore \quad d = \left|\frac{0.x + 0.y + 0.(-z) + 2}{\sqrt{1^2 + 0^2 + (-1)^2}}\right| = \left|\frac{2}{\sqrt{2}}\right| = \sqrt{2} \text{ units.}$$

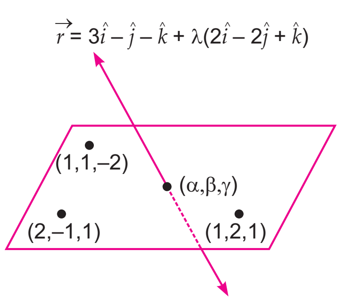
[**Note:** The distance of the point (α, β, γ) to the plane $ax + by + cz + d = 0$ is given by $\left|\frac{a\alpha + b\beta + c\gamma + d}{\sqrt{a^2 + b^2 + c^2}}\right|$.

10. Find the vector equation of the plane passing through three points with position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Also find the coordinates of the point of intersection of this plane and the line $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$. ***[CBSE Delhi 2013]***

Sol. The equation of plane passing through three points $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ *i.e.*, $(1, 1, -2)$, $(2, -1, 1)$ and $(1, 2, 1)$ is

$$\begin{vmatrix} x-1 & y-1 & z+2 \\ 2-1 & -1-1 & 1+2 \\ 1-1 & 2-1 & 1+2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} x-1 & y-1 & z+2 \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = 0$$



$$\Rightarrow \quad (x - 1)(-6 - 3) - (y - 1)(3 - 0) + (z + 2)(1 + 0) = 0$$

$$\Rightarrow \quad -9x + 9 - 3y + 3 + z + 2 = 0$$

$$\Rightarrow \quad 9x + 3y - z = 14 \qquad \ldots (i)$$

Its vector form is $\vec{r}.(9\hat{i} + 3\hat{j} - \hat{k}) = 14$

The given line is $\vec{r} = (3\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

Its cartesian form is

$$\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z+1}{1} \qquad \ldots (ii)$$

Let the line (ii) intersect plane (i) at (α, β, γ)

$\because$ (α, β, γ) lie on (*ii*)

$\Rightarrow$ $\dfrac{\alpha-3}{2} = \dfrac{\beta+1}{-2} = \dfrac{\gamma+1}{1} = \lambda$ (say) $\Rightarrow$ $\alpha = 2\lambda + 3;\ \beta = -2\lambda - 1;\ \gamma = \lambda - 1$

Also, point (α, β, γ) lie on plane (*i*)

$\Rightarrow$ $9\alpha + 3\beta - \gamma = 14$ $\Rightarrow$ $9(2\lambda+3) + 3(-2\lambda-1) - (\lambda-1) = 14$

$\Rightarrow$ $18\lambda + 27 - 6\lambda - 3 - \lambda + 1 = 14$ $\Rightarrow$ $11\lambda + 25 = 14$

$\Rightarrow$ $11\lambda = 14 - 25$ $\Rightarrow$ $11\lambda = -11$

$\Rightarrow$ $\lambda = -1$

Therefore, point of intersection $\equiv (1, 1, -2)$.

11. Find the direction ratios of the normal to the plane, which passes through the points (1, 0, 0) and (0, 1, 0) and makes angle $\frac{\pi}{4}$ with the plane $x + y = 3$. Also find the equation of the plane. **[*CBSE Patna 2015*]**

Sol. Let the equation of plane passing through the point (1, 0, 0) be

$$a(x-1) + b(y-0) + c(z-0) = 0$$

$\Rightarrow$ $ax - a + by + cz = 0$ $\Rightarrow$ $ax + by + cz = a$ ...(*i*)

Since, (*i*) also passes through (0, 1, 0)

$\Rightarrow$ $0 + b + 0 = a$ $\Rightarrow$ $b = a$ ...(*ii*)

Given, the angle between plane (*i*) and plane $x + y = 3$ is $\frac{\pi}{4}$.

$\therefore$ $\cos\dfrac{\pi}{4} = \left|\dfrac{a.1 + b.1 + c.0}{\sqrt{a^2+b^2+c^2}\sqrt{1^2+1^2}}\right|$ $\Rightarrow$ $\dfrac{1}{\sqrt{2}} = \left|\dfrac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{1+1}}\right|$

$\Rightarrow$ $\dfrac{1}{\sqrt{2}} = \left|\dfrac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}\right|$ $\Rightarrow$ $1 = \left|\dfrac{a+b}{\sqrt{a^2+b^2+c^2}}\right|$

$\Rightarrow$ $\sqrt{a^2+b^2+c^2} = \pm(a+b)$ $\Rightarrow$ $a^2+b^2+c^2 = (a+b)^2$

$\Rightarrow$ $a^2+b^2+c^2 = a^2+b^2+2ab$

$\Rightarrow$ $c^2 = 2ab$ $\Rightarrow$ $c^2 = 2a^2$ [From (*ii*)]

$\Rightarrow$ $c = \pm\sqrt{2}a$

Now, equation (*i*) becomes $ax + ay \pm \sqrt{2}az = a$.

$\Rightarrow$ $x + y \pm \sqrt{2}z = 1$, is the required equation of plane.

Therefore, required direction ratios are $1, 1, \pm\sqrt{2}$.

12. Find the equation of the plane which contains the line of intersection of the planes

$$\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0 \text{ and } \vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$$

and whose intercept on x-axis is equal to that of on y-axis. **[*CBSE (North) 2016*]**

Sol. Given planes are $\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$

These can be written in cartesian form as

$x - 2y + 3z - 4 = 0$ ...(*i*)

and $-2x + y + z + 5 = 0$ ...(*ii*)

Now the equation of plane containing the line of intersection of the planes (*i*) and (*ii*) is given by

$(x - 2y + 3z - 4) + \lambda(-2x + y + z + 5) = 0$ ...(*iii*)

$\Rightarrow$ $(1-2\lambda)x - (2-\lambda)y + (3+\lambda)z - 4 + 5\lambda = 0$ $\Rightarrow$ $(1-2\lambda)x - (2-\lambda)y + (3+\lambda)z = 4 - 5\lambda$

$$\Rightarrow \qquad \frac{x}{\frac{4-5\lambda}{1-2\lambda}} + \frac{y}{\frac{4-5\lambda}{-2+\lambda}} + \frac{z}{\frac{4-5\lambda}{3+\lambda}} = 1$$

According to question $\frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{-2+\lambda}$

$\Rightarrow \quad 1 - 2\lambda = -2 + \lambda \qquad \Rightarrow \quad 3\lambda = 3 \qquad \Rightarrow \quad \lambda = 1$

Putting the value of $\lambda = 1$ in (*iii*), we get

$$(x - 2y + 3z - 4) + 1\,(-2x + y + z + 5) = 0$$

$$-x - y + 4z + 1 = 0 \qquad \Rightarrow \qquad x + y - 4z - 1 = 0$$

Its vector form is $\vec{r}.(\hat{i} + \hat{j} - 4\hat{k}) - 1 = 0$

13. If $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$ and $n_1 + n_2 + n_3$ makes equal angles with them. ***[NCERT Exemplar]***

Sol. Let $\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$; $\quad \vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$; $\quad \vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$

$$\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$$

Also, let α, β and γ are the angles between $\vec{a}$ and $\vec{d}$, $\vec{b}$ and $\vec{d}$, $\vec{c}$ and $\vec{d}$.

$$\begin{aligned}
\therefore \qquad \cos\alpha &= l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3) \\
&= l_1^2 + l_1 l_2 + l_1 l_3 + m_1^2 + m_1 m_2 + m_1 m_3 + n_1^2 + n_1 n_2 + n_1 n_3 \\
&= (l_1^2 + m_1^2 + n_1^2) + (l_1 l_2 + l_1 l_3 + m_1 m_2 + m_1 m_3 + n_1 n_2 + n_1 n_3) \\
&= 1 + 0 = 1
\end{aligned}$$

$[\because\ l_1^2 + m_1^2 + n_1^2 = 1 \text{ and } l_1 \perp l_2, l_1 \perp l_3, m_1 \perp m_2, m_1 \perp m_3, n_1 \perp n_2, n_1 \perp n_3]$

Similarly, $\cos\beta = l_2(l_1 + l_2 + l_3) + m_2(m_1 + m_2 + m_3) + n_2(n_1 + n_2 + n_3)$

$= 1 + 0$ and $\cos\gamma = 1 + 0$

$\Rightarrow \qquad \cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \qquad \alpha = \beta = \gamma$

So, the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ makes equal angles with the three mutually perpendicular lines whose direction cosines are $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3 respectively.

14. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, –6. ***[CBSE (F) 2015] [HOTS]***

Sol. Let $Q(\alpha, \beta, \gamma)$ be the point on the given plane

$$x - y + z = 5 \qquad \ldots(i)$$

Since PQ is parallel to given line

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} \qquad \ldots(ii)$$

where $P(1, -2, 3)$ is the given point.

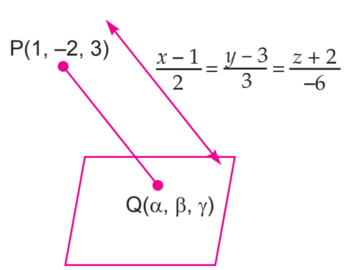
$\because \quad PQ$ is parallel to given line (*ii*).

$\therefore \quad \overrightarrow{PQ} \parallel \vec{b}$ (parallel vector of line).

$$\Rightarrow \qquad \frac{\alpha-1}{2} = \frac{\beta+2}{3} = \frac{\gamma-3}{-6} = \lambda$$

$\Rightarrow \qquad \alpha = 2\lambda + 1,\ \beta = 3\lambda - 2,\ \gamma = -6\lambda + 3$

Now, $\because Q(\alpha, \beta, \gamma)$ lie on plane (*i*)



$$\alpha - \beta + \gamma = 5 \Rightarrow \quad 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$-7\lambda + 6 = 5 \quad \Rightarrow -7\lambda = -1 \Rightarrow \quad \lambda = \frac{1}{7}$$

$$\alpha = 2\times\frac{1}{7} + 1 = \frac{9}{7}; \beta = 3\times\frac{1}{7} - 2 = -\frac{11}{7} \quad \text{and} \quad \gamma = -6\times\frac{1}{7} + 3 = \frac{15}{7}$$

Therefore required distance

$$PQ = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(-\frac{11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = \sqrt{1} = 1 \text{ unit.}$$

15. A plane meets the coordinate axes in *A, B, C,* such that the centroid of the triangle ABC is the point (a, b, g). Show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$. **[*HOTS*]**

Sol. Let the equation of required plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad \text{...}(i)$$

Then the coordinates of *A, B, C* are $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ respectively. So, the centroid of triangle *ABC* is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$. But the coordinates of the centroid are (α, β, γ) as given in problem.

$$\alpha = \frac{a}{3}, \beta = \frac{b}{3} \quad \text{and} \quad \gamma = \frac{c}{3} \Rightarrow a = 3\alpha, b = 3\beta, c = 3\gamma$$

Substituting the values of a, b and c in equation (*i*), we get the required equation of the plane as follows

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1 \quad \Rightarrow \quad \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3.$$

PROFICIENCY EXERCISE

■ **Objective Type Questions:** **[1 mark each]**

1. Choose and write the correct option in each of the following questions.

(*i*) The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the *x*-axis are given by [*NCERT Exemplar*]

(*a*) (2, 0, 0) (*b*) (0, 5, 0) (*c*) (0, 0, 7) (*d*) (0, 5, 7)

(*ii*) The co-ordinates of the foot of the perpendicular drawn from the point (–2, 8, 7) on the *XZ*-plane is

(*a*) (–2, –8, 7) (*b*) (2, 8, –7) (*c*) (–2, 0, 7) (*d*) (0, 8, 0)

(*iii*) The locus represented by $xy + yz = 0$ is

(*a*) a pair of perpendicular lines (*b*) a pair of parallel lines
(*c*) a pair of perpendicular planes (*d*) a pair of parallel planes

(*iv*) The vector equation of *XY*-plane is [*CBSE 2020 (65/3/1)*]

(*a*) $\vec{r}\,.\,\hat{k} = 0$ (*b*) $\vec{r}\,.\,\hat{j} = 0$ (*c*) $\vec{r}\,.\,\hat{i} = 0$ (*d*) $\vec{r}\,.\,\vec{n} = 1$

(*v*) The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is [*CBSE 2020 (65/5/1)*]

(*a*) $-\frac{2}{3}$ (*b*) $\frac{2}{3}$ (*c*) -2 (*d*) 2

(*vi*) The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{2}$

2. **Fill in the blanks.**

(*i*) If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}$ and $\frac{\pi}{4}$ with X, Y, Z axes respectively, then its direction cosines are ______________ .

(*ii*) The value of a for which the lines $\frac{x-1}{1} = \frac{y+1}{a} = \frac{z-2}{3}$ and $\frac{x+1}{2a} = \frac{y-1}{1} = \frac{z-3}{1}$ are perpendicular to each other, is ______________ .

(*iii*) The vector equation of the line through the points (3, 4, –7) and (1, –1, 6) is ______________ .

(*iv*) The equation of the plane $2x + 5y - 3z = 4$ in the vector form is ______________ .

(*v*) A plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4). The equation of the plane is ______________ .

■ Very Short Answer Questions: **[1 mark each]**

3. Cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$.
Write the direction ratios of a line parallel to AB. [*CBSE Sample Paper*]

4. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

5. If a line makes angles α, β, γ with the direction of the axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

6. If a line makes angles 90°, 135°, 45° with the X, Y and Z axes respectively, find its direction cosines. [*CBSE 2019 (65/1/1)*]

7. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$. [*CBSE 2019 (65/1/1)*]

8. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form. [*CBSE 2019 (65/2/1)*]

9. If a line has the direction ratios – 18, 12, – 4, then what are its direction cosines? [*CBSE 2019 (65/3/1)*]

10. Find the co-ordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the XY-plane. [*CBSE 2020 (65/2/1)*]

11. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin. [*CBSE (AI) 2012*]

12. Find the Cartesian equation of the line which passes through the point (–2, 4, – 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. [*CBSE Delhi 2013*]

13. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$. [*CBSE (AI) 2013*]

14. If the cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line. [*CBSE (AI) 2014*]

15. Find the angle between the lines $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. [*CBSE (F) 2014*]

16. Write the sum of intercepts cut off by the plane $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axes. [*CBSE North 2016*]

17. Find the vector equation of the plane with intercepts 3, – 4 and 2 on x, y and z axes. [*CBSE Central 2016*]

■ **Short Answer Questions–I:** **[2 marks each]**

18. Find the coordinates of the point where the line

$$\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4} \text{ meets the plane } x+y+4z=6.$$

19. If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), find the equation of the plane.

20. Find the distance of the point whose position vector is $(2\hat{i}+\hat{j}-\hat{k})$ from the plane $\vec{r}.(\hat{i}-2\hat{j}+4\hat{k})=9$

21. Write the unit vector normal to the plane

$x+2y+3z-6=0.$

22. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$. [*CBSE Delhi 2016*]

23. Find the equation of the plane passing through the line of intersection of the planes $2x+2y-3z=7$ and $2x+5y+3z=9$ the point (2, 1, 3).

24. Find the angle between the planes, whose vector equations are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.

■ **Short Answer Questions–II:** **[3 marks each]**

25. Find the shortest distance between the following pair of skew lines:

$$\frac{x-1}{2}=\frac{2-y}{3}=\frac{z+1}{4}, \frac{x+2}{1}=\frac{y-3}{2}=\frac{z}{3}$$ [*CBSE Sample Paper 2016*]

26. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect. Find their point of intersection. [*CBSE (East) 2016*]

27. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$ Hence find the image of the point A in the line BC. [*CBSE (North) 2016*]

28. Find the equation of plane passing through the points $A(3, 2, 1)$, $B(4, 2, -2)$ and $C(6, 5, -1)$ and hence find the value of λ for which $A(3, 2, 1)$, $B(4, 2, -2)$, $C(6, 5, -1)$ and $D(\lambda, 5, 5)$ are coplanar. [*CBSE (South) 2016*]

29. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$. [*CBSE (F) 2016*]

30. Show that the following two lines are coplanar:

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \text{ and } \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$$ [*CBSE Patna 2015*]

31. Find the acute angle between the plane $5x-4y+7z-13=0$ and the y-axis. [*CBSE Patna 2015*]

32. Find the equation of a plane which is at a distance of $3\sqrt{3}$ units from origin and the normal to which is equally inclined to the coordinate axes. [*CBSE (F) 2013*]

33. Let $P(3, 2, 6)$ be a point in the space and Q be a point on the line $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$, then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$. [*CBSE Chennai 2015*]

34. Find the vector and cartesian equations of the plane which bisects the line joining the points (3, –2, 1) and (1, 4, –3) at right angles. [*CBSE Chennai 2015*]

35. Find the distance of the point $P(3, 4, 4)$ from the point, where the line joining the points $A(3, -4, -5)$ and $B(2, -3, 1)$ intersect the plane $2x+y+z=7$. [*CBSE Allahabad 2015*]

36. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, –6. [*CBSE (F) 2015*]

37. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not. [*CBSE (South) 2016*]

38. Find the vector and cartesian equations of a line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1). [*CBSE Guwahati 2015*]

39. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$. [*CBSE (AI) 2011*]

40. Find the vector and cartesian equations of the line passing through the point P (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$. [*CBSE (F) 2012*]

■ **Long Answer Questions:** **[5 marks each]**

41. Find the equation of the plane passing through the point $P(1, 1, 1)$ and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that the plane contains the line
$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$. [*CBSE (AI) 2010*]

42. Find the vector and cartesian equations of the plane passing through the intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r}.(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point (1, 1, 1). [*CBSE Chennai 2015*]

43. Find the value of k for which the following lines are perpendicular to each other:
$$\frac{x+3}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1}; \frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}$$
Hence, find the equation of the plane containing the above lines. [*CBSE Guwahati 2015*]

44. Show that the lines:
$$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = 4\hat{j} + 2\hat{k} + \mu(2\hat{i} - \hat{j} + 3\hat{k}) \text{ are coplanar.}$$
Also, find the equation of the plane containing these lines. [*CBSE Panchkula 2015*]

45. Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are co-planar. Also, find the equation of the plane containing these lines. [*CBSE Sample Paper 2015*]

46. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\vec{r}(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$. Also, find image of P in the plane. [*CBSE (Central) 2016*]

47. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$ Also find the image of P in the plane. [*CBSE (East) 2016*]

48. Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x-intercept is twice its z-intercept.

Hence write the vector equation of a plane passing through the point (2, 3, –1) and parallel to the plane obtained above. [*CBSE (F) 2016*]

49. Find the coordinate of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane, passing through the points (2, 2, 1), (3, 0, 1) and (4, –1, 0). [*CBSE Delhi 2013*]

50. Show that lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection. [*CBSE Delhi 2014*]

51. Find the distance between the point (7, 2, 4) and the plane determine by the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$. [*CBSE Delhi 2014*]

52. Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect. Also, find the co-ordinate of the point of intersection and equation of the plane containing the two lines. [*CBSE 2020 (65/3/1)*]

Answers

1. (*i*) (*a*) (*ii*) (*c*) (*iii*) (*c*) (*iv*) (*a*) (*v*) (*a*) (*vi*) (*c*)

2. (*i*) $\pm\left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (*ii*) -1 (*iii*) $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda\left(-2\hat{i} - 5\hat{j} + 13\hat{k}\right)$

(*iv*) $\vec{r}.\left(2\hat{i} + 5\hat{j} - 3\hat{k}\right) = 4$ (*v*) $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

3. $1, -7, 2$ 4. $k = \frac{-10}{7}$ 5. 2 6. $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

7. $\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ 8. $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$

9. $l = \frac{-9}{11}, m = \frac{6}{11}, n = \frac{-2}{11}$ 10. (7, 10, 0) 11. $\frac{3}{13}$ 12. $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

13. 3 units 14. $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ 15. $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

16. $\frac{5}{2}$ 17. $\vec{r}.\left(\frac{\hat{i}}{3} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right) = 1$ 18. (1, 1, 1) 19. $3x - 2y + 6z - 27 = 0$

20. $\frac{13}{\sqrt{21}}$ 21. $\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$ 22. $\vec{r}.\left(\frac{2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}\right) = 5$

23. $38x + 68y + 3z = 153$ 24. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$ 25. $\frac{42}{\sqrt{390}}$ units 26. (4, 0, –1)

27. (–2, 1, 7); (–3, –6, 10) 28. $9x - 7y + 3z - 16 = 0$; $\lambda = 4$ 31. $\theta = \sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$

32. $x + y + z = 9$ 33. $\mu = \frac{1}{4}$ 34. $\vec{r}.(\hat{i} - 3\hat{j} + 2\hat{k}) + 3 = 0$; $x - 3y + 2z + 3 = 0$ 35. 7 units

36. 1 unit 37. $8x + y - 5z = 7$; yes the plane contains the given line

38. $\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$; $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$ 39. $\frac{8}{\sqrt{29}}$ units

40. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$, $\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$ 41. $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$

42. $20x + 23y + 26z - 69 = 0$; $\vec{r}.(20\hat{i} + 23\hat{j} + 26\hat{k}) = 69$ 43. $k = -1$, $4x + 31y + 7z = 54$

44. $\vec{r}.(-2\hat{i} - \hat{j} + \hat{k}) + 2 = 0$ 45. $x - 2y + z = 0$

46. $3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}$; $\sqrt{\frac{7}{2}}$ units, (4, 4, 7) 47. (3, 1, –1); $\sqrt{14}$ sq. units; (2, –1, –4)

48. $13x + 14y + 11z = 0$ or $7x + 11y + 14z - 15 = 0$, $\{\vec{r} - (2\hat{i} + 3\hat{j} - \hat{k})\}.(7\hat{i} + 11\hat{j} + 14\hat{k}) = 0$

49. (1, –2, 7) 50. (4, 0, –1) 51. $\sqrt{29}$ units 52. (1, –1, 2); $2x - y + z - 5 = 0$

SELF-ASSESSMENT TEST

Time allowed: 1 hour **Max. marks: 30**

1. Choose and write the correct option in the following questions. **(4 × 1 = 4)**

(*i*) The foot of perpendicular from (α, β, γ) on y-axis is

(*a*) $(\alpha, 0, 0)$ (*b*) $(0, \beta, 0)$ (*c*) $(0, 0, \gamma)$ (*d*) $(0, \beta, \gamma)$

(*ii*) Find the equation of the plane through the points (2, 1, –1), (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$

(*a*) $18x + 17y + 4z = 49$ (*b*) $20x - 12y + 3z = 11$

(*c*) $3x - 2y - 4z = 17$ (*d*) $7x - 2y - 3z = 0$

(*iii*) The coordinates of the point where the line through (3, – 4, – 5) and (2, –3, 1) crosses the plane passing through three points (2, 2, 1), (3, 0, 1) and (4, – 1, 0) are

(*a*) (0, – 2, 7) (*b*) (3, – 2, 5) (*c*) (1, – 2, – 7) (*d*) (1, – 2, 7)

(*iv*) The distance between the parallel planes $x + 2y - 3z = 2$ and $2x + 4y - 6z + 7 = 0$ is

(*a*) $\dfrac{2}{\sqrt{14}}$ unit (*b*) $\dfrac{11}{\sqrt{56}}$ unit (*c*) $\dfrac{7}{\sqrt{56}}$ unit (*d*) none of these

2. Fill in the blanks. **(2 × 1 = 2)**

(*i*) The distance of the plane $2x - 3y + 6z + 14 = 0$ from origin is ____________ units.

(*ii*) The Cartesian equation of the line joining the points (–2, 1, 3) and (3, 1, –2) is ____________ .

■ **Solve the following questions.** **(3 × 1 = 3)**

3. Find the angle between the line : $\dfrac{x-2}{3} = \dfrac{y+1}{-1} = \dfrac{z-3}{2}$ and the plane $3x + 4y + z + 5 = 0$.

4. Find the angle between the line: $\vec{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$ and the plane $\vec{r}.(3\hat{i} + 4\hat{j} + \hat{k}) + 5 = 0$.

5. Find the co-ordinates of the point where the line $\dfrac{x-1}{3} = \dfrac{y+4}{7} = \dfrac{z+4}{2}$ cuts the XY-plane.

■ **Solve the following questions.** **(2 × 2 = 4)**

6. If the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3), find the equation of the plane.

7. Find the distance of the point whose position vector is $(2\hat{i} + \hat{j} - \hat{k})$ from the plane $\vec{r}.(\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.

■ **Solve the following questions.** **(4 × 3 = 12)**

8. If a plane meets the coordinate axes in A, B, C such that the centroid of the ΔABC is the point (α, β, γ), then find the equation of the plane.

9. Find the distance of the point (2, 3, 4) from the plane $3x + 2y + 2z + 5 = 0$ measured parallel to the line $\dfrac{x+3}{3} = \dfrac{y-2}{6} = \dfrac{z}{2}$.

10. Find the distance of the point, whose position vector is $(2\hat{i} + \hat{j} - \hat{k})$ from the plane $\vec{r}.(\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.

11. Let $P(3, 2, 6)$ be a point in the space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$, then find the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$.

■ **Solve the following questions.** **(1 × 5 = 5)**

12. Find the equation of perpendicular from the point (3, –1, 11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also find the foot of the perpendicular and length of the perpendicular.

Answers

1. (*i*) (*b*) (*ii*) (*a*) (*iii*) (*d*) (*iv*) (*b*)

2. (*i*) 2 (*ii*) $\frac{x+2}{5} = \frac{y-1}{0} = \frac{z-3}{-5}$

3. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ **4.** $\sin^{-1}\left(\frac{5}{2\sqrt{91}}\right)$ **5.** (7, 10, 0)

6. $3x - 2y + 6z - 27 = 0$ **7.** $\frac{13}{\sqrt{21}}$ **8.** $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

9. 7 units **10.** $\frac{13}{\sqrt{21}}$ **11.** $\mu = \frac{1}{4}$ **12.** $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$, (2, 5, 7), $\sqrt{53}$ units