7 CHAPTER
Principal Stress-strain and Theories of Failure

Q.1 A shaft is subjected to torsion due to which it experiences a shear stress τ on the surface. The maximum principal stress on the surface which is at 45° to the axis will have a value

(a) $\tau \cos 45°$ (b) $2\tau \cos 45°$
(c) $\tau \cos^2 45°$ (d) $2\tau \sin 45° \cos 45°$

Q.2 The number of components in a stress tensor defining stress at a point in three dimension is

(a) 3 (b) 4
(c) 6 (d) 9

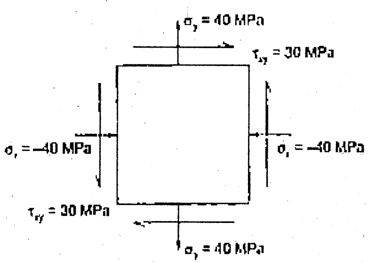
Q.3 A body is subjected to pure tensile stress of 300 units. What is the maximum shear stress produced in the body at some oblique plane due to the above tensile stress?

(a) 100 units (b) 300 units
(c) 150 units (d) 600 units

Q.4 In a material, one of the principal stress is twice the other. The maximum shear stress is τ_{max}. Then, what is the value of maximum principal stress?

(a) τ_{max} (b) $2\tau_{max}$
(c) $\frac{\tau_{max}}{2}$ (d) $4\tau_{max}$

Q.5 The state of stress at a point in a member is shown in figure. The magnitude of maximum shear stress is

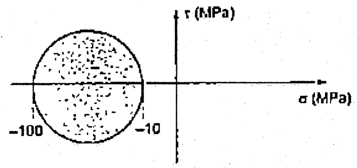


(a) 10 MPa (b) 50 MPa
(c) 100 MPa (d) 30 MPa

Q.6 A solid circular rod of 100 mm diameter is subjected to axial stress of 50 MPa. It is also subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is

(a) 41 MPa (b) 82 MPa
(c) 164 MPa (d) 204 MPa

Q.7 The Mohr's circle of plane stress for a point in a body is as shown. The design is done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is



(a) 45 MPa (b) 50 MPa
(c) 90 MPa (d) 100 MPa

Q.8 If the two principal strains at a point are 900×10^{-6} and -500×10^{-6}, then the maximum shear strain is

(a) 700×10^{-6} (b) 1400×10^{-6}
(c) 2100×10^{-6} (d) 2800×10^{-6}

Q.9 In a two-dimensional problem, the state of pure shear at a point is characterised by

(a) $\epsilon_x = \epsilon_y$ and $\gamma_{xy} = 0$
(b) $\epsilon_x = -\epsilon_y$ and $\gamma_{xy} = 0$
(c) $\epsilon_x = 2\epsilon_y$ and $\gamma_{xy} \neq 0$
(d) $\epsilon_x = 0.5\epsilon_y$ and $\gamma_{xy} = 0$

Q.10 Consider the case of bi-axial normal stresses, the normal stress at 45° plane is euqal to
(a) the sum of the normal stresses
(b) difference of the normal stresses
(c) half the sum of the normal stresses
(d) half the difference of the normal stresses

Q.11 Assertion (A): If the state at a point is pure shear, then the principal planes through that point making an angle of 45° with plane of shearing stress carries principal stresses whose magnitude is equal to that of shearing stress.
Reason (R): Complementary shear stresses are equal in magnitude, but opposite in direction.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

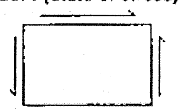
Q.12 For the state of stress with pure shear τ, the strain energy stored per unit volume in the elastic, homogeneous isotropic material having elastic constants E and ν is
(a) $\frac{\tau^2}{E}(1+\nu)$ (b) $\frac{\tau^2}{2E}(1+\nu)$
(c) $\frac{2\tau^2}{E}(1+\nu)$ (d) $\frac{\tau^2}{2E}(2+\nu)$

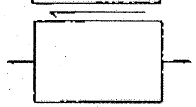
Q.13 Assertion (A): Circular shafts made of brittle material fail along helicoidal surface inclined at 45° to the axis when subjected to twisting moment.
Reason (R): The state of pure shear caused by torsion of the shaft is equivalent to one of tension at 45° to the shaft axis and equal compression in perpendicular direction.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

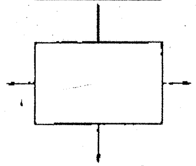
Q.14 Consider the following statements:
State of stres in two dimensions at a point in a loaded component can be completely specified by indicating the normal and shear stresses on
1. a plane containing the point
2. any two planes passing through the point
3. two mutually perpendicular planes passing through the point
Of these statements
(a) 1 and 3 are correct (b) 2 along is correct
(c) 1 alone is correct (d) 3 alone is correct

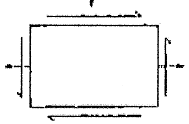
Q.15 Assertion (A): Mohr's circle of stress can be related to Mohr's circle of strain by some constant of proportionality.
Reason (R): The relationship is a function of yield strength of the material.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.16 Match List-I with List-II and select the correct answer, using the codes given below the lists:
List-I (State of stress)

A. 

B. 

C. 

D. 

List-II (Kind of loading)
1. Combined bending and torsion of circular shaft.
2. Torsion of circular shaft.
3. Thin cylinder subjected to internal pressure.
4. Tie bar subjected to tensile force.

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	3	4	1
(c)	2	4	3	1
(d)	3	4	1	2

Q.17 Assertion (A): A plane state of stress always results in a plane state of strain.
Reason (R): A uniaxial state of stress results in a three dimensional state of strain.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.18 Assertion (A): When an isotropic, linearly elastic material is loaded biaxially, the directions of principal stresses are different from those of principal strains.
Reason (R): For an isolrpic, linearly elastic material, the Hooke's law gives two independent material properties.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

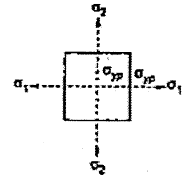
Q.19 The strain tensor is given as

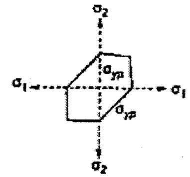
$$\begin{bmatrix} 0.002 & 0.003 & 0.004 \\ 0.003 & 0.001 & 0 \\ 0.004 & 0 & \end{bmatrix}$$

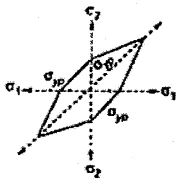
Assume G = 100 GPa
The shear stress, τ_{xy} is
(a) 200 MPa (b) 300 MPa
(c) 600 MPa (d) 800 MPa

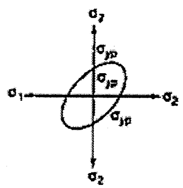
Q.20 If e_1 and e_2 ($e_1 > e_2$) are the maximum and minimum strains in the neighbourhood of a point in a stressed material of Young's modulus E and Poisson's ratio μ, then the maximum principal stress will be given by
(a) Ee_1 (b) $E(e_1 + e_2)$
(c) $E(e_1 + \mu e_2)/(1-\mu^2)$ (d) $E(e_2 + \mu e_1)/(1-\mu^2)$

Q.21 Which of the following diagram correctly represents the Rankine's or the maximum principal stress theory of failure?

(a) 

(b) 

(c) 

(d) 

Q.22 In a two dimensional stress system, it is assumed that the principal stresses σ_1 and σ_2 are such that $\sigma_1 > \sigma_2$; then according to the maximum shear stress theory, the failure occurs when

(a) $\frac{1}{E}[\sigma_1 - \mu\sigma_2]$

(b) $\left(\sigma_1^2 + \sigma_2^2 + 2\mu\sigma_1\sigma_2\right)$

(c) $(\sigma_1 - \sigma_2) \ge \sigma_y$

(d) $\left(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right) \ge \sigma_y$

Q.23 A certain steel in simple tension has proportionality limit of 300 N/mm². Under a three dimensional stress system, the principal stresses are 150 N/mm² (tensile), 75 N/mm² (tensile), 30 N/mm² (compressive) and $\mu = 0.3$. According to maximum principal stress theory, the factor of safety would be

(a) 1 (b) 2
(c) 4 (d) 5

Q.24 Consider the following statements:

1. In a member subjected to uniaxial tensile force, the maximum normal stress is the external load divided by the maximum cross sectional area.
2. When the structural member is subjected to uniaxial loading, the shear stress is zero on a plane where the normal stress is maximum.
3. In a member subjected to uniaxial loading, the normal stress on the planes of maximum shear stress is less than the maximum.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1, 2 and 3

Q.25 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
A. Shear centre	1. Tension
B. Principal plane	2. Slope
C. Fixed end	3. Shear stress
D. Middle third rule	4. Twisting

Codes:

	A	B	C	D
(a)	4	3	2	1
(b)	3	1	4	2
(c)	4	1	2	3
(d)	4	2	3	1

Q.26 Match the List-I with List-II and select the correct answer from the codes given below the lists:

List-I
A. Maximum shear stress theory
B. Maximum strain energy theory
C. Maximum shear strain energy theory
D. Maximum principal semimajor axis strain theory

List-II
1. Rhomboid
2. Ellipse with semi major axis $= \frac{\sigma}{\sqrt{1-\gamma}}$ and semi minor axis $= \frac{\sigma}{\sqrt{1+\gamma}}$
3. Ellipse with sami major axis $= \sqrt{2}\,\sigma$ and semi minor axis $= \sqrt{\frac{2}{3}}\,\sigma$
4. Hexagonal

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	2	3	1
(c)	1	3	2	4
(d)	4	3	2	1

Q.27 Assertion (A) : In a Mohr's circle, the vertical coordinates of the ends of any diameter are equal in magnitude and opposite in direction.

Reason (R): The shear stresses on two planes at right angles are equal in magnitude and tend to rotate the element in opposite directions.

(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.28 A material subjected to pure shear can fail by

(a) Rankine's theory only
(b) St. Venant's theory only
(c) Tresca's theory only
(d) any of the above theory

Q.29 For ductile material, the suitable theory of failure is

(a) maximum principal stress theory
(b) maximum shear stress theory
(c) both (a) and (b)
(d) none of these

Q.30 A body is subjected to two normal strains of magnitude $\epsilon_x = 0.003$ and $\epsilon_y = 0.002$. The shearing strain on a plane inclined at 30° with ϵ_x is

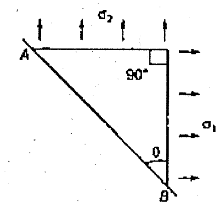
(a) $\frac{\sqrt{3}}{2} \times 10^{-3}$ (b) $\frac{\sqrt{3}}{4} \times 10^{-3}$

(c) $\frac{1}{2} \times 10^{-3}$ (d) $\frac{1}{4} \times 10^{-3}$

Q.31 In a rectangular strain gauge roselle, the strain recorded are $\epsilon_0 = 400\,\mu$ strain, $\epsilon_{45°} = 300\,\mu$ strain and $\epsilon_{90°} = 200\,\mu$ strain, what is the maximum principal strain at the point

(a) 500 μ strain (b) 400 μ strain
(c) 300 μ strain (d) 200 μ strain

Q.32 In the figure below, σ_1 and σ_2 are the maximum and minimum principal stresses. In order that the resultant stress on plane *AB* is $\sqrt{\left(\frac{\sigma_1+\sigma_2}{2}\right)^2+\left(\frac{\sigma_1-\sigma_2}{2}\right)^2}$, the value of θ should be



(a) 30° (b) 45°
(c) 60° (d) 75°

Q.33 All the theories of failure, will give nearly the same result when

(a) one of the principal stresses at a point is large in comparison to the other
(b) shear stresses act
(c) both the principal stresses are numerically equal
(d) For all situations of stress

Q.34 If permissible bending moment in a circular shaft under pure bending is *M*, according to maximum principal stress theory of failure, then according to maximum shear theory of failure, the permissible bending moment in the shaft is

(a) *M*/2 (b) *M*
(c) $\sqrt{2}M$ (d) 2*M*

Q.35 Which theory of failure is suitable for aluminium components under steady loading?

(a) Principal stress theory
(b) Principal strain theory
(c) Strain energy theory
(d) Maximum shear stress theory

Q.36 Match List-I (Theory of failure) with List-II (Predicted ratio of shear stress to direct stress at yield condition) and select correct answer using the code given below:

List-I
A. Maximum shear stress theory
B. Maximum distortion energy theory
C. Maximum principal stress theory
D. Maximum principal strain theory

List-II
1. 1.0
2. 0.577
3. 0.62
4. 0.50

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	4	3	1	2
(c)	1	3	4	2
(d)	4	2	1	3

Q.37 A transmission shaft subjected to bending loads must be designed on the basis of
(a) Maximum normal stress theory
(b) Maximum shear stress theory
(c) Maximum normal stress and maximum shear stress theories
(d) Fatigue strength

Answers **Principal Stress-strain and Theories of Failure**

1. (d)	2. (c)	3. (c)	4. (d)	5. (b)	6. (b)	7. (c)	8. (b)	9. (b)	10. (c)
11. (b)	12. (a)	13. (a)	14. (d)	15. (c)	16. (c)	17. (d)	18. (d)	19. (c)	20. (c)
21. (a)	22. (c)	23. (b)	24. (c)	25. (a)	26. (b)	27. (a)	28. (d)	29. (b)	30. (a)
31. (b)	32. (b)	33. (a)	34. (b)	35. (d)	36. (d)	37. (a)			

Explanations **Principal Stress-strain and Theories of Failure**

1. (d)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 45° + \tau_{xy}\sin 2\theta$$

$\sigma_x = \sigma_y = 0$

$\tau_{xy} = \tau$

$\theta = 45°$

$\therefore \quad \sigma_1 = \tau \sin 90° = 2\tau \sin 45° \cos 45°$

2. (c)

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz}$

3. (c)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - 0}{2} = 150 \text{ units}$$

4. (d)

$\sigma_1 = 2\sigma_2$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1 - \frac{\sigma_2}{2}}{2} = \frac{\sigma_1}{4}$$

$\therefore \quad \sigma_1 = 4\tau_{max}$

5. (b)

$$\tau_{max} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$= \sqrt{\left(\frac{-40-40}{2}\right)^2 + 30^2}$$

$= 50$ MPa

6. (b)

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 10 \times 10^6}{\pi \times (100)^3} \text{ MPa}$$

$= 50.93$ MPa

Maximum principal stress

$$= \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{50}{2} + \sqrt{\left(\frac{50}{2}\right)^2 + 50.93^2}$$

$= 81.73 \text{ MPa} \simeq 82$ MPa

7. (c)

$\sigma_1 = -10$ MPa

$\sigma_2 = -100$ MPa

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_y}{2}$$

$\Rightarrow \quad \sigma_y = \sigma_1 - \sigma_2$

$= -10 - (-100)$

$= 90$ MPa

8. (b)

Shear strain $= e_1 - e_2 = (900 - (-500)) \times 10^{-6}$

$= 1400 \times 10^{-6}$

10. (c)

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = 45°$ and $\tau_{xy} = 0$

$$\therefore \sigma_n = \frac{\sigma_x + \sigma_y}{2}$$

12. (a)

In pure shear state,

$$U = \frac{\tau^2}{2G} = \frac{\tau^2}{2E} \times 2(1+\nu) \qquad \left[\because G = \frac{E}{2(1+\nu)}\right]$$

$$= \frac{\tau^2}{E}(1+\nu)$$

18. (d)

The principal stress and strain directions are same.

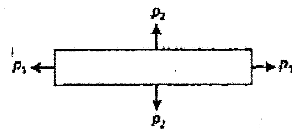
19. (c)

$\gamma_{xy} = 2\epsilon_{xy} = 2 \times 0.003 = 0.006$

$\tau_{xy} = G\gamma_{xy}$

$= 100 \times 10^3 \times 0.006 = 600$ MPa

20. (c)



$$e_1 = \frac{p_1}{E} - \mu\frac{p_2}{E}$$

$\Rightarrow \quad e_1E = p_1 - \mu p_2$

$$e_2 = \frac{p_2}{E} - \mu\frac{p_1}{E}$$

$\Rightarrow \quad e_2E = p_2 - \mu p_1$

$\Rightarrow \quad p_2 = e_2E + \mu p_1$

$\therefore \quad e_1E = p_1 - \mu(e_2E + \mu p_1)$

$\Rightarrow \quad E(e_1 + \mu e_2) = p_1 - \mu^2 p_1$

$$\Rightarrow \quad p_1 = \frac{E(e_1 + \mu e_2)}{1 - \mu^2}$$

$$= \frac{E}{1-\mu^2}(e_1 + \mu e_2)$$

23. (b)

$\sigma_1 = 150$ N/mm²

$$\therefore \text{ FOS} = \frac{f_y}{\sigma_1} = \frac{300}{150} = 2$$

24. (c)

In a member subjected to uniaxial tensile force, the maximum normal stress is the external load divided by the minimum cross sectional area.

25. (a)

Shear centre : It is also called centre of flexure and it is that point from which if load passes, then only bending takes place but no twisting.

Principal Plane: Shear stress is zero.

Fixed ends: Slope is zero.

Middle third rule: In a rectangular section of a column, if the load passes through middle third strip, then no tension develops.

29. (b)

For Brittle material → Maximum principal stress theory and maximum principal strain theory.

Ductile material → Maximum; principal strain theory, maximum shear strear stress theory, maximum strain energy theory and maximum shear strain energy theory.

30. (a)

Shearing strain $= (\epsilon_x - \epsilon_y)\sin 2\theta$

$= (0.003 - 0.002)\sin 60°$

$$= \frac{\sqrt{3}}{2} \times 10^{-3}$$

31. (b)

$\phi_{xy} = 2\epsilon_{45°} = (\epsilon_0 + \epsilon_{90°})$

$$= (2 \times 300 - (400 + 200))\mu$$

$$= (600 - 600)\mu = 0$$

$$\therefore \quad \epsilon_{1,2} = \frac{\epsilon_0 + \epsilon_{90}}{2} \pm \sqrt{\left(\frac{\epsilon_0 - \epsilon_{90}}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= 300\mu \pm \sqrt{(100)^2}\,\mu$$

$$= 300\,\mu \pm 100\,\mu \text{ strain}$$

$\therefore$ Maximum principal strain

$$= 400\,\mu \text{ strain}$$

33. (a)

When one of the principal stresses at a point is large in comparison to the other, the situation resembles uniaxial tension test. Therefore all theories give nearly the same results.

34. (b)

According to maximum principal stress theory,

$\sigma_1 = \sigma_y$.

According to maximum shear stress theory,

$\sigma_1 - \sigma_2 = \sigma_y$

Under pure bending, $\sigma_1 = \frac{M}{I} y$ or $\sigma_1 \propto M$ and

$\sigma_2 = 0$.

Therefore, in both the cases, permissible bending moment is M.

35. (d)

Aluminium is ductile is nature.

So maximum shear stress theory is suitable.

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