

# Chapter 16

# Electric Charges and Fields

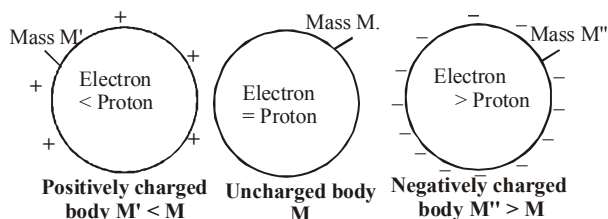
## ELECTRIC CHARGE

Charge is something associated with matter due to which it produces and experiences electric and magnetic effects.

There are **two types of charges** :

(i) *Positive charge* and (ii) *Negative charge*

**Positive and negative charges** : Positive charge means the deficiency of electrons while negative charge means excess of electrons. In any neutral body the net charge is equal to zero i.e., the sum of positive charges is equal to the sum of negative charges.



Charge is a scalar quantity and its **SI unit** is coulomb (C).

## CONDUCTORS AND INSULATORS

The materials which allow electric charge (or electricity) to flow freely through them are called **conductors**. Metals are very good conductors of electricity. Silver, copper and aluminium are some of the best conductors of electricity. Our skin is also a conductor of electricity. Graphite is the only non-metal which is a conductor of electricity.

All metals, alloys and graphite have '*free electrons*', which can move freely throughout the conductor. These free electrons make metals, alloys and graphite good conductor of electricity.

Aqueous solutions of electrolytes are also conductors.

The materials which do not allow electric charge to flow through them are called **nonconductors** or **insulators**.

For example, most plastics, rubber, non-metals (except graphite), dry wood, wax, mica, porcelain, dry air etc., are insulators.

Insulators can be charged but do not conduct electric charge. Insulators do not have '*free electrons*' that is why insulators do not conduct electricity.

Induced charge can be lesser or equal to inducing charge (but never greater) and its max. value is given by  $Q' = -Q (1 - 1/k)$ , where ' $Q$ ' is inducing charge and ' $k$ ' is the dielectric const. of the material of the uncharged body.

For metals  $k = \infty \Rightarrow Q' = -Q$ .

## METHODS OF CHARGING

(i) **By friction** : By rubbing two suitable bodies, given in box one is charged by +ve and another by -ve charge in equal amount.

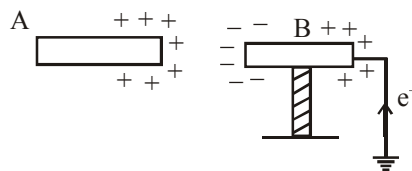
+ve	-ve
Glass rod	Silk
Fur	Ebonite rod
Dry hair	Comb
Wool	Amber

**Note :** Note : Electric charges remain confined only to the rubbed portion of a non-conductor but in case of a conductor, they spread up throughout the conductor.

(ii) **By conduction** : Charging a neutral body by touching it with a charged body is called charging by conduction.

- It is important to note that when the bodies are charged by conduction, a charged and an uncharged bodies are brought into contact and then separated, the two bodies may or may not have equal charges.
- If the two bodies are identical the charges on the two will be equal.
- If the two bodies are not identical, the charges will be different.
- The potential of the two bodies will always be the same.

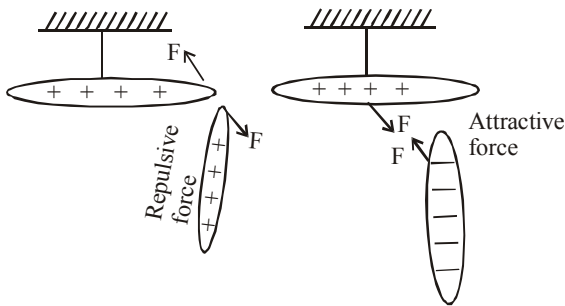
(iii) **By induction** : Charging a body without bringing it in contact with a charged body is called charging by induction.



First rearrangement of charge takes place in metal rod B. When the rod B is connected to earth, electrons flow from earth to the rod B thus making it -vely charged. The magnitude of elementary positive or negative charge (electron) is same and is equal to  $1.6 \times 10^{-19} \text{ C}$ .

## Properties of Electric Charge

- (i) **Similar charges repel and dissimilar charges attract each other.**



In rare situation you may find similar charged bodies attracting each other. Suppose a big positive charged body is placed near a small positively charged body then because of induction, opposite charge produced on the small body makes it to attract the other body.

- (ii) **A charged body attracts light uncharged bodies, due to polarisation of uncharged body.**

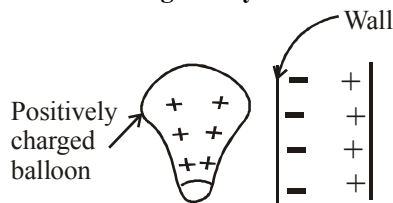
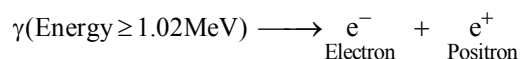


Fig : When a positively charged balloon is placed in contact with the wall, an opposite charge is induced with the wall, the balloon stick to the wall due to electrostatic attraction

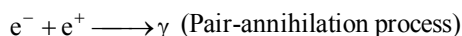
- (iii) **Charge is conserved** i.e., the charge can neither be created nor be destroyed but it may simply be transferred from one body to the other.

Thus we may say that the total charge in the universe is constant or we may say that charges can be created or destroyed in equal and opposite pair. For example



(Pair-production process)

- Positron** is an antiparticle of electron. It has same mass as that of electron but equal negative charge.



- (iv) **Charge is unaffected by motion.** This is also called charge invariance with motion

Mathematically,  $(q)_{\text{at rest}} = (q)_{\text{in motion}}$

- (v) **Quantisation of charge.** A charge is an aggregate of small unit of charges, each unit being known as fundamental or elementary charge which is equal to  $e = 1.6 \times 10^{-19} \text{ C}$ . This principle states that *charge on any body exists as integral multiple of electronic charge.*

i.e.  $q = ne$  where  $n$  is an integer.

According to the concept of quantisation of charges, the charge  $q$  cannot go below  $e$ . On macroscopic scale, this is as good as taking limit  $q_0 \rightarrow 0$ .

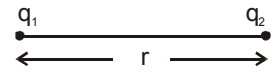
Quantisation of electric charge is a basic (unexplained) law of nature. It is important to note that there is no analogous law of quantisation of mass.

Recent studies on high energy physics have indicated the presence of graphs with charge  $2e/3$ ,  $e/3$ . But since these cannot be isolated and are present in groups with total charge, therefore the concept of elementary charge is still valid.

## COULOMB'S LAW

The force of attraction or repulsion between two point charges ( $q_1$  and  $q_2$ ) at finite separation ( $r$ ) is directly proportional to the product of charges and inversely proportional to the square of distance between the charges and is directed along the line joining the two charges.

$$\text{i.e., } F \propto \frac{q_1 q_2}{r^2} \text{ or } F = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2}$$



where  $\epsilon$  is the permittivity of medium between the charges.

If  $\epsilon_0$  is the permittivity of free space, then **relative permittivity of medium** or **dielectric constant (K)**, is given by

$$\epsilon_r \text{ (or } K) = \frac{\epsilon}{\epsilon_0}$$

The permittivity of free space

$$\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\text{and } \frac{1}{4\pi\epsilon_0} = \frac{1}{4 \times 3.14 \times 8.86 \times 10^{-2}} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}.$$

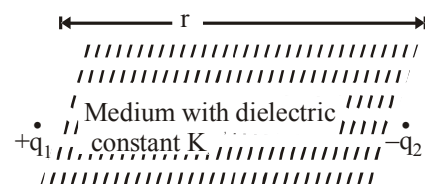
Also  $\frac{1}{4\pi\epsilon_0} = 1$  in CGS system of unit.

Coulomb's law may also be expressed as

$$F = \frac{1}{4\pi\epsilon_0\epsilon_r} \cdot \frac{q_1 q_2}{r^2}$$

Let  $F_0$  be the force between two charges placed in vacuum then

$$F_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$



$$\text{Hence } \frac{F_0}{F} = K \left( \because F = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \text{ and } \frac{\epsilon}{\epsilon_0} = K \right)$$

Therefore we can conclude that the force between two charges becomes  $1/K$  times when placed in a medium of dielectric constant  $K$ .

**The value of K for different media**

Medium	Dielectric Constant (K)
Air	1.006
Vacuum	1.00
Water	1.00026
Mica	3 to 6
Metals	$\infty$

**Dielectric :** A dielectric is an insulator. It is of two types -

- (i) Polar dielectric and
- (ii) Non-polar dielectric.

**Significance of Permittivity Constant or Dielectric Constant :** Permittivity constant is a measure of the inverse degree of permission of the medium for the charges to interact.

**Dielectric strength :** The maximum value of electric field that can be applied to the dielectric without its electric breakdown is called its dielectric strength.

**Difference between electrostatic force and gravitational force :**

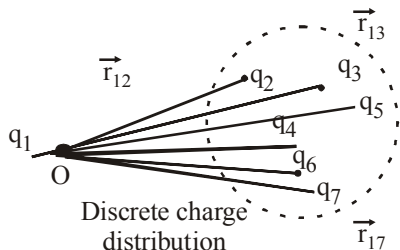
Electrostatic force	Gravitational force
1. Much stronger	1. Much weaker as compared to electrostatic force
2. Can be attractive or repulsive	2. Only attractive
3. Depends on the nature of medium between charges	3. Does not depend on the nature of medium between masses

**Note :** Both electric and gravitational forces follow inverse square law.

**Vector Form of Coulomb's Law :**

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0 k} \frac{q_1 q_2}{r^3} \vec{r}_{12} = \frac{1}{4\pi\epsilon_0 k} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

**SUPERPOSITION PRINCIPLE FOR DISCRETE CHARGE DISTRIBUTION : FORCE BETWEEN MULTIPLE CHARGES**



The electric force  $\vec{F}_1$  on  $q_1$  due to a number of charges placed in air or vacuum is given by

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_1 q_i}{r_{1i}^2} \hat{r}_{1i} = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + \dots \right]$$

**Note :** Coulomb's law is valid if  $r \geq 10^{-15}$  m and if charges are point charges

**FORCE FOR CONTINUOUS CHARGE DISTRIBUTION**

A small element having charge  $dq$  is considered on the body. The force on the charge  $q_1$  is calculated as follows :

$$d\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 dq}{r^2} \hat{r}$$

Now the total force  $\vec{F}_1$  is calculated by integrating  $d\vec{F}_1$  under proper limits.

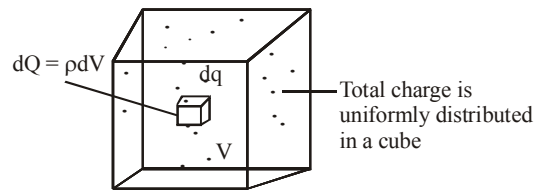
$$\text{i.e., } \vec{F}_1 = \int d\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \int \frac{q_1 dq}{r^2} \hat{r} = \frac{q_1}{4\pi\epsilon_0} \int \frac{dq}{r} \hat{r}$$

where  $\hat{r}$  is a variable unit vector which points from each  $dq$ , towards the location of charge  $q_1$  (where  $dq$  is a small charge element)

**Types of Charge Distribution**

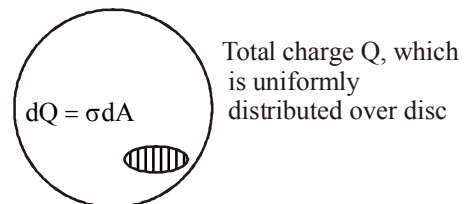
(i) **Volume charge distribution :** If a charge,  $Q$  is uniformly distributed through a volume  $V$ , the charge per unit volume  $\rho$  (volume charge density) is defined by

$$\rho = \frac{Q}{V}; \rho \text{ has unit coulomb/m}^3.$$



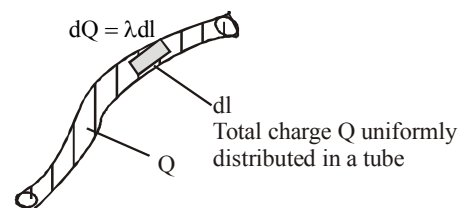
(ii) **Surface charge distribution :** If a charge  $Q$  is uniformly distributed on a surface of area  $A$ , the surface charge density  $\sigma$ , is defined by the following equation

$$\sigma = \frac{Q}{A}, \sigma \text{ has unit coulomb / m}^2$$



(iii) **Linear charge distribution :** If a charge  $q$  is uniformly distributed along a line of length  $\lambda$ , the linear charge density  $\lambda$ , is defined by

$$\lambda = \frac{Q}{\ell}, \lambda \text{ has unit coulomb/m.}$$



- If the charge is non uniformly distributed over a volume, surface, or line we would have to express the charge densities as

$$\rho = \frac{dQ}{dV}, \quad \sigma = \frac{dQ}{dA}, \quad \lambda = \frac{dQ}{dl},$$

where  $dQ$  is the amount of charge in a small volume, surface or length element.

- In general, when there is a distribution of direct and continuous charge bodies, we should follow the following steps to find force on a charge  $q$  due to all the charges :

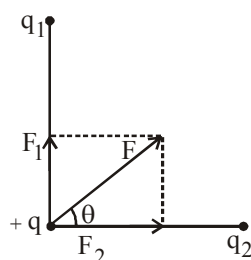
- (1) Fix the origin of the coordinate system on charge  $q$ .
- (2) Draw the forces on  $q$  due to the surrounding charges considering one charge at a time.
- (3) Resolve the force in  $x$  and  $y$ -axis respectively and find  $\Sigma F_x$  and  $\Sigma F_y$
- (4) The resultant force is  $F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$  and the

direction is given by  $\tan \theta = \frac{\Sigma F_x}{\Sigma F_y}$ .

#### Calculation of electric force in some situations :

##### (a) Force on one charge due to two other charges :

Resultant force on  $q$  due to  $q_1$  and  $q_2$  are obtained by vector addition of individual forces



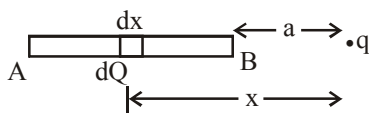
$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$|\vec{F}| = \sqrt{|\vec{F}_1|^2 + |\vec{F}_2|^2}$$

The direction of  $F$  is given by  $\tan \theta = \frac{|\vec{F}_1|}{|\vec{F}_2|}$

##### (b) Force due to linear charge distribution :

Let  $AB$  is a long (length  $\ell$ ) thin rod with uniform distribution of total charge  $Q$ .



We calculate force of these charges i.e.  $Q$  on  $q$  which is situated at a distance  $a$  from the edge of rod  $AB$ .

Let,  $dQ$  is a small charge element in rod  $AB$  at a distance  $x$  from  $q$ .

The force on  $q$  due to this element will be

$$dF = \frac{q}{4\pi\epsilon_0} \frac{dQ}{x^2} = \frac{q}{4\pi\epsilon_0} \frac{\mu dx}{x^2}$$

where  $\mu$  is linear charge density i.e.,  $\mu = Q / \ell$ .

$$\text{so, } F = \int_a^{\ell+a} dF = \int_a^{\ell+a} \frac{q}{4\pi\epsilon_0} \cdot \frac{\mu}{x^2} dx$$

$$= \frac{q\mu}{4\pi\epsilon_0} \int_a^{\ell+a} \frac{1}{x^2} dx \text{ newton}$$

#### Keep in Memory

1. When the distance between the two charges placed in vacuum or a medium is increased  $K$ -times then the force between them decreases  $K^2$ -times. i.e., if  $F_0$  and  $F$  be the initial and final forces between them, then

$$F = \frac{F_0}{K^2}$$

2. When the distance between the two charges placed in vacuum or a medium is decreased  $K$ -times then the force between them increases  $K^2$ -times. i.e., if  $F_0$  and  $F$  be the initial and final forces then

$$F = K^2 F_0$$

3. When a medium of dielectric constant  $K$  is placed between the two charges then the force between them decreases by  $K$ -times. i.e., if  $F_0$  and  $F$  be the forces in vacuum and the medium respectively, then

$$F = \frac{F_0}{K}$$

4. When a medium of dielectric constant  $K$  between the charges is replaced by another medium of dielectric constant  $K'$  then the force decreases or increases by  $(K/K')$  times according as  $K'$  is greater than  $K$  or  $K'$  is less than  $K$ .

#### Example 1.

If we supply a charge to a soap bubble then it will expand. Why?

#### Solution :

Since we know like charges repel and try to get away from each other which is at outer surface of the conductor. So a soap bubble expand.

#### Example 2.

Calculate the net charge on a substance consisting of (a)  $5 \times 10^{14}$  electrons (b) a combination of  $7 \times 10^{13}$  protons and  $4 \times 10^{13}$  electrons.

#### Solution :

- (a) The charge of one electron is  $-1.6 \times 10^{-19} \text{C}$ . So net charge on a substance consisting of  $5 \times 10^{14}$  electrons is

$$5 \times 10^{14} \times (-1.6 \times 10^{-19} \text{C}) = -8 \times 10^{-5} \text{C} = -80 \mu\text{C}.$$

- (b) Similarly the net charge on a substance consisting of a combination of  $7 \times 10^{13}$  protons and  $4 \times 10^{13}$  electron is

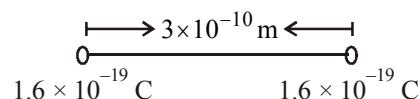
$$[7 \times 10^{13} \times (1.6 \times 10^{-19} \text{C})] + [4 \times 10^{13} (-1.6 \times 10^{-19} \text{C})] = +4.8 \mu\text{C}.$$

( $\therefore$  the charge on one proton is  $+1.6 \times 10^{-19} \text{C}$ )

#### Example 3.

Two protons in a molecule is separated by a distance  $3 \times 10^{-10} \text{m}$ . Find the electrostatic force exerted by one proton on the other.

#### Solution :



According to coulomb's law, the electrostatic force  $F$  between two charges  $q_1$  and  $q_2$ , which are separated by distance  $r$  is

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

Here,  $q_1 = q_2 = 1.6 \times 10^{-19} \text{ C}$ ,  $r = 3 \times 10^{-10} \text{ m}$

$$\text{so, } F = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{9 \times 10^{-20}} = 2.56 \times 10^{-9} \text{ C (Repulsive)}$$

#### Example 4.

When a piece of polythene is rubbed with wool, a charge of  $2 \times 10^{-7} \text{ C}$  is developed on polythene. What is the amount of mass, which is transferred to polythene?

#### Solution :

No. of electrons transferred,  $n = \frac{q}{e}$

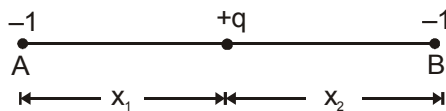
$$\begin{aligned} \text{Mass transferred} &= m_e \times n = m_e \times \left( \frac{q}{e} \right) \\ &= 9.1 \times 10^{-31} \times \left( \frac{2 \times 10^{-7}}{1.6 \times 10^{-19}} \right) \\ &= 11.38 \times 10^{-19} \text{ kg} \end{aligned}$$

#### Example 5.

Two negative charges of unit magnitude each and a positive charge  $q$  are placed along a straight line. At what position and for what value of  $q$  will the system be in equilibrium? Check whether it is stable, unstable or neutral equilibrium?

#### Solution :

Let the charge  $+q$  be held at a distance  $x_1$  from unit negative charge at A, and at a distance  $x_2$  from unit negative charge at B.



$$\text{For equilibrium of } q, \frac{q(-1)}{4\pi\epsilon_0 x_1^2} = \frac{q(-1)}{4\pi\epsilon_0 x_2^2}$$

$\therefore x_1 = x_2$  i.e.  $q$  must be equidistant from A and B.

For equilibrium of unit negative charge at B.

Force on B due to charge at A + force on B due to  $q = 0$

$$\begin{aligned} \frac{(-1)(-1)}{4\pi\epsilon_0 (x_1 + x_2)^2} + \frac{q(-1)}{4\pi\epsilon_0 x_2^2} &= 0 \\ \Rightarrow \frac{1}{4\pi\epsilon_0 (2x_2)^2} &= \frac{-(-q)}{4\pi\epsilon_0 x_2^2} \quad (\because x_1 = x_2) \end{aligned}$$

$$q = \frac{1}{4} \text{ i.e. } \frac{1}{4} \text{ th of the magnitude of either unit charge.}$$

**Stability :** If  $q$  is displaced slightly towards A, force of attraction due to A exceeds the force of attraction due to B. Therefore,  $q$  will get displaced further towards A. Hence the equilibrium of  $q$  is unstable.

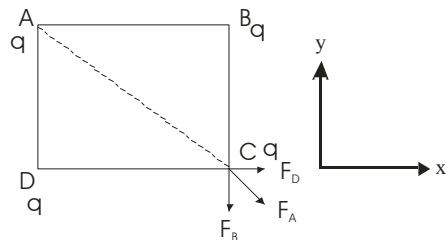
However, if  $q$  is displaced in a direction  $\perp$  to AB, net force would bring  $q$  back to its normal position. Therefore, the equilibrium will be stable.

#### Example 6.

Four identical point charges each of magnitude  $q$  are placed at the corners of a square of side  $a$ . Find the net electrostatic force on any of the charge.

#### Solution :

Let the concerned charge be at C then charge at C will experience the force due to charges at A, B and D. Let these forces respectively be  $\vec{F}_A$ ,  $\vec{F}_B$  and  $\vec{F}_D$  thus forces are given as



$$\vec{F}_A = \frac{1}{4\pi\epsilon_0} \frac{q^2}{AC^2} \text{ along AC} = \frac{q^2}{4\pi\epsilon_0 2a^2} \left( \frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}} \right)$$

$$\vec{F}_B = \frac{1}{4\pi\epsilon_0} \frac{q^2}{BC^2} \text{ along BC} = \frac{q^2}{4\pi\epsilon_0 a^2} (-\hat{j})$$

$$\vec{F}_D = \frac{1}{4\pi\epsilon_0} \frac{q^2}{DC^2} \text{ along DC} = \frac{q^2}{4\pi\epsilon_0 a^2} (\hat{i})$$

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_A + \vec{F}_B + \vec{F}_D \\ &= \frac{q^2}{4\pi\epsilon_0 a^2} \left[ \hat{i} \left( \frac{1}{2\sqrt{2}} + 1 \right) - \hat{j} \left( \frac{1}{2\sqrt{2}} + 1 \right) \right] \end{aligned}$$

$$|\vec{F}_{\text{net}}| = \sqrt{2} \left( \frac{1}{2\sqrt{2}} + 1 \right) \frac{q^2}{4\pi\epsilon_0 a^2} = \left( \frac{1}{2} + \sqrt{2} \right) \frac{q^2}{4\pi\epsilon_0 a^2}$$

#### Example 7.

Electric force between two point charges  $q$  and  $Q$  at rest is  $F$ . Now if a charge  $-q$  is placed next to  $q$  what will be the (a) force on  $Q$  due to  $q$  (b) total force on  $Q$ ?

#### Solution :

(a) As electric force between two body interaction, i.e., force between two particles, is independent of presence or absence of other particles, the force between  $Q$  and  $q$  will remain unchanged, i.e.,  $F$ .

(b) An electric force is proportional to the magnitude of charges, total force on  $Q$  will be given by

$$\frac{F'}{F} = \frac{Qq'}{Qq} = \frac{q'}{q} = \frac{0}{q} = 0 \quad [\text{as } q' = q + (-q) = 0]$$

i.e., the resultant or total force on  $Q$  will be zero.

#### ELECTRIC FIELD

The space around an electric charge, where it exerts a force on another charge is an **electric field**.

Electric force, like the gravitational force acts between the bodies that are not in contact with each other. To understand these forces, we involve the concept of force field. When a mass is present somewhere, the properties of space in vicinity can be considered to be so altered in such a way that another mass



brought to this region will experience a force there. The space where alteration is caused by a mass is called its Gravitational field and any other mass is thought of as interacting with the field and not directly with the mass responsible for it.

Similarly an electric charge produces an electric field around it so that it interacts with any other charges present there. One reason it is preferable not to think of two charges as exerting forces upon each other directly is that if one of them is changed in magnitude or position, the consequent change in the forces each experiences does not occur immediately but takes a definite time to be established. This delay cannot be understood on the basis of coulomb law but can be explained by assuming (using field concept) that changes in field travel with a finite speed. ( $\approx 3 \times 10^8$  m / sec).

Electric field can be represented by field lines or line of force.

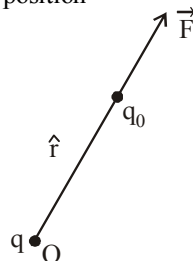
The direction of the field at any point is taken as the direction of the force on a positive charge at the point.

Electric field intensity due to a charge  $q$  at any position

( $\vec{r}$ ) from that charge is defined as

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E}(\vec{r}) = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon} \frac{q}{r^2} \hat{r}$$



where  $\vec{F}$  is the force experienced by a small positive test charge  $q_0$  due to charge  $q$ . Its **SI unit** is  $\text{NC}^{-1}$ . It is a vector quantity. If there are more charges responsible for the field, then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

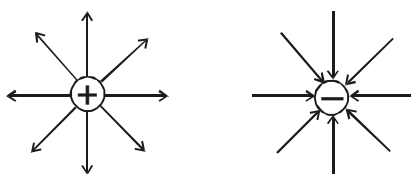
where  $\vec{E}_1, \vec{E}_2, \vec{E}_3, \dots$  are the electric field intensities due to charges  $q_1, q_2, q_3, \dots$  respectively.

### ELECTRIC LINES OF FORCE

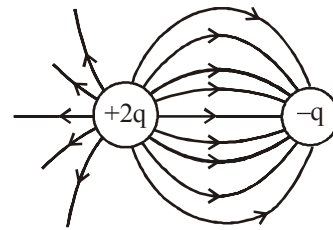
*These are the imaginary lines of force and the tangent at any point on the lines of force gives the direction of the electric field at that point.*

#### Properties of Electric Lines of Force

- (i) The lines of force diverge out from a positive charge and converge at a negative charge. i.e. the lines of force are always directed from higher to lower potential.



- (ii) The electric lines of force contract length wise indicating unlike charges attract each other and expand laterally indicating like charges repel each other.

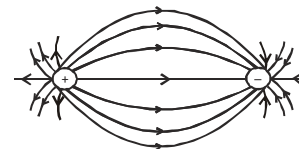


- (iii) The number of lines that originate from or terminate on a charge is proportional to the magnitude of charge.

$$\text{i.e., } \frac{|q_1|}{|q_2|} = \frac{N_1}{N_2}$$

- (iv) Two electric lines of force never intersect each other.

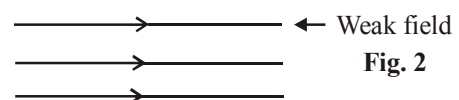
- (v) They begin from positive charge and end on negative charge i.e., they do not make closed loop (while magnetic field lines form closed loop).



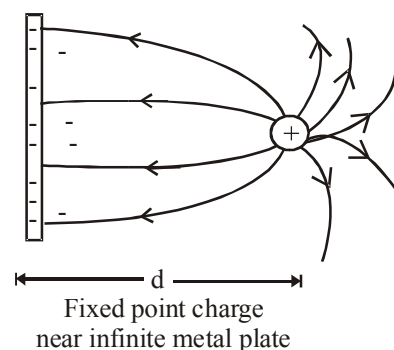
- (vi) **Where the electric lines of force are**

- (a) close together, the field is strong (see fig.1)  
(b) far apart, the field is weak (see fig.2)

Electric field lines



- (vii) Electric lines of force generate or terminate at charges /surfaces at right angles.

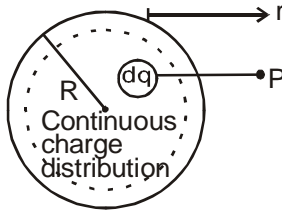


### Electric Field for Continuous Charge Distribution :

If the charge distribution is continuous, then the electric field strength at any point may be calculated by dividing the charge into infinitesimal elements. If  $dq$  is the small element of charge within the charge distribution, then the electric field  $d\vec{E}$  at point P at a distance  $r$  from charge element  $dq$  is

$$d\vec{E} = \frac{1}{4\pi\epsilon} \frac{dq}{r^2} \hat{r};$$

Non conducting sphere (dq is small charge element)



$$\begin{aligned} dq &= \lambda dl \text{ (line charge density)} \\ &= \sigma ds \text{ (surface charge density)} \\ &= \rho dv \text{ (volume charge density)} \end{aligned}$$

The net field strength due to entire charge distribution is given by

$$\vec{E} = \frac{1}{4\pi\epsilon} \int \frac{dq \hat{r}}{r^2}$$

where the integration extends over the entire charge distribution.

**Note :** Electric field intensity due to a point charge q, at a distance  $(r_1 + r_2)$  where  $r_1$  is the thickness of medium of dielectric constant  $K_1$  and  $r_2$  is the thickness of medium of dielectric constant  $K_2$  as shown in fig. is given by

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(r_1\sqrt{K_1} + r_2\sqrt{K_2})^2}$$

### CALCULATION OF ELECTRIC FIELD INTENSITY FOR A DISTRIBUTION OF DIRECT AND CONTINUOUS CHARGE

1. Fix origin of the coordinate system where electric field intensity is to be found.
2. Draw the direction of electric field intensity due to the surrounding charges considering one charge at a time.
3. Resolve the electric field intensity in x and y-axis respectively and find  $\Sigma E_x$  and  $\Sigma E_y$
4. The resultant intensity is  $E = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2}$  and  $\tan \theta = \frac{\Sigma E_y}{\Sigma E_x}$  where  $\theta$  is the angle between  $\vec{E}$  and x-axis.
5. To find the force acting on the charge placed at the origin, the formula  $F = qE$  is used.

**Energy density :** Energy in unit volume of electric field is called energy density and is given by  $u = \frac{1}{2} \epsilon_0 E^2$ ,

where  $E$  = electric field and  $\epsilon_0$  = permittivity of vacuum

### Electric Field due to Various Charge Distribution :

- (i) Electric Field due to an isolated point charge

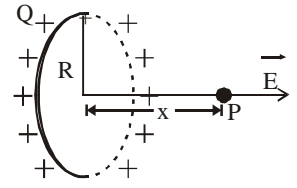
$$E = k \frac{q}{x^2}$$

- (ii) A circular ring of radius R with uniformly distributed charge

$$E = k \frac{Qx}{(R^2 + x^2)^{3/2}}$$

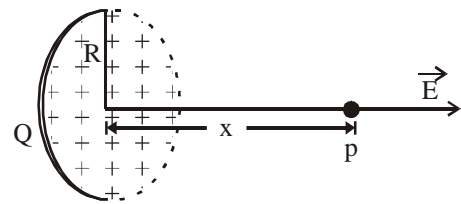
$$\text{When } x \gg R, E = k \frac{Q}{x^2}$$

[The charge on ring behaves as point charge]



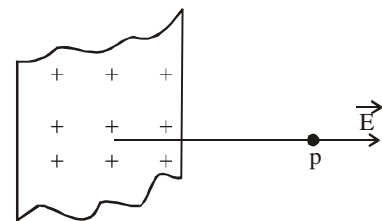
$$E \text{ is max when } x = \pm \frac{R}{\sqrt{2}}. \text{ Also } E_{\max} = \frac{Q}{6\sqrt{3}\pi\epsilon_0 R^2}$$

- (iii) A circular disc of radius R with uniformly distributed charge with surface charge density  $\sigma$



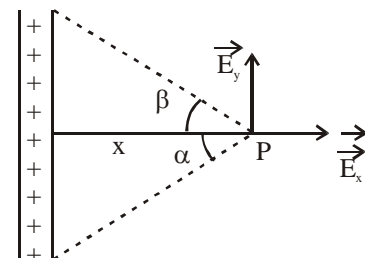
$$E = \frac{2kQ}{R^2} \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right] = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

- (iv) An infinite sheet of uniformly distributed charges with surface charge density  $\sigma$



$$E = \frac{\sigma}{2\epsilon_0}$$

- (v) A finite length of charge with linear charge density  $\lambda$



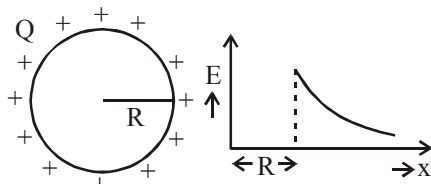
$$E_x = \frac{k\lambda}{x} [\sin \alpha + \sin \beta] \text{ and } E_y = \frac{k\lambda}{x} [\cos \alpha - \cos \beta]$$

### Special case :

For Infinite length of charge,  $\alpha = \beta = \frac{\pi}{2}$

$$\therefore E_x = \frac{2k\lambda}{x} \text{ and } E_y = 0$$

- (vi) Due to a spherical shell of uniformly distributed charges with surface charge density  $\sigma$

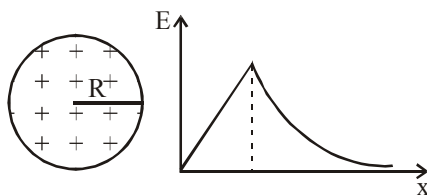


$$E_{in} = 0 \quad (x < R)$$

$$E_{surface} = k \frac{Q}{R^2} = \frac{\sigma}{\epsilon_0} \quad (x = R)$$

$$E_{out} = k \frac{Q}{x^2}$$

- (vii) Due to a solid non conducting sphere of uniformly distributed charges with charge density  $\rho$



$$E_{centre} = 0$$

$$E_{in} = k \frac{Qx}{R^3}$$

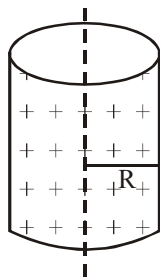
$$E_{surface} = k \frac{Q}{R^2}$$

$$E_{out} = k \frac{Q}{x^2}$$

- (viii) Due to a solid non-conducting cylinder with linear charge density  $\lambda$

$$E_{axis} = 0, E_{in} = k \frac{2\lambda x}{R^2},$$

$$E_{surface} = k \frac{2\lambda}{R}, E_{out} = k \frac{2\lambda}{x}$$



$$\text{In above cases, } k = \frac{1}{4\pi \epsilon_0}$$

### Keep in Memory

- If the electric lines of force are parallel and equally spaced, the field is uniform.
- If  $E_0$  and  $E$  be the electric field intensity at a point due to a point charge or a charge distribution in vacuum and in a medium of dielectric constant  $K$  then  

$$E = KE_0$$
- If  $E$  and  $E'$  be the electric field intensity at a point in the two media having dielectric constant  $K$  and  $K'$  then

$$\frac{E'}{E} = \frac{K'}{K}$$

- The electric field intensity at a point due to a ring with uniform charge distribution doesn't depend upon the radius of the ring if the distance between the point and the centre of the ring is much greater than the radius of the ring. The ring simply behaves as a point charge.
- The electric field intensity inside a hollow sphere is zero but has a finite value at the surface  $\left( = \frac{KQ}{R^2} \right)$  and outside it  $\left( = \frac{KQ}{x^2} \right)$ ;  $x$  being the distance of the point from the centre of the sphere).
- The electric field intensity at a point outside a hollow sphere (or spherical shell) does not depend upon the radius of the sphere. It just behaves as a point charge.
- The electric field intensity at the centre of a non-conducting solid sphere with uniform charge distribution is zero. At other points inside it, the electric field varies directly with the distance from the centre (i.e.  $E \propto x$ ;  $x$  being the distance of the point from the centre). On the surface, it is constant but varies inversely with the square of the distance from the centre (i.e.  $E \propto \frac{1}{x^2}$ ). Note that the field doesn't depend on the radius of the sphere for a point outside it. It simply behaves as a point charge.
- The electric field intensity at a point on the axis of non-conducting solid cylinder is zero. It varies directly with the distance from the axis inside it (i.e.  $E \propto x$ ). On the surface, it is constant and varies inversely with the distance from the axis for a point outside it (i.e.  $E \propto \frac{1}{x}$ ).

### MOTION OF A CHARGED PARTICLE IN AN ELECTRIC FIELD

Let a charged particle of mass  $m$  and charge  $q$  be placed in an uniform electric field  $\vec{E}$ , then electric force on the charge particle is

$$\vec{F} = q\vec{E}$$

$$\therefore \text{acceleration, } \vec{a} = \frac{q\vec{E}}{m} \text{ (constant)}$$

- (a) The velocity of the charged particle at time  $t$  is,

$$v = u + at = at = \frac{qE}{m} t \text{ (Particle initially at rest)}$$

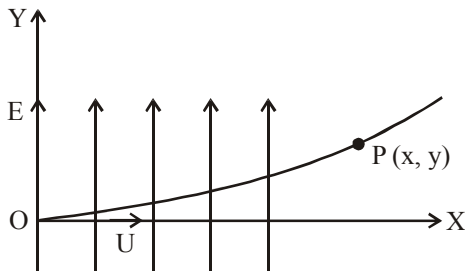
$$\text{or } v = \frac{qE}{m} t$$

- (b) Distance travelled by particle is  $S = \frac{1}{2} at^2 = \frac{1}{2} \frac{qE}{m} t^2$

- (c) Kinetic energy gained by particle,  $K = \frac{1}{2} mv^2 = \frac{q^2 E^2 t^2}{2m}$



**If a charged particle is entering the electric field in perpendicular direction.**



Let  $\vec{E} = E \hat{j}$  and the particle enters the field with speed  $u$  along  $x$ -axis.

Acceleration along  $Y$ -axis,  $a_y = \frac{qE}{m}$

The initial component of velocity along  $y$ -axis is zero. Hence the deflection of the particle along  $y$ -axis after time  $t$  is

$$y = u_y t + \frac{1}{2} \cdot \frac{qE}{m} t^2 = 0 + \frac{1}{2} a_y t^2;$$

$$\therefore y = \frac{1}{2} \cdot \frac{qE}{m} t^2 \quad \dots\dots (i)$$

Distance covered by particle in  $x$ -axis,  
 $x = ut \quad \dots\dots (ii) (\because \text{acceleration } a_x = 0)$

Eliminating  $t$  from equation (i) & (ii),

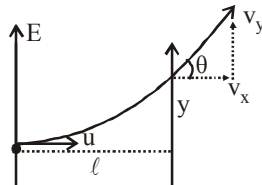
$$y = \frac{qE}{2m} \cdot \frac{x^2}{u^2} \text{ i.e. } y \propto x^2.$$

This shows that *the path of charged particle in perpendicular field is parabola.*

If the width of the region in which the electric field exists be  $\ell$  then

- (i) the particle will leave the field at a distance from its original path in the direction of field, given by

$$y = \frac{qE}{2m} \cdot \frac{\ell^2}{u^2}$$



- (ii) The particle will leave the region in the direction of the tangent drawn to the parabola at the point of escape.  
 (iii) The velocity of the particle at the point of escape is given by

$$v = \sqrt{v_x^2 + v_y^2}, \text{ where } v_x = u \text{ and}$$

$$v_y = u_y + a_y t = 0 + \frac{qE}{m} t$$

$$= \frac{qEt}{m} = \frac{qE\ell}{mu} \quad \left( \because t = \frac{\ell}{u} \right)$$

$$\therefore v = \sqrt{u^2 + \left( \frac{qE\ell}{mu} \right)^2}$$

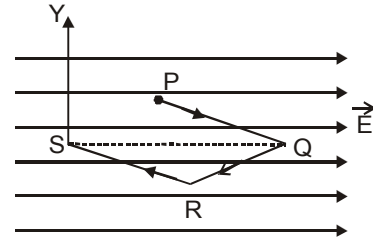
- (iv) The direction of the particle in which it leaves the field is given by

$$\tan \theta = \frac{v_y}{v_x} = \frac{qE\ell}{mu \cdot u} = \frac{qE\ell}{mu^2}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{qE\ell}{mu^2} \right)$$

### Example 8.

Point charge  $q$  moves from point  $P$  to point  $S$  along the path  $PQRS$  (as shown in fig.) in a uniform electric field  $E$  pointing co-parallel to the positive direction of  $X$ -axis. The coordinates of the points  $P, Q, R$  and  $S$  are  $(a, b, 0), (2a, 0, 0), (a, -b, 0)$  and  $(0, 0, 0)$  respectively.



The workdone by the field in the above case is given by the expression

- (a)  $q E A$  (b)  $-q E A$   
 (c)  $q E A \sqrt{2}$  (d)  $q E \sqrt{[(2a)^2 + b^2]}$

**Solution : (b)**

$$W = \vec{F} \cdot \vec{S} = q E \vec{i} \cdot \vec{S}$$

$$\text{Now } W_{P \rightarrow Q} = q E \vec{i} \cdot (a \vec{i} + b \vec{j}) = q E a$$

$$W_{Q \rightarrow R} = q E \vec{i} \cdot (-a \vec{i} + b \vec{j}) = -q E a$$

(workdone against field is taken as negative)

$$W_{R \rightarrow S} = q E \vec{i} \cdot (-a \vec{i} + b \vec{j}) = -q E a$$

$$W = W_{P \rightarrow Q} + W_{Q \rightarrow R} + W_{R \rightarrow S} \\ = q E a - q E a - q E a = -q E a$$

### Example 9.

Calculate the electric field strength required to just support a water drop of mass  $10^{-7}$  kg and having charge  $1.6 \times 10^{-19}$  C.

**Solution :**

Here,  $m = 10^{-7}$  kg,  $q = 1.6 \times 10^{-19}$  C

**Step 1 :** Let  $E$  be the electric field strength required to support the water drop.

Force acting on the water drop due to electric field  $E$  is

$$F = qE = 1.6 \times 10^{-19} E$$

Weight of drop acting downward,

$$W = mg = 10^{-7} \times 9.8 \text{ newton.}$$

**Step 2 :** Drop will be supported if  $F$  and  $W$  are equal and opposite.

$$\text{i.e., } 1.6 \times 10^{-19} E = 9.8 \times 10^{-7}$$

$$\text{or } E = \frac{9.8 \times 10^{-7}}{1.6 \times 10^{-19}} \\ = 6.125 \times 10^{12} \text{ N C}^{-1}.$$

### Example 10.

Can a metal sphere of radius 1cm hold a charge of 1 coulomb.

### Solution :

Electric field at the surface of the sphere.

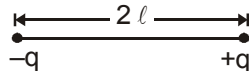
$$E = \frac{KQ}{R^2} = \frac{9 \times 10^9 \times 1}{(1 \times 10^{-2})^2} = 9 \times 10^{13} \frac{\text{V}}{\text{m}}$$

This field is much greater than the dielectric strength of air ( $3 \times 10^6 \text{ v/m}$ ), the air near the sphere will get ionised and charge will leak out. Thus a sphere of radius 1 cm cannot hold a charge of 1 coulomb in air.

### ELECTRIC DIPOLE

Two equal and opposite charges separated by a finite distance constitute an **electric dipole**. If  $-q$  and  $+q$  are charges at distance  $2\ell$  apart, then **dipole moment**,

$$p = q \times 2\ell$$



Its **SI unit** is coulomb metre.

Its direction is from  $-q$  to  $+q$ . It is a vector quantity.

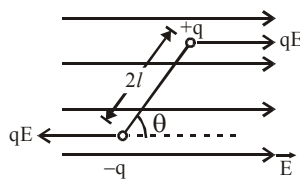
The **torque**  $\tau$  on a dipole in uniform electric field as shown in

figure is given by,  $\tau = qE \times 2\ell \sin \theta = \vec{p} \times \vec{E}$

So  $\tau$  is maximum, when dipole is  $\perp$  to field & minimum ( $=0$ ) when dipole is parallel or antiparallel to field.

If  $\vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$  and  $\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

$$\text{Then } \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p_x & p_y & p_z \\ E_x & E_y & E_z \end{vmatrix}$$



**The work done in rotating the dipole** from equilibrium through an angle  $d\theta$  is given by

$$dW = \tau d\theta = pE \sin \theta d\theta$$

and from  $\theta_1 \rightarrow \theta_2$ ,

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} pE \sin \theta d\theta$$

If  $\theta_1 = 0$  i.e., equilibrium position, then

$$W = \int_0^{\theta_2} pE \sin \theta d\theta = pE(1 - \cos \theta_2)$$

**Workdone in rotating an electric dipole** in uniform electric field from  $\theta_1$  to  $\theta_2$  is  $W = pE(\cos \theta_1 - \cos \theta_2)$

**Potential energy of an electric dipole** in an electric field is,

$$U = -\vec{p} \cdot \vec{E} \text{ i.e. } U = -pE \cos \theta$$

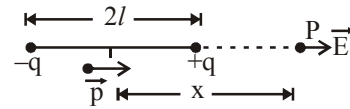
where  $\theta$  is the angle between  $\vec{E}$  and  $\vec{p}$ .

We can also write

$$U = p_x E_x + p_y E_y + p_z E_z$$

### Electric Field due to an Electric Dipole

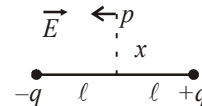
(a) *Along the axial line (or end-on position)*



$\vec{p}$  and  $\vec{E}$  are parallel

$$E_{ax} = \frac{1}{4\pi \epsilon_0} \cdot \frac{2px}{(x^2 - \ell^2)^2} \text{ when } x \gg \ell$$

(b) *Along equatorial line (or broad-side on position)*

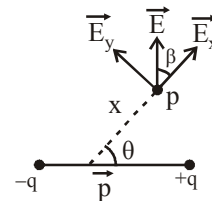


$$E_{eq} = \frac{1}{4\pi \epsilon_0} \cdot \frac{p}{(x^2 + \ell^2)^{3/2}} \text{ when } x \gg \ell$$

When  $\vec{p}$  and  $\vec{E}$  are anti parallel then,

$$E_{ax} = 2E_{eq}$$

(c) *At any point (from the dipole)*



$$E = \frac{1}{4\pi \epsilon_0} \frac{p}{x^3} \sqrt{3\cos^2 \theta + 1}; \tan \beta = \frac{1}{2} \tan \theta$$

Electric field intensity due to a point charge varies inversely as cube of the distance and in case of quadrupole it varies inversely as the fourth power of distance from the quadrupole.

### Electric Force between Two Dipoles

The electrostatic force between two dipoles of dipole moments  $p_1$  and  $p_2$  lying at a separation  $r$  is

$$F = \frac{1}{4\pi \epsilon_0} \frac{6p_1 p_2}{r^4} \text{ when dipoles are placed co-axially}$$

$$F = \frac{1}{4\pi \epsilon_0} \frac{3p_1 p_2}{r^4} \text{ when dipoles are placed perpendicular to each other.}$$

### Keep in Memory

1. The dipole moment of a dipole has a direction from the negative charge to the positive charge.
2. If the separation between the charges of the dipole is increased (or decreased)  $K$ -times, the dipole moment increases (or decreases) by  $K$ -times.
3. The torque experienced by a dipole placed in a uniform electric field has value always lying between zero and  $pE$ , where  $p$  is the dipole moment and  $E$ , the uniform electric field. It varies directly with the separation between the charges of the dipole.

- The work done in rotating a dipole in a uniform electric field varies from zero (minimum) to  $2pE$  (maximum). Also, it varies directly with the separation between the charges of the dipole.
- The potential energy of the dipole in a uniform electric field always lies between  $+pE$  and  $-pE$ .
- The electric field intensity at a point due to an electric dipole varies inversely with the cube of the distance of the point from its centre if the distance is much greater than the length of the dipole.
- The electric field at a point due to a small dipole in end-on position is double of its value in broad side-on position,  
i.e.  $E_{\text{End-on}} = 2E_{\text{Broad side-on}}$
- For a small dipole, the electric field tends from infinity at a point very close to the axis of the dipole to zero at a point at infinity.
- The force between two dipoles increases (or decreases) by  $K^4$ -times as the distance between them decreases (or increases) by  $K$ -times.
- Time period of a dipole** in uniform electric field is

$$T = 2\pi \sqrt{\frac{I}{pE}}$$

where  $I$  = moment of inertia of the dipole about the axis of rotation.

#### Example 11.

Calculate the electric field intensity due to a dipole of length 10 cm and having a charge of  $500 \mu\text{C}$  at a point on the axis distant 20 cm from one of the charges in air.

#### Solution :

Given :  $q = 500 \times 10^{-6}\text{C}$ ,  $a = 10\text{ cm}$  or  $a/2 = 5 \times 10^{-2}\text{ m}$ ,

$r = (20 + 5)\text{ cm} = 25 \times 10^{-2}\text{ m}$ ,

$p = q \times a = (500 \times 10^{-6}) \times (10 \times 10^{-2}) = 5 \times 10^{-5}\text{ C-m}$

The electric intensity on the axial line of the dipole is given by

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \times \frac{2rp}{\left[r^2 - \left(\frac{a}{2}\right)^2\right]^2}$$

$$E_{\text{axial}} = (9 \times 10^9) \times \frac{2 \times (25 \times 10^{-2}) \times (5 \times 10^{-5})}{10^{-8}[(25)^2 - (5)^2]^2}$$

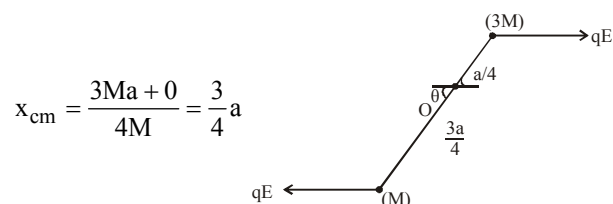
$$= 3.25 \times 10^7 \text{ NC}^{-1}$$

#### Example 12.

Consider two objects of masses  $M$  and  $3M$  separated by a distance  $\ell$ . Charge  $q$  and  $-q$  are placed on them respectively and they are lying in an electric field  $E$ .

Find the angular frequency of oscillation (S.H.M.)

#### Solution :



( $\therefore x_{\text{cm}}$  is the position of centre of mass of system)

Since net force is zero, the centre of mass will not move but the dipole will rotate about the centre of mass due to torque

$$\tau = qE(a/4)\sin\theta + qE(3a/4)\sin\theta$$

$$\tau = qEa\sin\theta \quad \dots\dots(1)$$

$$\text{Also, } \tau = I_{\text{cm}}\alpha = -I_{\text{cm}}\frac{d^2\theta}{dt^2} \quad \dots\dots(2)$$

For small oscillations  $\sin\theta = \theta$

$$\Rightarrow I_{\text{cm}}\frac{d^2\theta}{dt^2} = -qEa\sin\theta = -qEa\theta;$$

From eq. (1) & (2),

$$\frac{d^2\theta}{dt^2} = -\left(\frac{qEa}{I_{\text{cm}}}\right)\theta$$

$$I_{\text{cm}} = 3M\left(\frac{a}{4}\right)^2 + M\left(\frac{3a}{4}\right)^2$$

$$= \frac{3Ma^2 + 9Ma^2}{16} = \frac{12Ma^2}{16} = \frac{3}{4}Ma^2$$

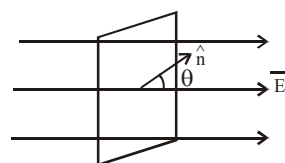
$$\therefore \omega^2 = \frac{qEa}{\frac{3}{4}Ma^2} = \frac{4qE}{3Ma} \quad \text{or} \quad \omega = \sqrt{\frac{4qE}{3Ma}}$$

## ELECTRIC FLUX

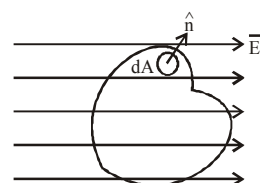
**Electric flux** is a measure of the number of electric field lines passing through the surface. If surface is not open & encloses some net charge, then net number of lines that go through the surface is proportional to net charge within the surface.

For uniform electric field when the angle between area vector ( $\vec{A}$ ) and electric field ( $\vec{E}$ ) has the same value throughout the

area,  $\phi = \vec{E} \cdot \vec{A} \Rightarrow \phi = EA\cos\theta$



For uniform electric field when the angle between the area vector and electric field is not constant throughout the area



$$d\phi = \vec{E} \cdot d\vec{A} \Rightarrow \phi = \int \vec{E} \cdot d\vec{A} \Rightarrow \phi = \int EdA\cos\theta = E \int dA\cos\theta$$

## Keep in Memory

- The electric flux is a scalar although it is a product of two vectors  $\vec{E}$  and  $\vec{A}$  (because it is a scalar product of the two).

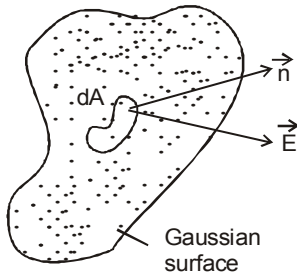
2. The electric flux has values lying between  $-EA$  and  $+EA$ , where  $E$  and  $A$  are the electric field and the area of cross-section of the surface.

### GAUSS'S LAW

It states that, *the net electric flux through a closed surface in vacuum is equal to  $1/\epsilon_0$  times the net charge enclosed within the surface.*

$$\text{i.e., } \phi = \oint_s \vec{E} \cdot d\vec{A} = \oint_s \vec{E} \cdot \vec{n} dA = \frac{Q_{in}}{\epsilon_0}$$

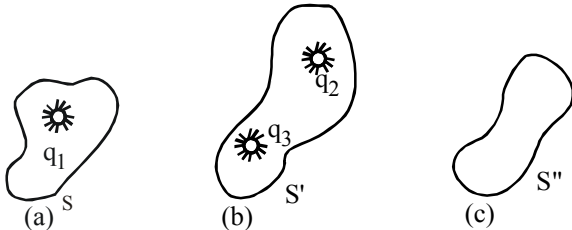
where  $Q_{in}$  represents the net charge inside the gaussian surface  $S$ .



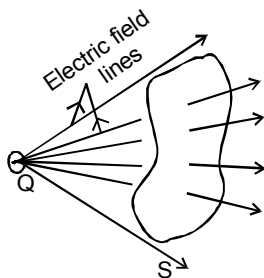
Closed surface of irregular shape which enclosed total charge  $Q_{in}$

In principle, Gauss's law can always be used to calculate the electric field of a system of charges or a continuous distribution of charge. But in practice it is useful only in a limited number of situation, where there is a high degree of symmetry such as spherical, cylindrical etc.

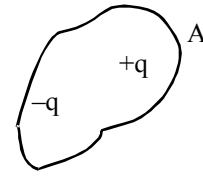
- (i) The net electric flux through any closed surface depends only on the charge inside that surface. In the figures, the net flux through  $S$  is  $q_1/\epsilon_0$ , the net flux through  $S'$  is  $(q_2 + q_3)/\epsilon_0$  and the net flux, through  $S''$  is zero.



- (ii) A point charge  $Q$  is located outside a closed surface  $S$ . In this case note that the number of lines entering the surface equals to the number of lines leaving the surface. In other words the net flux through a closed surface is zero, if there is no charge inside.



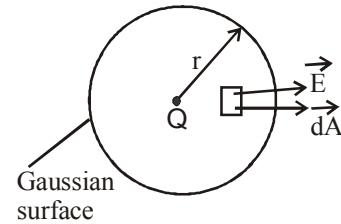
- (iii) The net flux across surface  $A$  is zero



$$\text{i.e., } \phi = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{in}}{\epsilon_0} = 0 \text{ because } Q_{in} = -q + q = 0$$

### Applications of Gauss's Law :

- (i) To determine electric field due to a point charge :



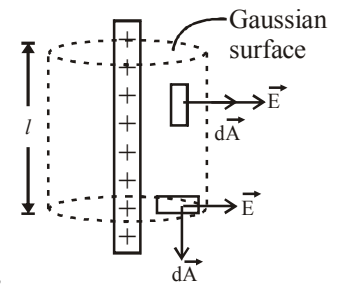
The point charge  $Q$  is at the centre of spherical surface shown in figure. Gaussian surface and  $\vec{E}$  is parallel to  $d\vec{A}$  (direction normal to Gaussian surface) at every point on the Gaussian surface.

$$\text{so, } \phi_c = \oint \vec{E} \cdot d\vec{A} = \oint E dA = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2} \quad (\because A = 4\pi r^2)$$

- (ii) To determine electric field due to a cylindrically symmetric charge distribution :

We calculate the electric field at a distance  $r$  from a uniform positive line charge of infinite length whose charge per unit length is  $\lambda = \text{constant}$ . The flux through the plane surfaces of the Gaussian cylinder is zero, since  $\vec{E}$  is



parallel to the plane of end surface ( $\vec{E}$  is perpendicular to  $d\vec{A}$ ). The total charge inside the Gaussian surface is  $\lambda \ell$ , where  $\lambda$  is linear charge density and  $\ell$  is the length of cylinder.

Now applying Gauss's law and noting  $\vec{E}$  is parallel to  $d\vec{A}$  everywhere on cylindrical surface, we find that

$$\phi_c = \oint \vec{E} \cdot d\vec{A} = (\text{flux})_{\text{both ends}} + (\text{flux})_{\text{cylindrical surface}}$$

Gaussian surface

$$= 0 + E \cdot 2A$$

$$2E \cdot \pi r \ell = \frac{Q_{in}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0} \text{ or } E = \frac{\lambda}{2\pi\epsilon_0 r}$$

### Keep in Memory

1. The closed imaginary surfaces drawn around a charge are called Gaussian surfaces.
2. If the flux emerging out of a Gaussian surface is zero then it is not necessary that the intensity of electric field is zero.
3. In the Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

$\vec{E}$  is the resultant electric field due to all charges lying inside or outside the Gaussian surface, but  $Q_{\text{in}}$  is the charge lying only inside the surface.

4. The net flux of the electric field through a closed surface due to all the charges lying inside or outside the surface is equal to the flux due to the charges only enclosed by the surface.
5. The electric flux through any closed surface does not depend on the dimensions of the surface but it depends only on the net charge enclosed by the surface.

#### Example 13.

A charge  $q$  is enclosed in a cube. What is the electric flux associated with one of the faces of cube?

#### Solution :

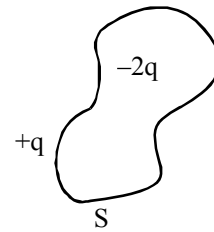
According to Gauss's theorem,

$$\begin{aligned} \text{Total electric flux } \phi &= \frac{1}{\epsilon_0} \times \text{total enclosed charge} \\ &= \frac{1}{\epsilon_0} \times q. \end{aligned}$$

Since cube has six faces, hence electric flux linked with each face  $= (1/6\phi) = q/6\epsilon_0$ .

#### Example 14.

The following figure shows a surface  $S$  which is enclosing  $-2q$  charge. The charge  $+q$  is kept outside the surface  $S$ . Calculate the net outward/ inward flux from the surface  $S$ .



#### Solution :

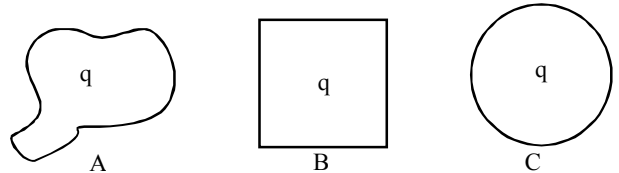
According to Gauss's law, the net flux is

$$\phi = \frac{1}{\epsilon_0} \times \text{net charge enclosed by closed surface} = \frac{-2q}{\epsilon_0}$$

(Because  $+q$  is outside the surface  $S$ , so net flux due to  $+q$  is zero)

#### Example 15.

In which of the following figures, the electric flux is maximum?



#### Solution :

According to Gauss's law, the electric flux linked with a closed surface depends only on net charge enclosed by that surface. It does not depend on the shape and size of that closed surface. Hence electric flux linked in above three figures are same i.e.,  $\phi_A = \phi_B = \phi_C$ .

#### Example 16.

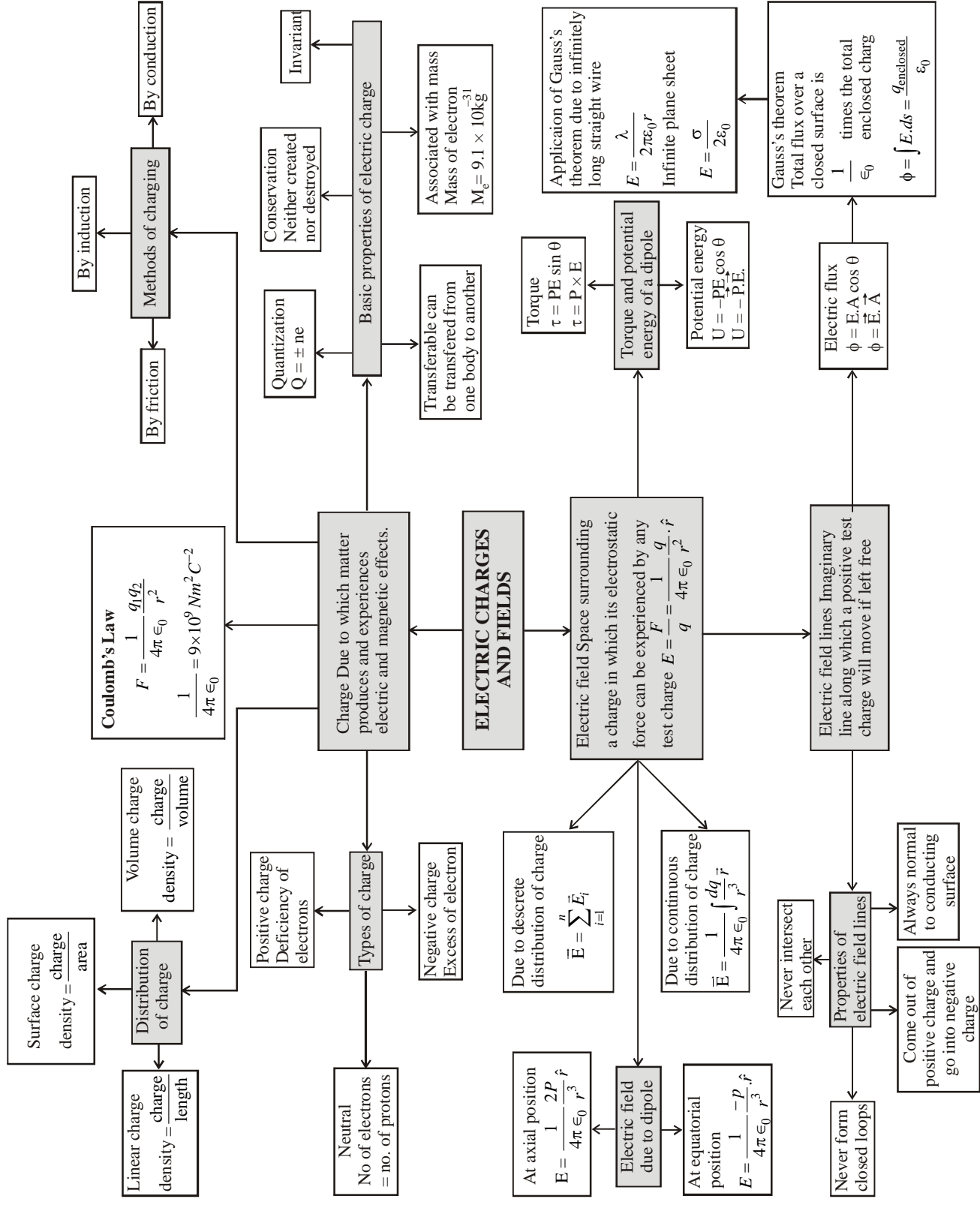
A point charge  $+q$  is placed at mid point of a cube of side 'L'. What is the electric flux emerging from the cube?

#### Solution :

According to Gauss's law,

$$\begin{aligned} \phi_{\text{net closed surface}} &= \frac{q}{\epsilon_0} \\ &= \frac{\text{Net charge enclosed by closed surface}}{\epsilon_0} \end{aligned}$$

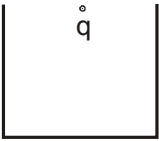
# CONCEPT MAP



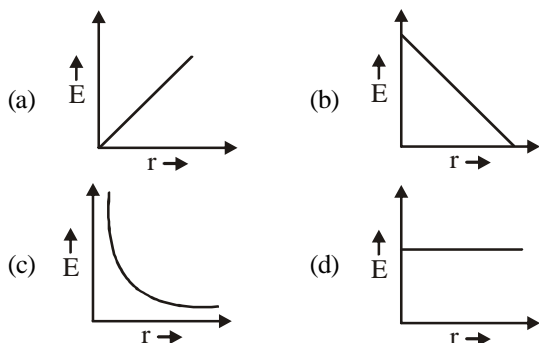


# EXERCISE - 1

## Conceptual Questions

- The electric field strength at a distance  $r$  from a charge  $q$  is  $E$ . What will be electric field strength if the distance of the observation point is increased to  $2r$ ?  
 (a)  $E/2$  (b)  $E/4$   
 (c)  $E/6$  (d) None of these
- The surface density on the copper sphere is  $\sigma$ . The electric field strength on the surface of the sphere is  
 (a)  $\sigma$  (b)  $\sigma/2$   
 (c)  $Q/2\epsilon_0$  (d)  $Q/\epsilon_0$
- What is the angle between the electric dipole moment and the electric field due to it on the axial line?  
 (a)  $0^\circ$  (b)  $90^\circ$   
 (c)  $180^\circ$  (d) None of these
- In a region of space having a uniform electric field  $E$ , a hemispherical bowl of radius  $r$  is placed. The electric flux  $\phi$  through the bowl is  
 (a)  $2\pi RE$  (b)  $4\pi R^2E$   
 (c)  $2\pi R^2E$  (d)  $\pi R^2E$
- A cylinder of radius  $R$  and length  $\ell$  is placed in a uniform electric field  $E$  parallel to the axis of the cylinder. The total flux over the curved surface of the cylinder is  
 (a) zero (b)  $\pi R^2E$   
 (c)  $2\pi R^2E$  (d)  $E/\pi R^2$
- An electric dipole when placed in a uniform electric field  $E$  will have minimum potential energy if the dipole moment makes the following angle with  $E$ .  
 (a)  $\pi$  (b)  $\pi/2$   
 (c) zero (d)  $3\pi/2$
- At the centre of a cubical box  $+Q$  charge is placed. The value of total flux that is coming out a wall is  
 (a)  $Q/\epsilon_0$  (b)  $Q/3\epsilon_0$   
 (c)  $Q/4\epsilon_0$  (d)  $Q/6\epsilon_0$
- If a charge is moved against the coulomb force of an electric field, then  
 (a) work is done by the electric field  
 (b) energy is used from some outside source  
 (c) the strength of the field is decreased  
 (d) the energy of the system is decreased
- The charge given to any conductor resides on its outer surface, because  
 (a) the free charge tends to be in its minimum potential energy state  
 (b) the free charge tends to be in its minimum kinetic energy state  
 (c) the free charge tends to be in its maximum potential energy state.  
 (d) the free charge tends to be in its maximum kinetic energy state
- Identify the wrong statement in the following. Coulomb's law correctly describes the electric force that  
 (a) binds the electrons of an atom to its nucleus  
 (b) binds the protons and neutrons in the nucleus of an atom  
 (c) binds atoms together to form molecules  
 (d) binds atoms and molecules together to form solids
- An infinite parallel plane sheet of a metal is charged to charge density  $\sigma$  coulomb per square metre in a medium of dielectric constant  $K$ . Intensity of electric field near the metallic surface will be  
 (a)  $E = \frac{\sigma}{\epsilon_0 K}$  (b)  $E = \frac{\sigma}{2\epsilon_0}$   
 (c)  $E = \frac{\sigma}{2\epsilon_0 K}$  (d)  $E = \frac{K\sigma}{2\epsilon_0}$
- In a medium of dielectric constant  $K$ , the electric field is  $\vec{E}$ . If  $\epsilon_0$  is permittivity of the free space, the electric displacement vector is  
 (a)  $\frac{K\vec{E}}{\epsilon_0}$  (b)  $\frac{\vec{E}}{K\epsilon_0}$   
 (c)  $\frac{\epsilon_0 \vec{E}}{K}$  (d)  $K\epsilon_0 \vec{E}$
- Two conducting spheres of radii  $r_1$  and  $r_2$  are charged to the same surface charge density. The ratio of electric fields near their surface is  
 (a)  $r_1^2/r_2^2$  (b)  $r_2^2/r_1^2$   
 (c)  $r_1/r_2$  (d)  $1:1$
- A charge  $q$  is placed at the centre of the open end of a cylindrical vessel. The flux of the electric field through the surface of the vessel is  

 (a) zero (b)  $q/\epsilon_0$   
 (c)  $q/2\epsilon_0$  (d)  $2q/\epsilon_0$
- Two thin infinite parallel sheets have uniform surface densities of charge  $+\sigma$  and  $-\sigma$ . Electric field in the space between the two sheets is  
 (a)  $\sigma/\epsilon_0$  (b)  $\sigma/2\epsilon_0$   
 (c)  $2\sigma/\epsilon_0$  (d) zero

16. The E-r curve for an infinite linear charge distribution will be



17. If a dipole of dipole moment  $\vec{p}$  is placed in a uniform electric field  $\vec{E}$ , then torque acting on it is given by

- (a)  $\vec{\tau} = \vec{p} \cdot \vec{E}$  (b)  $\vec{\tau} = \vec{p} \times \vec{E}$   
 (c)  $\vec{\tau} = \vec{p} + \vec{E}$  (d)  $\vec{\tau} = \vec{p} - \vec{E}$

18. A charge  $Q$  is enclosed by a Gaussian spherical surface of radius  $R$ . If the radius is doubled, then the outward electric flux will

- (a) increase four times (b) be reduced to half  
 (c) remain the same (d) be doubled

19. What is the value of  $E$  in the space outside the sheets?

- (a)  $\sigma/\epsilon_0$  (b)  $\sigma/2\epsilon_0$   
 (c)  $E \neq 0$  (d)  $2\sigma/\epsilon_0$

20. Two charges are at a distance  $d$  apart. If a copper plate of thickness  $\frac{d}{2}$  is kept between them, the effective force will be

- (a)  $F/2$  (b) zero  
 (c)  $2F$  (d)  $\sqrt{2} F$

21. When air is replaced by a dielectric medium of force constant  $K$ , the maximum force of attraction between two charges, separated by a distance

- (a) decreases  $K$ -times (b) increases  $K$ -times  
 (c) remains unchanged (d) becomes  $\frac{1}{K^2}$  times

22. A point  $Q$  lies on the perpendicular bisector of an electrical dipole of dipole moment  $p$ . If the distance of  $Q$  from the dipole is  $r$  (much larger than the size of the dipole), then the electric field at  $Q$  is proportional to

- (a)  $p^{-1}$  and  $r^{-2}$  (b)  $p$  and  $r^{-2}$   
 (c)  $p^2$  and  $r^{-3}$  (d)  $p$  and  $r^{-3}$

23. A particle of mass  $m$  and charge  $q$  is placed at rest in a uniform electric field  $E$  and then released. The kinetic energy attained by the particle after moving a distance  $y$  is

- (a)  $q E y^2$  (b)  $q E^2 y$   
 (c)  $q E y$  (d)  $q^2 E y$

24. Intensity of an electric field ( $E$ ) depends on distance  $r$ , due to a dipole, is related as

- (a)  $E \propto \frac{1}{r}$  (b)  $E \propto \frac{1}{r^2}$   
 (c)  $E \propto \frac{1}{r^3}$  (d)  $E \propto \frac{1}{r^4}$

25. The formation of a dipole is due to two equal and dissimilar point charges placed at a

- (a) short distance (b) long distance  
 (c) above each other (d) none of these

## EXERCISE - 2

### Applied Questions

1. The electric field intensity just sufficient to balance the earth's gravitational attraction on an electron will be: (given mass and charge of an electron respectively are  $9.1 \times 10^{-31} \text{ kg}$  and  $1.6 \times 10^{-19} \text{ C}$ .)

- (a)  $-5.6 \times 10^{-11} \text{ N/C}$  (b)  $-4.8 \times 10^{-15} \text{ N/C}$   
 (c)  $-1.6 \times 10^{-19} \text{ N/C}$  (d)  $-3.2 \times 10^{-19} \text{ N/C}$

2. The insulation property of air breaks down when the electric field is  $3 \times 10^6 \text{ Vm}^{-1}$ . The maximum charge that can be given to a sphere of diameter 5 m is approximately

- (a)  $2 \times 10^{-2} \text{ C}$  (b)  $2 \times 10^{-3} \text{ C}$   
 (c)  $2 \times 10^{-4} \text{ C}$  (d)  $2 \times 10^{-5} \text{ C}$

3. There is an electric field  $E$  in X-direction. If work done in moving a charge  $0.2 \text{ C}$  through a distance of  $2 \text{ m}$  along a line making an angle of  $60^\circ$  with X-axis is  $4.0 \text{ joule}$ , what is the value of  $E$ ?

- (a)  $\sqrt{3}$  newton per coulomb  
 (b) 4 newton per coulomb  
 (c) 5 newton per coulomb  
 (d) None of these

4. From a point charge, there is a fixed point A. At A, there is an electric field of  $500 \text{ V/m}$  and potential difference of  $3000 \text{ V}$ . Distance between point charge and A will be

- (a) 6m (b) 12m  
 (c) 16m (d) 24m

5. If electric field in a region is radially outward with magnitude  $E = Ar$ , the charge contained in a sphere of radius  $r$  centred at the origin is

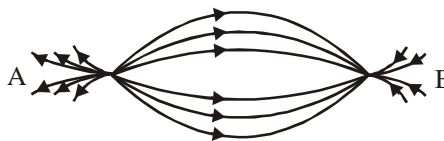
- (a)  $\frac{1}{4\pi\epsilon_0} Ar^3$  (b)  $4\pi\epsilon_0 Ar^3$   
 (c)  $\frac{1}{4\pi\epsilon_0} \frac{A}{r^3}$  (d)  $\frac{4\pi\epsilon_0 A}{r^3}$

6. A hollow insulated conduction sphere is given a positive charge of  $10 \mu\text{C}$ . What will be the electric field at the centre of the sphere if its radius is 2 metres?
- (a) Zero (b)  $5 \mu\text{Cm}^{-2}$   
(c)  $20 \mu\text{Cm}^{-2}$  (d)  $8 \mu\text{Cm}^{-2}$
7. An electric dipole has the magnitude of its charge as  $q$  and its dipole moment is  $p$ . It is placed in uniform electric field  $E$ . If its dipole moment is along the direction of the field, the force on it and its potential energy are respectively.
- (a)  $q \cdot E$  and max. (b)  $2q \cdot E$  and min.  
(c)  $q \cdot E$  and min (d) zero and min.
8. If the dipole of moment  $2.57 \times 10^{-17} \text{ cm}$  is placed into an electric field of magnitude  $3.0 \times 10^4 \text{ N/C}$  such that the fields lines are aligned at  $30^\circ$  with the line joining P to the dipole, what torque acts on the dipole?
- (a)  $7.7 \times 10^{-13} \text{ Nm}$  (b)  $3.855 \times 10^{-13} \text{ Nm}$   
(c)  $3.855 \times 10^{-15} \text{ Nm}$  (d)  $7.7 \times 10^{-15} \text{ Nm}$
9. An electric dipole is placed at an angle of  $30^\circ$  with an electric field of intensity  $2 \times 10^5 \text{ NC}^{-1}$ . It experiences a torque of  $4 \text{ Nm}$ . Calculate the charge on the dipole if the dipole length is 2 cm.
- (a)  $8 \text{ mC}$  (b)  $4 \text{ mC}$   
(c)  $8 \mu\text{C}$  (d)  $2 \text{ mC}$
10. Charge  $Q$  is distributed to two different metallic spheres having radii  $R$  and  $2R$  such that both spheres have equal surface charge density, then charge on large sphere is
- (a)  $\frac{4Q}{5}$  (b)  $\frac{Q}{5}$   
(c)  $\frac{3Q}{5}$  (d)  $\frac{5Q}{4}$
11. Two point charges  $q_1 = 4 \mu\text{C}$  and  $q_2 = 9 \mu\text{C}$  are placed 20 cm apart. The electric field due to them will be zero on the line joining them at a distance of
- (a) 8 cm from  $q_1$  (b) 8 cm from  $q_2$   
(c)  $\frac{80}{13} \text{ cm}$  from  $q_1$  (d)  $\frac{80}{13} \text{ cm}$  from  $q_2$
12. Three charge  $q$ ,  $Q$  and  $4q$  are placed in a straight line of length  $l$  at points distant  $0$ ,  $\frac{1}{2}l$  and  $l$  respectively from one end. In order to make the net force on  $q$  zero, the charge  $Q$  must be equal to
- (a)  $-q$  (b)  $-2q$   
(c)  $-\frac{q}{2}$  (d)  $q$
13. Among two discs A and B, first have radius 10 cm and charge  $10^{-6} \mu\text{C}$  and second have radius 30 cm and charge  $10^{-5} \mu\text{C}$ . When they are touched, charge on both  $q_A$  and  $q_B$  respectively will, be
- (a)  $q_A = 2.75 \mu\text{C}$ ,  $q_B = 3.15 \mu\text{C}$   
(b)  $q_A = 1.09 \mu\text{C}$ ,  $q_B = 1.53 \mu\text{C}$   
(c)  $q_A = q_B = 5.5 \mu\text{C}$   
(d) None of these

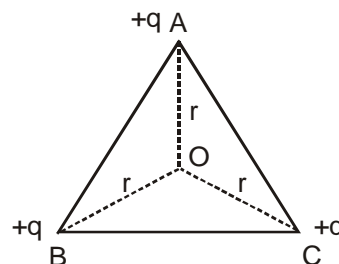
14. Two particle of equal mass  $m$  and charge  $q$  are placed at a distance of 16 cm. They do not experience any force. The value of  $\frac{q}{m}$  is

- (a) 1 (b)  $\sqrt{\frac{\pi\epsilon_0}{G}}$   
(c)  $\sqrt{\frac{G}{4\pi\epsilon_0}}$  (d)  $\sqrt{4\pi\epsilon_0 G}$

15. The spatial distribution of electric field due to charges (A, B) is shown in figure. Which one of the following statements is correct ?

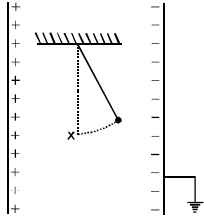


- (a) A is +ve and B -ve,  $|A| > |B|$   
(b) A is -ve and B +ve,  $|A| = |B|$   
(c) Both are +ve but  $A > B$   
(d) Both are -ve but  $A > B$
16. A drop of oil of density  $\rho$  and radius  $r$  carries a charge  $q$  when placed in an electric field  $E$ , it moves upwards with a velocity  $v$ . If  $\rho_0$  is the density of air,  $\eta$  be the viscosity of the air, then which of the following force is directed upwards?
- (a)  $qE$  (b)  $6\pi\eta r v$   
(c)  $\frac{4}{3}\pi r^3 (\rho - \rho_0)g$  (d) None of these
17. ABC is an equilateral triangle. Charges  $+q$  are placed at each corner as shown as fig. The electric intensity at centre O will be



- (a)  $\frac{1}{4\pi\epsilon_0} \frac{q}{r}$  (b)  $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$   
(c)  $\frac{1}{4\pi\epsilon_0} \frac{3q}{r^2}$  (d) zero
18. An electric dipole is placed along the x-axis at the origin O. A point P is at a distance of 20 cm from this origin such that OP makes an angle  $\pi/3$  with the x-axis. If the electric field at P makes an angle  $\theta$  with the x-axis, the value of  $\theta$  would be
- (a)  $\frac{\pi}{3}$  (b)  $\frac{\pi}{3} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$   
(c)  $\frac{2\pi}{3}$  (d)  $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

19. A simple pendulum has a length  $\ell$ , mass of bob  $m$ . The bob is given a charge  $q$ . The pendulum is suspended between the vertical plates of the charged parallel plate capacitor. If  $E$  is the field strength between the plates, then time period  $T$  equal to

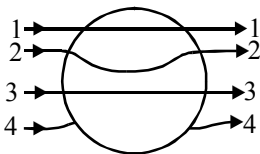


- (a)  $2\pi\sqrt{\frac{\ell}{g}}$  (b)  $2\pi\sqrt{\frac{\ell}{g + \frac{qE}{m}}}$   
 (c)  $2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}}$  (d)  $2\pi\sqrt{\frac{\ell}{g^2 + \left(\frac{qE}{m}\right)^2}}$

20. Two identical thin rings, each of radius  $a$  meter, are coaxially placed at a distance  $R$  meter apart. If  $Q_1$  coulomb and  $Q_2$  coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge  $q$  coulomb from the centre of one ring to that of the other is

- (a) Zero (b)  $\frac{q(Q_1 - Q_2)(\sqrt{2} - 1)}{4\sqrt{2}\pi\epsilon_0 a}$   
 (c)  $\frac{q\sqrt{2}(Q_1 + Q_2)}{4\pi\epsilon_0 a}$  (d)  $\frac{q(Q_1 + Q_2)(\sqrt{2} + 1)}{4\sqrt{2}\pi\epsilon_0 a}$

21. A metallic sphere is placed in a uniform electric field. The line of force follow the path (s) shown in the figure as



- (a) 1 (b) 2  
 (c) 3 (d) 4

22. A soap bubble is given negative charge, its radius will  
 (a) increase (b) decrease  
 (c) remain unchanged (d) fluctuate

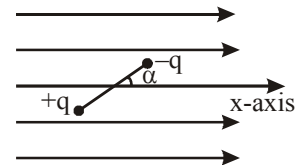
23. A and B are two identically spherical charged bodies which repel each other with force  $F$ , kept at a finite distance. A third uncharged sphere of the same size is brought in contact with sphere B and removed. It is then kept at mid point of A and B. Find the magnitude of force on C.

- (a)  $\frac{F}{2}$  (b)  $\frac{F}{8}$   
 (c)  $F$  (d) Zero

24. An electric dipole of moment  $\vec{P}$  is placed in a uniform electric field  $\vec{E}$  such that  $\vec{P}$  points along  $\vec{E}$ . If the dipole is slightly rotated about an axis perpendicular to the plane containing  $\vec{E}$  and  $\vec{P}$  and passing through the centre of the dipole, the dipole executes simple harmonic motion. Consider  $I$  to be the moment of inertia of the dipole about the axis of rotation. What is the time period of such oscillation?

- (a)  $\sqrt{(pE/I)}$  (b)  $2\pi\sqrt{(I/pE)}$   
 (c)  $2\pi\sqrt{(I/2pE)}$  (d) None of these

25. There exists a non-uniform electric field along  $x$ -axis as shown in the figure below. The field increases at a uniform rate along  $+ve$   $x$ -axis. A dipole is placed inside the field as shown. Which one of the following is correct for the dipole?



- (a) Dipole moves along positive  $x$ -axis and undergoes a clockwise rotation  
 (b) Dipole moves along negative  $x$ -axis and undergoes a clockwise rotation  
 (c) Dipole moves along positive  $x$ -axis and undergoes an anticlockwise rotation  
 (d) Dipole moves along negative  $x$ -axis and undergoes an anticlockwise rotation

26. The electric dipole is situated in an electric field as shown in fig 1. The dipole and electric field are both in the plane of the paper. The dipole is rotated about an axis perpendicular to plane of paper passing through A in anticlockwise direction. If the angle of rotation ( $\theta$ ) is measured with respect to the direction of electric field, then the torque ( $\tau$ ) experienced by the dipole for different values of the angle of rotation  $\theta$  will be represented in fig. 2, by

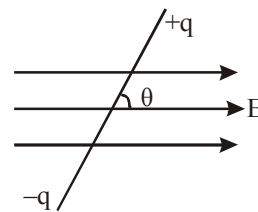


Fig. 1

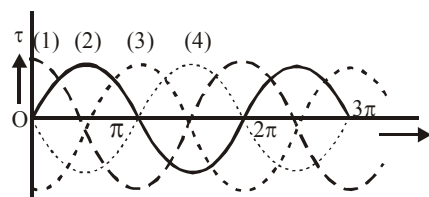


Fig. 2

- (a) curve (1) (b) curve (2)  
 (c) curve (3) (d) curve (4)

27. Force between two identical charges placed at a distance of  $r$  in vacuum is  $F$ . Now a slab of dielectric of dielectric constant 4 is inserted between these two charges. If the thickness of the slab is  $r/2$ , then the force between the charges will become

- (a)  $F$  (b)  $\frac{3}{5}F$   
(c)  $\frac{4}{9}F$  (d)  $\frac{F}{2}$

28. A charge  $+q$  is at a distance  $L/2$  above a square of side  $L$ . Then what is the flux linked with the surface?

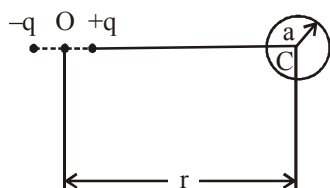
- (a)  $\frac{q}{4\epsilon_0}$  (b)  $\frac{2q}{3\epsilon_0}$   
(c)  $\frac{q}{6\epsilon_0}$  (d)  $\frac{6q}{\epsilon_0}$

29. Positive and negative point charges of equal magnitude

are kept at  $\left(0, 0, \frac{a}{2}\right)$  and  $\left(0, 0, -\frac{a}{2}\right)$  respectively. The work done by the electric field when another positive point charge is moved from  $(-a, 0, 0)$  to  $(0, a, 0)$  is

- (a) positive  
(b) negative  
(c) zero  
(d) depends on the path connecting the initial and final positions

30. A short electric dipole of dipole moment  $\vec{p}$  is placed at a distance  $r$  from the centre of a solid metallic sphere of radius  $a$  ( $a \ll r$ ) as shown in the figure. The electric field intensity at the centre of sphere  $C$  due to induced charge on the sphere is

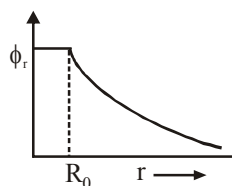


- (a) zero (b)  $\frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$  along CO  
(c)  $\frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$  along OC (d)  $\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$  along CO

31. The electrostatic potential ( $\phi_r$ ) of a spherical symmetric system kept at origin, is shown in the figure, and given as

$$\phi_r = \frac{q}{4\pi\epsilon_0 r} \quad (r \geq R_0)$$

$$\phi_r = \frac{q}{4\pi\epsilon_0 R_0} \quad (r \leq R_0)$$



Which of the following option(s) is/are incorrect

- (a) For spherical region  $r \leq R_0$  total electrostatic energy stored is zero

- (b) Within  $r = 2R_0$  total charge is  $q$ .

- (c) There will be no charge anywhere except at  $r = R_0$

- (d) None of these

32. A solid spherical conductor of radius  $R$  has a spherical cavity of radius  $a$  ( $a < R$ ) at its centre. A charge  $+Q$  is kept at the centre. The charge at the inner surface, outer and at a position  $r$  ( $a < r < R$ ) are respectively

- (a)  $+Q, -Q, 0$  (b)  $-Q, +Q, 0$   
(c)  $0, -Q, 0$  (d)  $+Q, 0, 0$

33. A glass rod rubbed with silk is used to charge a gold leaf electroscope and the leaves are observed to diverge. The electroscope thus charged is exposed to X-rays for a short period. Then

- (a) the divergence of leaves will not be affected  
(b) the leaves will diverge further  
(c) the leaves will collapse  
(d) the leaves will melt

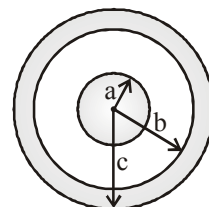
34. A solid conducting sphere of radius  $a$  has a net positive charge  $2Q$ . A conducting spherical shell of inner radius  $b$  and outer radius  $c$  is concentric with the solid sphere and has a net charge  $-Q$ . The surface charge density on the inner and outer surfaces of the spherical shell will be

(a)  $-\frac{2Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$

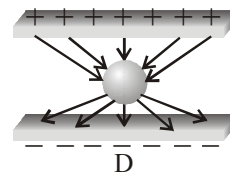
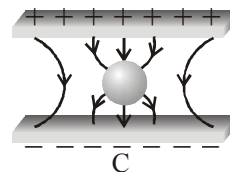
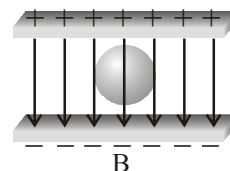
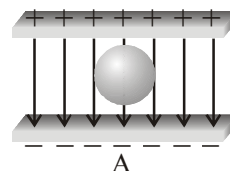
(b)  $-\frac{Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$

(c)  $0, \frac{Q}{4\pi c^2}$

- (d) None of these



35. An uncharged sphere of metal is placed in between charged plates as shown. The lines of force look like



- (a) A  
(c) C

- (b) B  
(d) D

36. If a charge  $q$  is placed at the centre of the line joining two equal charges  $Q$  such that the system is in equilibrium then the value of  $q$  is

- (a)  $Q/2$  (b)  $-Q/2$   
(c)  $Q/4$  (d)  $-Q/4$

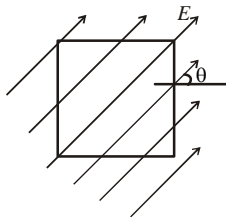
37. The electric intensity due to a dipole of length 10 cm and having a charge of  $500 \mu\text{C}$ , at a point on the axis at a distance 20 cm from one of the charges in air, is

- (a)  $6.25 \times 10^7 \text{ N/C}$  (b)  $9.28 \times 10^7 \text{ N/C}$   
(c)  $13.1 \times 10^{11} \text{ N/C}$  (d)  $20.5 \times 10^7 \text{ N/C}$

38. Two positive ions, each carrying a charge  $q$ , are separated by a distance  $d$ . If  $F$  is the force of repulsion between the ions, the number of electrons missing from each ion will be ( $e$  being the charge of an electron)

- (a)  $\frac{4\pi\epsilon_0 F d^2}{e^2}$  (b)  $\sqrt{\frac{4\pi\epsilon_0 F e^2}{d^2}}$   
(c)  $\sqrt{\frac{4\pi\epsilon_0 F d^2}{e^2}}$  (d)  $\frac{4\pi\epsilon_0 F d^2}{q^2}$

39. A square surface of side  $L$  meter in the plane of the paper is placed in a uniform electric field  $E$  (volt/m) acting along the same plane at an angle  $\theta$  with the horizontal side of the square as shown in Figure. The electric flux linked to the surface, in units of volt.  $m$ , is



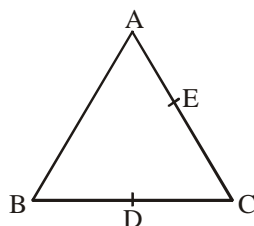
- (a)  $EL^2$  (b)  $EL^2 \cos \theta$   
(c)  $EL^2 \sin \theta$  (d) zero

40. The electric potential  $V$  at any point  $(x, y, z)$ , all in meters in space is given by  $V = 4x^2$  volt. The electric field at the point  $(1, 0, 2)$  in volt/meter is

- (a) 8 along positive X-axis (b) 16 along negative X-axis  
(c) 16 along positive X-axis (d) 8 along negative X-axis

41. Three charges, each  $+q$ , are placed at the corners of an isosceles triangle ABC of sides BC and AC,  $2a$ . D and E are the mid points of BC and CA. The work done in taking a charge  $Q$  from D to E is

- (a)  $\frac{3qQ}{8\pi\epsilon_0 a}$   
(b)  $\frac{qQ}{4\pi\epsilon_0 a}$   
(c) zero  
(d)  $\frac{3qQ}{4\pi\epsilon_0 a}$



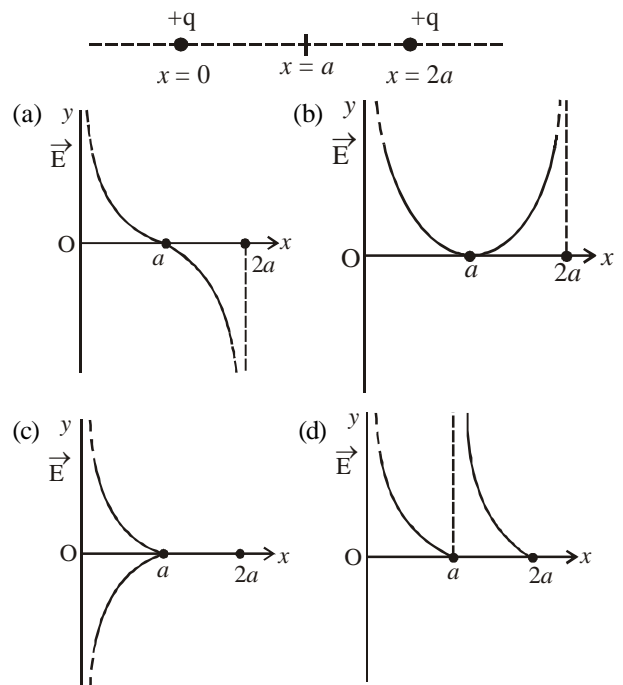
42. Two metallic spheres of radii 1 cm and 3 cm are given charges of  $-1 \times 10^{-2} \text{ C}$  and  $5 \times 10^{-2} \text{ C}$ , respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is

- (a)  $2 \times 10^{-2} \text{ C}$  (b)  $3 \times 10^{-2} \text{ C}$   
(c)  $4 \times 10^{-2} \text{ C}$  (d)  $1 \times 10^{-2} \text{ C}$

43. An electric dipole of moment ' $p$ ' is placed in an electric field of intensity ' $E$ '. The dipole acquires a position such that the axis of the dipole makes an angle  $\theta$  with the direction of the field. Assuming that the potential energy of the dipole to be zero when  $\theta = 90^\circ$ , the torque and the potential energy of the dipole will respectively be

- (a)  $p E \sin \theta, -p E \cos \theta$  (b)  $p E \sin \theta, -2 p E \cos \theta$   
(c)  $p E \sin \theta, 2 p E \cos \theta$  (d)  $p E \cos \theta, -p E \cos \theta$

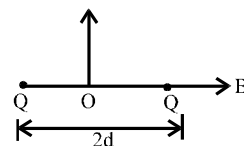
44. Figure shows two charges of equal magnitude separated by a distance  $2a$ . As we move away from the charge situated at  $x = 0$  to the charge situated at  $x = 2a$ , which of the following graphs shows the correct behaviour of electric field ?



**DIRECTIONS (for Qs. 45 to 50) :** Each question contains **STATEMENT-1** and **STATEMENT-2**. Choose the correct answer (**ONLY ONE** option is correct) from the following-

- (a) Statement-1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1  
(b) Statement-1 is true, Statement-2 is true; Statement -2 is NOT a correct explanation for Statement-1  
(c) Statement-1 is true, Statement-2 is false  
(d) Statement-1 is false, Statement-2 is true

45. **Statement-1 :** Consider two identical charges placed distance  $2d$  apart, along  $x$ -axis.



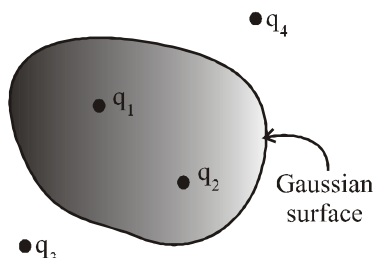
The equilibrium of a positive test charge placed at the point O midway between them is stable for displacements along the  $x$ -axis.

**Statement-2:** Force on test charge is zero.



46. **Statement-1** : A deuteron and an  $\alpha$ -particle are placed in an electric field. If  $F_1$  and  $F_2$  be the forces acting on them and  $a_1$  and  $a_2$  be their accelerations respectively then,  $a_1 = a_2$ .  
**Statement-2** : Forces will be same in electric field.

47. **Statement-1** : Four point charges  $q_1, q_2, q_3$  and  $q_4$  are as shown in figure. The flux over the shown Gaussian surface depends only on charges  $q_1$  and  $q_2$ .



**Statement-2** : Electric field at all points on Gaussian surface depends only on charges  $q_1$  and  $q_2$ .

48. **Statement-1** : The positive charge particle is placed in front of a spherical uncharged conductor. The number of lines of forces terminating on the sphere will be more than those emerging from it.

**Statement-2** : The surface charge density at a point on the sphere nearest to the point charge will be negative and maximum in magnitude compared to other points on the sphere.

49. **Statement-1** : A uniformly charged disc has a pin hole at its centre. The electric field at the centre of the disc is zero.

**Statement-2** : Disc can be supposed to be made up of many rings. Also electric field at the centre of uniformly charged ring is zero.

50. **Statement-1** : When a conductor is placed in an external electrostatic field, the net electric field inside the conductor becomes zero after a small instant of time.

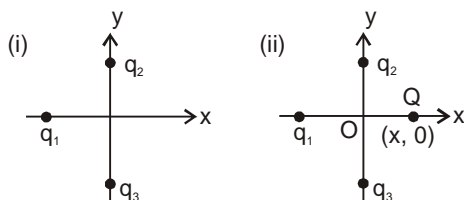
**Statement-2** : It is not possible to set up an electric field inside a conductor.

## EXERCISE - 3

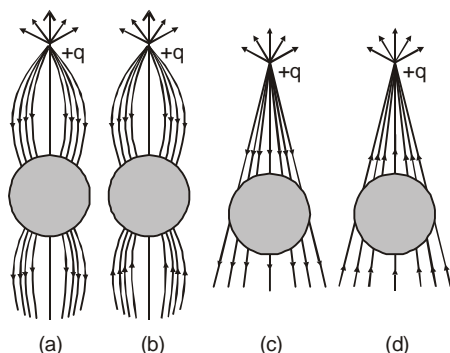
### Exemplar & Past Years NEET/AIPMT Questions

#### Exemplar Questions

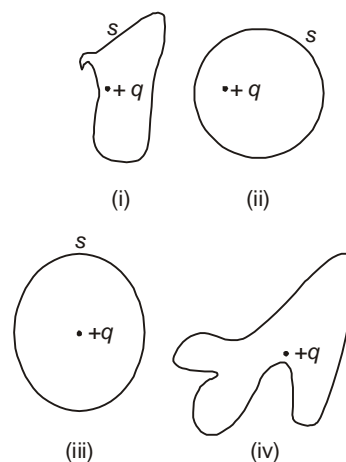
1. In figure two positive charges  $q_2$  and  $q_3$  fixed along the y-axis, exert a net electric force in the + x-direction on a charge  $q_1$  fixed along the x-axis. If a positive charge  $Q$  is added at  $(x, 0)$ , the force on  $q_1$



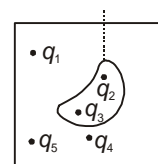
- (a) shall increase along the positive x-axis  
 (b) shall decrease along the positive x-axis  
 (c) shall point along the negative x-axis  
 (d) shall increase but the direction changes because of the intersection of  $Q$  with  $q_2$  and  $q_3$
2. A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by



3. The electric flux through the surface

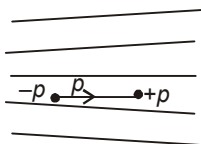


- (a) in Fig. (iv) is the largest  
 (b) in Fig. (iii) is the least  
 (c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)  
 (d) is the same for all the figures
4. Five charges  $q_1, q_2, q_3, q_4$ , and  $q_5$  are fixed at their positions as shown in Figure,  $S$  is a Gaussian surface. The Gauss' law is given by  $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$ . Which of the following statements is correct?



- (a) E on the LHS of the above equation will have a contribution from  $q_1$ ,  $q_5$  and  $q_1$ ,  $q_5$  and  $q_3$  while q on the RHS will have a contribution from  $q_2$  and  $q_4$  only
- (b) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from  $q_2$  and  $q_4$  only
- (c) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from  $q_1$ ,  $q_3$  and  $q_5$ .
- (d) Both E on the LHS and q on the RHS will have contributions from  $q_2$  and  $q_4$  only

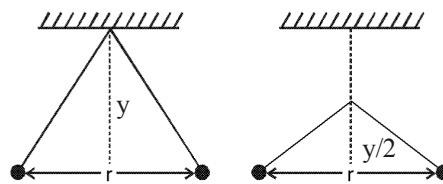
5. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct?



- (a) The dipole will not experience any force
- (b) The dipole will experience a force towards right
- (c) The dipole will experience a force towards left
- (d) The dipole will experience a force upwards
6. A point charge +q is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is
- (a) directed perpendicular to the plane and away from the plane
- (b) directed perpendicular to the plane but towards the plane
- (c) directed radially away from the point charge
- (d) directed radially towards the point charge.
7. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed
- (a) perpendicular to the diameter
- (b) parallel to the diameter
- (c) at an angle tilted towards the diameter
- (d) at an angle tilted away from the diameter

#### NEET/AIPMT (2013-2017) Questions

8. Two pith balls carrying equal charges are suspended from a common point by strings of equal length. The equilibrium separation between them is r. Now the strings are rigidly clamped at half the height. The equilibrium separation between the balls now become [2013]



- (a)  $\left(\frac{r}{\sqrt[3]{2}}\right)$
- (b)  $\left(\frac{2r}{\sqrt{3}}\right)$
- (c)  $\left(\frac{2r}{3}\right)$
- (d)  $\left(\frac{r}{\sqrt{2}}\right)^2$

9. A charge 'q' is placed at the centre of the line joining two equal charges 'Q'. The system of the three charges will be in equilibrium if 'q' is equal to [NEET Kar. 2013]

- (a)  $Q/2$
- (b)  $-Q/4$
- (c)  $Q/4$
- (d)  $-Q/2$

10. An electric dipole of dipole moment p is aligned parallel to a uniform electric field E. The energy required to rotate the dipole by  $90^\circ$  is [NEET Kar. 2013]

- (a)  $pE^2$
- (b)  $p^2E$
- (c)  $pE$
- (d) infinity

11. The electric field in a certain region is acting radially outward and is given by  $E = Aa$ . A charge contained in a sphere of radius 'a' centred at the origin of the field, will be given by [2015]

- (a)  $A \epsilon_0 a^2$
- (b)  $4 \pi \epsilon_0 Aa^3$
- (c)  $\epsilon_0 Aa^3$
- (d)  $4 \pi \epsilon_0 Aa^2$

12. Two identical charged spheres suspended from a common point by two massless strings of lengths l, are initially at a distance d ( $d \ll l$ ) apart because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. Then v varies as a function of the distance x between the spheres, as : [2016]

- (a)  $v \propto x^{\frac{1}{2}}$
- (b)  $v \propto x$
- (c)  $v \propto x^{-\frac{1}{2}}$
- (d)  $v \propto x^{-1}$

13. Suppose the charge of a proton and an electron differ slightly. One of them is  $-e$ , the other is  $(e + \Delta e)$ . If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then  $\Delta e$  is of the order of [Given mass of hydrogen  $m_h = 1.67 \times 10^{-27}$  kg] [2017]

- (a)  $10^{-23}$  C
- (b)  $10^{-37}$  C
- (c)  $10^{-47}$  C
- (d)  $10^{-20}$  C

# Hints & Solutions

## EXERCISE - 1

1. (b) As new distance = 2 r and electric field due to single

$$\text{charge, } E \propto \frac{1}{r^2},$$

therefore, new intensity = E/4.

2. (d) According to Gauss's theorem,

$$\therefore E = \frac{q/4\pi R^2}{\epsilon_0} \quad [\because q/4\pi R^2 = \sigma]$$

$$\text{or } E = \sigma/\epsilon_0$$

3. (a) Electric field  $\vec{E}$  is along the dipole axis.

$$\therefore \theta = 0^\circ.$$

4. (c)  $\phi = E(ds) \cos \theta = E(2\pi r^2) \cos 0^\circ = 2\pi r^2 E.$

5. (a) For the curved surface,  $\theta = 90^\circ$

$$\therefore \phi = E ds \cos 90^\circ = 0.$$

6. (c)  $U_p = -\vec{p} \cdot \vec{E} = -pE \cos \theta$

$$(U_p)_{\text{minimum}} = -pE, \text{ when } \theta = 0^\circ$$

7. (d) According to Gauss' Law

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\text{enclosed by closed surface}}}{\epsilon_0} = \text{flux}$$

$$\text{so total flux} = Q/\epsilon_0$$

Since cube has six face, so flux coming out through one wall or one face is  $Q/6\epsilon_0$ .

8. (b) To overcome electrostatic repulsion forces, work will have to be done by external agency.

9. (a)

10. (b) Nuclear force binds the protons and neutrons in the nucleus of an atom.

11. (c) Electric field intensity due to thin infinite plane sheet of charge,

$$E = \frac{\sigma}{2\epsilon} = \frac{\sigma}{2\epsilon_0 K}$$

$$[\text{Where dielectric constant, } K = \frac{\epsilon}{\epsilon_0}]$$

12. (d) Electric displacement vector,  $\vec{D} = \epsilon \vec{E}$

$$\text{As, } \epsilon = \epsilon_0 K \quad \therefore \vec{D} = \epsilon_0 K \vec{E}$$

13. (d) As  $\sigma_1 = \sigma_2$

$$\therefore \frac{Q_1}{4\pi r_1^2} = \frac{Q_2}{4\pi r_2^2} \text{ or } \frac{Q_1}{4\pi \epsilon_0 r_1^2} = \frac{Q_2}{4\pi \epsilon_0 r_2^2}$$

$$\therefore E_1 = E_2 \text{ or } E_1/E_2 = 1 \Rightarrow E_1:E_2 = 1:1$$

14. (a) The flux is zero according to Gauss' Law because it is a open surface which enclosed a charge q.

$$15. (a) E = E_1 - E_2 = \frac{\sigma}{2\epsilon_0} - \left( \frac{-\sigma}{2\epsilon_0} \right) = \sigma/\epsilon_0$$

The field intensity in between sheets having equal and opposite uniform surface densities of charge becomes constant. ie, an uniform electric field is produced and it is independent of the distance between the sheets.

16. (c) The field due to infinite linear charge distribution

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} \Rightarrow E \propto \frac{1}{r} \quad \text{So hyperbola.}$$

17. (b) **Given :** Dipole moment of the dipole =  $\vec{p}$  and uniform

electric field =  $\vec{E}$ . Torque ( $\tau$ ) = Either force  $\times$  perpendicular distance between the two forces =  $qaE \sin \theta$  or  $\tau = pE \sin \theta$  or  $\vec{\tau} = \vec{p} \times \vec{E}$  (vector form)

18. (c) By Gauss's theorem,  $\phi = \frac{Q_{\text{in}}}{\epsilon_0}$

Thus, the net flux depends only on the charge enclosed by the surface. Hence, there will be no effect on the net flux if the radius of the surface is doubled.

19. (c) Inside a charged conducting surface  $E = 0$ , but on or outside the surface  $E \neq 0$ .

$\therefore$  Electric intensity is discontinuous across a charged conducting surface.

20. (b) The dielectric constant for metal is infinity, the force between the two charges would be reduced to zero.

21. (a) In air,  $F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$$\text{In medium, } F_m = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{K r^2}$$

$$\therefore \frac{F_m}{F_{\text{air}}} = \frac{1}{K} \Rightarrow F_m = \frac{F_{\text{air}}}{K} \text{ (decreases K-times)}$$

22. (d)  $E = \frac{p}{4\pi\epsilon_0 r^3}$

$$\text{Apparently, } E \propto p \text{ and } E \propto \frac{1}{r^3} \propto r^{-3}.$$

23. (c) K.E. acquired = work done  
= force  $\times$  distance =  $qE \times y = qEy$

24. (c) Intensity of electric field due to a Dipole

$$E = \frac{p}{4\pi\epsilon_0 r^3} \sqrt{3\cos^2\theta + 1} \Rightarrow E \propto \frac{1}{r^3}$$

25. (a) Dipole is formed when two equal and unlike charges are placed at a short distance.

### EXERCISE - 2

1. (a)  $-eE = mg$

$$\bar{E} = -\frac{9.1 \times 10^{-31} \times 10}{1.6 \times 10^{-19}} = -5.6 \times 10^{-11} \text{ N/C}$$

2. (b)  $E = \frac{kQ}{r^2} \Rightarrow Q = \frac{E \times r^2}{k} = \frac{3 \times 10^6 \times (2.5)^2}{9 \times 10^9}$   
 $= 2 \times 10^{-3} \text{ C}$

3. (d) Work done,  $W = F s \cos\theta = (qE) s \cos\theta$

4. (a) Given : Electric field ( $E$ ) = 500 V/m  
and potential difference ( $V$ ) = 3000 V.  
We know that electric field

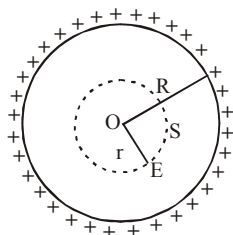
$$(E) = 500 = \frac{V}{d} \text{ or } d = \frac{3000}{500} = 6 \text{ m}$$

[where  $d$  = Distance between point charge and A]

5. (b)  $E = \frac{q}{4\pi\epsilon_0 r^2} \Rightarrow Ar = \frac{q}{4\pi\epsilon_0 r^2} \Rightarrow q = 4\pi\epsilon_0 Ar^3$

6. (a) Charge resides on the outer surface of a conducting hollow sphere of radius  $R$ . We consider a spherical surface of radius  $r < R$ .

By Gauss theorem

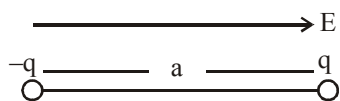


$$\int_s \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \times \text{charge enclosed or } E \times 4\pi r^2 = \frac{1}{\epsilon_0} \times 0$$

$$\Rightarrow E = 0$$

i.e electric field inside a hollow sphere is zero.

7. (d) When the dipole is in the direction of field then net force is  $qE + (-qE) = 0$



and its potential energy is minimum  $= -p \cdot E$

$$= -qaE$$

8. (b)  $\tau = (2.57 \times 10^{-17} \text{ Cm}) \left( 3.0 \times 10^4 \frac{\text{N}}{\text{C}} \right) \left( \frac{1}{2} \right)$   
 $= 3.855 \times 10^{-13} \text{ Nm}$

9. (d) Torque,  $\vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$

$$4 = p \times 2 \times 10^5 \times \sin 30^\circ$$

$$\text{or, } p = \frac{4}{2 \times 10^5 \times \sin 30^\circ} = 4 \times 10^{-5} \text{ Cm}$$

Dipole moment,  $p = q \times l$

$$q = \frac{p}{l} = \frac{4 \times 10^{-5}}{0.02} = 2 \times 10^{-3} \text{ C} = 2 \text{ mC}$$

10. (a) Let  $q$  and  $q'$  be the charges on spheres of radii  $R$  and  $2R$  respectively.

$$\text{Given } q + q' = Q \quad \dots(i)$$

Surface charge densities are

$$\sigma = \frac{q}{4\pi R^2} \text{ and } \sigma = \frac{q'}{4\pi(2R)^2}$$

Given  $\sigma = \sigma'$

$$\therefore \frac{q}{4\pi R^2} = \frac{q'}{4\pi(2R)^2} \text{ or, } q' = 4q$$

$$\text{From eq. (i), } q' = Q - q \text{ or, } 4q = Q - q$$

$$\text{or, } Q = 5q \quad \dots(ii)$$

$$\therefore q' = Q - q = Q - \frac{Q}{5} = \frac{4Q}{5}$$

11. (a)  $E_p = 0$

$$\Rightarrow \frac{4}{x^2} = \frac{9}{(20-x)^2} \Rightarrow \frac{20-x}{x} = \frac{3}{2}$$

$$\Rightarrow 40 - 2x = 3x \Rightarrow x = 8 \text{ cm}$$

12. (a)  $(F_{\text{net}})_q = 0$

$$\Rightarrow k \frac{Qq}{\left(\frac{\ell}{2}\right)^2} + k \frac{4q^2}{\ell^2} = 0$$



$$\text{where } k = \frac{1}{4\pi\epsilon_0}$$

$$\Rightarrow 4Qq + 4q^2 = 0 \Rightarrow Q = -q$$

13. (c) The charge on disc A is  $10^{-6} \mu\text{C}$ . The charge on disc B is  $10 \times 10^{-6} \mu\text{C}$ . The total charge on both =  $11 \mu\text{C}$ . When touched, this charge will be distributed equally i.e.  $5.5 \mu\text{C}$  on each disc.

14. (d) They will not experience any force if  $|\vec{F}_G| = |\vec{F}_e|$

$$\Rightarrow G \frac{m^2}{(16 \times 10^{-2})^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(16 \times 10^{-2})^2} \Rightarrow \frac{q}{m} = \sqrt{4\pi\epsilon_0 G}$$

15. (a) Since lines of force starts from A and ends at B, so A is +ve and B is -ve. Lines of forces are more crowded near A, so  $A > B$ .
16. (a) Force due to electric field ( $F = qE$ ) acts upwards.
17. (d) Unit positive charge at O will be repelled equally by three charges at the three corners of triangle.

By symmetry, resultant  $\vec{E}$  at O would be zero.

18. (b) From figure,  $\theta = \frac{\pi}{3} + \alpha$ , where

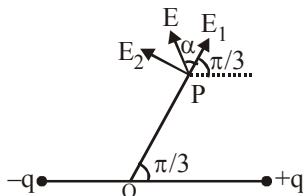
$$\tan \alpha = \frac{E_2}{E_1} = \left( \frac{p \sin \theta}{4\pi\epsilon_0 r^3} \right) \left( \frac{4\pi\epsilon_0 r^3}{2p \cos \theta} \right) = \frac{1}{2} \tan \theta$$

$$\tan \left( \theta - \frac{\pi}{3} \right) = \frac{1}{2} \tan \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\text{or } \theta - \frac{\pi}{3} = \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$$

$$\text{or } \alpha = \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$$

$$\text{or } \theta = \frac{\pi}{3} + \tan^{-1} \left( \frac{\sqrt{3}}{2} \right)$$



19. (d) 20. (b) 21. (d)
22. (a) For a soap bubble (electrified)

$$\text{excess pressure} = P = P_i - P_0 = \frac{4T}{r}$$

$T$  = surface tension,  $r$  = radius.

Force due to excess pressure balances surface tension.

When bubble is charged

$$P_{\text{excess}} = P_{\text{electrostatic}} + \frac{4T}{r}$$

$$P_{\text{electrostatic}} = -\frac{\sigma^2}{2\epsilon_0} = -\frac{1}{2\epsilon_0} \frac{q^2}{16\pi^2 r^4}$$

$$P_{\text{excess}} = \frac{4}{r} \left[ T - \frac{q^2}{128\pi^2 \epsilon_0 r^3} \right] \dots\dots (A)$$

Surface tension decreases after electrification of

bubble and therefore pressure decreases.  $\left( P \propto \frac{1}{r} \right)$

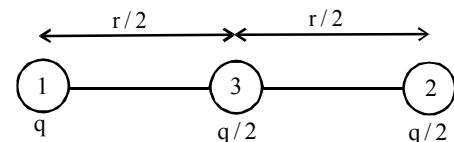
means radius increases. In equation (A),  $q^2$  is not affected by positive or negative charge hence, whether it is given a positive or a negative charge it always expands in radius.

23. (c) Initial force between the two spheres carrying charge (say  $q$ ) is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \quad (r \text{ is the distance between them})$$

Further when an uncharged sphere is kept in touch with the sphere of charge  $q$ , the net charge on both

become  $\frac{q+0}{2} = \frac{q}{2}$ . Force on the 3rd charge, when placed at center of the 1st two



$$F_3 = \frac{1}{4\pi\epsilon_0} \frac{q \left( \frac{q}{2} \right)}{\left( \frac{r}{2} \right)^2} - \frac{1}{4\pi\epsilon_0} \frac{\left( \frac{q}{2} \right)^2}{\left( \frac{r}{2} \right)^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} [2 - 1] = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = F$$

24. (b) The dipole experiences a torque  $pE \sin \theta$  tending to bring itself back in the direction of field.

Therefore, on being released (i.e. rotated) the dipole oscillates about an axis through its centre of mass and perpendicular to the field. If  $I$  is the moment of inertia of the dipole about the axis of rotation, then the

equation of motion is  $I d^2\theta / dt^2 = -pE \sin \theta$

For small amplitude  $\sin \theta \approx \theta$

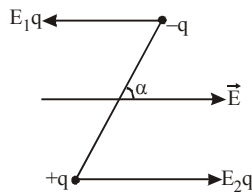
Thus  $d^2\theta / dt^2 = -(pE / I) \cdot \theta = -\omega^2 \theta$

where  $\omega = \sqrt{(pE / I)}$ .

This is a S.H.M., whose period of oscillation is

$$T = 2\pi / \omega = 2\pi \sqrt{(I / pE)}$$

25. (d) The dipole is placed in a non-uniform field, therefore a force as well as a couple acts on it. The force on the negative charge is more ( $F \propto E$ ) and is directed along negative x-axis. Thus the dipole moves along negative x-axis and rotates in an anticlockwise direction.



26. (b)  $\tau = pE \sin \theta$ , this is given by the second curve.

27. (c) In vacuum,  $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$  ... (i)

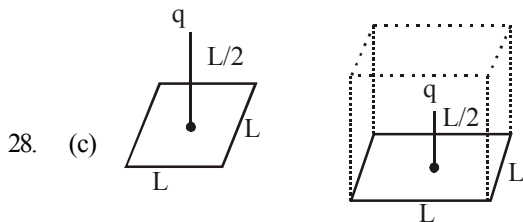
Suppose, force between the charges is same when charges are  $r'$  distance apart in dielectric.

$$\therefore F' = \frac{1}{4\pi\epsilon_0} \frac{q^2}{kr^2} \quad \dots (ii)$$

From (i) and (ii),  $kr^2 = r'^2$  or,  $r = \sqrt{kr'}$

In the given situation, force between the charges would be

$$F' = \frac{1}{4\pi\epsilon_0} \frac{q^2}{\left(\frac{r}{2} + \sqrt{4}\frac{r}{2}\right)^2} = \frac{4}{9} \frac{q^2}{4\pi\epsilon_0 r^2} = \frac{4F}{9}$$



28. (c)

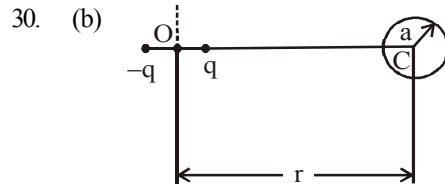
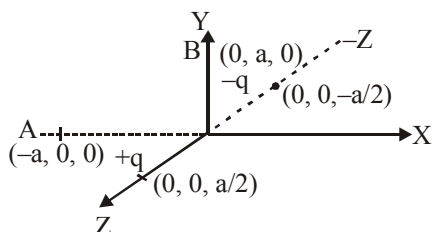
The given square of side  $L$  may be considered as one of the faces of a cube with edge  $L$ . Then given charge  $q$  will be considered to be placed at the centre of the cube. Then according to Gauss's theorem, the magnitude of the electric flux through the faces (six) of the cube is given by

$$\phi = q/\epsilon_0$$

Hence, electric flux through one face of the cube for the given square will be

$$\phi' = \frac{1}{6} \phi = \frac{q}{6\epsilon_0}$$

29. (c) The charges make an electric dipole. A and B points lie on the equatorial plane of the dipole. Therefore potential at A = potential at B = 0  
 $W = Q(V_A - V_B) = q \times 0 = 0$



30. (b)

Electric field at C due to electric dipole

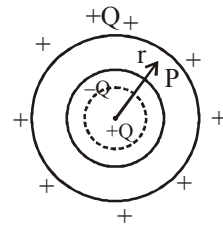
$$= \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \text{ along OC}$$

Electric field at C due to induced charge must be equal and opposite to electric field due to dipole as net field at C is zero.

31. (a, b, c, d)

The potential shown is for charged spherical conductor.

32. (b) The charge at the inner surface, outer surface and inside the conductor at  $P = (-Q, +Q, 0)$  as shown in the figure

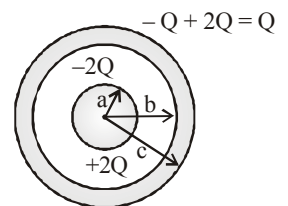


33. (b) Charge on glass rod is positive, so charge on gold leaves will also be positive. Due to X-rays, more electrons from leaves will be emitted, so leaves become more positive and diverge further.

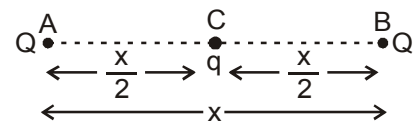
34. (a) Surface charge density ( $\sigma$ ) =  $\frac{\text{Charge}}{\text{Surface area}}$

$$\text{So } \sigma_{\text{inner}} = \frac{-2Q}{4\pi b^2}$$

$$\text{and } \sigma_{\text{Outer}} = \frac{Q}{4\pi c^2}$$



35. (c) Electric lines of force never intersect the conductor. They are perpendicular and slightly curved near the surface of conductor.
36. (d) Let  $q$  charge is situated at the mid position of the line AB. The distance between AB is  $x$ . A and B be the positions of charges  $Q$  and  $Q$  respectively.



$$\text{Let } AC = \frac{x}{2}, BC = \frac{x}{2}$$



The force on A due to charge q at C,

$$\vec{F}_{CA} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{(x/2)^2} \text{ along } \vec{AC}$$

The force on A due to charge Q at B

$$\vec{F}_{AB} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{x^2} \text{ along } \vec{BA}$$

The system is in equilibrium, then two oppositely directed force must be equal, i.e., total force on A is equal to zero.

$$\vec{F}_{CA} + \vec{F}_{AB} = 0 \Rightarrow \vec{F}_{CA} = -\vec{F}_{AB}$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{4Q \cdot q}{x^2} = \frac{-1}{4\pi\epsilon_0} \cdot \frac{Q^2}{x^2} \Rightarrow q = -\frac{Q}{4}$$

37. (a) **Given :** Length of the dipole ( $2l$ ) = 10cm = 0.1m or  $l = 0.05$  m

Charge on the dipole ( $q$ ) = 500  $\mu$ C =  $500 \times 10^{-6}$  C and distance of the point on the axis from the mid-point of the dipole ( $r$ ) = 20 + 5 = 25 cm = 0.25 m.

We know that the electric field intensity due to dipole on the given point (E)

$$= \frac{1}{4\pi\epsilon_0} \times \frac{2(q \cdot 2l)r}{(r^2 - l^2)^2}$$

$$= 9 \times 10^9 \times \frac{2(500 \times 10^{-6} \times 0.1) \times 0.25}{[(0.25)^2 - (0.05)^2]^2}$$

$$= \frac{225 \times 10^3}{3.6 \times 10^{-3}} = 6.25 \times 10^7 \text{ N/C (k=1 for air)}$$

38. (c) Let  $n$  be the number of electrons missing.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{d^2} \Rightarrow q = \sqrt{4\pi\epsilon_0 d^2 F} = ne$$

$$\therefore n = \sqrt{\frac{4\pi\epsilon_0 F d^2}{e^2}}$$

39. (d) Electric flux,  $\phi = EA \cos \theta$ , where  $\theta$  = angle between  $E$  and normal to the surface.

$$\text{Here } \theta = \frac{\pi}{2}$$

$$\Rightarrow \phi = 0$$

40. (d)  $\vec{E} = -\left[\frac{dv}{dx}\hat{i} + \frac{dv}{dy}\hat{j} + \frac{dv}{dz}\hat{k}\right] = -8 \times \hat{i} \text{ volt/meter}$

$$\therefore \vec{E}_{(1,0,2)} = -8\hat{i} \text{ V/m}$$

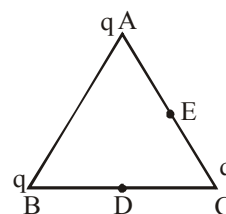
41. (c)  $AC = BC$

$$V_D = V_E$$

We have,

$$W = Q(V_E - V_D)$$

$$\Rightarrow W = 0$$



42. (b) At equilibrium potential of both sphere becomes same if charge of sp here one  $x$  and other sphere  $Q - x$  then

$$\text{where } Q = 4 \times 10^{-2} \text{ C}$$

$$v_1 = v_2$$

$$\frac{kx}{1 \text{ cm}} = \frac{k(Q - x)}{3 \text{ cm}}$$

$$3x = Q - x \Rightarrow 4x = Q$$

$$x = \frac{Q}{4} = \frac{4 \times 10^{-2}}{4} \text{ C} = 1 \times 10^{-2}$$

$$Q' = Q - x = 3 \times 10^{-2} \text{ C}$$

43. (a) The torque on the dipole is given as

$$\tau = PE \sin \theta$$

The potential energy of the dipole in the electric field is given as

$$U = -PE \cos \theta$$

44. (a) For the distances close to the charge at  $x = 0$  the field is very high and is in positive direction of  $x$ -axis. As we move towards the other charge the net electric field becomes zero at  $x = a$  thereafter the influence of charge at  $x = 2a$  dominates and net field increases in negative direction of  $x$ -axis and grows unboundedly as we come closer and closer to the charge at  $x = 2a$ .

45. (b) If +ve charge is displaced along  $x$ -axis, then net force will always act in a direction opposite to that of displacement and the test charge will always come back to its original position.

46. (c)  $q_d = e$ ,  $m_d = 2m_p = 2m$

$$q_\alpha = 2e$$

$$m_\alpha = 4m_p = 4m$$

$$F_1 = F_\alpha = eE$$

$$F_2 = F_\alpha = 2eE \neq F_1$$

$$\text{Further, } a_1 = \frac{F_1}{2m} = \frac{eE}{2m}$$

$$\text{and } a_2 = \frac{F_2}{2m} = \frac{2eE}{4m} = \frac{eE}{2m} = a_1$$

47. (c) Electric field at any point depends on presence of all charges.

48. (d) No. of lines entering the surface = No. of lines leaving the surface.

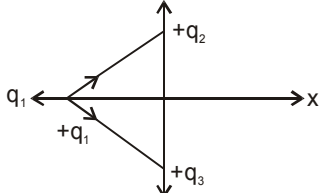
49. (a) The electric field due to disc is superposition of electric field due to its constituent ring as given in statement-2. Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1.

50. (c) Statement-1 is correct. The induced field cancels the external field. Statement-2 is false. When a current is set up in a conductor, there exists an electric field inside it.

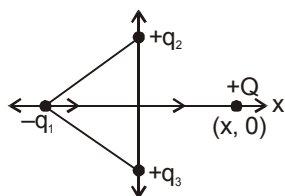
## EXERCISE - 3

### Exemplar Questions

1. (a) The force on  $q_1$  depend on the force acting between  $q_1$  and  $q_2$  and  $q_1$  and  $q_3$  so that the net force acting on  $q_1$  by  $q_2$  and  $q_1$  by  $q_3$  is along the + x-direction, so the force acting between  $q_1$ ,  $q_2$  and  $q_1$ ,  $q_3$  is attractive force as shown in figure :



The attractive force between these charges states that  $q_1$  is a negative charge (since,  $q_2$  and  $q_3$  are positive). Then the force acting between  $q_1$  and charge  $Q$  (positive) is also known as attractive force and then the net force on  $q_1$  by  $q_2$ ,  $q_3$  and  $Q$  are along the same direction as shown in the figure.

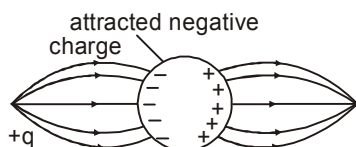


The figure shows that the force on  $q_1$  shall increase along the positive x-axis due to the positive charge  $Q$ .

2. (a) If a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive charge and electric field passes through a charged body. This leaves an excess of positive charge on the (right) surface of sphere due to the induction process.

Both type of charges are bound in the (isolated conducting) sphere and cannot escape. They, therefore, reside on the surface.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as shown in figure.



An electric field lines start from positive charge and ends at negative charge.

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity shown in figure (a).

3. (d) **By Gauss's law :** The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity i.e.,  $\phi = \frac{Q}{\epsilon_0}$ .

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface. So all the given figures have same electric flux as all of them also has same single positive charge.

4. (b) Gauss's law states that total electric flux of an enclosed surface is given by,  $\oint_s E \cdot dS = \frac{q}{\epsilon_0}$ , includes the sum of all charges enclosed by the surface.

The charges may be located anywhere inside the surface, and outside the surface. Then, the electric field on the left side of equation is due to all the charges, both inside and outside  $S$ .

So,  $E$  on LHS of the above equation will have a contribution from all charges while  $q$  on the RHS will have a contribution from  $q_2$  and  $q_4$  only.

5. (c) The electric field lines, are directed away from positively charged source and directed toward negatively charged source. In electric field force are directly proportional to the electric field strength hence, higher the electric field strength greater the force and vice-versa.

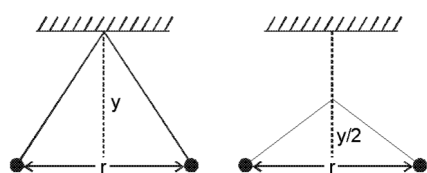
The space between the electric field lines is increasing, from left to right so strength of electric field decreases with the increase in the space between electric field lines. Then the force on charges also decreases from left to right.

Thus, the force on charge  $-q$  is greater than force on charge  $+q$  in turn dipole will experience a force towards left.

6. (a) When a positive point charge  $+q$  is placed near an isolated conducting plane, some negative charge develops on the surface of the plane towards the charge and an equal positive charge develops on opposite side of the plane. This is called induction process and the electric field on a isolated conducting plane at point is directly projected in a plane perpendicular to the field and away from the plane.
7. (a) Consider a point on diameter away from the centre of hemisphere uniformly positively charged, then the electric field is perpendicular to the diameter and the component of electric intensity parallel to the diameter cancel out.

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8. (a)



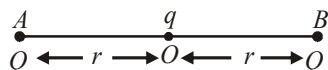
$$\text{From figure, } \tan \theta = \frac{F_e}{mg} \Rightarrow \frac{r/2}{y} = \frac{kq^2}{r^2 mg}$$

$$[\because F = \frac{kq^2}{r^2} \text{ from coulomb's law}]$$

$$\Rightarrow r^3 \propto y \Rightarrow r^3 \propto \frac{y}{2} \Rightarrow \frac{r'}{r} = \frac{1}{2^{1/3}}$$

$$\Rightarrow r' = \frac{r}{\sqrt[3]{2}}$$

9. (b) The system of three charges will be in equilibrium.



For this, force between charge at A and B + force between charge at point O and either at A or B is zero.

$$\text{i.e., } \frac{KQ^2}{r^2} + \frac{KQq}{(r/2)^2} = 0$$

By solving we get,

$$q = -\frac{Q}{4}.$$

10. (c) When electric dipole is aligned parallel  $\theta = 0^\circ$  and the dipole is rotated by  $90^\circ$  i.e.,  $\theta = 90^\circ$ .

Energy required to rotate the dipole

$$W = U_f - U_i = (-pE \cos 90^\circ) - (-pE \cos 0^\circ) = pE.$$

11. (b) Net flux emitted from a spherical surface of radius a according to Gauss's theorem

$$\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\text{or, } (Aa)(4\pi a^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\text{So, } q_{\text{in}} = 4\pi\epsilon_0 A a^3$$

12. (c) From figure  $\tan \theta = \frac{F_e}{mg}$

$$\frac{kq^2}{x^2 mg} = \frac{x}{2\ell}$$

$$\text{or } x^3 \propto q^2 \dots (1)$$

$$\text{or } x^{3/2} \propto q \dots (2)$$

Differentiating eq. (1) w.r.t. time

$$3x^2 \frac{dx}{dt} \propto 2q \frac{dq}{dt} \text{ but } \frac{dq}{dt} \text{ is constant}$$

$$\text{so } x^2(v) \propto q \text{ Replace } q \text{ from eq. (2)}$$

$$x^2(v) \propto x^{3/2} \text{ or } v \propto x^{-1/2}$$

13. (b) According to question, the net electrostatic force ( $F_E$ ) = gravitational force ( $F_G$ )

$$F_E = F_G$$

$$\text{or } \frac{1}{4\pi\epsilon_0} \frac{\Delta e^2}{d^2} = \frac{Gm^2}{d^2}$$

$$\Rightarrow \Delta e = m \sqrt{\frac{G}{K}} \left( \frac{1}{4\pi\epsilon_0} = k = 9 \times 10^9 \right)$$

$$= 1.67 \times 10^{-27} \sqrt{\frac{6.67 \times 10^{-11}}{9 \times 10^9}}$$

$$\Delta e \approx 1.436 \times 10^{-37} \text{ C}$$

