39. Alternating Current

Short Answer

Answer.1

Infinite.

The reactance of a capacitor is given by $X_c = \frac{1}{\omega C}$... (i), where ω = angular frequency of oscillation of current, C = capacitance.

For a constant DC source, there are no oscillations in current and hence ω = 0.

Therefore, from (i), reactance X_c becomes $Xc = \frac{1}{0 \times C}$ = infinite. (Ans)

Answer.2

Zero. Given:

Voltage

$$V = V_0 \cos \omega t$$

Current $i = i_0 \sin \omega t$.

Formula used:

The power dissipated in a series AC circuit is given by

$P = V_{rms} i_{rms} \cos\phi$... (i),

where V_{rms} = root mean square value of voltage,

i_{rms} = root mean square value of current,

ϕ = phase difference between voltage and current

Now, $i(current) = i_0 sin\omega t = i_0 cos(\omega t - \frac{\pi}{2})$... (ii), where i_0 = peak value of current, ω = angular frequency of oscillation, t = time

Since $V = V_0 \cos \omega t$ we can say that the phase difference between the voltage and the current is $\phi = \frac{\pi}{2}$

Substituting this value in (i), we get

$$P = V_{rms} i_{rms} \cos\left(\frac{\pi}{2}\right)$$

P = 0 (since $\cos\left(\frac{\pi}{2}\right) = 0$)

Hence, the power dissipated in the given ac circuit is zero. (Ans)

Answer.3

Equal.

The rms value of current is given by $i_{rms} = \frac{i}{\sqrt{2}}$, where i_0 = peak value of current.

Since i_1 and i_2 have the same peak value of current = i_0, their rms values will also be equal. (Ans)

Answer.4

Yes.

Let the LCR circuit be connected across an AC supply of voltage $V = V_0 sin\omega t$ ... (i)

Now, the impedance in an AC circuit is given by

$$Z = \sqrt{r^2 + (X_L - X_c)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \text{ .... (ii)}$$

where R = resistance, X_L = inductive reactance,

X_c = capacitive reactance,

ω = angular frequency of oscillation of current,

L = inductance,

C = capacitance

Hence, the current in the circuit is given by $I = \frac{V}{Z}$ ... (iii),

where V = voltage,

Z = impedance

$\Rightarrow I = \frac{V_0 sin\omega t}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$... (iv)

Now, at resonance, $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ --------------(v)

Substituting (v) in (iv), we get

Current at resonance : $I_{res} = \frac{V_0 sin\omega t}{R}$... (vi)

Hence, at resonance, voltage across the inductor

$V_L = I_{res} \times X_L \Rightarrow V_L = \frac{V_0 sin\omega t}{R} \frac{sin\omega t}{R} x \times \omega L \frac{V}{R} x\, \omega L$ ... (vii)

where I_{res} = current at resonance, X_L = inductive reactance, V = source voltage = V_0sinωt, where V_0 = peak voltage, ω = angular frequency, t = time, R = resistance, L = inductance

Since ω = angular frequency is always greater than equal to 1,

Therefore, voltage across the inductor V_L will be greater than the source voltage V if $\frac{L}{R}$ > 1. (Ans)

Answer.5

No.

Ohm's law states that the current flowing through a circuit is directly proportional to the potential difference applied across its ends.

Therefore, V = IR, where V = voltage, I = current, R = resistance.

Now, Ohm's law is valid only in purely resistive circuits (without inductors or capacitors), where there is a linear relationship between voltage and current.

However, Ohm's law is not valid in case of circuits with non-linear elements, for example those containing inductors or capacitors or a combination of both.

Hence, in the given circuit, Ohm's law is not consistent. (Ans).

Answer.6

Increase.

The reactance of a capacitor is given by $X_C = \frac{1}{\omega C}$ ... (i)

where ω = angular frequency of oscillation of current, C = capacitance.

Now, the capacitance of a parallel plate capacitor is given by

$C = \frac{K \epsilon_0 A}{d}$ (ii), where k = dielectric constant, A = area of plates,

ϵ_0 = electric permittivity of vacuum, d = distance between plates.

For vacuum, the dielectric constant k = 1. Let the capacitance in vacuum be $C = C_0 = \frac{A\epsilon_0}{d}$... (iii) (from (ii))

For any other medium, k>1. Hence, capacitance of this slab is given by $C = \frac{K \epsilon_0 A}{d} = kC_0$ ... (iv), where k = dielectric constant, A = area of plates, ϵ_0= electric permittivity of vacuum, d = distance between plates, C_0= capacitance in vacuum

Hence, the reactance of the capacitor in vacuum will be

$X_{C1} = \frac{1}{\omega C_0}$ ... (v), where ω = angular frequency, C_0 = capacitance in vacuum

And, the reactance of the capacitor in the dielectric slab will be $X_{C2} = \frac{1}{\omega C} = \frac{1}{\omega k C_0}$... (vi) (from (iv)), where k = dielectric constant of slab

Since the dielectric constant k is greater than 1, it becomes clear that $X_{C1} > X_{C2}$, where X_{C1} = reactance of capacitor in vacuum, X_{C2} = reactance of capacitor in dielectric slab

Now, the rms value of current is given by

$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_C}$... (vii), where i_0 = peak value of current, V_0 = peak value of voltage, X_C = capacitive reactance

Let the rms value of current initially be i_1 and then be i_2 after insertion of dielectric slab.

Therefore, $i_1 = \frac{i_{01}}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_{C1}}$, where i_{01} = peak value of current initially, V_0 = peak value of voltage, X_{C1} = reactance of capacitor in vacuum

and $i_2 = \frac{i_{02}}{\sqrt{2}} = \frac{V_0}{\sqrt{2}X_{C2}}$, where i_{02} = peak value of current after insertion of slab, V_0 = peak value of voltage, X_{C1} = reactance of capacitor after insertion of slab

Since we found out that $X_{C1} > X_{C2}$, it is obvious that

$\frac{1}{X_{C1}} < \frac{1}{X_{C2}} \Rightarrow i_2 > i_1$.

Therefore, the rms current increases. (Ans)

Answer.7

No, current will flow through both of them.

At resonance, we know that $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$ ... (i), where X_L = inductive reactance, X_C = capacitive reactance, ω = angular frequency, L = inductance, C = capacitance

Current through an LCR circuit is given by $ii_0 = \frac{VV_0}{\sqrt{RR^2 + (\frac{1}{\omega C} - \omega L)^2}}$... (ii), where i_0 = peak value of current, V_0 = peak value of voltage, R = resistance, ω = angular frequency, L = inductance, C = capacitance

From (i) and (ii), at resonance, the peak value of current is given by $i_{res} = \frac{V_0}{R}$ ... (iii)

This current will flow through all circuit elements. However, since the inductive reactance and the capacitive reactance are equal, the potential difference across the inductor and capacitor will be equal and opposite and they will cancel each other out. (Ans)

Answer.8

No.

When an AC source is connected to a capacitor, there is a steady-state current in the circuit which transfers charges smoothly between the plates of the capacitor. This results in a potential difference across the plates of the capacitor. The direction of current is alternatively reversed every half-cycle and this is leads to alternating charging and discharging of capacitor.

Answer.9

Same thermal energy, principle of superposition is obeyed.

Given:

Current $i_1 = i_0 \sin \omega t$

Current $i_2 = -i_0 \sin \omega t$

Formula used:

The thermal energy produced in one time period due to a current i is given by $H = (i_{rms})^2 \times R \times \frac{2\pi}{\omega}$... (i), where i_{rms} = rms value of current, R = resistance, ω = angular frequency of oscillation of current

Now, the rms current i_{rms} in both cases is given by $\frac{i_0}{\sqrt{2}}$, where i_0 is the peak current.

Therefore, for current i_1, thermal energy produced is

$H_1 = \frac{{i_0}^2}{2} \times R \times \frac{2\pi}{\omega}$ and that produced for current i_2 is also $H_2 = \frac{{i_0}^2}{2} \times R \times \frac{2\pi}{\omega}$ where i_0 = peak value of current, R = resistance, ω = angular frequency of oscillation

Hence, the same thermal energy is produced due to both the currents individually. (Ans)

Since i_1 and i_2 have peak values i_0 and $-i_0$, they are equal and opposite in value. Hence, the net current through the resistor will be 0 when both pass through the resistor simultaneously. In this case, the thermal energy produced will be 0. (Ans)

Yes, the principle of superposition is obeyed in this case. (Ans)

Answer.10

No.

When a transformer steps up the voltage, the voltage increases but the current decreases in the process since the supplied power remains constant.

Now, the power is given by $P = VI$

where V = voltage, I = current.

Since the voltage and current increase and decrease in proportion to each other, the value of power remains constant.

Hence, energy is not produced. Energy = power x time remains constant. (Ans)

Answer.11

Ideally, we can consider a transformer to be a purely inductive circuit with inductance L.

Hence, the voltage in this case is given by $V = L\,\frac{di}{dt}$, where i = current, t = time, L = inductance.

⇒

Integrating on both sides, we get

$\int di = \int \frac{V}{L} dt \Rightarrow i = \frac{Vt}{L}$

For a DC source, the current across the inductor increases with time and after a certain amount of time, can reach a very large value. This burns the transformer. (Ans)

Answer.12

(a) No. (b) No.

For an LCR circuit with angular frequency ω, the impedance is given by the formula $Z = \sqrt{RR^2 + (\omega L - \frac{1}{\omega C})^{2}}$

where R = resistance, L = inductance, C = capacitance.

Now, the phase difference between the current and the voltage is given by the formula $\tan \emptyset = \omega L - \frac{1}{\omega C}$, where ∅ = phase difference, ω = angular frequency, L = inductance, C = capacitance, R = resistance.

Since there are no restrictions on the values of L, C or R, it is obvious that $\tan \emptyset$ can take any value between -∞ to +∞, that is ∅ can take any value between -90^0 to $+90^0$ (since $\tan 90^0 = \infty$)

Since both 120^0 and 180^0 fall beyond the permitted range of values, we cannot have an AC circuit with any of the given phase differences. (Ans)

Answer.13

Power after addition of capacitor will decrease. Power after addition of inductor will increase.

The impedance of the circuit after introduction of capacitor is given by $Z_1 = \sqrt{R^2 + X_C{}^2}$... (i), where R = resistance, X_C = capacitive reactance.

Now, average power is given by $P = i_{rms}{}^2 \times R$ ... (ii), where i_{rms} = rms value of current, R = resistance

Since i_{rms} decreases with increase in impedance, hence on the introduction of a capacitor, the average power absorbed by the resistance will also decrease. (Ans)

Now, the impedance of an LCR circuit is given by $Z_2 = \sqrt{RR^2 + (X_C - X_L)^2}$ ... (iii), where R = resistance, X_L = inductive reactance, X_C = capacitive reactance.

Hence, compared to Z_1, the value of Z_2 is less and the rms value of current is greater in this case. Therefore, if a small inductance is also introduced in the circuit, the average power absorbed increases. (Ans)

Answer.14

(i) Yes (ii) No

A hot wire ammeter only measures the root mean square(rms) value of alternating current. Hence, when it is used to measure a direct current, it will not show the fluctuating value of ac, but only show a constant current equal to the rms value of the current. Thus, it can be used to measure direct current having constant value. (Ans)

No, we do not need to change the graduations since the rms value of current is the same as the direct current.

Objective I

Answer.1

The resistance of a capacitor is given by $X_C = \frac{1}{\omega C}$, where ω = angular frequency of oscillation of current C = capacitance.

Now, in case of DC current, ω = 0.

Hence, the resistance for DC becomes $X_C = \frac{1}{0 \times C}$ = infinite. (Ans)

Answer.2

Given: Emf

$$\in=\in_0 [\cos(100\pi s^{-1})t + \cos(500\pi s^{-1})t]$$

Steady state current $i = i_1 \cos[(100\pi S^{-1})t + \emptyset_1] + i_2\cos[(500\pi S^{-1})t + \emptyset_2]$

Formula used:

Charge in steady state will be given by

$Q = C \in= C \in_0 [\cos(100\pi s^{-1})t + \cos(500\pi s^{-1})t]$ ... (i),

where C = capacitance, ε = emf, t = time

Hence, current is given by $i = \frac{dQ}{dt}$ ... (ii), where Q = charge, t = time

$\Rightarrow i\ i = -100C\varepsilon \in_0 \pi sin(100\pi ss^{-1})t - 500\pi sin(500\pi ss^{-1})t$, from (i)

Comparing this with $i = i_1 \cos[(100\pi S^{-1})t + \emptyset_1] + i_2\cos[(500\pi S^{-1})t + \emptyset_2]$, we get $i_1 = 100\pi C\varepsilon_0$ and $i_2 = 500\pi C\varepsilon_0$.

Hence, we find that $i_1 < i_2$ (Ans).

Answer.3

Given:

Rms value of AC source voltage V_{rms} = 220 V

Hence, peak value of voltage = $\sqrt{2}V_{rms} = (\sqrt{2} \times 220)$V which is around 310V. (Ans)

Answer.4

The frequency of the AC source is 50 Hz.

which means the time period of the wave is, 1/50 = 002 sec

Now the average voltage of the wave at the time interval 0.01 sec

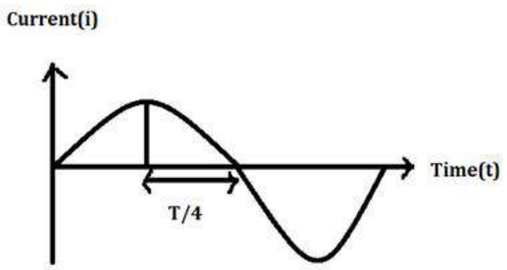
If the interval lies between π/2 and 3π/2, the average voltage will be zero. In all the other case it will not be zero.

Hence, it may be zero. Thus B is the correct answer.

Answer.5

The magnetic field energy in an inductor is given by $E = \frac{Li^2}{2}$... (i), where L = inductor, i = current.

The graph of alternating current is given by:



From (i), we can see that the magnetic field energy reached its maximum and minimum value when the current is maximum and 0 respectively.

Also, from the graph we can see that the time taken by the current to change from its maximum to zero value is T/4, where T = time period.

Now, from the given problem, $\frac{T}{4}$ = 5 ms = 0.005 s

Hence, time period T = (0.005 x 4) s = 0.02 s

Therefore, frequency of oscillation is equal to $\frac{1}{T}$= $\frac{1}{0.02} Hz$ = 50 Hz. (Ans)

Answer.6

The reactance of a series LC combination is given by

$$X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f c}$$

where X_L = resistance of inductor, X_C = resistance of capacitor, f = frequency, L = inductance, C = capacitance.

Also, we can see that when X = 0(point intersecting on the graph), $X_L = X_C$.

This is correctly represented in graph (d). (Ans)

Answer.7

The impedance of an AC circuit is given by $Z = \sqrt{R^2 + X^2}$... (i), where R = resistance, X = impedance.

Given: Resistance R = 4 Ω and Impedance X = 3 Ω

Substituting these values in (i), we get

Impedance Z = $\sqrt{4^2 + 3^2}\Omega = \sqrt{16+9} = \sqrt{25}$ = 5 Ω (Ans)

Answer.8

Transformers only work in AC circuits where they step up or step down the voltage.

In a DC circuit, there is no change in flux with time across the coils of the conductor, since the current is constant. So, there will be no induced emf in the secondary coil due to the change in flux from changing current in primary coil. For this reason, DC circuits do not obey the principle of transformers.

Answer.9

The rms value of current is given by

$i_{rms}^2 = \frac{\int_0^T i_0^2 \sin^2 \omega t \, dt}{\int_0^T i dt}$... (i), where

i = current = $i_1 \cos \omega t + i_2 \sin \omega t$ .... (ii)(given), t = time, ω = angular frequency, T = time period.

Now, squaring on both sides of (ii), we get

$i^2 = i_1^2\cos^2\omega t + 2i_1i_2\sin\omega t\cos\omega t + i_2^2\sin^2\omega t$ ... (iii)

$$= \frac{\left[\int_0^T \frac{i_1^2}{2(\cos 2\omega t + 1)dt} + \int_0^T i_1 i_2 \sin 2\omega t\, dt + \int_0^T \frac{i_1^2}{2(1 - \cos 2\omega t)dt}\right]}{T} =$$

$$\frac{\left(\left\{\frac{i_1^2}{2\left[\frac{\sin 2\omega t}{2\omega} + t\right]_0^T} - i_1 i_2\left[\frac{\cos 2\omega t}{2\omega}\right] [^T]_0 + \frac{i_2^2}{2\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T}\right\}\right)}{T}$$

ow, $\omega T = \omega \times \frac{2\pi}{\omega} = 2\pi$ ... (v), where ω = angular frequency, T = time period = 2π/ω

Therefore, (iv) becomes:

=>$\frac{i_1^2 + i_2^2}{2}$ [since sin nπ = 0, cos 0 = cos 2nπ = 1]

Hence, rms value of current i is $\sqrt{\frac{i_1^2 + i_2^2}{2}}$ (Ans)

Answer.10

To produce the same heating effect, the constant current required(i) will be the root mean square(rms) current(irms)

Hence, $i = i_{rms} = \frac{i_0}{2}$ where i_0 = peak current = 14 A

=>i = (14/√2) A = 9.899 A which is about 10 A. (Ans)

D. undefined for a direct current.

Answer.11

The rms current is equal to the value of the constant current.

Hence, the rms current is also 2.8 A. (Ans)

Objective II

Answer.1

The reactance of an inductor is given by $X_L = 2\pi f L$, where f = frequency, L = inductance.

Hence, if the frequency is increased, the reactance of the inductor also increases. Therefore, option (A) is correct. (Ans)

The resistance remains unchanged with change in frequency. Hence, option (B) is incorrect.

The reactance of a capacitor is given by $X_C = \frac{1}{2\pi f C}$, where f = frequency, C = capacitance.

Hence, if the frequency increases, the reactance of the capacitor decreases. Hence, option (C) is incorrect.

Now, the reactance of the circuit is given by

$$X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f c}$$

Since on increasing the frequency, X_L increases but X_C decreases, their overall difference, that is reactance of the circuit, also increases. Hence option (D) is also correct. (Ans)

Answer.2

The reactance of a circuit is given by $X = X_L - X_C$, where X_L = reactance of inductor, X_C = reactance of capacitor.

X can be 0 in two cases:

(i) When $X_L = X_C$, that is both inductor and capacitor are present. Hence option (A) is correct. (Ans)

(ii) When both X_L and $X_C = 0$, that is, there is neither an inductor nor a capacitor. Hence, option (D) is correct. (Ans)

When either an inductor or a capacitor is present, either X_L or X_C are non-zero and hence the reactance cannot be 0. Therefore, options (B) and (C) are incorrect.

Answer.3

In a pure inductive circuit, the voltage leads the current by a phase difference of 90^0. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (A) is correct. (Ans)

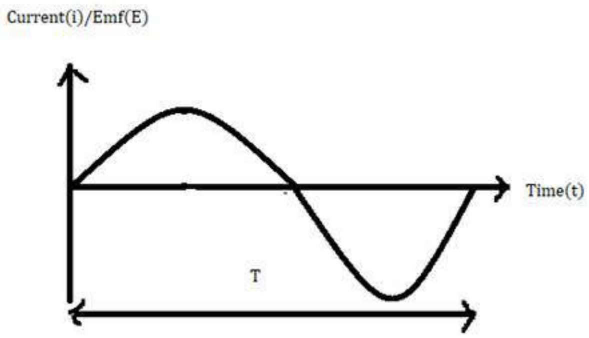
In a pure capacitive circuit, the voltage lags behind the current by a phase difference of 90^0. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (B) is correct. (Ans)

In case of a combination of an inductor and a capacitor also, the current may lead or lag behind the voltage by 90^0, depending on whether the voltage across the inductor or capacitor is greater. Hence, when the instantaneous voltage is maximum, the current is zero and vice versa. Hence, option (D) is correct. (Ans)

Option (C) is incorrect because in a pure resistor circuit, the current and voltage are in phase with each other. Hence, when the voltage is maximum, the current is also maximum and vice versa.

Answer.4

For an inductor-coil circuit with some resistance, the current and the induced emf in the inductor are of the sinusoidal form. It is shown in the diagram below:



Here, T is the time period.

From the graph, we can see that the average value of current or induced emf over a cycle is 0.

Mathematically, we can see it in the following way:

Let the emf be of the form E = E_0sinωt, where E_0 = peak value of emf, ω = angular frequency, t = time.

Average emf over an entire cycle

$$E_{avg} = \frac{\int_0^T E dt}{\int_0^T dt} = \frac{\int_0^T E_0 sin\omega t\, dt}{T} = -\frac{E_0}{\omega T[cos\omega t] \,[\!]_0^T} = -\frac{E_0}{\omega T[\cos 2\pi - \cos 0]}$$

$$= 0$$

Where T = time period

Similarly, we can also show that the average value of current over a full time period is also 0.

Hence, options (A) and (B) are correct.

Joule heat is given by H = i_{rms}^2 x R, where i_{rms} = rms value of current, and R = resistance, which is non zero. Hence, option (C) is incorrect.

Magnetic energy stored in inductor is given by $E = \frac{Li_{rms}^2}{2}$, where L = inductance, i_{rms} = rms value of current, which is non zero. Hence, option (D) is incorrect.

Answer.5

Only a hot-wire voltmeter can be used to measure AC voltage across a resistance. Normal ammeters cannot be used for this purpose due to the changing value and direction of alternating current. All other devices can measure only the DC voltage. Hence only option (B) is correct. (Ans)

Answer.6

Both DC and AC dynamo can be used to convert mechanical energy into electrical energy using wire coils rotating in a magnetic field. Hence, options (A) and (B) are correct. (Ans)

A motor is used to convert electrical energy to mechanical energy and not the other way around. Hence, option (C) is incorrect.

A transformer is used to step up or step down the voltage or to transfer electrical energy only. It cannot be used for transformation of energy from one type to another. Hence, option (D) is also incorrect.

Answer.7

Given:

Rms value of voltage(V_{rms}) = 100 V

Rms value of current(i_{rms}) = 10 A

Formula used:

The average power is given by $P = V_{rms} i_{rms} cos\Phi$, where V_{rms} = rms value of voltage, i_{rms} = rms value of current, $\emptyset$ = phase difference between current and voltage.

Substituting the given values, we get $P = 1000cos\emptyset$.

Now, cos$\emptyset$ can have any value between 0 to 1. (since $\emptyset$can have any value between $-\frac{\pi}{2}\ to\ +\frac{\pi}{2}$)

Therefore, the range of values for P is:

$0 \leq P \leq 1000$ W.

Therefore, the power can be less than 1000 W, or may be equal to 1000 W. Hence, options (B) and (D) are correct. (Ans)

Options (A) and (C) are incorrect since P can have any value between 0 and 1000 W, and can never be greater than 1000 W.

Exercises

Answer.1

I is current at any time 't',

I_0 is maximum value of the current in the circuit,

F is the frequency of the alternating current=

Current at any time is given as

$$I = I_0 \sin 2\pi ft$$

$$\text{Since, } I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$\text{hence , } \frac{I_0}{\sqrt{2}} = I_0 \sin 2\pi ft$$

$$\frac{1}{\sqrt{2}} = \sin 2\pi ft$$

$$= 2\pi ft = \frac{\pi}{4}$$

$$= 2 \times 50 \times t = \frac{1}{4}$$

$$= t = \frac{1}{400}$$

$$t = 2.5 \times 10^{-3} \text{sec}$$

Answer.2

Given that $E_{rms} = 220V$

Frequency= 50 Hz

Then, Peak voltage (E_0) is given as

$$E_0 = \sqrt{2} \times E_{rms}$$

$$E_0 = \sqrt{2} \times 220 = 1.414 \times 220$$

$$E_0 = 311.08V \approx 311V$$

Now, time taken for the current to reach the peak value= time taken to reach the zero value from rms

$$I = \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t$$

$$\omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f}$$

$$t = \frac{\pi}{8\pi \times 50} = \frac{1}{400} = 2.5 \times 10^{-3} \text{ sec.}$$

Where, f is the frequency, t is time taken and ω is angular velocity.

Answer.3

Given that: Power (P) = 60W

Alternate Voltage (V) =220V

Now, power can be expressed as

$$P = \frac{V^2}{R}$$

Therefore, $R = \frac{V^2}{P} = \frac{220 \times 220}{60} = 806.67$

Then instantaneous voltage is

$$\varepsilon_0 = \sqrt{2} \times V = \sqrt{2} \times 220 = 311.08$$

Thus, the maximum instantaneous current through the filament

$$I_0 = \frac{\varepsilon_0}{R} = \frac{311.08}{806.67} = 0.385 = 0.39 \text{ A}$$

Answer.4

Given that an electric bulb is designed to operate at voltage = 12V.

If the bulb is connected to an AC source and given normal brightness, then the peak voltage will be

$$E_0 = \sqrt{2} \times E$$

$$E_0 = \sqrt{2} \times 12$$

$= 1.414 \times 12 = 16.97$

Peak voltage $= 17V$

Answer.5

Given that: Peak power (P_0) = 80 W

Then, instantaneous power is

$$P_{rms} = \frac{P_0}{2} = 40W$$

The energy consumed by the coil in time t=100 seconds will be

= P × t = 40 × 100

= 4000J

Energy consumed = 4.0KJ

Answer.6

Given: Dielectric strength of air (E) = 3.0 × 10^6 V/m,

Area (A) =20 cm^2 and separation width (d) = 0.10 mm.

Potential difference (V) across the capacitor is

$$V = E \times d = 3.0 \times 10^6 \times 1 \times 10^{-4}$$

$$V = 300V$$

The maximum rms voltage of an AC source which can be safely connected to this capacitor will be given as

$$V_{rms} = \frac{V}{\sqrt{2}}$$

$$V_{rms} = \frac{300}{\sqrt{2}} = 212V$$

Answer.7

The current in a discharging LR circuit is given by

$I = I_0 e^{-t/T}$

Then, the rms current for the period t=0 to t= T can be obtained by:

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 e^{-2t/T}\,dt = \frac{I_0^2}{T}\int_0^T e^{-2t/T}\,dt$$

$$= \frac{I_0^2}{T} \times \left[\frac{T}{2} e^{-2t/T}\right]_0^T = \frac{I_0^2}{T} \times \frac{T}{2} \times [e^{-2T/T} - 1]$$

$$I_{rms}^2 = \frac{I_0^2}{T} \times \left(1 - \frac{1}{e^2}\right)$$

So the rms current is

$$I_{rms} = \frac{I_0}{e}\left(\sqrt{\frac{e^2 - 1}{2}}\right)$$

Answer.8

Capacitance of the capacitor C=10 μF,

Output voltage of the oscillator ϵ = (10V) sin ωt.

On comparing the output voltage of the oscillator with

$\epsilon = \epsilon_0$

We get Peak voltage $\epsilon_0 = 10V$

For a capacitive circuit,

Reactance, $X_c = \frac{1}{\omega C}$

Here, ω is angular frequency,

C is capacitance of capacitor,

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

{a} At $\omega = 10\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/10 \times 10^{-5}}$$

$I_0 = 10^{-3}\ A$

{b} At $\omega = 100\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/100 \times 10^{-5}}$$

$I_0 = 10^{-2}\ A$

{c} At $\omega = 500\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/\omega C}$$

$$I_0 = \frac{10}{1/500 \times 10^{-5}}$$

$I_0 = 5 \times 10^{-2}\ A = 0.05A$

{d} At $\omega = 1000\ s^{-1}$

Peak current $I_0 = \frac{\epsilon_0}{X_c}$

$$I_0 = \frac{\epsilon_0}{1/_{\omega C}}$$

$$I_0 = \frac{10}{1/_{1000 \times 10^{-5}}}$$

$$I_0 = 10^{-1}\, A = 0.1A$$

Answer.9

Given: Inductance of a coil= 5.0 mH and

Peak voltage ϵ_0= $10V$

{a} At ω= 100 s^{-1}

Reactance of a coil is given by X_L

Then, $X_L = \omega L$

$$X_L = 0.005 \times 100$$

$$X_L = 0.5\ ohm$$

Here ω is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{0.5} = 20A$$

{b} At ω= 500 s^{-1}

Reactance of a coil is given by X_L

Then, $X_L = \omega L$

$$X_L = 0.005 \times 500$$

$X_L = 2.5\ ohm$

Here ω is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{2.5} = 4A$$

{c} At ω= 1000 s^{-1}

Reactance of a coil is given by X_L

Then, $X_L = \omega L$

$X_L = 0.005 \times 1000$

$X_L = 5\ ohm$

Here ω is angular velocity

Peak current

$$I_0 = \frac{\epsilon_0}{X_L}$$

$$I_0 = \frac{10}{5} = 2A$$

Answer.10

Given: Resistance (R) = 10 Ω,

Inductance (L) = 0.4 Henry,

It is connected to an AC source (E) of 6.5 V and

Frequency (f) = 30/π Hz.

Then, Impedance (Z) of a coil is given by

$$Z = \sqrt{\{R^2 + X_L^2\}}$$

$Z = \sqrt{\{R^2 + (2\pi fL)^2\}}$

Hence, the rms current will be $I_{rms} = \frac{6.5}{Z}$

And $cos\, \emptyset = \frac{R}{Z}$

Thus, the average power consumed in the circuit will be

$$Power = V_{rms} I_{rms} \cos \emptyset$$

$$= 6.5 \times \frac{6.5}{Z} \times \frac{R}{Z}$$

$$= \frac{(6.5)^2 \times R}{\left(\sqrt{\{R^2 + (2\pi f L)^2\}}\right)^2}$$

$$= \frac{6.5 \times 6.5 \times 10}{(10 \times 10) + \left(2\pi \times 0.4 \times \frac{30}{\pi}\right)^2}$$

$$= \frac{6.5 \times 65}{100 + 576} = \frac{422.5}{676}$$

$$= 0.625 \text{ W}$$

Answer.11

Given: Resistance (R) =100 Ω

Source emf ϵ = (12 V) sin (250 π s^{-1}) t

T=1.0 ms =10^{-3} s.t

Then, energy dissipated (E) will be

$$E = \int_0^t \frac{\epsilon^2}{R}$$

$$E = \int_0^t \frac{[(12\, V) sin\, (250\, \pi\, t)]^2\, dt}{R}$$

Since, $sin^2\, \theta = (1 - cos^2\, \theta)\frac{1}{2}$

$$E = \frac{144}{100}\int_0^t \frac{1 - cos\, 2 \times 250\pi t}{2}\, dt$$

$$E = \frac{144}{100}(1 - cos\, 500\pi t\,)dt$$

On integrating,

$$E = \frac{144}{100}\left[t - \frac{sin\, 500\pi t}{500\pi}\right]$$

$At\ t = 10^{-3}s$

$$E = \frac{144}{100}\left[10^{-3} - \frac{1}{500\pi}\right]$$

$$= \frac{144}{100}\left[\frac{1}{1000} - \frac{1}{500 \times 3.14}\right]$$

$$E = 2.61 \times 10^{-4} J$$

Answer.12

Given: Resistance (R) = 300Ω,

Capacitance (C) = 25 μF =$25 \times 10^{-6}\ F$,

Rms voltage (ϵ_0) = 50 V and

Frequency (v) = 50/π Hz.

Then, Reactance (X_C) will be

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{2\pi v C}$$

$$= \frac{1}{2\pi \times 25 \times 10^{-6} \times \frac{50}{\pi}} = \frac{10^4}{25}$$

Now, Impedance (Z) will be

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{300^2 + \left[\frac{10^4}{25}\right]^2}$$

$$Z = \sqrt{300^2 + 400^2}$$

$$Z = 500$$

So, Peak current (I_0) will be

$$I_0 = \frac{\epsilon_0}{Z}$$

$$I_0 = \frac{50}{500} = 0.1\ A$$

And average power dissipated will be

$$\text{Power} = \mathrm{V_{rms}I_{rms}} \cos \emptyset$$

$$= \frac{\mathrm{E_0}}{\sqrt{2}} \times \frac{\mathrm{E_0}}{\sqrt{2} \times \mathrm{Z}} \times \frac{\mathrm{R}}{\mathrm{Z}}$$

$$= \frac{\mathrm{E_0^2 \times R}}{2 \times \mathrm{Z}^2}$$

$$= \frac{50 \times 50 \times 300}{2 \times 500 \times 500} = \frac{3}{2} = 1.5\mathrm{W}$$

Answer.13

Given: Power (P) = 55W,

Bulb operated at voltage (V) = 110V,

Voltage supplied (E) = 220V

Resistance (R) will be

$$R = \frac{V^2}{P} = \frac{110 \times 110}{55} = 220\ ohm$$

Frequency (f) =50Hz,

Angular velocity (ω) will be

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi$$

Current (I) in the circuit will be

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (\omega L)^2}}$$

Voltage drop across the resistor will be

$$V = IR = \frac{ER}{\sqrt{R^2 + (\omega L)^2}}$$

$$110 = \frac{220 \times 220}{\sqrt{(220)^2 + (100\pi L)^2}}$$

$$220 \times 2 = \sqrt{(220)^2 + (100\pi L)^2}$$

$$(440)^2 = (220)^2 + (100\pi L)^2$$

$$48400 + 10^4(\pi L)^2 = 193600$$

$$10^4(\pi L)^2 = 193600 - 48400$$

$$L^2 = \frac{142500}{10^4\pi^2}$$

$$L^2 = 1.4726$$

$$L = \sqrt{1.4726} = 1.2135$$

$$L = 1.2\ Hz$$

Where, L is the inductance of the coil for which the bulb gets correct voltage.

Answer.14

Given: resistance R = 300Ω,

Capacitance of capacitor C = 20 μF,

Inductance of inductor L = 1.0 Henry,

Voltage across the circuit ϵ_{rms} = 50 V and

Frequency v = 50/π Hz

{a} rms current (I_{rms}) in the circuit will be

$$I_{rms} = \frac{\epsilon_{rms}}{Z}$$

Where Z is impedance in the circuit

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi f C} - 2\pi f L\right)^2\right\}}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1\right)^2\right\}}$$

$$Z = \sqrt{\left\{(300)^2 + \left(\frac{10^4}{20} - 100\right)^2\right\}}$$

$$Z = 500$$

Then, $I_0 = \frac{50}{500} = 0.1\ A$

{b} potential difference across the capacitor will be

$$V_c = I_0 \times X_c = 0.1 \times 500 = 50V$$

Potential difference across the resistor will be

$$V_R = I_0 \times R = 0.1 \times 300 = 30V$$

Potential difference across the inductor will be

$$V_L = I_0 \times X_L = 0.1 \times 100 = 10V$$

Net sum of all potential drops = $V_c + V_L + R = 50 + 10 + 30 = 90V$

Rms voltage, $\epsilon_{rms} = 50V$

Hence, sum of all potential drops > rms potential applied.

Answer.15

Given: resistance R = 300Ω,

Capacitance of capacitor C = 20 μF,

Inductance of inductor L = 1.0 Henry,

Voltage across the circuit ϵ_{rms} = 50 V and

Frequency v = 50/π Hz

Then the rms current (I_{rms}) across the circuit

$$I_{rms} = \frac{\epsilon_{rms}}{Z}$$

Where Z is impedance in the circuit

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi fC} - 2\pi fL\right)^2\right\}}$$

$$Z = \sqrt{\left\{300^2 + \left(\frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} - 2\pi \times \frac{50}{\pi} \times 1\right)^2\right\}}$$

$$Z = \sqrt{\left\{(300)^2 + \left(\frac{10^4}{20} - 100\right)^2\right\}}$$

$$Z = 500$$

Then, $I_0 = \frac{50}{500} = 0.1\ A$

Electric energy (E_c) stored in capacitor will be

$$E_C = \frac{1}{2}CV^2$$

$$E_C = \frac{1}{2} \times 20 \times 10^{-6} \times 50 \times 50$$

$$E_C = 25 \times 10^{-3} J = 25 mJ$$

Magnetic field energy (E_M) stored in the coil will be

$$E_M = \frac{1}{2} L I_0^2$$

$$E_M = \frac{1}{2} \times 1 \times (0.1)^2 \times 5 \times 10^{-3} J$$

$$E_M = 5 mJ$$

Answer.16

{a} for current to be maximum in a circuit

$$X_L = X_c \; (resonant\ condition)$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC} = \frac{1}{2 \times 18 \times 10^{-6}} = \frac{10^6}{36}$$

$$\omega = \frac{10^3}{6} = 2\pi f$$

$$f = \frac{1000}{6 \times 2\pi}$$

$$f = 26.537\ Hz = 27\ Hz$$

Where, f is frequency, ω is angular velocity, L is inductance of inductor, C is capacitance of capacitor, X_L is resonance across inductor and X_c is resonance across capacitor.

{b} maximum current (I) will be

$$I = \frac{E}{R}$$

$$I = \frac{20}{10\times10^3} = \frac{2}{10^3} \text{ A}$$

$$I = 2\ mA$$

Answer.17

Given that rms voltage E_{rms} =24V

Internal resistance, r= 4 ohm

Rms current I_{rms} =6A

Then, rms resistance will be

$$R = \frac{E_{rms}}{I_{rms}}$$

$$R = \frac{24}{6} = 4\ ohm$$

If this inductor coil is connected to a battery of emf E= 12 V and internal resistance r' = 4.0Ω, then the steady current will be

$$I = \frac{E}{R'}$$

Since, net resistance R' = (R + r') = 4+4 =8 ohm.

Therefore, the steady current (I) is

$$I = \frac{12}{8} = 1.5\ A$$

Answer.18

Given: Voltage V_1=10×10^{-3}V,

Resistance (R) =1×10^{3} ohm,

Capacitance (C) =10×10^{-9} F

Angular velocity (ω) will be

$\omega = 2\pi f$

{a} At frequency (f) =10kHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-4}} = \frac{10^4}{2\pi}$$

$$X_c = \frac{5000}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{5000}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}$$

Then, Current (I_0) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}}$$

Thus, Output voltage (V_0) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}} \times \frac{5000}{\pi}$$

$$V_0 = 16.124V = 16.1mV$$

{b} At frequency (f) =100kHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^5 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$$

$$X_c = \frac{500}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{500}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}$$

Then, Current (I_0) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}}$$

Thus, Output voltage (V_0) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}} \times \frac{500}{\pi}$$

$$V_0 = 1.6124V = 1.6mV$$

{c} At frequency (f) =1 MHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^6 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10^{-2}} = \frac{10^2}{2\pi}$$

$$X_c = \frac{50}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{50}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}$$

Then, Current (I_0) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}}$$

Thus, Output voltage (V_0) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}} \times \frac{50}{\pi}$$

$$V_0 = 0.16124V = 0.16mV$$

{a} At frequency (f) =10 MHz

Reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_c = \frac{1}{2\pi \times 10^7 \times 10 \times 10^{-9}}$$

$$X_c = \frac{1}{2\pi \times 10} = \frac{10}{2\pi}$$

$$X_c = \frac{5}{\pi}$$

Impedance, $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(1 \times 10^3)^2 + \left(\frac{5}{\pi}\right)^2}$$

$$Z = \sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}$$

Then, Current (I_0) will be

$$I_0 = \frac{V_1}{Z}$$

$$I_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}}$$

Thus, Output voltage (V_0) will be

$$V_0 = I_0 X_c$$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}} \times \frac{5}{\pi}$$

$$V_0 = 0.016124V = 16\mu V$$

Answer.19

Given that a transformer has 50 turns in the primary and 100 in the secondary.

If the primary is connected to a 220 V DC supply, the voltage across the secondary is zero because a transformer does not work on DC.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and the transformer will not work.