RECTILINEAR MOTION

Contents

JEE (ADVANCED) SYLLABUS

Kinematics in one Dimensions.

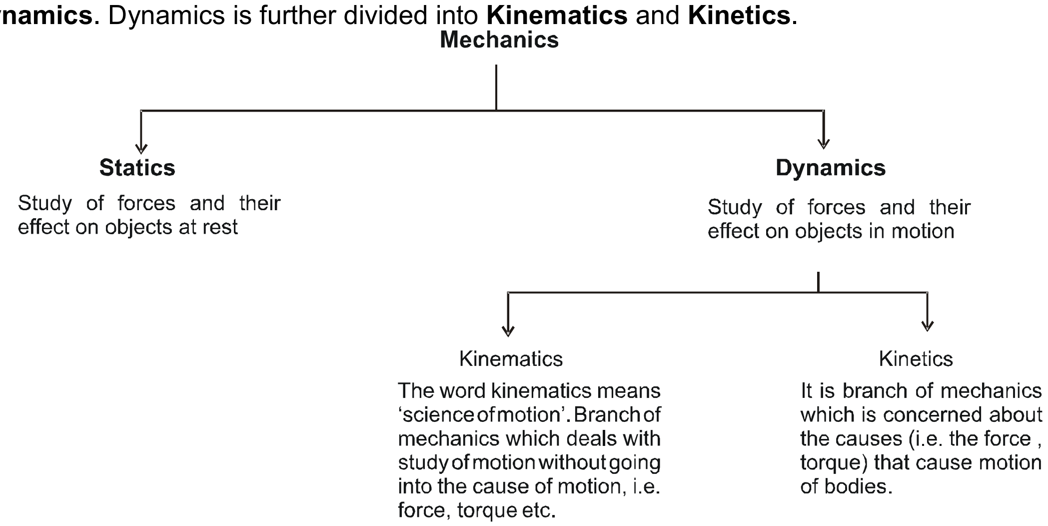
JEE (MAIN) SYLLABUS

Motion in a straight line : Position time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity Uniformly acceleration motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

RECTILINEAR MOTION

1. MECHANICS

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely **Statics** and **Dynamics**. Dynamics is further divided into **Kinematics** and **Kinetics**.



2. MOTION AND REST

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to a observer, if its position changes with respect to that observer. It may happen by both ways either observer moves or object moves.

3. RECTILINEAR MOTION

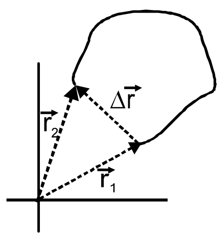
Rectilinear motion is motion, along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

3.1 Position

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question – "where is the particle at a particular moment of time?"

3.2 Displacement

The change in the position of a moving object is known as displacement. It is the vector joining the initial position ($\vec{r}_1$) of the particle to its final position ($\vec{r}_2$) during an interval of time.



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

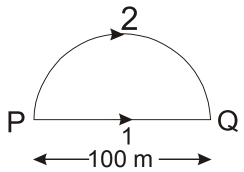
Displacement can be negative positive or zero.

3.3 Distance

The length of the actual path travelled by a particle during a given time interval is called as distance. The distance travelled is a scalar quantity which is quite different from displacement. In general, the distance travelled between two points may not be equal to the magnitude of the displacement between the same points.

Solved Example

Example 1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).



(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?

Solution :

(a) Distance travelled by Ram = 100 m
Distance travelled by Shyam = π(50 m) = 50π m

(b) Displacement of Ram = 100 m
Displacement of Shyam = 100 m

3.4 Average Velocity (in an interval) :

The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the lapsed time.

Average Velocity = $\dfrac{\text{displacement}}{\text{time interval}}$

for straight line motion, along x-axis, we have

$$\mathbf{v_{av}} = \bar{v} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The dimension of velocity is [LT^{-1}] and its SI unit is m/s.

The average velocity is a vector in the direction of displacement. For motion in a straight line, directional aspect of a vector can be taken care of by +ve and -ve sign of the quantity.

3.5 Instantaneous Velocity (at an instant) :

The velocity at a particular instant of time is known as instantaneous velocity. The term “velocity” usually means instantaneous velocity.

$$V_{inst.} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

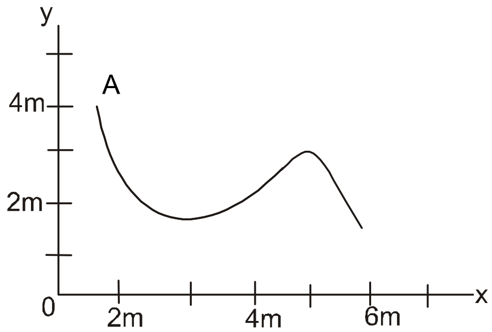
In other words, the instantaneous velocity at a given moment (say, t) is the limiting value of the average velocity as we let Δt approach zero. The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the derivative of x with respect to t.

Note :

- The magnitude of instantaneous velocity and instantaneous speed are equal.
- The determination of instantaneous velocity by using the definition usually involves calculation of derivative. We can find v = $\dfrac{dx}{dt}$ by using the standard results from differential calculus.
- Instantaneous velocity is always tangential to the path.

Solved Example

Example 1. A particle starts from a point A and travels along the solid curve shown in figure. Find approximately the position B of the particle such that the average velocity between the positions A and B has the same direction as the instantaneous velocity at B.

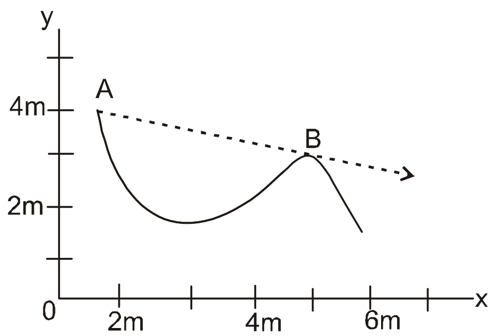


Answer : x = 5m, y = 3m

Solution : The given curve shows the path of the particle starting at y = 4 m.

Average velocity = $\frac{\text{displacement}}{\text{time taken}}$ **;** where displacement is straight line distance between points.

Instantaneous velocity at any point is the tangent drawn to the curve at that point.



Now, as shown in the graph, line AB shows displacement as well as the tangent to the given curve. Hence, point B is the point at which direction of AB shows average as well as instantaneous velocity.

3.6 Average Speed (in an interval)

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

$$\textbf{Average Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

The dimension of velocity is $[LT^{-1}]$ and its SI unit is m/s.

Note :

- Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.
- If the motion of a particle is along a straight line and in same direction then, average velocity = average speed.
- Average speed is, in general, greater than the magnitude of average velocity.

Solved Example

Example 1. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find

(a) Average speed of Ram and Shyam?

(b) Average velocity of Ram and Shyam?

Solution :

(a) Average speed of Ram = $\frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam = $\frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram = $\frac{100}{4}$ m/s = 25 m/s (From P to Q)

Average velocity of Shyam = $\frac{100}{5}$ m/s = 20 m/s (From P to Q)

Example 2. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution : Let total distance travelled by the particle be 2s.

Time taken to travel first half = $\frac{s}{v_1}$

Time taken to travel next half = $\frac{s}{v_2}$

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$ (harmonic progression)

Example 3. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Answer : $v = \frac{v_1+v_2}{2}$ (Arithmetic progression)

Solution :

As shown, the person travels from A to B through a distance S, where first part S_1 is travelled in time t/2 and next S_2 also in time t/2.

So, according to the condition : $v_1 = \frac{S_1}{t/2}$ and $v_2 = \frac{S_2}{t/2}$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}} = \frac{S_1+S_2}{t} = \frac{\frac{v_1t}{2}+\frac{v_2t}{2}}{t} = \frac{v_1+v_2}{2}$

3.7 Average acceleration (in an interval):

The average acceleration for a finite time interval is defined as :

Average acceleration = $\frac{\text{change in velocity}}{\text{time interval}}$

Average acceleration is a vector quantity whose direction is same as that of the change in velocity.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Since for a straight line motion the velocities are along a line, therefore

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

(where one has to substitute v_f and v_i with proper signs in one dimensional motion)

3.8 Instantaneous Acceleration (at an instant):

The instantaneous acceleration of a particle is its acceleration at a particular instant of time. It is defined as the derivative (rate of change) of velocity with respect to time. We usually mean instantaneous acceleration when we say “acceleration”. For straight motion we define instantaneous acceleration as :

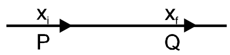
$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) \qquad \text{and in general} \quad \vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right)$$

The dimension of acceleration is $[LT^{-2}]$ and its SI unit is m/s^2.

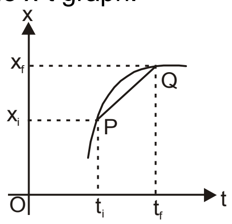
4. GRAPHICAL INTERPRETATION OF SOME QUANTITIES

4.1 Average Velocity

If a particle passes a point P (x_i) at time $t = t_i$ and reaches Q (x_f) at a later time instant $t = t_f$, its average velocity in the interval PQ is $V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$



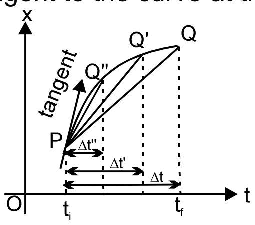
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.



4.2 Instantaneous Velocity

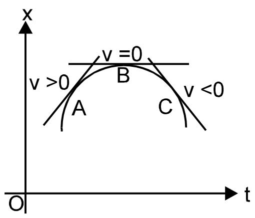
Consider the motion of the particle between the two points P and Q on the x-t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δt, $\Delta t'$, $\Delta t''$,......) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ′, PQ″......).

As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \to 0$, V_{av} ($=\Delta x/\Delta t$) $\to V_{inst.}$



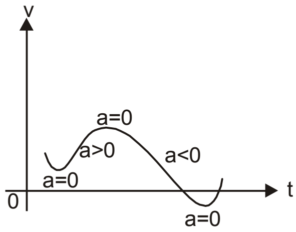
Geometrically, as $\Delta t \to 0$, chord PQ $\to$ tangent at P.

Hence the instantaneous velocity at P is the slope of the tangent at P in the x – t graph. When the slope of the x – t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.



4.3 Instantaneous Acceleration :

The derivative of velocity with respect to time is the slope of the tangent in velocity time (v–t) graph.



Solved Example

Example 1. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Solution : Velocity; $v = \frac{dx}{dt} = 10t + 4$

At $t = 2$ s

$v = 10(2) + 4$

$v = 24$ m/s

Acceleration; $a = \frac{d^2x}{dt^2} = 10$

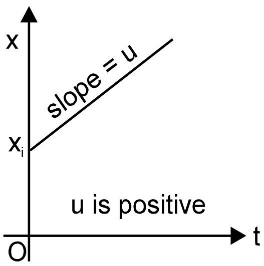
Acceleration is constant, so at t = 2 s $a = 10 \text{ m/s}^2$

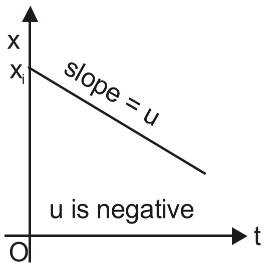
5. MOTION WITH UNIFORM VELOCITY

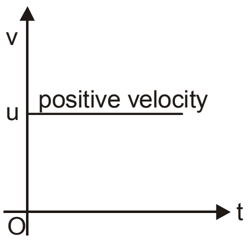
Consider a particle moving along x-axis with uniform velocity u starting from the point $x = x_i$ at $t = 0$.

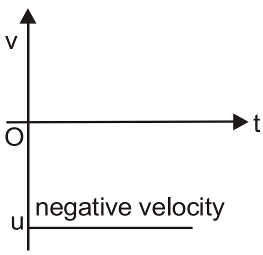
Equations of x, v, a are : $x(t) = x_i + ut$; $v(t) = u$; $a(t) = 0$

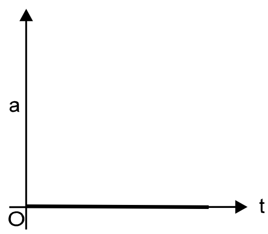
- x-t graph is a straight line of slope u through x_i.
- as velocity is constant, v – t graph is a horizontal line.
- a-t graph coincides with time axis because a = 0 at all time instants.







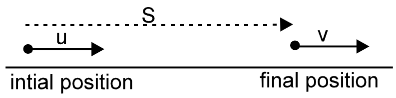




6. UNIFORMLY ACCELERATED MOTION :

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x-axis) during a time interval of t seconds, the following important results can be used.



(i) $a = \frac{v-u}{t}$

(ii) $V_{av} = \frac{v+u}{2}$

(iii) $S = (v_{av})t$

(iv) $S = \left(\frac{v+u}{2}\right)t$

(v) $v = u + at$

(vi) $s = ut + 1/2\ at^2$; $s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(vii) $v^2 = u^2 + 2as$

(viii) $s_n = u + a/2\ (2n - 1)$

u = initial velocity (at the beginning of interval)

a = acceleration

v = final velocity (at the end of interval)

s = displacement ($x_f - x_i$)

x_f = final coordinate (position)

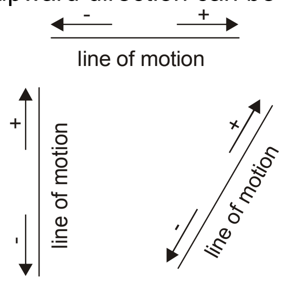
x_i = initial coordinate (position)

s_n = displacement during the n^{th} sec

7. DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

- For example, if a particle is moving in a horizontal line (x-axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.
- For vertical or inclined motion, upward direction can be taken +ve and downward as –ve



- For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$ i.e., $a = -9.8\ m/s^2$ (–ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

Note :

- If acceleration is in same direction as velocity, then speed of the particle increases.
- If acceleration is in opposite direction to the velocity then speed decreases i.e., the particle slows down. This situation is known as retardation.

Solved Example

Example 1. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Solution : Let the total distance be 2x.

$\therefore$ distance upto midpoint = x

Let the velocity at the mid point be v and acceleration be a.

From equations of motion

$v^2 = 10^2 + 2ax$ .....(1)

$30^2 = v^2 + 2ax$ .....(2)

(2) – (1) gives

$v^2 - 30^2 = 10^2 - v^2$

$\Rightarrow$ $v^2 = 500$ $\Rightarrow$ $v = 10\sqrt{5}$ m/s

Example 2. Mr. Sharma brakes his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200 m.

(a) How much time elapses during this interval?

(b) What is the acceleration?

(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Solution :

(a) We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at t = 0, and the final velocity and position are $v_x = +15$ m/s and x = 200 m at time t. Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

$\therefore$ $v_{av,\,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20$ m/s.

The average velocity can also be expressed as $v_{av,\,x} = \frac{\Delta x}{\Delta t}$. With $\Delta x = 200$ m

and $\Delta t = t - 0$, we can solve for t : $t = \frac{\Delta x}{v_{av,x}} = \frac{200}{20} = 10$ s.

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$a_x = \frac{v_x - u_x}{t} = \frac{15 - 25}{10} = -1$ m/s^2.

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity $u_x = 25$ m/s to $v_x = 0$. Solving for t, we find

$t = \frac{v_x - u_x}{a_x} = \frac{0 - 25}{-1} = 25$ s.

The total distance covered is $x = x_i + u_x t + \frac{1}{2} a_x t^2$

$= 0 + (25)(25) + 1/2\,(-1)(25)^2 = 625 - 312.5 = 312.5$ m.

Additional distance covered = 312.5 – 200 = 112.5 m.

Example 3. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

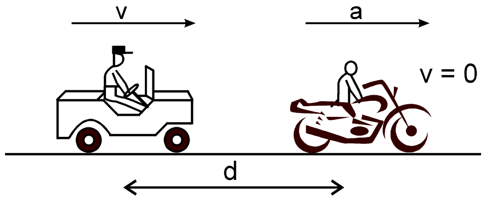
Solution : Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \qquad \ldots\ldots(1)$$

During this interval the jeep travels a distance

$$s + d = vt \qquad \ldots\ldots(2)$$

By (1) and (2),

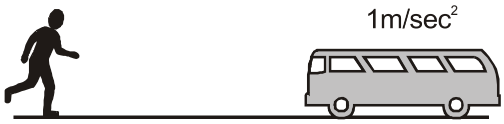


$$\frac{1}{2}at^2 + d = vt \quad \text{or,} \quad t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive. This will be possible if

$$v^2 \geq 2ad \quad \text{or,} \quad v \geq \sqrt{2ad}$$

Example 4. A man is standing 40 m behind the bus. Bus starts with 1 m/sec^2 constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by man to catch the bus.



Solution : Let after time 't' man will catch the bus

For bus

$x = x_0 + ut + \frac{1}{2}at^2$, $\qquad x = 40 + 0(t) + \frac{1}{2}(1)t^2$

$x = 40 + \frac{t^2}{2}$ ....(i)

For man, $\quad x = 9t$ ....(ii)

From (i) & (ii)

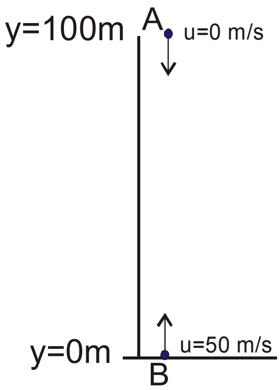
$40 + \frac{t^2}{2} = 9t$ or $t = 8$ s or $t = 10$s.

Example 5. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet ? (take $g = 10\ m/s^2$)

Solution : Let the upward direction is positive.

Let the particles meet at a distance y from the ground.

For particle A,



$y_0 = + 100$ m
u = 0 m/s
a = – 10 m/s²

$y = 100 + 0(t) - \frac{1}{2} 10 \times t^2 [y = y_0 + ut + \frac{1}{2}at^2]$

$= 100 - 5t^2$ .....(1)

For particle B,
$y_0 = 0$ m
u = + 50 m/s
a = – 10 m/s²
$y = 50(t) - 10t^2$
$= 50t - 5t^2$ ......(2)

According to the problem;

$50t - 5t^2 = 100 - 5t^2$

t = 2 sec

Putting t = 2 sec in eqn. (1),

y = 100 – 20 = 80 m

Hence, the particles will meet at a height 80 m above the ground.

Example 6. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower ? (take g = 10 m/s²)

Solution : Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n - 1)$ $\Rightarrow$ $45 = 0 + \frac{10}{2}(2n - 1)$

n = 5 sec

Height of tower ; $\frac{1}{2}h = gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m

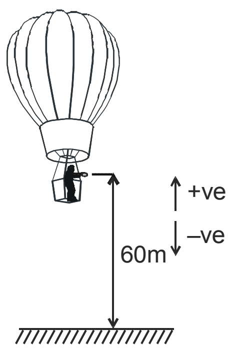
8. REACTION TIME

When a situation demands our immediate action. It takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Solved Example

Example 1. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s².

Solution : $S = ut + \frac{1}{2}at^2$



$-60 = 5(t) + \frac{1}{2}(-10)t^2$

$-60 = 5t - 5t^2$

$5t^2 - 5t - 60 = 0$

$t^2 - t - 12 = 0$

$t^2 - 4t + 3t - 12 = 0$

$(t - 4)(t + 3) = 0$

$\therefore$ t = 4

Height of balloon from ground at this instant

= 60 + 4 × 5 = 80 m

Example 2. A balloon is rising with constant acceleration 2 m/sec^2. Two stones are released from the balloon at the interval of 2 sec. Find out the distance between the two stones 1 sec. after the release of second stone.

Solution : Acceleration of balloon = 2 m/sec^2

Let at t = 0, y = 0 when the first stone is released.

By the question, $y_1 = 0\, t_1 + \frac{1}{2} g t_1^2$ (taking vertical upward as – ve and downward as + ve)

$\therefore$ Position of Ist stone = $\frac{9}{2}g$

(1 second after release of second stone will be the 3rd second for the 1st stone)

For second stone $y_2 = ut_2 + \frac{1}{2} g t_2^2$

$u = 0 + at = -2 \times 2 = -4$m/s (taking vertical upward as – ve and downward as + ve)

$\therefore$ $y_2 = -4 \times 1 + \frac{1}{2} g \times (1)^2$ (t_2 = 1 second)

The 2nd stone is released after 2 second

$\therefore$ $y = -\frac{1}{2} at^2 = -\frac{1}{2} \times 2 \times 4 = -4$

So, Position of second stone from the origin = $-4 + \frac{1}{2} g - 4$

Distance between two stones = $\frac{1}{2} g \times 9 - \frac{1}{2} g \times 1 + 8 = 48$ m.

Note :

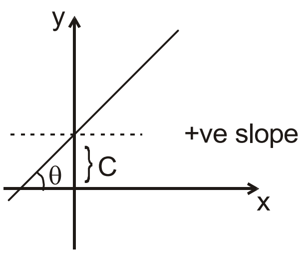
- As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

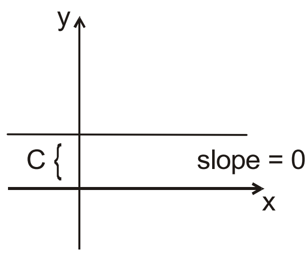
9. STRAIGHT LINE-EQUATION, GRAPH, SLOPE (+VE, –VE, ZERO SLOPE).

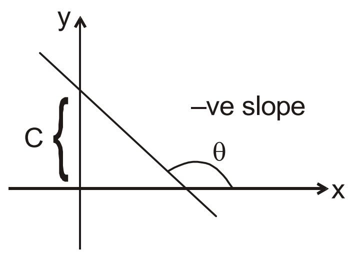
If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by m = tan θ. If θ is 90°, m does not exist, but the line is parallel to the y-axis. If θ = 0, then m = 0 & the line is parallel to the x-axis.

Slope – intercept form : y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

$$m = \text{slope} = \tan\theta = \frac{dy}{dx}$$

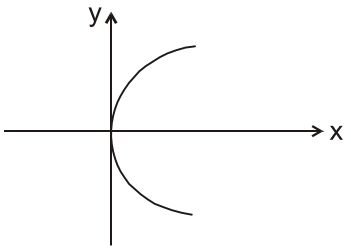




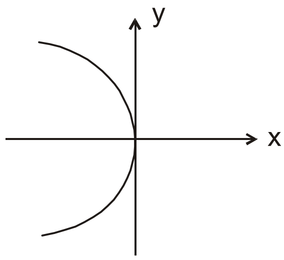


10. PARABOLIC CURVE-EQUATION, GRAPH (VARIOUS SITUATIONS UP, DOWN, LEFT, RIGHT WITH CONDITIONS)

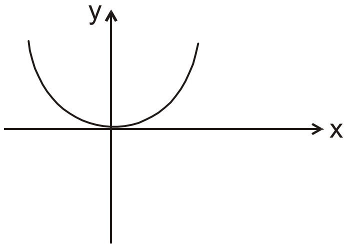
$y^2 = kx$



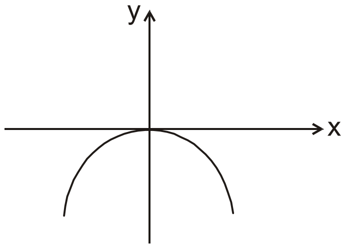
$y^2 = -kx$



$x^2 = ky$



$x^2 = -ky$



Where k is a positive constant.

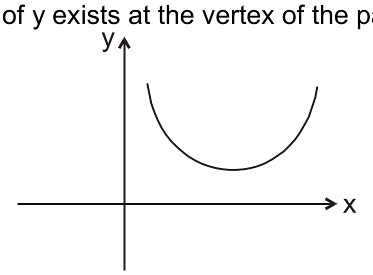
Equation of parabola :

Case (i) : $y = ax^2 + bx + c$

For $a > 0$

The nature of the parabola will be like that of the of nature $x^2 = ky$

Minimum value of y exists at the vertex of the parabola.

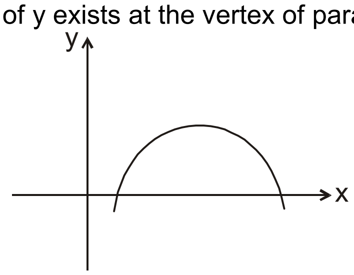


$y_{min} = \frac{-D}{4a}$ where $D = b^2 - 4ac$; Coordinates of vertex $= \left(\frac{-b}{2a}, \frac{D}{4a}\right)$

Case (ii) : $a < 0$

The nature of the parabola will be like that of the nature of $x^2 = -ky$

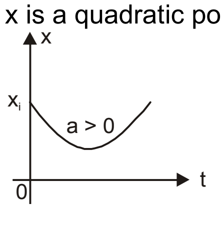
Maximum value of y exists at the vertex of parabola.

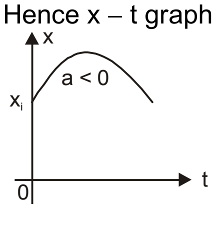


$y_{max} = D/4a$ where $D = b^2 - 4ac$

11. GRAPHS IN UNIFORMLY ACCELERATED MOTION ($A \neq 0$)

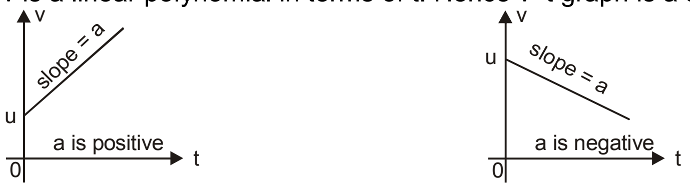
- x is a quadratic polynomial in terms of t. Hence x – t graph is a parabola.





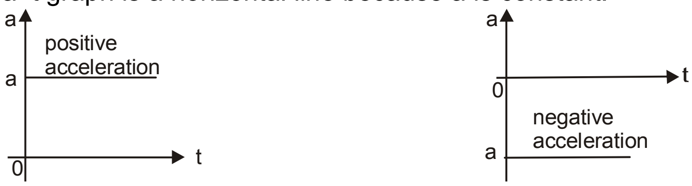
x-t graph

- v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.



v-t graph

- a–t graph is a horizontal line because a is constant.



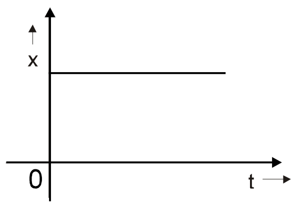
a-t graph

12. INTERPRETATION OF SOME MORE GRAPHS

12.1 Position vs Time graph

12.1.1 Zero Velocity

As position of particle is fixed at all the time, so the body is at rest.



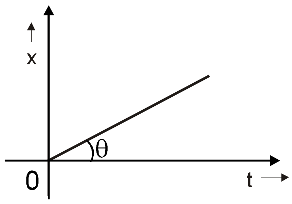
Slope; $dx/dt = \tan\theta = \tan 0^\circ = 0$

Velocity of particle is zero

12.1.2 Uniform Velocity

Here $\tan\theta$ is constant $\tan\theta = dx/dt$

$\therefore$ dx/dt is constant.



$\therefore$ Velocity of particle is constant.

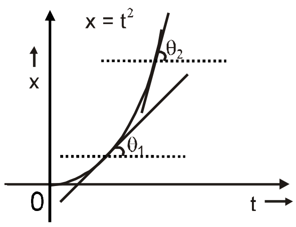
12.1.3 Non uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

$\therefore$ $dx/dt = \tan\theta$ is also increasing

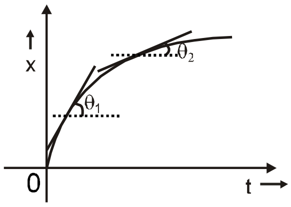
Hence, velocity of particle is increasing.



12.1.4 Non uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.



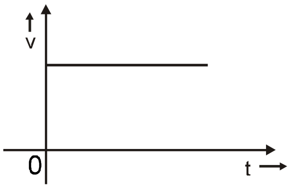
$$\therefore \quad \frac{dx}{dt} = \tan\theta \text{ also decreases.}$$

Hence, velocity of particle is decreasing.

12.2 Velocity vs time graph

12.2.1 Zero acceleration

Velocity is constant.



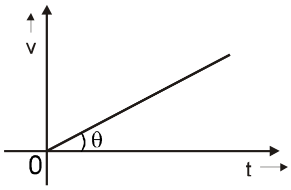
$\tan\theta = 0$

$$\therefore \quad \frac{dv}{dt} = 0$$

Hence, acceleration is zero.

12.2.2 Uniform acceleration

$\tan\theta$ is constant.



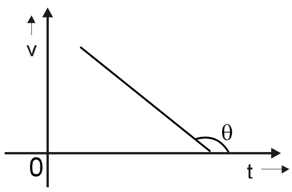
$$\therefore \quad \frac{dv}{dt} = \text{constant}$$

Hence, it shows constant acceleration.

12.2.3 Uniform retardation

Since $\theta > 90^\circ$

$\therefore$ $\tan\theta$ is constant and negative.

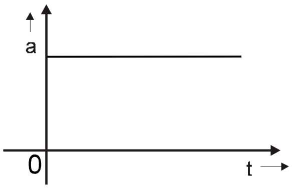


$$\therefore \quad \frac{dv}{dt} = \text{negative constant}$$

Hence, it shows constant retardation.

12.3 Acceleration vs time graph

12.3.1 Constant acceleration



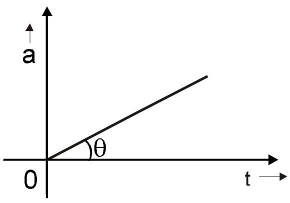
$\tan\theta = 0$

$\therefore$ $da/dt = 0$

Hence, acceleration is constant.

12.3.2 Uniformly increasing acceleration

θ is constant.



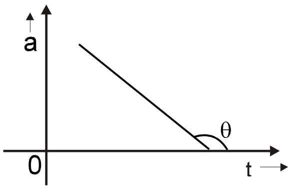
$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$\therefore$ $da/dt = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time.

12.3.3 Uniformly decreasing acceleration

Since $\theta > 90^\circ$



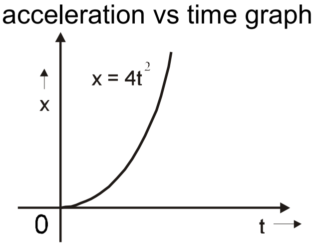
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore$ da/dt = negative constant

Hence, acceleration is uniformly decreasing with time

Solved Example

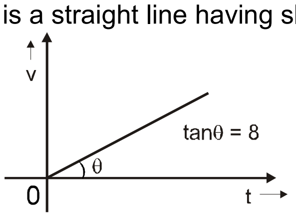
Example 1. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Solution : $x = 4t^2$

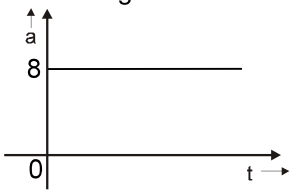
$v = dx/dt = 8t$

Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.

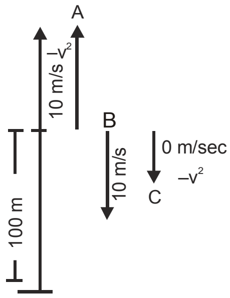


$$a = \frac{dv}{dt} = 8$$

Hence, acceleration is constant throughout and is equal to 8.



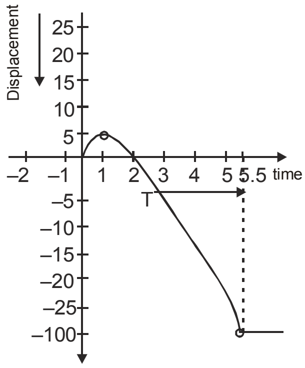
Example 2. At the height of 100 m, a particle A is thrown up with V = 10 m/s, B particle is thrown down with V = 10m/s and C particle released with V = 0 m/s. Draw graphs of each particle.



(i) Displacement–time (ii) Speed–time (iii) Velocity–time (iv) Acceleration–time

Solution : **For particle A :**

(i) Displacement vs time graph is



$y = ut + \frac{1}{2}at^2$

$u = +10$ m/sec^2

$y = 10t - \frac{1}{2} \times 10t^2 = 10t - 5t^2$

$v = \frac{dy}{dt} = 10 - 10t = 0$

t = 1 ; hence, velocity is zero at t = 1

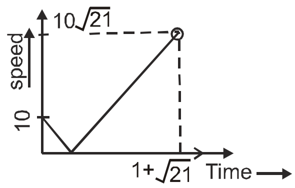
$10t - 5t^2 = -100$

$t^2 - 2t - 20 = 0$

t = 5.5 sec.

i.e., particle travels up till 5.5 seconds.

(ii) Speed vs time graph :



Particle has constant acceleration = g ↓ throughout the motion, so v-t curve will be straight line.

when moving up, v = u + at

0 = 10 – 10 t or t = 1 is the time at which speed is zero.

there after speed increases at constant rate of 10 m/s^2 .

Resulting Graph is : (speed is always positive).

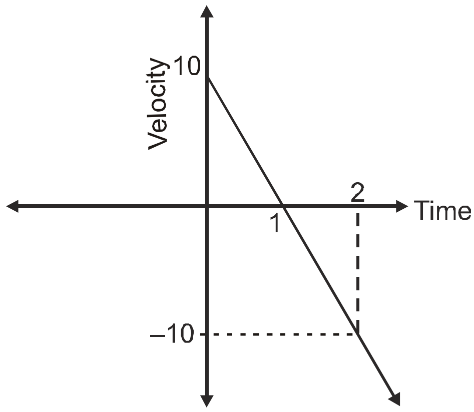
This shows that particle travels till a time of

$1 + \sqrt{21}$ seconds

(iii) Velocity vs time graph :

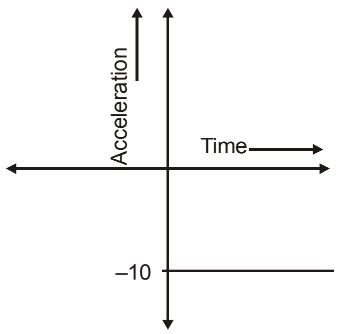
V = u + at

V = 10 – 10t ; this shows that velocity becomes zero at t = 1 sec and thereafter the velocity is negative with slope g.



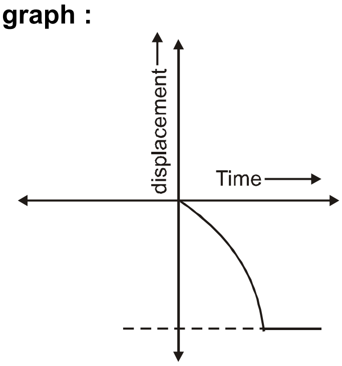
(iv) Acceleration vs time graph :

throughout the motion, particle has constant acceleration = –10 m/s^2.



For particle B : u = – 10 m/s. $y = -10t - \frac{1}{2}(10)t^2 = -10t - 5t^2$

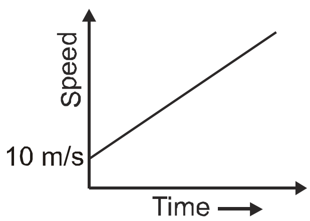
(i) Displacement time graph :



$y = 10t - 5t^2$; $\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$

this shows that slope becomes more negative with time.

(ii) Speed time graph :

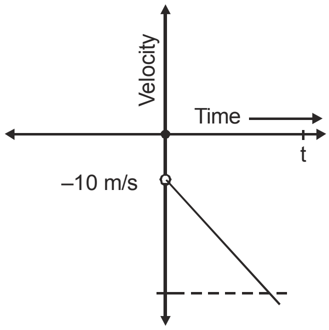


$\frac{dy}{dt} = -10t - 5t^2 = -10 - 10t$

hence, speed is directly proportional to time with slope of 10 initial speed = 10 m/s

(iii) Velocity time graph :

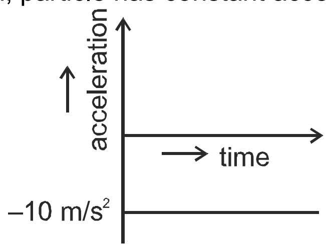
$\frac{dy}{dt}$ = – 10t – 5t² = – 10 – 10t



hence, velocity is directly proportional to time with slope of –10. Initial velocity = –10 m/s

(iv) Acceleration vs time graph :

throughout the motion, particle has constant acceleration = $-10\ m/s^2$.

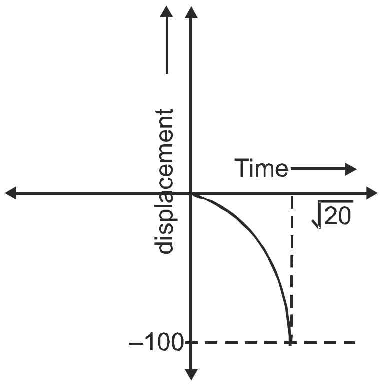


$a = \frac{dv}{dt} = -10$

For Particle C :

(i) Displacement time graph :

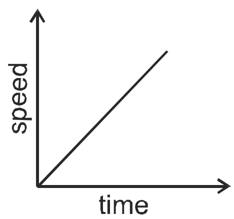
$u = 0,\ y = -\frac{1}{2} \times 10t^2 = -5t^2$



this shows that slope becomes more negative with time.

(ii) Speed vs time graph :

$v = \frac{dy}{dt} = -10\ t$

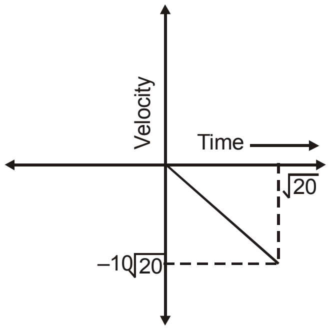


hence, speed is directly proportional to time with slope of 10.

(iii) Velocity time graph :

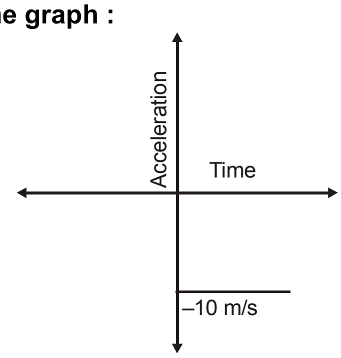
V = u + at

V = – 10t ;



hence, velocity is directly proportional to time with slope of –10.

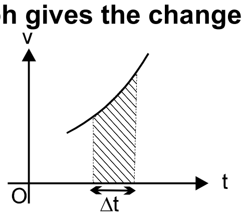
(iv) Acceleration vs time graph :



throughout the motion, particle has constant acceleration = –10 m/s^2.

13. DISPLACEMENT FROM V-T GRAPH & CHANGE IN VELOCITY FROM A -T GRAPH

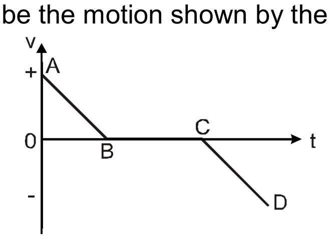
Displacement = Δx = area under v-t graph. Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that $\Delta v = a\ \Delta t$ leads to the conclusion that **area under a – t graph gives the change in velocity Δv during that interval.**



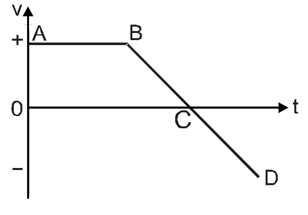
Solved Example

Example 1. Describe the motion shown by the following velocity-time graphs.

(a)



(b)



Solution :

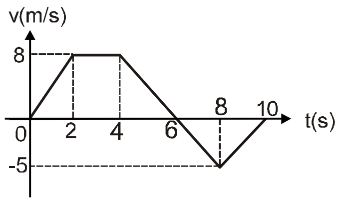
(a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Example 2. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Solution : Distance travelled = Area under v-t graph (taking all areas as +ve.)

Distance travelled = Area of trapezium + Area of triangle

$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5 = 32 + 10 = 42$ m

Displacement = Area under v-t graph (taking areas below time axis as –ve.)

Displacement = Area of trapezium – Area of triangle

$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5 = 32 - 10 = 22$ m

Hence, distance travelled = 42 m and displacement = 22 m.

14. MOTION WITH NON-UNIFORM ACCELERATION (USE OF DEFINITE INTEGRALS)

$\Delta x = \int_{t_i}^{t_f} v(t)dt$ (displacement in time interval $t = t_i$ to t_f)

The expression on the right hand side is called the definite integral of v(t) between $t = t_i$ and $t = t_f$. Similarly change in velocity

$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)dt$

14.1 Solving Problems which Involves Non uniform Acceleration

(i) Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t,

$\int_{v_0}^{v} dv = \int_0^t a(t)dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$.

On integrating, we get a relation between v and t, and then

using $\int_{x_0}^{x} dx = \int_0^t v(t)dt$, x and t can also be related.

(ii) Acceleration depending on velocity v or position x

$a = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dx}\frac{dx}{dt} \Rightarrow a = \frac{dx}{dt}\frac{dv}{dx} \Rightarrow a = v\frac{dv}{dx}$

This is another important expression for acceleration. If a is in terms of x, $\int_{v_0}^{v} vdv = \int_{x_0}^{x} a(x)dx$.

If a is in terms of v,

On integrating, we get a relation between x and v.

Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

Solved Example

Example 1. An object starts from rest at t = 0 and accelerates at a rate given by a = 6t. What is
(a) its velocity and
(b) its displacement at any time t?

Solution : As acceleration is given as a function of time, $\therefore \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$

Here $t_0 = 0$ and $v(t_0) = 0$ $\therefore$ $v(t) = \int_0^t 6tdt = 6\left(\frac{t^2}{2}\right)\Big|_0^t = 6\left(\frac{t^2}{2} - 0\right) = 3t^2$

So, $v(t) = 3t^2$

As $\Delta x = \int_{t_0}^{t} v(t)dt$ $\therefore$ $\Delta x = \int_0^t 3t^2dt = 3\left(\frac{t^3}{3}\right)\Big|_0^t = 3\left(\frac{t^3}{3} - 0\right) = t^3$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$.

Example 2. For a particle moving along v + x-axis, acceleration is given as a = x. Find the position as a function of time? Given that at t = 0 , x = 1 v = 1.

Solution : $a = x \Rightarrow \frac{vdv}{dx} = x \Rightarrow \frac{v^2}{2} = \frac{x^2}{2} + C$

t = 0, x = 1 and v = 1

$\therefore C = 0 \Rightarrow v^2 = x^2$

$v = \pm x$ but given that x = 1 when v = 1

$\therefore \quad v = x \Rightarrow dx/dt = x \Rightarrow dx/x = dt$

$\ell nx = t + C \Rightarrow 0 = 0 + C \Rightarrow \ell nx = t \Rightarrow x = e^t$

Example 3. For a particle moving along x-axis, acceleration is given as a = v. Find the position as a function of time ? Given that at t = 0, x = 0 v = 1.

Solution : $a = v \Rightarrow \frac{dv}{dt} = v \Rightarrow \int \frac{dv}{v} = \int dt$

$\ell nv = t + c \Rightarrow 0 = 0 + c$

$v = e^t \Rightarrow \frac{dx}{dt} = e^t \Rightarrow \int dx = \int e^t dt$

$\Rightarrow x = e^t + c \Rightarrow 0 = 1 + c \Rightarrow x = e^t - 1$

MISCELLANEOUS SOLVED PROBLEMS

Problem 1 A particle covers 3/4 of total distance with speed v_1 and next 1/4 with v_2. Find the average speed of the particle?

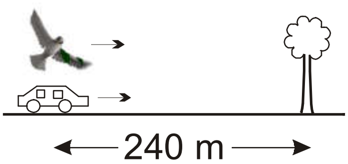
Answer : $\frac{4v_1v_2}{v_1 + 3v_2}$

Solution : Let the total distance be s

average speed (< v >) = $\frac{\text{Total distance}}{\text{Total time taken}}$

$$< v > = \frac{s}{\frac{3s}{4v_1} + \frac{s}{4v_2}} = \frac{1}{\frac{3}{4v_1} + \frac{1}{4v_2}} = \frac{4v_1v_2}{v_1 + 3v_2}$$

Problem 2 A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Answer : 360 m

Solution : Time taken by a car to reaches the tree (t) = $\frac{240 \text{ m}}{60 \text{ km/hr}} = \frac{0.24}{60}$hr

Now, the distance travelled by the bird during this time interval (s)

$= 90 \times \frac{0.24}{60}$ = 0.12 × 3 km = 360 m.

Problem 3 The position of a particle moving on X-axis is given by $x = At^3 + Bt^2 + Ct + D$. The numerical values of A, B, C, D are 1, 4, –2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4s, (d) the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t = 4 s.

Answer : **(a)** $[A] = [LT^{-3}]$, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$; **(b)** 78 m/s ; **(c)** 32 m/s^2 ; **(d)** 30 m/s ; **(e)** 20 m/s^2

Solution : As $x = At^3 + Dt^2 + Ct + D$

(a) Dimensions of A, B, C and D,

$[At^3] = [x]$ (by principle of homogeneity)

$[A] = [LT^{-3}]$

Similarly, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) As $v = dx/dt = 3At^2 + 2Bt + C$

Velocity at t = 4 sec.

$v = 3(1)(4)^2 + 2(4)(4) - 2 = 78$ m/s.

(c) Acceleration (a) = dv/dt = 6At + 2B ; a = 32 m/s^2

(d) Average velocity as $x = At^3 + Bt^2 + Ct = D$

position at t = 0, is x = D = 5m.

Position at t = 4 sec is (1)(64) + (4)(16) – (2) (4) + 5 = 125 m

Thus the displacement during 0 to 4 sec. is 125 – 5 = 120 m

$\therefore$ < v > = 120 / 4 = 30 m/s

(e) $v = 3At^2 + 20t + C$, velocity at t = 0 is c = – 2 m/s

velocity at t = 4 sec is 78 m/s $\therefore < a > = \frac{v_2 - v_1}{t_2 - t_1} = 20$ m/s^2

Problem 4 For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At t = 0, particle is at origin. Find the position as a function of time?

Solution : $v = 2t^2 + \sin t \quad \Rightarrow \quad dx/dt = 2t^2 + \sin t$

$$\int_0^x dx = \int_0^t (2t^2 + \sin t)dt = x = \frac{2}{3}t^3 - \cos t + 1 \quad \textbf{Ans.}$$

Problem 5 A car decelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution : v = 0 u = 20 m/s s = 100 m $\Rightarrow$ as $v^2 = u^2 + 2as$

$0 = 400 + 2a \times 100 \quad \Rightarrow \quad a = -2$ m/s

$\therefore$ Acceleration = **2 m/s²** **Ans.**

Problem 6 A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

Solution : $v^2 = u^2 + 2as$

$25 \times 25 = 0 + 100\,a$

$a = \frac{25}{4}$ m/s²

Now, for time taken by the back end of the train to pass the worker

we have $v'^2 = v^2 + 2al = (25)^2 + 2 \times 25/48 \times 150$

$v'^2 = 25 \times 25 \times 4$

$v' =$ **50 m/s.** **Ans.**

Problem 7 A particle is thrown vertically with velocity 20 m/s. Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Answer : 25m, 15m

Solution : Highest point say B

$V_B = 0$

$v = u + gt$

$0 = 20 - 10\,t$

t = 2 sec.

$\therefore$ distance travel in first 2 seconds.

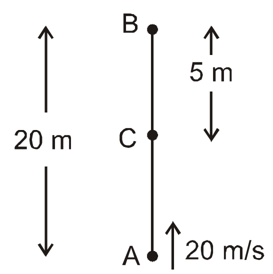
s = s(t =0 to 2sec) + s (2sec. to 3sec.)

$s = [ut + 1/2\,at^2]_{t = 0 \text{ to } t = 2s} + [ut + 1/2at^2]_{t = 2 \text{ to } t = 3s}$

$s = 20 \times 2 - 1/2 \times 10 \times 4 + 1/2 \times 10 \times 1^2$

= (40 – 20) + 5 = **25 m.**

and displacement = 20 – 5 = **15 m.**



Problem 8 A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t. Find the maximum velocity acquired by the car.

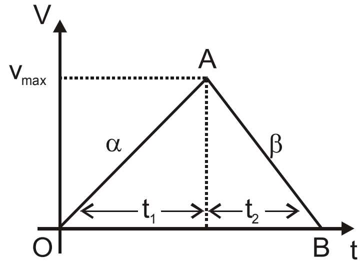
Solution : $t = t_1 + t_2$

slope of OA curve = $\tan\theta = \alpha = \frac{v_{max}}{t_1}$

slope of AB curve = $\beta = \frac{v_{max}}{t_2}$

$t = t_1 + t_2$

$\Rightarrow \quad t = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} \quad \Rightarrow \quad v_{max} = \left(\frac{\alpha\beta}{\alpha + \beta}\right)t$



Problem 9 In the above question find total distance travelled by the car in time 't' .

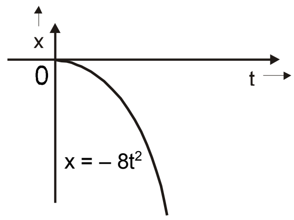
Solution : $v_{max} = \frac{\alpha\beta}{(\alpha+\beta)}t \quad \Rightarrow \quad t_1 = \frac{v_{max}}{\alpha} = \frac{\beta t}{(\alpha+\beta)} \quad \Rightarrow \quad t_2 = \frac{v_{max}}{\beta} = \frac{\alpha t}{(\alpha+\beta)}$

$\therefore$ Total distance travelled by the car in time 't'

$$= \frac{1}{2}\alpha t_1^2 + v_{max}\, t_2 - \frac{1}{2}\beta t_2^2 = \frac{1}{2}\frac{\alpha\beta^2 t^2}{(\alpha+\beta)^2} + \frac{\alpha^2\beta t^2}{(\alpha+\beta)^2} - \frac{1}{2}\frac{\beta\alpha^2 t^2}{(\alpha+\beta)^2}$$

Area under graph (directly) = $\frac{1}{2}\frac{\alpha\beta t^2}{(\alpha+\beta)} = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$ **Ans.**

Problem 10 The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



Upwards direction is taken as positive, downwards direction is taken as negative.

Solution :

(a) The equation of motion is : $x = -8t^2$

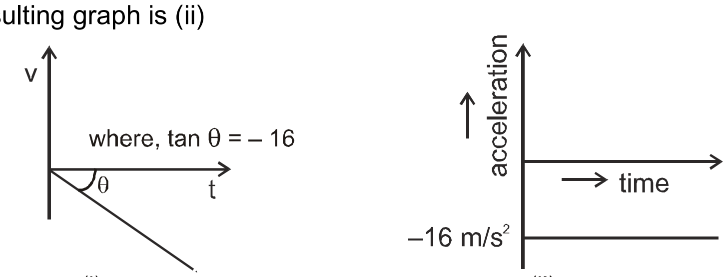
$\therefore$ $v = \frac{dx}{dt} = -16\,t$; this shows that velocity is directly proportional to time and slope of velocity-time curve is negative i.e., – 16.

Hence, resulting graph is (i)

(b) Acceleration of particle is : $a = \frac{dv}{dt} = -16$.

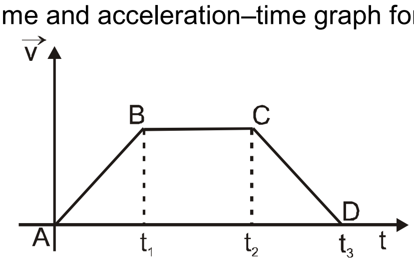
This shows that acceleration is constant but negative.

Resulting graph is (ii)



(i) (ii)

Problem 11 Draw displacement–time and acceleration–time graph for the given velocity–time graph.



Solution : **Part AB :** v-t curve shows constant slope

i.e. constant acceleration or Velocity increases at constant rate with time.

Hence, s-t curve will show constant increase in slope

and a-t curve will be a straight line.

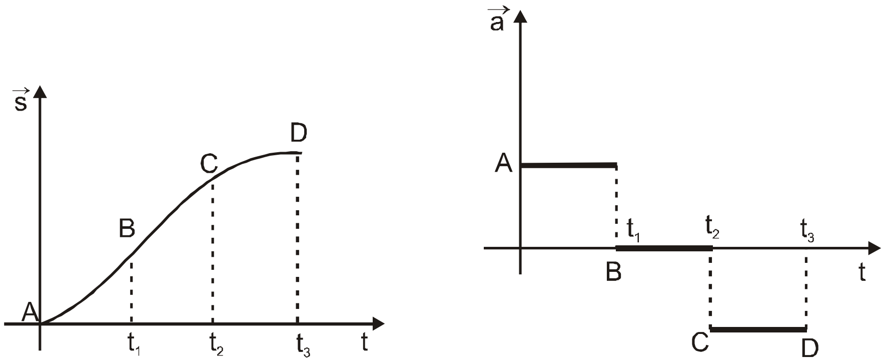
Part BC : v-t curve shows zero slope i.e. constant velocity. So, s-t curve will show constant slope and acceleration will be zero.

Part CD : v-t curve shows negative slope i.e. velocity is decreasing with time or acceleration is negative.

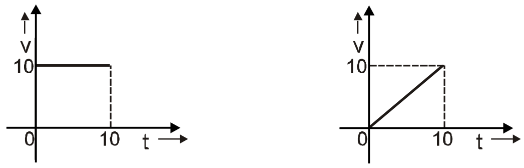
Hence, s-t curve will show decrease in slope becoming zero in the end.

and a-t curve will be a straight line with negative intercept on y-axis.

RESULTING GRAPHS ARE :



Problem 12 For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



(a) (b)

Solution :

(a) Distance area under the v - t curve

$\therefore$ distance = $10 \times 10 = 100$ m **Ans.**

(b) Area under v – t curve

$\therefore$ distance = $\frac{1}{2} \times 10 \times 10 = 50$ m **Ans.**

Problem 13 For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at $x = 0$, find speed as a function of x.

Solution : $a = 2v^2 \Rightarrow$ or $\frac{dv}{dt} = 2v^2$ or $\frac{dv}{dx} \times \frac{dx}{dt} = 2v^2$

$v\frac{dv}{dx} = 2v^2 \Rightarrow \frac{dv}{dx} = 2v$

$\int_{v_0}^{v} \frac{dv}{v} = \int_0^x 2\ dx \Rightarrow [\ell nv]_{v_0}^{v} = [2x]_0^x$

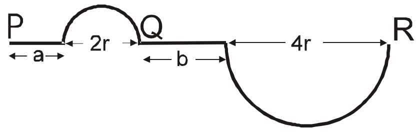
$\ell n\frac{v}{v_0} = 2x \Rightarrow v = v_0e^{2x}$ **Ans.**

Exercise-1

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Distance and Displacement

A-1. A car starts from P and follows the path as shown in figure. Finally car stops at R. Find the distance travelled and displacement of the car if a = 7 m, b = 8 m and r = $\frac{11}{\pi}$ m ?



[Take $\pi = \frac{22}{7}$]

A-2. A man moves to go 50 m due south, 40 m due west and 20 m due north to reach a field.
(a) What distance does be have to walk to reach the field ?
(b) What is his displacement from his house to the field?

Section (B) : Average speed and average velocity

B-1. When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km. During journey he stay for 15 minute at midway.
(a) What is the average speed of the car during this period ?
(b) What is the average velocity?

B-2. A particle covers each 1/3 of the total distance with speed v_1, v_2 and v_3 respectively. Find the average speed of the particle ?

Section (C) : Velocity, Acceleration, Average acceleration

C-1. The position of a body is given by $x = At + 4Bt^3$, where A and B are constants, x is position and t is time. Find (a) acceleration as a function of time, (b) velocity and acceleration at t = 5 s.

C-2. An athelete takes 2s to reach his maximum speed of 18 km/h after starting from rest. What is the magnitude of his average accleration?

C-3 A boy start towards east with uniform speed 5m/s. After t = 2 second he turns right and travels 40 m with same speed. Again he turns right and travels for 8 second with same speed. Find out the displacement; average speed, average velocity and total distance travelled.

Section (D) : Equations of motion and motion under gravity

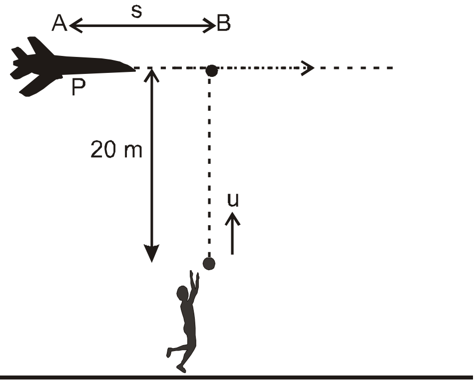
D-1. A car accelerates from 36 km/h to 90 km/h in 5 s on a straight rod. What was its acceleration in m/s^2 and how far did it travel in this time? Assume constant acceleration and direction of motion remains constant.

D-2. A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute after applying breaks. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed. (Assume retardation to be constant)

D-3. A car travelling at 72 km/h decelerates uniformly at 2 m/s^2. Calculate (a) the distance it goes before it stops, (b) the time it takes to stop, and (c) the distance it travels during the first and third seconds.

D-4. A ball is dropped from a tower. In the last second of its motion it travels a distance of 15 m. Find the height of the tower. [Take g = 10m/sec^2]

D-5. A toy plane P starts flying from point A along a straight horizontal line 20 m above ground level starting with zero initial velocity and acceleration 2 m/s^2 as shown. At the same instant, a man P throws a ball vertically upwards with initial velocity 'u'. Ball touches (coming to rest) the base of the plane at point B of plane's journey when it is vertically above the man. 's' is the distance of point B from point A. Just after the contact of ball with the plane, acceleration of plane increases to 4 m/s^2. Find:



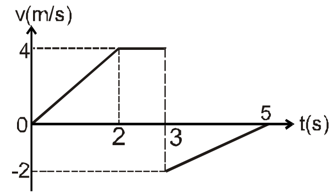
(i) Initial velocity 'u' of ball.

(ii) Distance 's'.

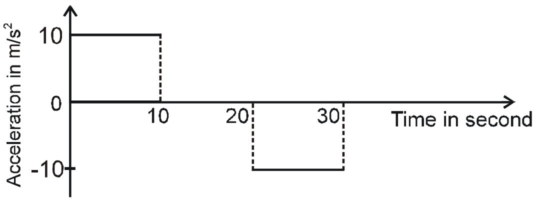
(iii) Distance between man and plane when the man catches the ball back. (g = 10 m/s^2) (Neglect the height of man)

Section (E) : Graph related questions

E-1. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle? Also find the average velocity of the particle in intervel 0 to 5 second.



E-2. A cart started at t = 0, its acceleration varies with time as shown in figure. Find the distance travelled in 30 seconds and draw the position-time graph.



E-3. Two particles A and B start from rest and move for equal time on a straight line. The particle A has an acceleration a for the first half of the total time and 2a for the second half. The particle B has an acceleration 2a for the first half and a for the second half. Which particle has covered larger distance?

E-4 A tiger running 100 m race, accelerates for one third time of the total time and then moves with uniform speed. Then find the total time taken by the tiger to run 100 m if the acceleration of the tiger is 8 m/s^2.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Distance and Displacement

A-1. A hall has the dimensions 10 m × 10 m × 10 m. A fly starting at one corner ends up at a farthest corner. The magnitude of its displacement is:

(A) $5\sqrt{3}$ m (B) $10\sqrt{3}$ m (C) $20\sqrt{3}$ m (D) $30\sqrt{3}$ m

Section (B) : Average speed and average velocity

B-1. A car travels from A to B at a speed of 20 km h^{-1} and returns at a speed of 30 km h^{-1}. The average speed of the car for the whole journey is :
(A) 5 km h^{-1} (B) 24 km h^{-1} (C) 25 km h^{-1} (D) 50 km h^{-1}

B-2. A person travelling on a straight line without changing direction moves with a uniform speed v_1 for half distance and next half distance he covers with uniform speed v_2. The average speed v is given by
(A) $v = \frac{2v_1v_2}{v_1+v_2}$ (B) $v = \sqrt{v_1v_2}$ (C) $\frac{v_1+v_2}{2}$ (D) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

B-3. A body covers first 1/3 part of its journey with a velocity of 2 m/s, next 1/3 part with a velocity of 3 m/s and rest of the journey with a velocity 6m/s. The average velocity of the body will be
(A) 3 m/s (B) $\frac{11}{3}$ m/s (C) $\frac{8}{3}$ m/s (D) $\frac{4}{3}$ m/s

B-4. A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap? (π = 3.14)
(A) velocity 10m/s, speed 10 m/s (B) velocity zero, speed 10 m/s
(C) velocity zero, speed zero (D) velocity 10 m/s, speed zero

Section (C) : Velocity, Acceleration and Average acceleration

C-1. The displacement of a body is given by $2s = gt^2$ where g is a constant. The velocity of the body at any time t is:
(A) gt (B) gt/2 (C) $gt^2/2$ (D) $gt^3/6$

C-2. A stone is thrown vertically upward with an initial speed u from the top of a tower, reaches the ground with a speed 3u. The height of the tower is:
(A) $\frac{3u^2}{g}$ (B) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$

C-3. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the particle in the last two seconds is :
(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$ (C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(2n+1)}{n}$

Section (D) : Equations of motion and motion under gravity

D-1. A body starts from rest and is uniformly acclerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as
(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

D-2. A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m height of a window some distance from the top of the building. If the speed of the ball at the top and at the bottom of the window are v_T and v_B respectively, then (g = 9.8 m/sec^2)
(A) $v_T + v_B$ = 12 ms^{-1} (B) $v_T - v_B$ = 4.9 m s^{-1} (C) v_Bv_T = 1 ms^{-1} (D) $\frac{v_B}{v_T}$ = 1 ms^{-1}

D-3. A stone is released from an elevator going up with an acceleration a and speed u. The acceleration and speed of the stone just after the release is
(A) a upward, zero (B) (g-a) upward, u (C) (g-a) downward, zero (D) g downward, u

D-4. The initial velocity of a particle is given by u (at t = 0) and the acceleration by f, where f = at (here t is time and a is constant). Which of the following relation is valid?
(A) $v = u + at^2$ (B) $v = u + at^2/2$ (C) $v = u + at$ (D) $v = u$

D-5. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after dropping the stone at which the splash is heard is given by

(A) T = 2h/v (B) $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$ (C) $T = \sqrt{\frac{2h}{g}} + \frac{h}{2v}$ (D) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

D-6. A student determined to test the law of gravity for himself walks off a sky scraper 320 m high with a stopwatch in hand and starts his free fall (zero initial velocity). 5 second later, superman arrives at the scene and dives off the roof to save the student. What must be superman's initial velocity in order that he catches the student just before reaching the ground ? [Assume that the superman's acceleration is that of any freely falling body.] ($g = 10\ m/s^2$)

(A) 98 m/s (B) $\frac{275}{3}$ m/s (C) $\frac{187}{2}$ m/s (D) It is not possible

D-7. In the above question, what must be the maximum height of the skyscraper so that even superman cannot save him.

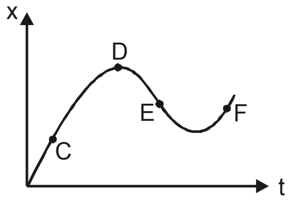
(A) 65 m (B) 85 m (C) 125 m (D) 145 m

D-8. Two particles held at different heights a and b above the ground are allowed to fall from rest. The ratio of their velocities on reaching the ground is :

(A) a : b (B) $\sqrt{a} : \sqrt{b}$ (C) $a^2 : b^2$ (D) $a^3 : b^3$

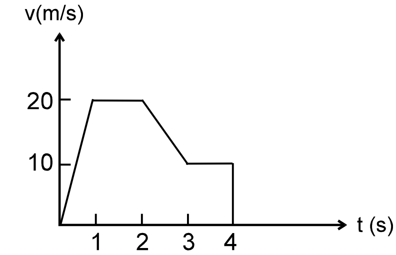
Section (E) : Graph related questions

E-1. In the displacement–time graph of a moving particle is shown, the instantaneous velocity of the particle is negative at the point :



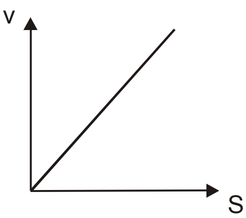
(A) C (B) D (C) E (D) F

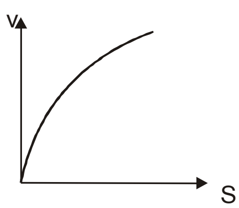
E-2. The variation of velocity of a particle moving along a straight line is shown in the figure. The distance travelled by the particle in 4 s is :

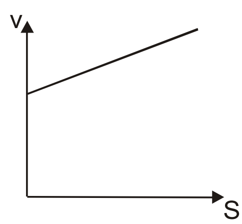


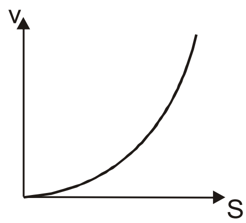
(A) 25 m (B) 30 m (C) 55 m (D) 60 m

E-3. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is :

(A) 

(B) 

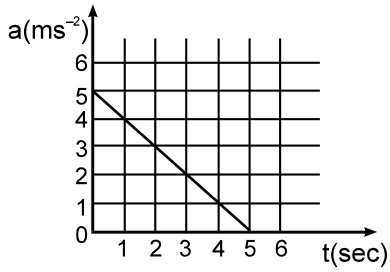
(C) 

(D) 

E-4. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30° and 60° with the time axis. If the velocity of A is v_A and that of B is v_B, then the value of v_A/v_B is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{1}{3}$

E-5. Starting from rest at t = 0, a car moves in a straight line with an acceleration given by the accompanying graph. The speed of the car at t = 3 s is :

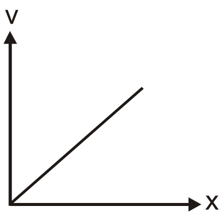


(A) 1 m s^{-1} (B) 2 m s^{-1} (C) 6.0 m s^{-1} (D) 10.5 m s^{-1}

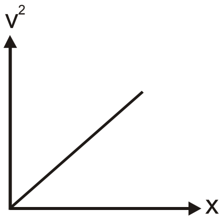
PART - III : MATCH THE COLUMN

1. Column I gives some graphs for a particle moving along x-axis in positive x–direction. The variables v, x and t represent velocity of particle, x–coordinate of particle and time respectively. Column II gives certain resulting interpretation. Match the graphs in Column I with the statements in Column II.

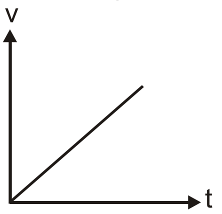
Column I

(A)


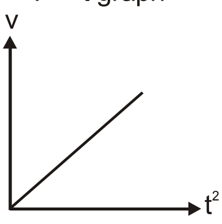
v - x graph

(B)


v^2 - x graph

(C)


v - t graph

(D)


v - t^2 graph

Column II

(p) Acceleration of particle is uniform

(q) Acceleration of particle is nonuniform

(r) Acceleration of particle is directly proportional to ‘t’

(s) Acceleration of particle is directly proportional to ‘x’.

2.✍ **Match the following :**

Column-I	**Column-II**
(A) Rate of change of displacement	(p) Magnitude of average velocity
(B) Average speed is always greater than or equal to	(q) Initial to final position
(C) Displacement has the same direction as that of	(r) Velocity
(D) Motion under gravity is considered as the case of	(s) Uniform acceleration

Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. In the one-dimensional motion of a particle, the relation between position x and time t is given by $x^2 + 2x = t$ (here x > 0). Choose the correct statement :

(A) The retardation of the particle is $\frac{1}{4(x+1)^3}$

(B) The uniform acceleration of the particle is $\frac{1}{(x+1)^3}$

(C) The uniform velocity of the particle is $\frac{1}{(x+1)^3}$

(D) The particle has a variable acceleration of 4t + 6.

2.✍ Two balls of equal masses are thrown upward, along the same vertical line at an interval of 2 seconds, with the same initial velocity of 40 m/s. Then these collide at a height of (Take g = 10 m/s^2)

(A) 120 m (B) 75 m (C) 200 m (D) 45 m

3. A body is released from the top of a tower of height h metre. It takes T seconds to reach the ground. Where is the ball at the time T/2 seconds ?

(A) at h/4 metre from the ground (B) at h/2 metre from the ground

(C) at 3h/4 metre from the ground (D) depends upon the mass of the ball

4.✍ A ball is thrown vertically upwards from the top of a tower of height h with velocity v. The ball strikes the ground after time.

(A) $\frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$ (B) $\frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$ (C) $\frac{v}{g}\left(1+\frac{2gh}{v^2}\right)^{1/2}$ (D) $\frac{v}{g}\left(1-\frac{2gh}{v^2}\right)^{1/2}$

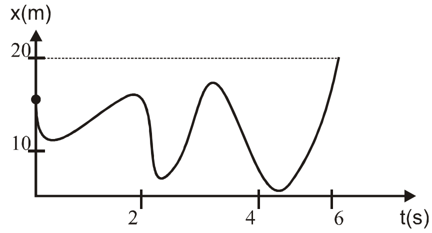
5.✍ A balloon is moving upwards with velocity 10 ms^{-1}. It releases a stone which comes down to the ground in 11 s. The height of the balloon from the ground at the moment when the stone was dropped is :

(A) 495 m (B) 592 m (C) 460 m (D) 500 m

6.✍ Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ? (Take g = 10 ms^{-2})

(A) $\frac{5}{4}$ m (B) 4 m (C) $\frac{5}{2}$ m (D) $\frac{15}{4}$ m

7. Figure shows the position of a particle moving on X-axis as function of time.



(A) The particle has come to rest 5 times

(B) Initial speed of particle was zero

(C) The velocity remains positive for t = 0 to t = 6 s

(D) The average velocity for the total period shown is negative.

PART - II : NUMERICAL VALUE

1. A particle moving in straight line, traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time and with velocity v_2 for the other half of the time. Mean velocity of the particle averaged over the whole time of motion comes out to be $av_0\left(\frac{v_1 + v_2}{b\,v_0 + v_1 + v_2}\right)$, where a and b are positive integers. Find a + b.

2. The displacement of a particle moving on a straight line is given by $x = 16t - 2t^2$. Distance travelled by the particle during the first 2 sec. is S_1 and during first 6 sec. is S_2. Find $\frac{3S_2}{S_1}$

3. A healthy youngman standing at a distance of 6 m from a 11.5 m high building sees a kid slipping from the top floor. With what uniform acceleration in m/s^2 (starting from rest) should he run to catch the kid at the arms height (1.5 m)? Take $g = 10\ m/s^2$.

4. A body freely falling from rest has a velocity v after it falls through distance 2m. The distance it has to fall down further in m for its velocity to become double is :

5. Two objects moving along the same straight line are leaving point A with an acceleration a, 2a & velocity 2 u, u respectively at time t = 0. The distance moved by the object with respect to point A when one object overtakes the other is $\alpha u^2/a$. Here α is an integer. Find α :

6. A particle is thrown upwards from ground. It experiences a constant air resistance which can produce a retardation of $2\ m/s^2$ opposite to the direction of velocity of particle. The ratio of time of ascent to the time of descent is $\sqrt{\frac{\alpha}{\beta}}$. Where α and β are integers. Find minimum value of $\alpha + \beta$ [$g = 10\ m/s^2$]

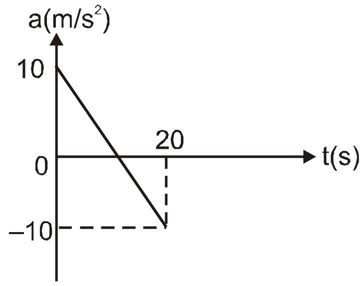
7. A police jeep is chasing a culprit going on a moter bike. The motor bike crosses a turn at a speed of 72 km/h. The jeep follows it at a speed of 108 km/h, crossing the turn 10 seconds later than bike (keeping constant speed). After crossing the turn, jeep acclerates with constant accleration $2\ m/s^2$. Assuming bike travels at constant speed, after travelling a distance 20α m. from the turn, the jeep catches the bike. Where α is an integer. Find α.

8. A body starts with an initial velocity of 10 m/s and moves along a straight line with a constant acceleration. When the velocity of the particle becomes 50 m/s the acceleration is reversed in direction without changing magnitude. Find the speed of the particle in m/s when it reaches the starting point.

9. A lift starts from the top of a mine shaft and descends with a constant speed of 10 m/s. 4 s later a boy throws a stone vertically upwards from the top of the shaft with a speed of 30 m/s. If stone hits the lift at a distance x below the shaft write the value of x/3 (in m) [Take: $g = 10\ m/s^2$] (Give value of $20\sqrt{6} = 49$)

PART - III : ONE OR MORE THAN ONE OPTION CORRECT TYPE

1. The acceleration time plot for a particle (starting from rest) moving on a straight line is shown in figure. For given time interval :



(A) The particle has zero average acceleration

(B) The particle has never turned around.

(C) The particle has zero displacement

(D) The average speed in the interval 0 to 10s is the same as the average speed in the interval 10s to 20s.

2. The acceleration of a particle is zero at t = 0

(A) Its velocity must be constant.

(B) The speed at t = 0 may be zero.

(C) If the acceleration is zero from t = 0 to t = 5 s, the speed is constant in this interval.

(D) If the speed is zero from t = 0 to t = 5 s the acceleration is also zero in the interval.

3. Mark the correct statements for a particle going on a straight line (x–position coordinate, v–velocity, a–acceleration) :

(A) If v and a have opposite sign, the object is slowing down.

(B) If x and v have opposite sign, the particle is moving towards the origin.

(C) If v is zero at an instant, then a should also be zero at that instant.

(D) If v is zero for a time interval, then a is zero at every instant within the time interval.

4. The displacement of a moving particle is proportional to the square of the time. For this particle **[REE 1994]**

(A) the velocity is constant (B) the velocity is variable

(C) the acceleration is constant (D) the acceleration is variable

5. A particle moves along the Y-axis and its y-coordinate(y) changes with time(t) as $y = u(t-2) + a(t-2)^2$

(A) the initial velocity (at t = 0) of the particle is u (B) the acceleration of the particle is a

(C) the acceleration of the particle is 2a (D) at t = 2s particle is at the origin

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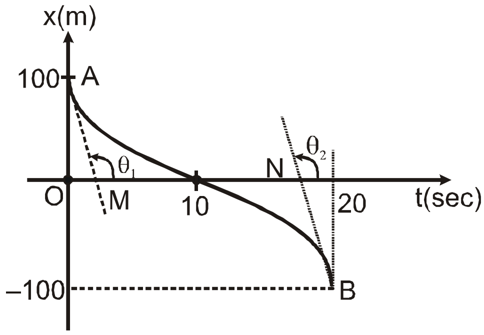
PART - IV : COMPREHENSION

Comprehension-1

Read the following write up and answer the questions based on that.

The graph below gives the coordinate of a particle travelling along the X-axis as a function of time. AM is the tangent to the curve at the starting moment and BN is tangent at the end moment ($\theta_1 = \theta_2 = 120°$).



1 The average velocity during the first 20 seconds is

(A) – 10 m/s (B) 10 m/s (C) zero (D) 20 m/s

2. The average acceleration during the first 20 seconds is

(A) – 1 m/s² (B) 1 m/s² (C) zero (D) 2 m/s²

3. The direction ($\hat{i}$ or $-\hat{i}$) of acceleration during the first 10 seconds is _____________ .

4. Time interval during which the motion is retarded.

(A) 0 to 20sec. (B) 10 to 20sec. (C) 0 to 10sec. (D) None of these

Comprehension # 2

The position of a particle is given by $x = 2(t - t^2)$ where t is expressed in seconds and x is in meter. Positive direction is towards right.

5. The acceleration of the particle is

(A) 0 (B) 4 m/s² (C) –4 m/s² (D) None of these.

6. The maximum value of position co-ordinate of particle on positive x-axis is

(A) 1 m (B) 2 m (C) 1/2 m (D) 4 m

7. The particle

(A) never goes to negative x-axis

(B) never goes to positive x-axis

(C) starts motion from the origin then goes upto x = 1/2 in the positive x-axis then goes to negative x-axis

(D) final velocity of the particle is zero

8. The total distance travelled by the particle between t = 0 to t = 1 s is :

(A) 0 m (B) 1 m (C) 2 m (D) $\frac{1}{2}$ m

Exercise-3

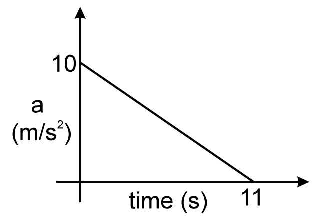
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. A block is moving down a smooth inclined plane starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval t = n – 1 to t = n. The ratio $\frac{S_n}{S_{n+1}}$ is **[JEE (Scr.), 2004, 3/84, –1]**

(A) $\frac{2n-1}{2n}$ (B) $\frac{2n-1}{2n+1}$ (C) $\frac{2n+1}{2n-1}$ (D) $\frac{2n}{2n-1}$

2. A particle is initially at rest, It is subjected to a linear acceleration a, as shown in the figure. The maximum speed attained by the particle is **[JEE (Scr.) 2004; 3/84, –1]**



(A) 605 m/s (B) 110 m/s (C) 55 m/s (D) 550 m/s

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

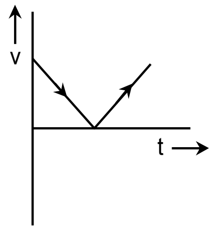
1. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{d\upsilon}{dt} = -2.5\sqrt{\upsilon}$, where υ is the instantaneous speed. The time taken by the object, to come to rest, would be :**[AIEEE 2011; 4/120, –1]**

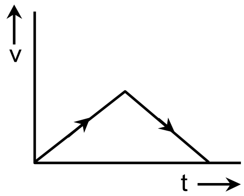
(1) 1 s (2) 2 s (3) 4 s (4) 8 s

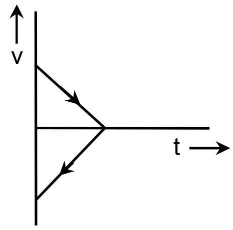
2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE (Main) 2014; 4/120, –1]**

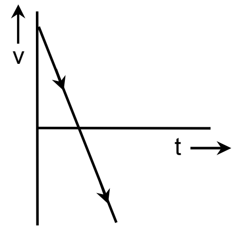
(1) $2gH = n^2u^2$ (2) $gH = (n-2)^2u^2$ (3) $2gH = nu^2(n-2)$ (4) $gH = (n-2)u^2$

3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ? **[JEE (Main) 2017; 4/120, –1]**

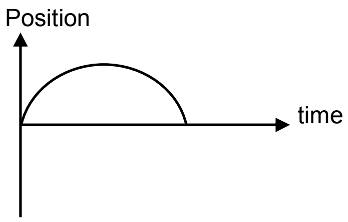
(1)


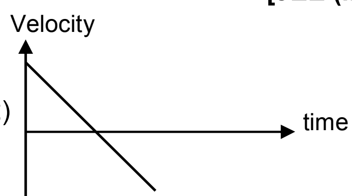
(2)


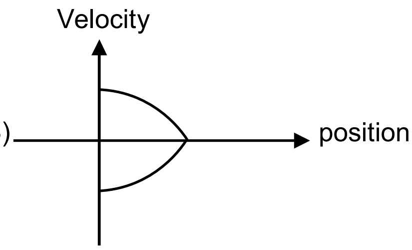
(3)


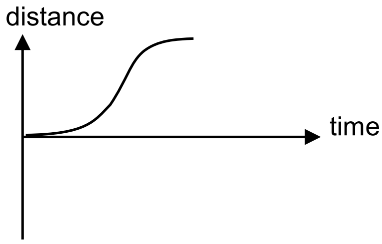
(4)


4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE (Main) 2018; 4/120, –1]**

(1)


(2)


(3)


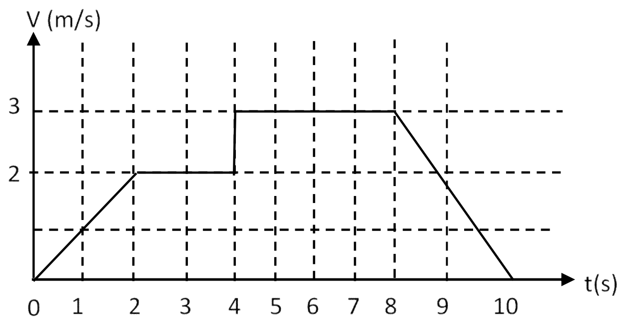
(4)


5. A particle is moving with speed $\upsilon = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0). **[JEE (Main) 2019 April; 4/120, –1]**

(1) $b^2\tau$ (2) $\frac{b^2\tau}{4}$ (3) $\frac{b^2\tau}{\sqrt{2}}$ (4) $\frac{b^2\tau}{2}$

6. A particle starts from origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?

[JEE (Main) 2019, January; 4/120, –1]



(1) 6 m (2) 9 m (3) 3 m (4) 16 m

7. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is …….

[JEE (Main) 2020, 09 January; 4/100]

8. A particle starts from the origin at t = 0 with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32m is D meters. The value of D is : **[JEE (Main) 2020, 09 January; 4/100, –1]**

(1) 50 (2) 40 (3) 32 (4) 60

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. Distance travelled by the car = 48 m, Displacement of the car = 36 m

A-2. (a) 110 m (b) 50 m, $\tan^{-1} 4/3$ west of south

Section (B) :

B-1. (a) 32 km/h (b) zero

B-2. $\dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_1v_3}$

Section (C) :

C-1. (a) 24 Bt ; (b) A + 300 B, 120 B

C-2. 5/2 = 2.5 m/s^2

C-3 50m at 53° S of W, 5m/s, 25/9 m/s at 53° S of W, 90 m

Section (D) :

D-1. a = 3 m/s^2 ; $\dfrac{175}{2}$ = 87.5 m

D-2. (a) 2700 m = 2.7 km, (b) 60 m/s, (c) 225 m and 2.25 km

D-3. (a) 100 m ; (b) 10 s ; (c) 19 m, 15 m

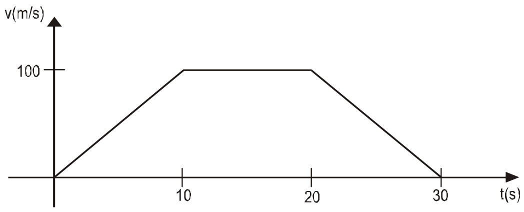
D-4. 20m

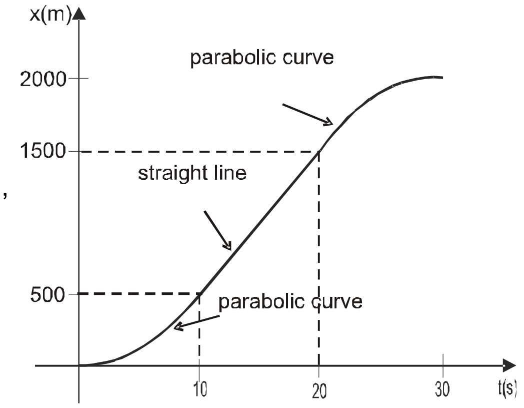
D-5. (i) 20 m/s (ii) 4 m (iii) $\sqrt{656}$ m.

Section (E) :

E-1. Distance travelled = 10m; displacement = 6m; average velocity = 6/5 = 1.2 m/s

E-2. 2000 m,





E-3. Particle B **E-4** $3\sqrt{5}$ m /s

PART - II

Section (A) :

A-1. (B)

Section (B) :

B-1. (B) **B-2.** (A) **B-3.** (A)
B-4. (B)

Section (C) :

C-1. (A) **C-2.** (B) **C-3.** (A)

Section (D) :

D-1. (C) **D-2.** (A) **D-3.** (D)
D-4. (B) **D-5.** (B) **D-6.** (B)
D-7. (C) **D-8.** (B)

Section (E) :

E-1. (C) **E-2.** (C) **E-3.** (B)
E-4. (D) **E-5.** (D)

PART - III

1. (A) q, s ; (B) p ; (C) p ; (D) q, r
2. (A) r ; (B) p ; (C) q ; (D) s

EXERCISE-2

PART - I

1. (A) **2.** (B) **3.** (C)
4. (A) **5.** (A) **6.** (D)
7. (A)

PART - II

1. 4 **2.** 5 **3.** 6
4. 6 **5.** 6 **6.** 5
7. 20 **8.** 70 **9.** 43

PART - III

1. (ABD) **2.** (BCD) **3.** (ABD)
4. (BC) **5.** (CD)

PART - IV

1 (A) **2.** (C) **3.** $\hat{i}$
4. (C) **5.** (C) **6.** (C)
7. (C) **8.** (B)

EXERCISE-3

PART - I

1. (B) **2.** (C)

PART - II

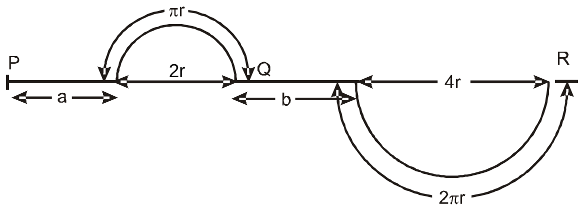
1. (2) **2.** (3) **3.** (4)
4. (4) **5.** (4) **6.** (2)
7. 3 **4.** (4)

SOLUTIONS OF RECTILINEAR MOTION

EXERCISE–1
PART - I

Section (A)

A-1. a = 7m, b = 8m, $r = \frac{11}{\pi}\left[\pi = \frac{22}{7}\right]$



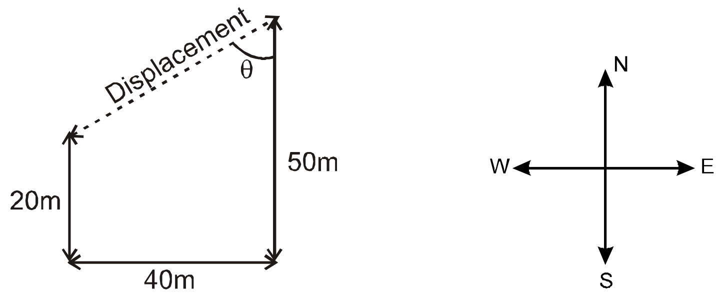
Distance travelled by the car from P to R

$= a + \pi r + b + 2\pi r = a + b + 3\pi r = 7 + 8 + 3\pi \times \frac{11}{\pi} =$ **48 m** **Ans**

Displacement of the car from P to R

$= a + 2r + b + 4r = a + b + 6r = 7 + 8 + 6 \times \frac{11}{\pi} = 15 + 6 \times \frac{11}{22} \times 7 =$ **36 m Ans**

A-2.



(a) Distance covered by the man to reach the field.

= 50 + 40 + 20 = **110 m Ans**

(b) Displacement of man from his house to the field

$= \sqrt{(40)^2 + (30)^2} = \sqrt{1600 + 900} = \sqrt{2500}$ **= 50 m Ans**

Direction of displacement can be known by finding θ

$\tan\theta = \frac{40}{30}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$ West of South **Ans**

Section (B) :

B-1. Initial reading of meter = 12352 km

Final reading of meter = 12416 km

Time taken by car = 2 hr

(a) Distance covered by car

= (Final reading– Initial reading)

= (12416 – 12352) = 64 km

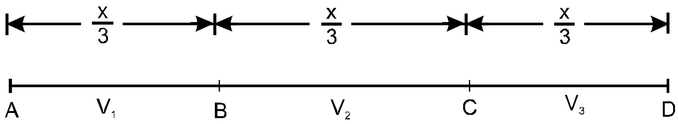
Average speed of car = 64/2 = 32 km/h **Ans**

(b) As the car returns to the initial point after whole journey,

hence, displacement of car = 0

Therefore, average velocity = 0 **Ans**

B-2.



Suppose the total distance covered by the particle i.e., AD = x

Particle covers first one – third distance AB with speed V_1

second one – third distance BC with speed V_2 and

third one – third distance CD with speed V_3.

Average speed of the particle = $\frac{\text{Total distance covered by the particle}}{\text{Total time taken by the particle}}$

$$= \frac{X}{\frac{X/3}{V_1} + \frac{X/3}{V_2} + \frac{X/3}{V_3}} = \frac{1}{\frac{1}{3V_1} + \frac{1}{3V_2} + \frac{1}{3V_3}} = \frac{3V_1V_2V_3}{V_1V_2 + V_2V_3 + V_3V_1} \quad \textbf{Ans}$$

Section (C)

C-1. The position of a body is given as $x = At + 4B\,t^3$

(a) $x = At + 4\,Bt^3$

$V = \frac{dx}{dt} = A + 12\,B\,t^2$, So, $a = \frac{dV}{dt} = 24\,Bt$ **Ans**

(b) At t = 5 s, $V = A + 12\,B\,(5)^2$,

i.e., $V = A + 300\,B$ **Ans**

At t = 5 s, a = 24 B (5) i.e., a = 120 B **Ans**

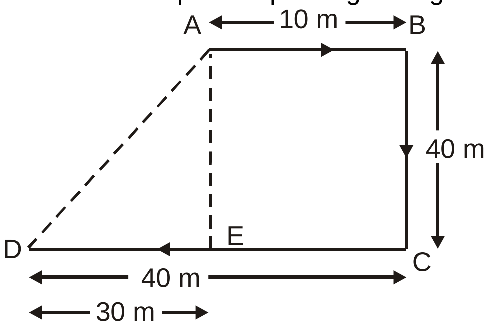
C-2. Maximum speed V = 18 km/h

= 18 × 5/18 = 5 m/s

Avg. acc. = $\frac{0 + V_{max}}{2} = \frac{0+5}{2}$,

So average acceleration = $\frac{5}{2}$ m/s^2 **Ans**

C-3 The particle starts from point A & reaches point D passing through B & C as shown in the figure.



Now, AE = 40 m & DE = 30 m

$\therefore$ Displacement = AD = $\sqrt{AE^2 + DE^2} = \sqrt{40^2 + 30^2}$ = 50 m

Total time taken in the motion = $t_{AB} + t_{BC} + t_{CD}$

$$= 2 + \frac{40}{5} + 8 = 18 \text{ s}$$

Total distance travelled = AB + BC + CD = 10 + 40 + 40 = 90m

Average velocity = $\frac{\text{Displacement}}{\text{time}} = \frac{50}{18} = \frac{25}{9}$ m/s

Average speed = $\frac{\text{Distance}}{\text{time}} = \frac{90}{18}$ = 5 m/s.

Section (D)

D-1. $u = 36 \text{ km/h} = 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s}$

$V = 90 \text{ km/h} = 90 \times \frac{5}{18} \text{ m/s} = 25 \text{ m/s}$

From the equation of motion;

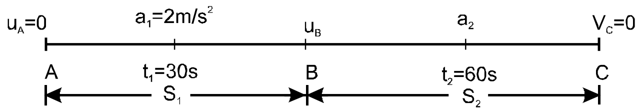
$V = u + at$ putting $t = 5s$

$25 = 10 + a(5)$, i.e., $a = \frac{25-10}{5} \Rightarrow a = 3 \text{ m/s}^2$ **Ans**

For distance travelled by the car in 5 sec, we use

$s = ut + \frac{1}{2}at^2 = 10 \times 5 + \frac{1}{2} \times 3\,(5)^2 = \frac{100+75}{2} = \frac{175}{2}$ i.e., = 87.5 m **Ans**

D-2.



(a) For motion from A to B :

$u_B = u_A + a_1t_1 = 0 + 2\,(30) = 60 \text{ m/s}$

Also, $S_1 = u_A\,t_1 + \frac{1}{2}a_1t_1^2 = 0 + \frac{1}{2}(2)\,(30)^2$ $\Rightarrow$ $S_1 = 900$ m

For motion from B to C :

$V_C = u_B - a_2\,t_2$; $0 = 60 - a_2\,(60)$;

i.e., $a_2 = \frac{60}{60} = 1 \text{ m/s}^2$, Also $V_C^2 = u_B^2 - 2a_2s_2$

$\Rightarrow (0)^2 = (60)^2 - 2(1)\,S_2$, i.e, $S_2 = \frac{60 \times 60}{2}$, i.e, $S_2 = 1800$ m

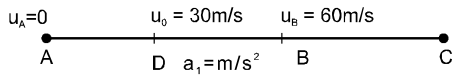
Now, total distance moved by the train

$S = S_1 + S_2 = 900 + 1800 \Rightarrow$ S = 2700 m **Ans**

(b) Maximum speed attained by the train will be at the point B, as after this point train starts retarding

So, $V_{max} = V_B = 60$ m/s **Ans**

(c) There will be two positions at which the train will be at half the maximum speed for motion from A to B.



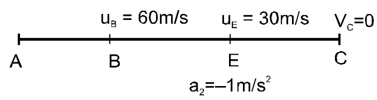
Let D be a point where $u_0 = \frac{u_B}{2} = \frac{60}{2}$; $u_D = 30$ m/s

$u_D^2 = u_A^2 + 2a_1\,(AD)$

$\Rightarrow AD = \frac{u_D^2 - u_A^2}{2a_1}$ $\Rightarrow$ $AD = \frac{(30)^2-(0)^2}{2 \times 2}$

$\Rightarrow AD = \frac{900}{4}$ $\Rightarrow$ AD = 225 m **Ans**

For motion from B to C



Let E be a point where $u_E = \frac{u_B}{2} = \frac{60}{2} = 30$ m/s.

$u_E^2 = u_B^2 + 2a_2\,(BE)$ $\Rightarrow$ $(30)^2 = (60)^2 + 2\,(-1)\,BE$

$\Rightarrow$ $BE = \frac{(60)^2-(30)^2}{2}$ $\Rightarrow$ $BE = \frac{2700}{2}$

$\Rightarrow$ BE = 1350 m

Hence, the position of point from initial point (A)

$\Rightarrow$ AE = AB + BE = 900 + 1350 = 2250 m AE = 2.25 km **Ans**

D-3. Given u = 72 km/h

$= 72 \times \frac{5}{18} = 20$ m/s & $a = -2$ m/s^2

(a) V = 0, s = ?

From the equation of motion;

$V^2 = u^2 + 2as$

$(0)^2 = (20)^2 + 2\times(-2)$ S, i.e., 4s = 400, or S = 100 m **Ans**

(b) V = 0, t = ?

From the equation of motion;

V = u + at

$0 = 20 + (-2)t \Rightarrow 2t = 20 \Rightarrow t = 10$ s **Ans**

(c) Distance travelled during the first second

$[s_t = u + \frac{1}{2}(-2)(2 \times t - 1)]$

$S_1 = 20 + \frac{1}{2}(-2)(2 \times 1 - 1) \Rightarrow S_1 = 20 - 1$

$\Rightarrow S_1 = 19$ m **Ans**

Distance travelled during the third second

$S_3 = 20 + 1/2\ (-2)\ (2 \times 3 - 1)$; or $S_3 = 20 - 5$; or, $S_3 = 15$ m **Ans**

Alternatively :

as चूंकि u = 72 × 5/18 = 20 m/s, $a = -2$ m/s^2.

(a) $v^2 = u^2 + 2as$

$(0)^2 = 400 + 2 \times -2 \times s$, s = 100m

(b) v = u + at , 0 = 20 – 2t t = 10 sec.

(c) $D_n = u + a/2\ (2n - 1)$

$\Rightarrow$ In First second

$D_1 = 20 - 2/2\ (2\times1 - 1) = 19$m

$\Rightarrow$ In Third second

$D_3 = 20 - 2/2\ (2 \times 3 - 1) = 15$m

D-4. Let h be the height of the tower and t be the total time taken by the ball to reach the ground.

Distance covered in tth (last second) second = 15 m

$[s_t = u + 1/2\ g\ (2t - 1)]$

$0 + 1/2\ g\ (2t - 1) = 15$ or, $1/2\ (10)\ (2t - 1) = 15$;

or, 2t – 1 = 3 or t = 2 sec

Now, height of the tower is given by

$h = ut + 1/2gt^2$; $h = 0 + 1/2\ (10)\ (2)^2$; ; i.e., h = 20 m **Ans**

D-5. (i) Maximum height reached by ball = 20 m.

So, taking upward direction as positive, $v^2 = u^2 + 2as$

So, $0 = u^2 - 2 \times 10 \times 20$

or u = 20 m/sec **Ans.**

Also time taken by ball = t = u/g = 20/10 = 2 sec. (for touching the plane)

(ii) Horizontal distance travelled by plane in this time $t = s = u_x t + 1/2\ a_x\ t^2$

where, u_x = initial velocity of plane, a_x = acceleration of plane.

So, $s = 0 \times 2 + 1/2 \times 2 \times 2^2 = 4$ m

(iii) Man catches the ball back 2 seconds after it touches the plane.

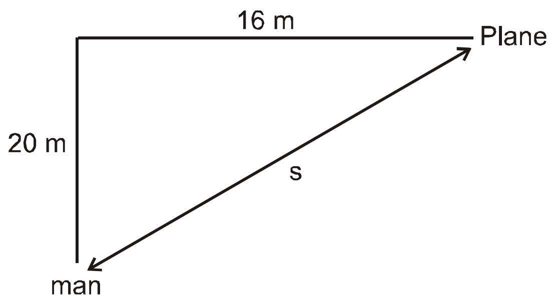
Velocity of plane when ball touches it

$\Rightarrow v_x = u_x + a_x t = 0 + 2 \times 2 = 4$ m/sec.

Now, acceleration of plane becomes : $a_x' = 4$ m/sec^2

so, s_x' = horizontal distance travelled by plane after touch with ball $= u_x' + 1/2\ a_x' t^2$

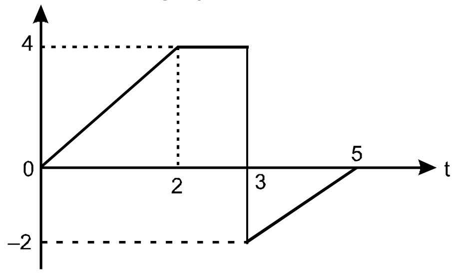
$= 4 \times 2 + 1/2 \times 4 \times 4 = 8 + 8 = 16$ m



Final distance between man and plane = s = $\sqrt{(20)^2 + (16)^2}$ = $\sqrt{656}$ m

Section (E)

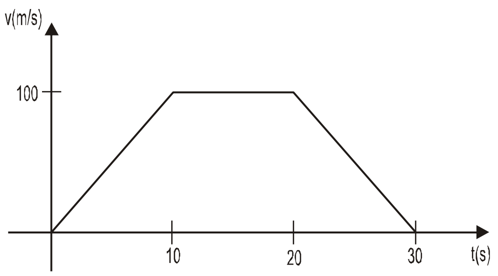
E-1. For a particle moving along x – axis, v – t graph is as shown.



Distance travelled by the particle = sum of areas under V – t graph
= 1/2 (3 + 1)4 + 1/2× 2 × 2 = 8 + 2 = 10 m
Displacement of the particle = area above t-axis – area below t-axis
= 1/2 (3 + 1) 4 – 1/2 × 2 × 2 = 8 – 2 = 6 m

Average velocity = $\frac{\text{Displacement}}{\text{time} - \text{interval}} = \frac{6}{5}$ = 1.2 m/s

E-2.



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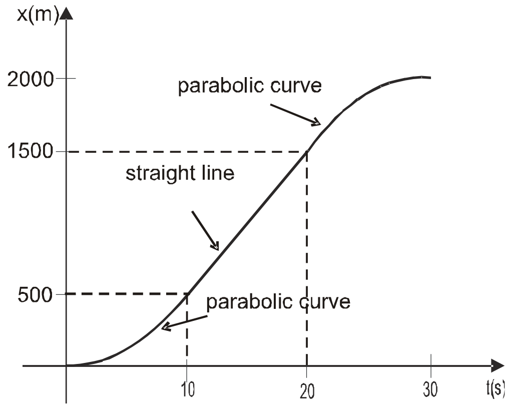
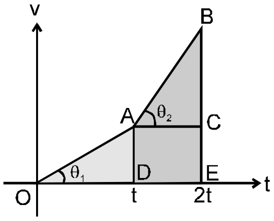


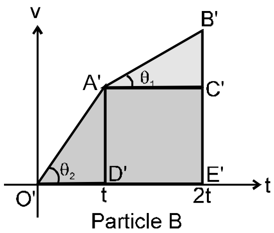
Distance travelled = area under V-t curve
= 2000 m

E-3. v - t diagram for the two situations is shown below



Particle A

$\tan\theta_1 = a$



Particle B

$\tan\theta_2 = 2a$

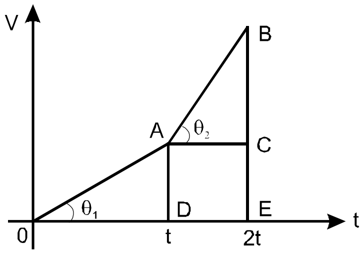
In v - t graph, distance travelled = area under the graph

Area (AOD) = Area (A'B'C')
Area (ABC) = Area (O'A'D')
Area (ACED) < Area (A'C'E'D')

∴ particle B has covered larger distance.

Aleternate Solution:

For particle A :



For v – t graph, slope = acceleration, Suppose the slope of OA, i.e., tan θ_1 = m ; hence, the slope of AB, i.e.,

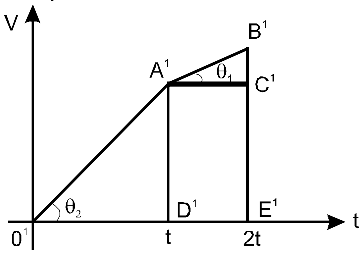
tan θ_2 = 2m, AD = mt, BC = 2mt

Distance travelled by the particle A

S_A = 1/2 (t) (mt) + 1/2 (mt + 3mt)t

S_A = 2.5 mt

For particle B



slope of OA^1 tan θ_2 = 2m

slope of A^1B^1 tan θ_1 = m

AD = 2mt, BC = mt

Distance travelled by the particle B

S_B = 1/2 (t) (2mt) + 1/2 (2mt + 3mt) t

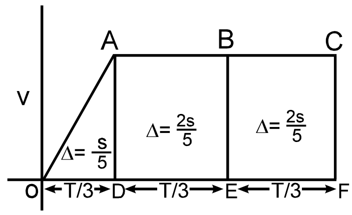
S_B = 3.5 mt

Therefore; $S_B > S_A$

E-4 Let the total time of race be T seconds and the distance be S = 100 m.

The velocity vs time graph is

Area of ΔOAD



Δ = s/5

∴ $\frac{s}{5} = \frac{1}{2} a \left(\frac{T}{3}\right)^2 = \frac{1}{2} 8 \left(\frac{T}{3}\right)^2$

or $T = 3\sqrt{5}$ m /s

PART - II

Section (A)

A-1. Dimension of hall, length of any side = 10 m = a (say) **(B) Ans**

Magnitude of displacement = Length of diagonal = $a\sqrt{3}$ = $10\sqrt{3}$ m

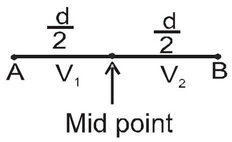
Section (B)

B-1. Suppose AB = x km

Average speed = $\dfrac{\text{Total distance covered}}{\text{Total time taken}}$

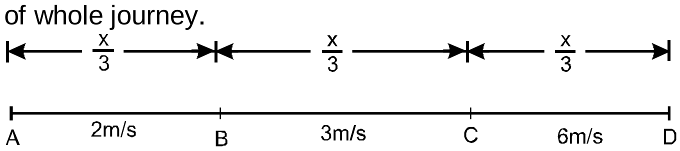
$= \dfrac{2x}{\frac{x}{20}+\frac{x}{30}} = \dfrac{2}{\frac{1}{20}+\frac{1}{30}} = \dfrac{20\times 60}{20+30}$ = 24 km/h = 24 kmh^{-1} **(B) Ans**

B-2. Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time interval}}$



$\Rightarrow \quad V = \dfrac{d}{\frac{d}{2v_1}+\frac{d}{2v_2}} = \dfrac{2V_1V_2}{V_1+V_2}$ **(A) Ans**

B-3. Let x be the length of whole journey.



Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time taken}}$

$= \dfrac{X}{\frac{x/3}{2}+\frac{x/3}{3}+\frac{x/3}{6}} = \dfrac{1}{\frac{1}{6}+\frac{1}{9}+\frac{1}{18}} = \dfrac{18}{3+2+1}$ = 3 m/s **(A) Ans**

B-4. Average speed = $\dfrac{\text{Total distance}}{\text{Total time taken}} = \dfrac{2\pi r}{62.8}$

$= \dfrac{2\times 3.14\times 100}{62.8}$ = 10m/s

Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{0}{62.8}$ = zero

Hence option (B) is correct.

Section (C)

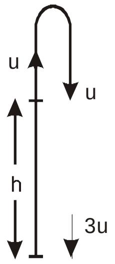
C-1. The displacement of a body is given as $2s = gt^2$

Differentiating both sides w.r.t. 't'

$\Rightarrow \quad 2\dfrac{ds}{dt} = 2\,gt \quad \Rightarrow \quad 2\,V = 2gt \quad \Rightarrow \quad V = gt$ **(A) Ans**

C-2. I mothod – Let downward direction is taken as +ve. Initial vel is –ve = – u (say)

$\therefore$ From the equation ; $v^2 - u^2 = 2as$ we get $(3u)^2 - (-u)^2 = 2hg$



$\Rightarrow$ $h = \frac{4u^2}{g}$ **"B" Ans.**

The stone is thrown vertically upward with an initial velocity u from the top of a tower it reaches the highest point and returns back and reaches the top of tower with the same velocity u vertically downward.

Now, from the equation, $V^2 = u^2 + 2gh$

$\Rightarrow (3u)^2 = u^2 + 2\,gh$ $\Rightarrow 2gh = 9u^2 - u^2$ $\Rightarrow h = \frac{8u^2}{2g}$ $\Rightarrow h = \frac{4u^2}{g}$ **"B" Ans.**

C-3. u = 0,
Acceleration = a
t = n sec,
The velocity after n sec is
n sec
V = u + at
V = 0 + a(n)
V = an
a = V/n ....(i)
The displacement of the body in the last two seconds [$S = ut + 1/2\, at^2 = 1/2\, at^2$]

$S_2 = S_n - S_{n-2}$

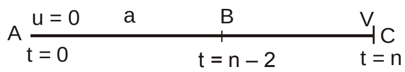
$= \frac{1}{2} an^2 - \frac{1}{2} a\,(n-2)^2 = \frac{1}{2} a\,[n^2 - (n-2)^2]$

$= \frac{1}{2}\, a\,[n^2 - n^2 - 4 + 4\,n]$

$S_2 = 2a(n-1)$

From equation (i) $S_2 = \frac{2V(n-1)}{n}$ **"A" Ans**

Aliter :



BC = ?
BC = AC – AB

$= [0 \times n + \frac{1}{2} an^2] - (0 \times (n-2) + \frac{1}{2} a\,(n-2)^2).$

$BC = \frac{a}{2}\,[n^2 - n^2 - 4 + 4n] = \frac{4a}{2}\,[n-1]$

BC = 2 a (n – 1) ....(1)

For AC AC के लिए

V = u + at
V = 0 + an
a = V/n ......(2)

From (1) and (2) $BC = \frac{2V}{n}(n-1)$

Section (D)

D-1. $u = 0$, Let acceleration $= a$

Total time $t = 30$ s

X_1 = distance travelled in the first 10 s.

Using , $S = ut + \frac{1}{2}at^2$, we get

$X_1 = 0 + \frac{1}{2}a(10)^2$, i.e., $X_1 = 50a$

Similarly,

X_2 = distance travelled in the next 10 s

So, $X_2 = (0 + 10a)\,10 + \frac{1}{2}a(10)^2$

So, $X_2 = 100a + 50a$

or, $X_2 = 150a$

and, X_3 = distance travelled in the last 10 s

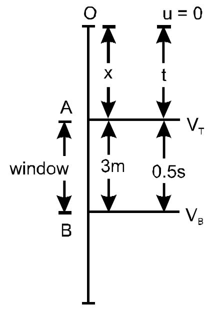
So, $X_3 = (10a + 10a)\,10 + \frac{1}{2}a(10)^2$

or या, $X_3 = 200a + 50a$

or, $X_3 = 250a$

Hence, $X_1 : X_2 : X_3 = 50a : 150a : 250a = 1 : 3 : 5$ **"C" Ans**

D-2. Let x be the distance of the top of window from the top of building and t be the time taken by the ball from the top of building to the top of window.



(i) Since, acceleration is constant = g

So, $S = \frac{u+v}{2}t$ (across the window)

$3 = \frac{v_T + v_B}{2}t \Rightarrow \quad 3 = \frac{v_T + v_B}{2}0.5$

So, $v_T + v_B = 12$ m/sec.

Aliter :

For motion from O to A

$V_T^2 = u^2 + 2gx = (0)^2 + 2gx$

$V_T^2 = 2gx$ ....(i)

$V_T = u + gt = 0 + gt$

$V_T = gt$ ....(ii)

For motion from O to B

$V_B^2 = u^2 + 2g(x + 3)$

$V_B^2 = (0)^2 + 2g(x + 3)$

$V_B^2 = 2g(x + 3)$ ....(iii)

$V_B = u + g(t + 0.5)$

$V_B = 0 + g(t + 0.5)$

$V_B = g(t + 0.5)$ ....(iv)

From equations (ii) and (iv)

$V_B - V_T = g(0.5)$ ....(v)

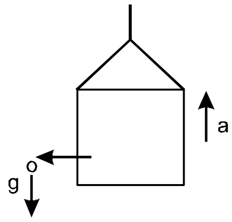
From equations (i) and (iii)

$V_B^2 - V_T^2 = 2g(3)$ ....(vi)

From equations (v) and (vi)

$\frac{V_B^2 - V_T^2}{V_B - V_T} = \frac{2g(3)}{g(0.5)} \Rightarrow \frac{(V_B - V_T)\ (V_B + V_T)}{(V_B - V_T)} = 12 \qquad \Rightarrow \qquad V_T + V_B = 12\ ms^{-1}$ **(A) Ans**

D-3.



After the release of stone from the elevator going up with an acceleration a, stone will move freely under gravity (g), hence the acceleration of the stone will be g towards downwards.

"D" Ans

Aliter :

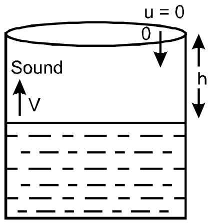
Acceleration of stone = g downward [free fall under gravity]

D-4. Initial velocity = u, acceleration = f = at

f = at $\qquad$ dV/dt = at $\qquad$ dV = at dt

Integrating both sides

$\Rightarrow \quad \int_u^v dV = \int_0^t at\,dt \qquad \Rightarrow \qquad V - u = \frac{at^2}{2} \qquad \Rightarrow \qquad V = u + \frac{at^2}{2}$ **"B" Ans**



D-5.

Suppose, t_1 = time taken by stone to reach the level of water

t_2 = time taken by sound to reach the top of well

so, $\quad T = t_1 + t_2$

For t_1 : $u = 0$

$h = ut + \frac{1}{2}gt^2 \qquad h = 0 + \frac{1}{2}gt_1^2 \qquad t_1 = \sqrt{\frac{2h}{g}}$

For t_2 : As the velocity of sound is constant

$h = Vt_2 \qquad \Rightarrow \qquad t_2 = \frac{h}{V}$

Therefore, $\quad T = \sqrt{\frac{2h}{g}} + \frac{h}{V}$ **"B" Ans**

Aliter :

T= Time taken by stone from top to level water. (T_1) + Time taken by sound from level water to top of the well. (T_2)

for downward journey of stone :

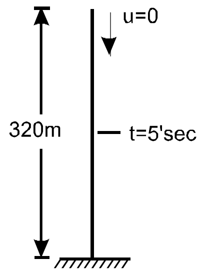
$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad h = 0 + \frac{1}{2}gT_1^2 \quad \Rightarrow \quad T_1 = \sqrt{\frac{2h}{g}}$

for upward journey of sound, Time $(T_2) = \frac{h}{v}$

$\therefore \quad T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$

Hence option (B) correct.

D-6.



Let t be the time taken by the superman to reach the student for saving the students life just before reaching the ground. Hence, the time taken by the student to reach the ground = (t + 5) s

For motion of student

u = 0, h = 320 m, g = 10 m/s^2

From equation, $h = ut + \frac{1}{2}gt^2$,

i.e., $320 = 0 + \frac{1}{2}(10)(t + 5)^2$

i.e., $(t + 5)^2 = 64$; or $t + 5 = 8$; i.e., t = 3 sec

For motion of superman

Let initial velocity u = V, h = 320 m, g = 10m/s^2

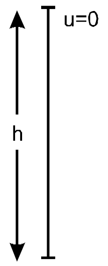
from equation $h = ut + \frac{1}{2}gt^2$

i.e., $320 = V(3) + \frac{1}{2}(10)(3)^3$,

i.e., 320 = 3 V + 45,

or 3V = 320 – 45, or $V = \frac{275}{3}$ m/s **"B" Ans**

D-7.



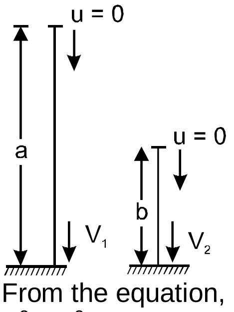
In the above problem, if height of the skyscraper is such that student covers the full height within 5 sec then superman will be unable to save him.

u = 0, t = 5 sec, g = 10 m/s^2

Hence; from equation $h = ut + 1/2\ gt^2$,

or $h = 0 + 1/2\ (10)(5)^2$, i.e, h = 125 m **"C" Ans**

D-8.



From the equation,

$V^2 = u^2 + 2gh$

$V_1^2 = 0 + 2ga$

$V_1^2 = 2ga$ ....(i)

$V_2^2 = 2gb$ ....(ii)

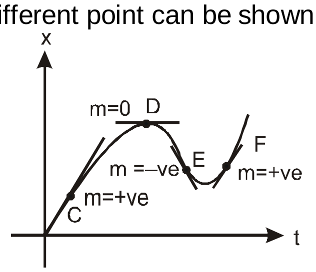
From the equations (i) and (ii)

we get $\frac{V_1^2}{V_2^2} = \frac{2ga}{2gb} = \frac{a}{b}$ i.e., $\frac{V_1}{V_2} = \frac{\sqrt{a}}{\sqrt{b}}$ **(B) Ans**

$\therefore$ option (B) is correct

Section (E)

E-1. The slope of position–time (x–t) graph at any point shows the instantaneous velocity at that point. The slope of given x – t graph at different point can be shown as



Obviously the slope is negative at the point E as the angle made by tangent with +ve X–axis is obtuse, hence the instantaneous velocity of the particle is negative at the point E i.e., **"C" Ans**

Aliter : As Instantaneous velocity is negative where slope of x–t curve is negative .

At. point C = slope is positive

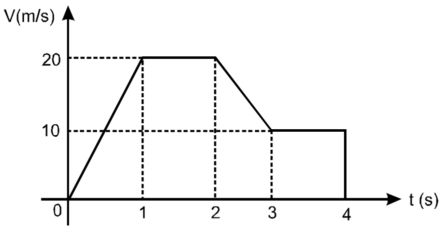
At. point D = slope is zero

At. point E = slope is negative

At. point F = slope is positive

Hence, option (C) is correct

E-2.



The distance travelled by the particle in 4s

= Sum of areas under V–t graph

$= 1/2 \times 1 \times 20 + 1 \times 20 + 1/2\,(20 + 10) \times 1 + 1 \times 10 = 55$ m

E-3. $u = 0$, a = Constant = k (let)

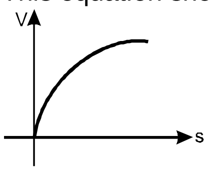
From equation of motion;

$V^2 = u^2 + 2as$

$V^2 = (0)^2 + 2ks$

$V^2 = 2\,ks$

This equation shows a parabola with S-axis as its axis. Hence, its graph can be shown as



i.e., **"B" Ans**

E-4. As the slope of displacement - time (x – t) graph shows the velocity, the ratio of velocities of two particles A and B is given by

$\frac{V_A}{V_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$ i.e., **"D" Ans**

E-5. $V_{t=3} - V_{t=0}$ = area under a – t curve

$\therefore$ $V_{t=3} = 10.5$ m/s

PART - III

1. From graph (a) $\Rightarrow$ v = kx
where k is positive constant

Acceleration = $v\dfrac{dv}{dx}$ = kx . k = k^2x

$\therefore$ Acceleration is non uniform and directly proportional to x.

$\therefore$ a $\rightarrow$ Q, S

From graph (b) $\Rightarrow$ v^2 = kx.

Differentiating both sides with respect to x.

$2v\dfrac{dv}{dx} = k$ or $v\dfrac{dv}{dx} = \dfrac{k}{2}$

Hence acceleration is uniform.

$\therefore$ b $\rightarrow$ P

From graph (c) $\Rightarrow$ v = kt

Acceleration = dv/dt = k

Hence acceleration is uniform $\Rightarrow$ c $\rightarrow$ P

From graph (d) $\Rightarrow$ v = kt^2

Acceleration = dv/dt = 2kt

Hence acceleration is non uniform and directly proportional to t.

$\therefore$ d $\rightarrow$ Q, R

EXERCISE–2
PART - I

1. **(A)** Relation between position x and time t is given as

$x^2 + 2x = t$

Differentiating both sides w.r.t 't'

$2x\dfrac{dx}{dt} + 2\dfrac{dx}{dt} = 1$, i.e., $2(x+1)V = 1$ $\Rightarrow$ $V = \dfrac{1}{2(x+1)}$

Again differentiation both sides w.r.t 't'

$\Rightarrow$ $\dfrac{dV}{dt} = \dfrac{-1}{2(x+1)^2}\left(\dfrac{dx}{dt}\right)$ $\Rightarrow$ $a = \dfrac{-V}{2(x+1)^2}$ $\Rightarrow$ $a = \dfrac{-1}{4(x+1)^3}$

Hence, the retardation of the particle is $\dfrac{1}{4(x+1)^3}$

[Note : As v and a are oppositely directed, so particle is retarding]

Aliter : $x^2 + 2x = t$

By Differentiation w. r. t. time

2x v + 2v = 1

or $xv + v = \dfrac{1}{2}$ ....(1) $v = \dfrac{1}{2}(x+1)$

or Again differentiate eq ....(1)

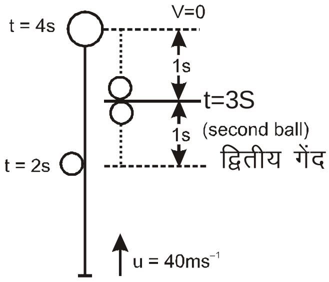
we have

$x\dfrac{dv}{dt} + v\dfrac{dx}{dt} + \dfrac{dv}{dt} = 0$

$xa + v^2 + a = 0$

$a = -\dfrac{v^2}{(x+1)}$ $a = -\dfrac{1}{4(x+1)^3}$

2.



u = 40 m/s , g = 10 m/s^2

Let t be time taken by the first ball to reach the highest point.

V = u – gt   0 = 40 – 10 t   t = 4 s

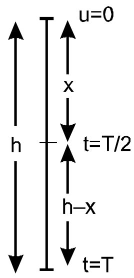
From figure second ball will collide with first ball after 3 second, therefore the height of collision point = height gained by the second ball in 3 sec

= 40 (3) – 1/2 (10) (3)^2

= 120 – 45 = 75 m   **"B" Ans**

3. u = 0, t = T ; $h = ut + \frac{1}{2}gt^2$; $h = \frac{1}{2}gT^2$

$h = \frac{1}{2}gT^2$ ...(i)



Let x be the distance covered by the body in t = T/2

$x = 0 + \frac{1}{2}g\,(T/2)^2$

$x = \frac{1}{8}gT^2$ ...(ii)

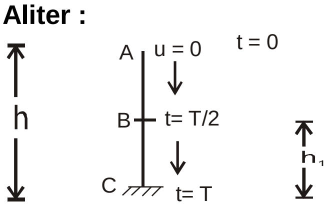
From equations (i) and (ii)

$\frac{h}{x} = \frac{1/2\ \ gT^2}{1/8\ \ gT^2}$ $\frac{h}{x} = \frac{4}{1}$ $\Rightarrow$ $x = \frac{h}{4}$

Therefore height of that point from ground

$= h - x\ = h - \frac{h}{4} = \frac{3h}{4}$ **"C" Ans**

Aliter :



Let at $t = \frac{T}{2}$ body is at point B.

For AC   For AB

s = ut +1/2 at^2 s = ut +1/2 at^2

$-h = -\frac{1}{2}\ g\ T^2$ $-(h - h_1) = -\frac{1}{2}g\left(\frac{T}{2}\right)^2$

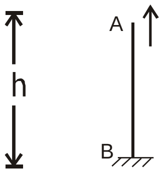
$h = g\frac{T^2}{2}$ ....(1) $h - h_1 = g\frac{T^2}{2\times 4}$ ....(2)

From (1) and (2) , we have

$h - h_1 = h/4$

$h - \frac{h}{4} = h_1$ or $h_1 = \frac{3h}{4}$ from the ground

4.



For AB

$s = ut + \frac{1}{2}at^2.$

$-h = vt - \frac{g}{2}t^2$

$\frac{g}{2}t^2 - vt - h = o$

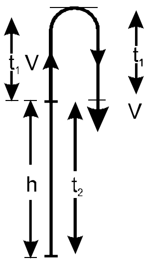
$$t = \frac{v \pm \sqrt{v^2 + 4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}}$$

$$t = \frac{v \pm \sqrt{v^2 + 2gh}}{g}$$

$t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$, $t = \frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$ [as time cannot be negative so we neglect it]

$\therefore \quad t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$

Aliter



Let t_1 be the time taken by ball from top of tower to the highest point then it will take again t_1 time to return back to the top of tower Let t_2 be the time taken be ball from top of tower to the ground.

For t_1 : From equation

$V = u - gt$ i.e., $0 = V - gt_1$ or, $t_1 = V/g$

For t_2 : From equation

$h = ut + \frac{1}{2}gt^2$ $h = Vt^2 + \frac{1}{2}gt_2^2$; or, $gt_2^2 + 2Vt_2 - 2h = 0$, or, $t_2 = \frac{-2V \pm \sqrt{4V^2 + 8gh}}{2g}$

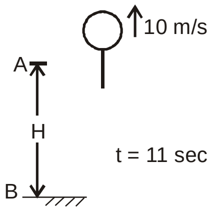
Taking (+) sign only (as we are interested in time projection i.e., t = 0) $t_2 = \frac{-V + \sqrt{V^2 + 2gh}}{g}$

Note that, –ve time indicate time before the projection.

Hence, the time after which the ball strikes ground $T = 2t_1 + t_2$ $\Rightarrow T = \frac{2V}{g} + \frac{-V + \sqrt{V^2 + 2gh}}{g}$

$T = \frac{V + \sqrt{V^2 + 2gh}}{g}$ $\Rightarrow$ $T = \frac{V}{g}\left[1 + \sqrt{1 + \frac{2gh}{V^2}}\right]$

5.



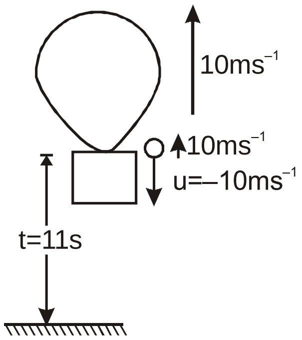
As $s = ut + \ at^2$

$- H = 10 \times 11 - 5 \times (11)^2$

$- H = 110 - 605$

$H = 495$ m

Aliter :



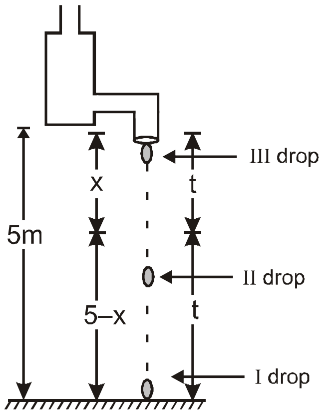
At the time of release, velocity of stone will be same as that of balloon, hence

$u = -10\ ms^{-1}$, $t = 11$ s

$h = ut + \frac{1}{2}gt^2$

$= (-10) \times 11 + \frac{1}{2}\ (10)\ (11)^2 = -110 + 605 = 495$ m **"A" Ans**

6.



Let t be the time interval between two successive drops. For the first drop :

From equation, $h = ut + \frac{1}{2}gt^2$

$5 = 0 + \frac{1}{2}g(2t)^2$ $\Rightarrow$ $5 = \frac{1}{2}g\ (2t)^2$ ....(i)

For the second drop :

From equation , $h = ut + \frac{1}{2}gt^2$

$x = 0 + \frac{1}{2}gt^2 \quad \Rightarrow \quad x = \frac{1}{2}gt^2$ ....(ii)

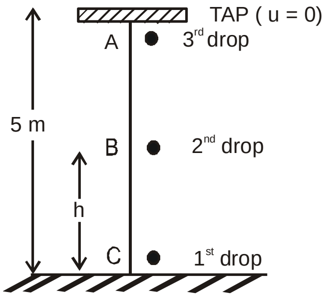
From the equations (i) and (ii);

$$\frac{5}{x} = \frac{\frac{1}{2}g(2t)^2}{\frac{1}{2}g\,2t^2} \Rightarrow \frac{5}{x} = \frac{4}{1} \Rightarrow \quad x = \frac{5}{4} = 1.25 \text{ m}$$

The distance of the second drop from the ground

$= 5 - x = 5 - 1.25 = 3.75 = \frac{15}{4}$ m

Aliter :



Let the time interval b/w two consecutive drops be t.

$\therefore$ Time b/w 1st and 3rd drop = 2t.

For AC

$s = ut + \frac{1}{2}at^2.$

$-5 = 0 + \frac{1}{2} \times -10 \times (2t)^2 \qquad \frac{1}{2} = t^2, \quad t = \frac{1}{2}$ sec.

$\therefore$ height of second drop.

$s = ut + \frac{1}{2}at^2 \qquad -(5 - h) = 0 + \frac{1}{2} \times -10 \times \frac{1}{4}$

$5 - h = \frac{10}{8} \qquad h = 5 - \frac{5}{4} = 3.75 \text{ m} = \frac{15}{4}$ m

7. (A) the given x – t graph has 5 points at which the slope of tangent is zero i.e, velocity becomes zero 5 times.

As we know that particle is at rest when its position does not change withe time. Clearly, from x–t graph, particle is at rest 5 times

$\therefore$ option (A) is correct .

(B) Slope is not zero at t = 0.

$\therefore$ option (B) is incorrect .

(C) Velocity is positive, when slope of x-t curve is positive. Slope changes from positive to negative and negative to zero.

$\therefore$ option (C) is incorrect

(D) Average velocity = $\frac{\text{Total Displacement}}{\text{Total Time taken}}$.

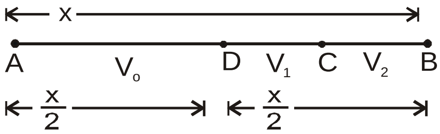
Total Displacement is positive

$\therefore$ Average velocity = positive

$\therefore$ option (D) is incorrect .

PART - II

1.



Let T is the time to cover DB.

$\therefore$ Time in DC = CB = $\frac{T}{2}$.

As mean velocity = $\frac{\text{Total Displacement (T.D)}}{\text{Total time taken.(TTT)}}$

T.D = x

T.T.T = $T_{AD} + T_{DC} + T_{CB} = \frac{x}{2V_o} + \frac{T}{2} + \frac{T}{2} = \frac{x}{2V_O} + T$ ......(1)

Now , BD = DC + CB

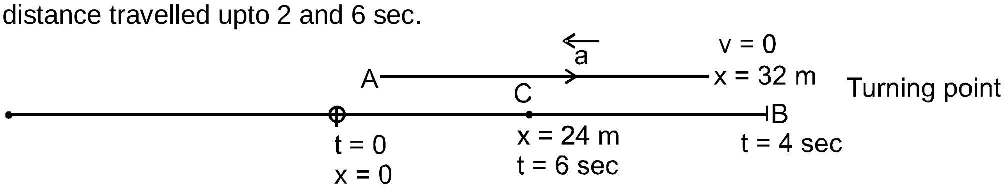
or $\frac{x}{2} = \frac{V_1T}{2} + \frac{V_2T}{2}$ or $x = T(V_1 + V_2)$. or $T = \frac{x}{V_1+V_2}$. ....(2)

From (1) and (2)

T.T.T = $\frac{x}{2V_o} + \frac{x}{V_1+V_2}$.

or Mean velocity = $\frac{x}{\frac{x}{2V_o}+\frac{x}{V_1+V_2}} = \frac{2V_o(V_1+V_2)}{(2V_o+V_1+V_2)}$

2. distance travelled upto 2 and 6 sec.



As $x = 16t - 2t^2$

At t = 0, x = 0

Now, $V = 16 - 4t = 0$ [$a = -4\ m/s^2$]

t = 4 sec.

At t = 4 sec, x = 16 × 4 – 2 × 16 = 32 m

Now, At t = 6 sec, x = 16 × 6 – 2 × 36 = 96 –72 = 24 m

$\therefore$ Distance upto 2 sec. = Displacemnent in 2 sec = 24 m.

[As turning point is at t = 4 sec]

and distance in 6 sec = AB + BC = 32 + (32 – 24) = 32 + 8 = 40 m.

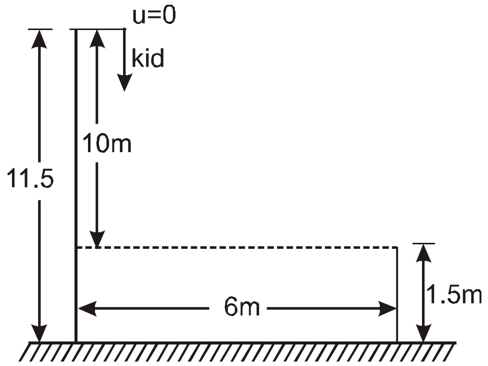
Aliter :

The distance travelled upto 6s

$= |x|_{t=4} + \left|\int_4^6 V dt\right| = |16(4) - 2(4)^2| + \left|\int_4^6 (16-4t)dt\right|$

$= 64 - 32 + \left|[16t - 2t^2]_4^6\right| = 32 + |32 - 40| = 32 + 8 = 40$ m **Ans**

3.



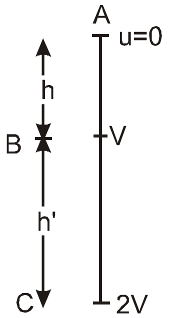
Let a be the acceleration of the youngman.
As the youngman catches the kid at the arms height (1.5 m) then the time taken by kid to fall through 10 m will be same as the time taken by the youngman to run 6 m on horizontal ground.
For motion of kid.
$u = 0$, $g = 10\ m/s^2$, $h = 10\ m$
For motion of kid.
$u = 0$, $g = 10\ m/s^2$, $h = 10\ m$

From the equation $h = ut + \frac{1}{2}gt^2 \Rightarrow 10 = 0 + \frac{1}{2}(10)t^2$

For motion of youngman

$6 = 0 + \frac{1}{2}at^2$ substitute value of t ; $a = 6\ m/s^2$.

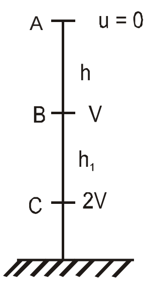
4.



For motion from A to B $u = 0$
From equation $V^2 = u^2 + 2gh$ $V^2 = (0)^2 + 2gh$ $V^2 = 2gh$ ...(i)
For motion from B to C $u = V$
From equation $V^2 = u^2 + 2gh$ $(2V)^2 = V^2 + 2gh'$ $4V^2 = V^2 + 2gh' \Rightarrow 3V^2 = 2gh'$ ...(ii)

From equations (i) and (ii) $\frac{V^2}{3V^2} = \frac{2gh}{2gh'} \Rightarrow h' = 3h$ **(C) Ans**

Aliter :

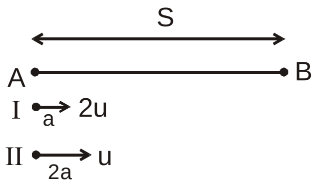


Let h_1 = distance it has to travel down further to double its velocity i.e., 2V

For AB
$V^2 = 2gh$ ......(1)

For BC
$(2v)^2 = (v)^2 + 2gh_1$
$3V^2 = 2gh_1$
$\therefore\ 3h = h_1$

5.



Suppose at point B (displacement S) II particle overtakes particle I

For I particle $S = (2u)\,t + \frac{1}{2}a\,t^2$ .... (1)

For II particle

$S = u\,t + \frac{1}{2}(2a)\,t^2$ ....(2)

$\therefore\ 2ut + \frac{1}{2}a\,t^2 = ut + \frac{1}{2}(2a)\,t^2$

$ut = \frac{1}{2}a\,t^2$

$t = \frac{2u}{a}$

Putting this value in equation (1) we get

$S = 2u \times \frac{2u}{a} + \frac{1}{2} \times a \times \left(\frac{2u}{a}\right)^2$

$= \frac{4u^2}{a} + \frac{2u^2}{a} = \frac{6u^2}{a}$

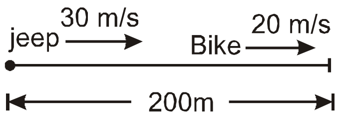
6. Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and time of descent respectively.

If the particle rises upto a height h

then $h = \frac{1}{2}(g + a)\,t_a^2$ and $h = \frac{1}{2}(g - a)\,t_d^2$

$\therefore\ \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$ **Ans.** $\sqrt{\frac{2}{3}}$

7.



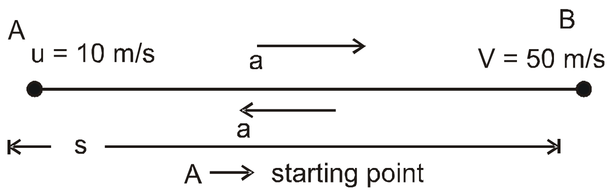
$200 = 10\,(t) + \frac{1}{2}(2)t^2$

$t^2 + 10\,t - 200 = 0$

t = 10 seconds

Distance = 200 + 200 = 400 m **Ans.**

8.



For AB

$V^2 = u^2 + 2as$

2400 = 2as or as = 1200 (1)

Now, for BA

$V_A^2 = (50)^2 + 2\ (-a)\ (-s)$

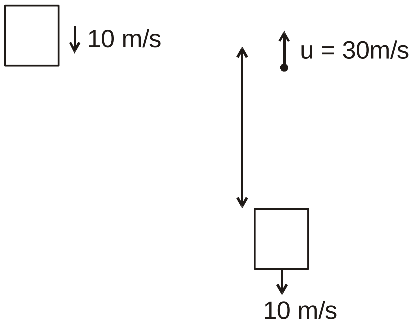
$V_A^2 = 2500 + 2 \times 1200$

$V_A = \sqrt{4900}$

$V_A = 70$ m/s

∴ velocity of particle when it reaches the starting point is 70 m/s.

9.



Let the time be t after which the thrown stone hits the lift at a depth d below the top of shaft

$d = ut + \frac{1}{2}gt^2$

$d = -30\ t + \frac{1}{2}\ (10)\ t^2$ .....(1)

for lift

$d = 40 + 10\ t$ .....(2)

(1) = (2)

$-30\ t + 5\ t^2 = 40 + 10\ t$

$5t^2 - 40\ t - 40 = 0$

$t^2 - 8t - 8 = 0$

$t = \frac{8 \pm \sqrt{64 + 32}}{2} = \frac{8 \pm \sqrt{96}}{2} = \frac{8 \pm 4\sqrt{6}}{2}$

$t = 4 + 2\sqrt{6}$

Net time after to hit the lift start desending

$= 4 + 4 + 2\sqrt{6} = 8 + 2\sqrt{6}$ sec

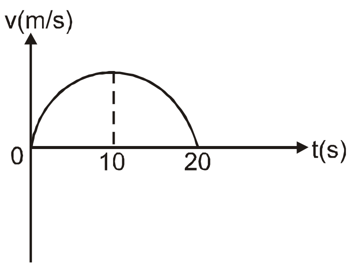
Putting value of t in equation (2)

$d = 40 + (4 + 2\sqrt{6}\ )\ 10$

$= 40 + 40 + 20\sqrt{6} = 129$ m

PART - III

1.



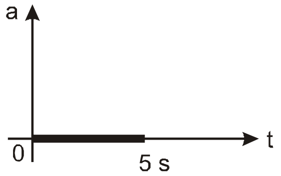
$a_{Avg} = \frac{\Delta \vec{v}}{t} = \frac{0}{20} = 0$

From 0 to 20 time interval velocity of particle doesn't change it's direction.

Area under v–t curve is not zero.

As the magnitude of area under v – t graph from t = 0 to 10 is same as from t = 10 to 20, hence the average speed in both the intervals will be same. **'D" is correct i.e., A & D Ans**

2.

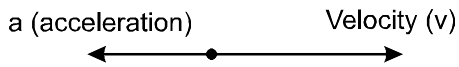


If the acceleration a is zero from t = 0 to 5 s, then speed is constant from t = 0 to 5s and as the speed is zero at t = 0. Hence speed is zero from t = 0 to t = 5 s.

If the speed is zero for a time interval from t = 0 to t = 5 s, as the speed is constant in this interval hence the acceleration is also zero in this interval.

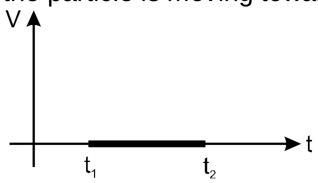
Because zero speed = object is not moving = velocity = constant (= 0) $\Rightarrow$ acceleration = 0

3.



If the velocity (u) and acceleration (a) have opposite directions, then velocity (v) will decrease, therefore the object is slowing down.

If the position (x) and velocity (u) have opposite sign the position (x) reduces to become zero. Hence the particle is moving towards the origin.



If $\vec{a} \cdot \vec{v} > 0$ speed will increase.

If velocity V = 0, $t_1 < t < t_2$

Hence; acceleration a = $\dfrac{\Delta V}{\Delta t}$ = 0 ; $t_1 < t < t_2$

Therefore if the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval. **(D) is correct**

[acc, a = $\dfrac{dv}{dt}$ $\Rightarrow$ v = u + at]

Now, v = 0 $\Rightarrow$ a = 0 $\Rightarrow$ a = – u/t $\Rightarrow$ acceleration may not be zero when vel. 'V' = 0, 'c' is incorrect.

4. $s \propto t^2$

$\therefore$ $s = ct^2$ where c = constant

(i) $v = \dfrac{ds}{dt} = 2\,ct$

$\therefore$ $v \propto t$

(ii) $a = \dfrac{dv}{dt} = 2c$

so, a = constant.

5. $y = u\,(t-2) + a(t-2)^2$

Velocity of particle at time t

$$\frac{dy}{dt} = u + 2a\,(t-2)$$

Velocity at t = 0 $\dfrac{dy}{dt} = u - 4a$

acceleration of particle

$$\frac{d^2y}{dt^2} = 2a$$

$y_{t=2} = 0$

So correct answer is (C) and (D).

PART - IV

1 to 4.

(1) $\langle \vec{v} \rangle = \frac{x_f - x_i}{\Delta t} = \frac{-100-100}{20} = -10$m/s

(2) **(C)** $\langle \vec{a} \rangle = \frac{v_f - v_i}{\Delta t} = \frac{\tan\theta_2 - \tan\theta_1}{20} = 0$ (since $\theta_2 = \theta_1$)

(3) during first 10 sec, speed decreases

$\therefore$ acceleration is opposite to the velocity

$\therefore$ acceleration is in $\hat{i}$

(4) **(C)** during first 10 sec., the slope of x-t curve decreases in negative direction

$\therefore$ Motion is retarded.

t = 0 to t = 10 s

Ans. (1) – 10m/s (2) 0 (3) $\hat{i}$ (4) t = 0 to t = 10 s

5. $x = 2(t - t^2)$

velocity = $\frac{dx}{dt} = 2 - 4t$

acceleration = $\frac{d^2x}{dt^2} = -4$ $\Rightarrow$ (C) is correct.

6. velocity = $\frac{dx}{dt} = 2 - 4t$ $\upsilon = 0$ $\Rightarrow$ $t = \frac{1}{2}$

After $t = \frac{1}{2}$ sec., particle moves to left

Position at $t = \frac{1}{2}$ sec $x = 2\left(\frac{1}{2} - \frac{1}{4}\right) = 2 \times \frac{1}{4} = \frac{1}{2}$ m. (C) is correct

7. (C) is correct

8. u = 0 at $t = \frac{1}{2}$ s

$\therefore$ position at $t = \frac{1}{2}$ s $\Rightarrow x = \frac{1}{2}$

position at t = 1 s $\Rightarrow x = 0$

$\therefore$ distance moved = $\left|\frac{1}{2} - 0\right| + \left|1 - \frac{1}{2}\right|$

= 1 m **Ans.**

EXERCISE–3
PART - I

1. Distance travelled in t^{th} second is,

$s_t = u + at - \frac{1}{2}a \; ; \; u + \frac{a}{2}(2t-1)$

Given : u = 0

$\therefore \quad \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$

Hence, the correct option is (B).

2. Area under acceleration-time graph gives the change in velocity.

Hence, $v_{max} = \frac{1}{2} \times 10 \times 11 = 55$ m/s

Therefore, the correct option is (C)

PART - II

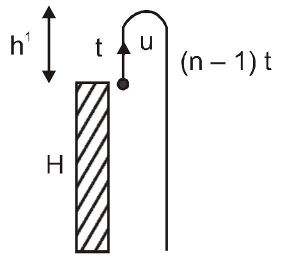
1. $\int_{6.25}^{0} \frac{dv}{\sqrt{v}} = -2.5\int_{0}^{t} dt$

$\left|2\sqrt{\upsilon}\right|_{6.25}^{0} = -2.5\ t$

$2.\sqrt{6.25} = 2.5\ t$

t = 2 sec. **Ans.**

2.



$t = u/g$ .....(1)

$h^1 = \frac{u^2}{2g}$ .....(2)

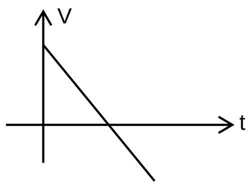
$h^1 + H = \frac{1}{2} g\ (n-1)^2\ t^2$

$\frac{u^2}{2g} + H = \frac{1}{2} g\ (n-1)^2 \frac{u^2}{g^2}$

$H = \frac{(n-1)^2 u^2}{2g} - \frac{u^2}{2g} \quad \Rightarrow \quad H = \frac{u^2}{2g}\ [n^2 - 2n]$

3. a = –g = constant
dv/dt = constant
slop of V – t curve is

constant & –ve



4. As in distance vs time graph slope is equal to speed In the given graph slope increase initially which is incorrect

5. $v = bx^{1/2}$

$\frac{dx}{dt} = bx^{1/2}$

$\int_0^x \frac{dx}{x^{1/2}} = \int_0^t b\,dt$

$2\sqrt{x} = bt$

$x = \frac{b^2t^2}{4} \Rightarrow v = \frac{dx}{dt} = \frac{b^2t}{2}$

6. Area = $\left(\frac{1}{2}\times 2\times 2\right)$ + (2 × 2) + (1 × 3)

Displacement = 2 + 4 + 3 = 9m

7. $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + ax = a$

$ax = a - \left(\frac{at+b}{x}\right)^2$

$a = \frac{a(at^2+2bt+c)-(at+b)^2}{x^3}$

$a = \frac{ac-b^2}{x^3}$

$a \propto x^{-3}$

8. $S_y = u_y t + \frac{1}{2}a_y t^2$

$32 = 0 + \frac{1}{2}\times 4t^2 \qquad \Rightarrow \qquad t = 4$ sec

$S_x = u_x t + \frac{1}{2}a_x t^2$

$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$

= 60 m.

HIGH LEVEL PROBLEMS
SUBJECTIVE QUESTIONS

1. Velocity of car on highway = v

Velocity of car on field = v/η

Let CD = x and AD = b

$T = t_{AC} + t_{CB} = \frac{b-x}{v} + \frac{\sqrt{\ell^2+x^2}}{(v/\eta)}$

$\frac{dT}{dx} = 0 \qquad \Rightarrow \qquad -\frac{1}{v} + \frac{\eta}{v}\left(\frac{2x}{2\sqrt{\ell^2+x^2}}\right) = 0$

$\Rightarrow \qquad x = \frac{\ell}{\sqrt{\eta^2-1}}$

2. (a) $V = \alpha\sqrt{x}$

$\frac{dx}{dt} = \alpha\sqrt{x} \qquad \Rightarrow \qquad \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$

$\therefore \qquad x = \frac{\alpha^2 t^2}{4}$

$\therefore \qquad \frac{dx}{dt} = V = \frac{\alpha^2 t}{2}$

Also $\qquad a = \frac{dv}{dt} = \frac{\alpha^2}{2}$.

(b) Let t_0 be the time taken to cover the first s metre

$\therefore \quad s = \dfrac{\alpha^2 t_0^{\,2}}{4} \quad \Rightarrow \quad t_0 = \dfrac{2\sqrt{s}}{\alpha}$

$$\therefore \quad \langle v \rangle = \frac{\int_0^{t_0} v\, dt}{\int_0^{t_0} dt}$$

$$\therefore \quad \langle v \rangle = \frac{\int_0^{t_0} \frac{\alpha^2 t}{2}\, dt}{t_0}$$

$$= \frac{\alpha^2}{2} \cdot \frac{1}{2} t_0$$

$$= \frac{\alpha^2}{4} \cdot \frac{2\sqrt{s}}{\alpha} = \frac{\alpha\sqrt{s}}{2}.$$

Aliter:

$v = \alpha\sqrt{x}, \qquad v^2 = \alpha^2 x$

$= 2\dfrac{\alpha^2}{2} x$

Comparing with $v^2 = 2ax$

$$\therefore \quad a = \frac{\alpha^2}{2} \text{ and } \quad \langle v \rangle = \frac{S}{t} = \frac{S}{\sqrt{2S/a}} = \frac{S\sqrt{a}}{\sqrt{2S}} = \sqrt{\frac{S^2}{2S}\frac{\alpha^2}{2}} = \frac{\alpha}{2}\sqrt{S}.$$

3. **(a)** From graph, obviously engine stopped at its highest velocity i.e., 190 ft/s. **Ans**

(b) The engine burned upto the instant it reached to its maximum velocity. Hence it burned for 2s. **Ans**

(c) The rocket reached its highest point for the time upto which the velocity is positive. Hence, from graph, rocket reached its highest point in 8 s.

$y_{max} \Rightarrow dy/dt = 0$

$\Rightarrow$ Velocity in y direction = v_y = 0 m/s.

(d) When the parachute opened up, the velocity of rocket starts increasing. Hence, at t = 10.85 (from graph), parachute was opened up. At that moment the velocity of the rocket falling down was 90 ft/s.

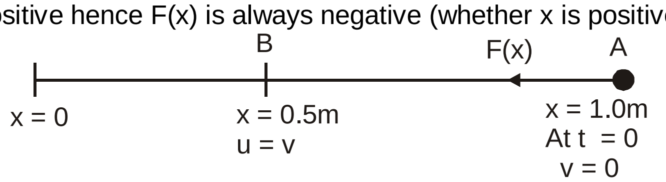
(e) The rocket starts falling when its velocity becomes negative. From the graph hence time taken by rocket to fall before the parachute opened will be (10.8 – 8) s = 2.8 s.

(f) Rocket's acceleration was greatest when the slope of tangent in V – t graph was maximum. As t = 2 sec, the tangent is vertical i.e, slope is infinity hence the rocket's acceleration was greatest at t = 2 s.

(g) The acceleration is constant when V – t graph is linear. Hence, the acceleration was constant between 2 and 10.8 s. Its value is given by slope = $-\dfrac{190}{8-2}$ = – 32 ft/s^2 (nearest to integer) **Ans**

4. **(a)** $F(x) = \dfrac{-k}{2x^2}$

k and x^2 both are positive hence F(x) is always negative (whether x is positive or negative .)



$$mv\frac{dv}{dx} = -\frac{k}{2x^2}$$

$$m\int_0^{v} v\,dv = \frac{-k}{2}\int_1^{0.5} \frac{1}{x^2}\, dx$$

$$m\left[\frac{v^2}{2}\right]_0^v = \frac{-k}{2}\left[\frac{-1}{x}\right]_1^{0.5}$$

$v^2 = 1$

$v = \pm 1$

but v is along –ve x direction so $v = -1\ \hat{i}$

(b) $m\int_0^v dv = \frac{-k}{2}\int_1^x \frac{1}{x^2}\ dx$

$v^2 = \left[\frac{1}{x} - \frac{1}{1}\right]$

$v^2 = \frac{1-x}{x}$

$v = \sqrt{\frac{1-x}{x}}$

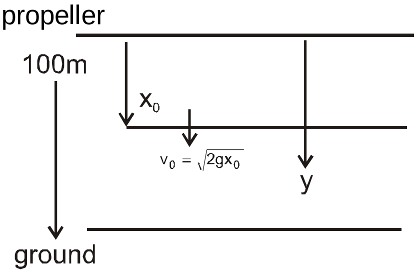
but $\quad v = -\left(\frac{dx}{dt}\right) = \sqrt{\frac{1-x}{x}}$

$\therefore \quad \sqrt{\frac{x}{1-x}}dx = -\,dt$

or $\quad \int_1^{0.25}\sqrt{\frac{x}{1-x}}dx = -\int_0^t dt$

Solving this, we get $t = 1.48$s

5. After switching on parachuite propeller



$v\frac{dv}{dy} = -2v$

$\int_{\sqrt{2gx_0}}^{0} dv = -2\int_{x_0}^{100} dy$

$\sqrt{2gx_0} = 2(100 - x_0)$

$x_0^2 - 205x_0 + 10000 = 0$

$x_0 = 80$m

$\Rightarrow \quad$ time of free fall $t = \sqrt{\frac{2(80)}{10}} = 4$ sec

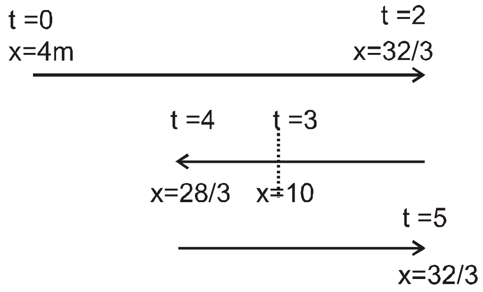
6. $x = t^3/3 - 3t^2 + 8t + 4$

$v = t^2 - 6t + 8 = (t-2)(t-4)$

$a = 2(t-3)$

		2	3	4
V	+	–	–	+
a	–	–	+	+

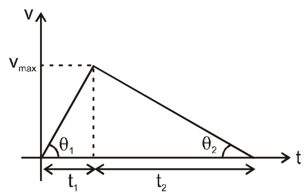
$S_1 = \left(\frac{32}{3} - 4\right) + \left(\frac{32}{3} - \frac{28}{3}\right) + \left(\frac{32}{3} - \frac{28}{3}\right) = \frac{20}{3} + \frac{8}{3} = \frac{28}{3}$ m.



$S_2 = \left(\frac{32}{3} - 4\right) + \left(10 - \frac{28}{3}\right) = \frac{20}{3} + \frac{2}{3} = \frac{22}{3}$ m

$\frac{s_1}{s_2} = \frac{28}{22} = \frac{14}{11}$

7.



here, v_{max} = v is the maximum velocity which can be achieved for the given path

from I[st] part, $\tan\theta_1 = 10 = \frac{v}{t_1} \Rightarrow t_1 = \frac{v}{10}$

from II[nd] part, $\tan\theta_2 = 5 = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{5}$

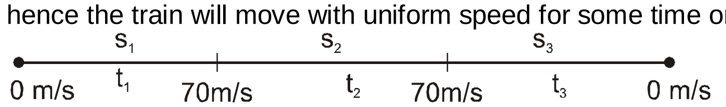
now, area under the graph is equal to total displacement

so, $\frac{1}{2}v\left[t_1 + t_2\right] = 1000$

$\frac{1}{2}v\left[\frac{v}{10} + \frac{v}{5}\right] = 1000$

so, $v_{max} = v = \frac{100\sqrt{2}}{\sqrt{3}}$ m/s = 81.6 m/s (approx)

The maximum speed is 70 m/s which is lesser than maximum possible speed v, hence the train will move with uniform speed for some time on the path.



The motion of train will be as shown

Let I[st] part of path has length s_1

then, by $v^2 = u^2 + 2as$, we get

$70^2 = 0^2 + 2 \times 10 \times s_1$, so s_1 = 245 m

Similarly by III[rd] equation of motion

$0^2 = 70^2 - 2 \times 5 \times s_3$, so s_3 = 490 m

Hence, s_2 = 1000 – (490 + 245) = 265 m

for part 1 of the path, time taken = t_1

from v = u + at, we get

$70 = 0 + 10\, t_1$ so, t_1 = 7 seconds

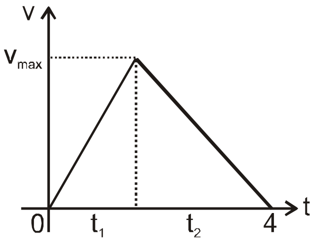
for part 2 of the path, time taken = $t_2 = \frac{s_2}{70} = \frac{265}{70} = \frac{53}{14}$ seconds

for 3rd part of the path, $0 = 70 - 5 \times t_3$

so, t_3 = 14 seconds.

Total time taken = $t_1 + t_2 + t_3 = 7 + \frac{53}{14} + 14 = \frac{347}{14}$ seconds

8.



Area of v-t curve is displacment which is equal to 2

$$\frac{1}{2} \times v_{max} \times 4 = 2$$

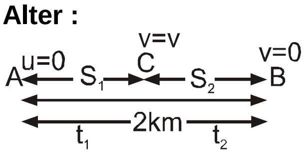
$v_{max} = 1$

Also $t_1 + t_2 = 4$

$$\frac{v_{max}}{x} + \frac{v_{max}}{y} = 4$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = 4$$

Alter :



Given, $S_1 + S_2 = 2$ ....(i)

$t_1 + t_2 = 4$ ....(ii)

For motion from A to C:

From, $V = u + at$

$V = 0 + xt,$

$t_1 = V/x$

From $V^2 = u^2 + 2as$

$V^2 = 0 + 2xS_1$

$$S_1 = \frac{V^2}{2x}$$

Similarly for motion from C to B

$t_2 = V/y$

$S_2 = V^2/2y$

From eqn.(i)

$$\frac{V^2}{2x} + \frac{V^2}{2y} = 2$$

$$\frac{V^2}{2}\left(\frac{1}{x} + \frac{1}{y}\right) = 2 \qquad \text{................(iii)}$$

From eqn. (ii)

$$V\left(\frac{1}{x} + \frac{1}{y}\right) = 4 \qquad \text{................(iv)}$$

Solving (iii) & (iv) we get,

$$\frac{1}{x} + \frac{1}{y} = 4$$

9. (a) $h_A = \frac{1}{2}g\left(\frac{t_0}{2}\right)^2$

$h_A = \frac{gt_0^2}{8}$

(b) $h = ut + gt^2$ ...(i)

$h = -u\,(t + t_0) + \frac{1}{2}\,g\,(t + t_0)^2$ ....(ii)

(i) × $(t + t_0)$ and (ii) × t

$h(t + t_0) = u(t)\,(t + t_0) + \frac{1}{2}\,gt^2\,(t + t_0)$

$h(t) = -u\,(t + t_0)\,(t) + \frac{1}{2}\,g\,(t + t_0)^2\,t$

$h(2t + t_0)\,\frac{1}{2} = gt\,(t + t_0)\,(2t + t_0)$

$h_T = \frac{1}{2}\,gt\,(t + t_0)$

Ans. (a) $h_A = \frac{gt_0}{8}$ (b) $h_T = \frac{1}{2}\,gt\,(t + t_0)$

10. $a = v\frac{dv}{dx} = cx + d$

Let at x = 0 v = u

$\therefore \int_u^v v\,dv = \int_0^x (cx + d)\,dx$

or $v^2 = cx^2 + 2dx + u^2$

v shall be linear function of x if $cx^2 + 2dx + u^2$ is perfect square

$\therefore \sqrt{\frac{d^2}{c}} = 3$