# QUADRATIC EQUATIONS

## 1. QUADRATIC EXPRESSION/EQUATION

The general form of a quadratic expression in x is,
$f(x) = ax^2 + bx + c$, where a, b, c $\in$ R & $a \neq 0$.
and general form of a quadratic equation in x is,
$ax^2 + bx + c = 0$, where a, b, c $\in$ R & $a \neq 0$.

## 2. ROOTS OF QUADRATIC EQUATION

**(a) The solution of the quadratic equation,**

$ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The expression $D = b^2 - 4ac$ is called the discriminant of the quadratic equation.

## 3. RELATION BETWEEN ROOT AND COEFFICIENTS

**(a) If $\alpha$ & $\beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then ;**

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$

(iii) $|\alpha - \beta| = \dfrac{\sqrt{D}}{|a|}$.

**(b) A quadratic equation whose roots are $\alpha$ & $\beta$ is $(x - \alpha)(x - \beta) = 0$ i.e.**

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ i.e.

$x^2$ – (sum of roots) x + product of roots = 0.

**NOTES :**

$$y = (ax^2 + bx + c) \equiv a(x - \alpha)(x - \beta) = a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a}$$

## 4. NATURE OF ROOTS

**(a) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ R & $a \neq 0$ then;**

(i) $D > 0 \Leftrightarrow$ roots are real & distinct (unequal).

(ii) $D = 0 \Leftrightarrow$ roots are real & coincident (equal).

(iii) $D < 0 \Leftrightarrow$ roots are imaginary.

(iv) If p + i q is one root of a quadratic equation, then the other must be the conjugate p – i q & vice versa. $(p,\ q \in R\ \&\ i = \sqrt{-1})$.

**(b) Consider the quadratic equation $ax^2 + bx + c = 0$ where a, b, c $\in$ Q & $a \neq 0$ then;**

(i) D > 0 and is a perfect square then the roots are rational and distinct.

(ii) D > 0, and is not a perfect square then the roots are conjugate surds i.e, $\alpha \pm \sqrt{\beta}.(\beta \neq 0)$

(iii) D = 0, then the roots are equal & rational

(iv) D < 0, then the roots are non-real conjugate complex numbers., i.e, $\alpha \pm i\beta$ .

**NOTES :**

Remember that a quadratic equation cannot have three different roots & if it has, it becomes an identity.

$ax^2 + bx + c = 0$ will be an identity (or can have more than two solutions) if a = 0, b = 0, c = 0

## 5. NATURE OF THE ROOTS OF TWO QUADRATIC EQUATION:

If $\Delta_1$ *and* $\Delta_2$ are the discriminants of two quadratic equations $P(x) = 0$ and $Q(x) = 0$, such that

(i) $\Delta_1 + \Delta_2 \geq 0$ then there will be at least two real roots for the equation $P(x) = 0$ or $Q(x) = 0$.

(ii) If $\Delta_1 + \Delta_2 < 0$, then there will be atleast two imaginary roots for the equation $P(x) = 0$ or $Q(x) = 0$.

(iii) If $\Delta_1.\Delta_2 < 0$, then the equation $P(x).Q(x) = 0$ will have two real roots and two imaginary roots.

(iv) If $\Delta_1.\Delta_2 > 0$, then the equation $P(x).Q(x) = 0$ has either four real roots or no real roots.

(v) If $\Delta_1.\Delta_2 = 0$ such that $\Delta_1 > 0$ and $\Delta_2 = 0$ or $\Delta_1 = 0$ *and* $\Delta_2 > 0$ then the equation $P(x).Q(x) = 0$ will have two equal roots and two distinct roots.

(vi) If $\Delta_1.\Delta_2 = 0$ where $\Delta_1 < 0$ *and* $\Delta_2 = 0$ or

$\Delta_1 = 0$ *and* $\Delta_2 < 0$ then the equation $P(x).\ Q(x) = 0$ will have two equal real roots and two non-real roots.

(vii) If $\Delta_1\Delta_2 = 0$ such that $\Delta_1 = 0$ *and* $\Delta_2 = 0$ then the equation $P(x)\,Q(x) = 0$ will have two pairs of equal roots.

## 6. GRAPH OF QUADRATIC EXPRESSION

Consider the quadratic expression, $y = ax^2 + bx + c$, $a \neq 0$ & $a, b, c \in R$ then ;

(i) The graph between x, y is always a parabola. If $a > 0$ then the shape of the parabola is concave upwards & if $a < 0$ then the shape of the parabola is concave downwards.

(ii) $y > 0\ \forall\ x \in R$, only if $a > 0$ & $D < 0$

(iii) $y < 0\ \forall\ x \in R$, only if $a < 0$ & $D < 0$

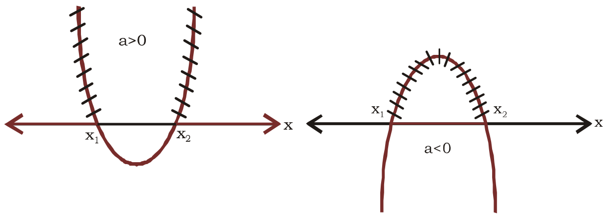
## 7. INEQUATIONS

$ax^2 + bx + c > 0\ (a \neq 0)$.

(i) If $D > 0$, then the equation $ax^2 + bx + c = 0$ has two different roots $(x_1 < x_2)$.

Then $a > 0 \Rightarrow x \in (-\infty, x_1) \cup (x_2, \infty)$

$a < 0 \Rightarrow x \in (x_1, x_2)$



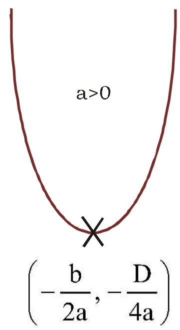

(ii) Inequalities of the form $\dfrac{P(x)}{Q(x)} \gtrless 0$ can be quickly solved using the method of intervals (wavy curve).

## 8. RANGE OF QUADRATIC EXPRESSION

Maximum & Minimum Value of $y = ax^2 + bx + c$ occurs at $x = -(b/2a)$ according as :

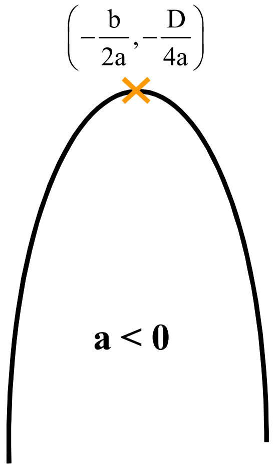
**For a > 0, we have :**

$$y \in \left[\frac{4ac - b^2}{4a}, \infty\right)$$




$$y_{min} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{max} \to \infty$$

**For a < 0, we have :**




$$y \in \left(-\infty, \frac{4ac-b^2}{4a}\right]$$

$$y_{max} = \frac{-D}{4a} \text{ at } x = \frac{-b}{2a}, \text{ and } y_{min} \to -\infty$$

## 9. POLYNOMIAL

If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the $n^{th}$ degree polynomial equation :

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0$$

where $a_0, a_1, \ldots\ldots\ a_n$ are all real & $a_0 \neq 0$,

Then,

$$\sum \alpha_1 = -\frac{a_1}{a_0};$$

$$\sum \alpha_1\,\alpha_2 = +\frac{a_2}{a_0};$$

$$\sum \alpha_1\,\alpha_2\,\alpha_3 = -\frac{a_3}{a_0};$$

............

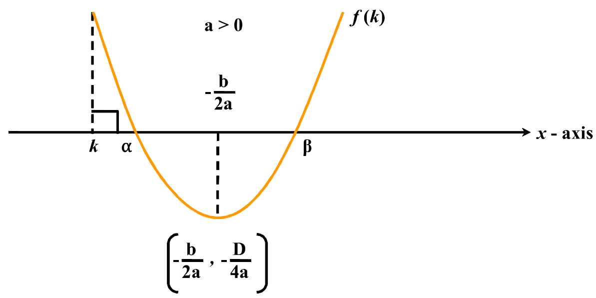
$$\alpha_1\,\alpha_2\,\alpha_3\ldots\ldots\alpha_n = (-1)^n\,\frac{a_n}{a_0}$$

## 10. LOCATION OF ROOTS OF QUADRATIC EQUATIONS

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

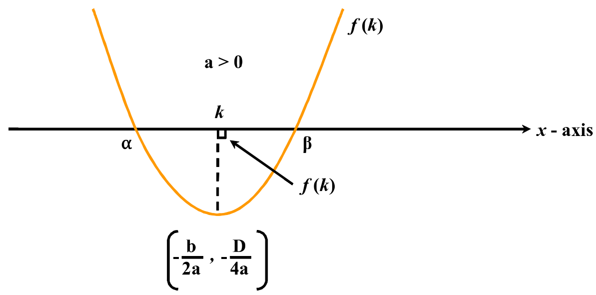
(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'k' are :

$D \geq 0$ & $f(k) > 0$ & $(-b/2a) > k$.



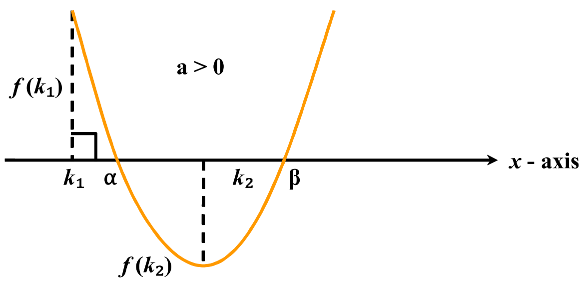

(ii) Conditions for both roots of $f(x) = 0$ to lie on either side of the number 'k' (in other words the number 'k' lies between the roots of $f(x) = 0$ is:

$a\,f(k) < 0$ and $D > 0$.



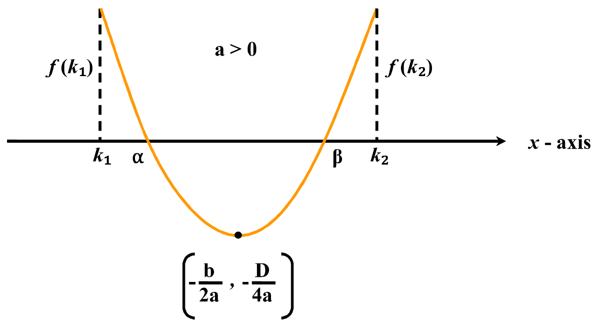

(iii) Conditions for exactly one root of $f(x) = 0$ to lie in the interval $(k_1, k_2)$ i.e. $k_1 < x < k_2$ are :

$D > 0$ & $f(k_1)\,.\,f(k_2) < 0$.



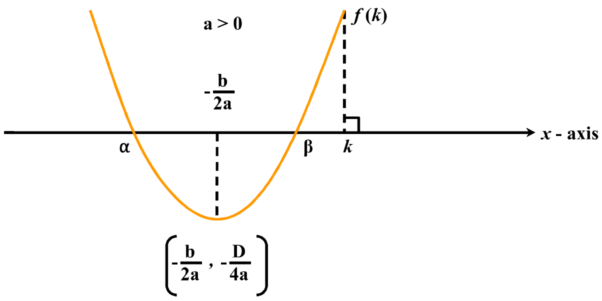
(iv) Conditions that both roots of f(x) = 0 to be confined between the numbers $k_1$ & $k_2$ are $(k_1 < k_2)$ :

$D \geq 0$ & $f(k_1)>0$ & $f(k_2)>0$ & $k_1<(-b/2a)<k_2$.




(v) Conditions for both the roots of f(x) = 0 to be less than a specified number 'k' are :

$$D \geq 0, f(k) > 0 \, and \, \frac{-b}{2a} < k$$




**NOTES :**

**Remainder Theorem :** If $f(x)$ is a polynomial, then $f(h)$ is the remainder when $f(x)$ is divided by x – h.

**Factor theorem :** If x = h is a root of equation $f(x) = 0$, then x–h is a factor of $f(x)$ and conversely.

## 11. RANGE OF RATIONAL FUNCTIONS

Here we shall find the values attained by a rational expresion of the form $\frac{a_1x^2+b_1x+c_1}{a_2x^2+b_2x+c_2}$ for real values of x.

If $f(x)=\frac{ax^2+bx+c}{ax^2-bx+c}(or)\,f(x)=\frac{ax^2-bx+c}{ax^2+bx+c}$, $(b^2-4ac<0)$, then the minimum and maximum values of f(x) are given by $f\left(\pm\sqrt{\frac{c}{a}}\right)$.

## 12. COMMON ROOTS

### (a) Only One Common Root

Let α be the common root of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$, such that a, a' ≠ 0 and a b' ≠ a'b.

Then, the condition for one common root is :

$(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$.

### (b) Two Common Roots

Let α, β be the two common roots of $ax^2 + bx + c = 0$ & $a'x^2 + b'x + c' = 0$, such that a, a' ≠ 0.

Then, the condition for two common roots is :

$$\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$$

## 13. FACTORS OF A SECOND DEGREE EQUATION

The condition that a quadratic function

$f(x, y) = ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c$

may be resolved into two linear factors is that ;

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

OR $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

## 14. FORMATION OF A POLYNOMIAL EQUATION

If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are the roots of the $n^{th}$ degree polynomial equation, then the equation is

$x^n - S_1x^{n-1} + S_2x^{n-2} - S_3x^{n-3} + \ldots\ldots + (-1)^n S_n = 0$

where $S_k$ denotes the sum of the products of roots taken k at a time.

**Particular Cases**

**(a) Quadratic Equation** if $\alpha$, $\beta$ be the roots of the quadratic equation, then the equation is :

$x^2 - S_1x + S_2 = 0$ *i.e.* $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

**(b) Cubic Equation** if $\alpha, \beta, \gamma$ be the roots of the cubic equation, then the equation is :

$x^3 - S_1x^2 + S_2x - S_3 = 0$ *i.e.*

$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$

(i) If $\alpha$ is a root of equation $f(x) = 0$, the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$. In other words, $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of nth degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If there be any two real numbers 'a' & 'b' such that f (a) & f (b) are of opposite signs, then $f(x) = 0$ must have atleast one real root between 'a' and 'b'.

(iv) Every equation $f(x) = 0$ of degree odd has atleast one real root of a sign opposite to that of its last term.

## 15. TRANSFORMATION OF EQUATIONS

(i) To obtain an equation whose roots are reciprocals of the roots of a given equation, it is obtained by replacing x by $1/x$ in the given equation

(ii) Transformation of an equation to another equation whose roots are negative of the roots of a given equation–replace x by $-x$.

(iii) Transformation of an equation to another equation whose roots are square of the roots of a given equation–replace x by $\sqrt{x}$ .

(iv) Transformation of an equation to another equation whose roots are cubes of the roots of a given equation-replace x by $x^{1/3}$.

(v) Transformation of an equation to another equation whose roots are'k' times the roots of given equation replace x by $\frac{x}{k}$ .

(vi) Transformation of an equation to another equation whose roots are 'k' times more than the roots of given equation-replace x by '$x - k$'.

# SOLVED EXAMPLES

**Example – 1**

Solve the equation

(i) $15.2^{x+1} + 15.2^{2-x} = 135$

(ii) $3^{x-4} + 5^{x-4} = 34$

(iii) $5^x \sqrt[x]{8^{x-1}} = 500$

**Sol.** (i) The equation can be rewritten in the form

$$30.2^x + \frac{60}{2^x} = 135$$

Let $t = 2^x$

then $30t^2 - 135t + 60 = 0$

$6t^2 - 27t + 12 = 0$

$\Rightarrow$ $6t^2 - 24t - 3t + 12 = 0$

$\Rightarrow$ $(t-4)(6t-3) = 0$

then $t_1 = 4$ and $t_2 = \frac{1}{2}$

$$2^x = 4 \text{ and } 2^x = \frac{1}{2}$$

then $x = 2$ and $x = -1$

Hence roots of the original equation are $x_1 = 2$ and $x_2 = -1$

**NOTES :**

An equation of the form

$a^{f(x)} + b^{f(x)} = c$

where $a, b, c \in R$

and a, b, c satisfies the condition $a^2 + b^2 = c$

then solution of the equation is $f(x) = 2$ and no other solution of this equation.

(ii) Here, $3^2 + 5^2 = 34$, then given equation has a solution $x - 4 = 2$

$\therefore$ $x = 6$ is a root of the original equation

**NOTES :**

An equation of the form $\{f(x)\}^{g(x)}$ is equivalent to the equation

$\{f(x)\}^{g(x)} = 10^{g(x)\log f(x)}$ where $f(x) > 0$

(iii) We have $5^x \sqrt[x]{8^{x-1}} = 500$

$\Rightarrow$ $5^x \sqrt[x]{8^{x-1}} = 5^3 . 2^2$

$\Rightarrow$ $5^x . 8^{\left(\frac{x-1}{x}\right)} = 5^3 . 2^2$

$\Rightarrow$ $5^x . 2^{\frac{3x-3}{x}} = 5^3 . 2^2$

$\Rightarrow$ $5^{x-3} . 2^{\left(\frac{x-3}{x}\right)} = 1$

$\Rightarrow$ $(5.2^{1/x})^{(x-3)} = 1$

is equivalent to the equation

$$10^{(x-3)\log\left(5.2^{1/x}\right)} = 1$$

$\Rightarrow$ $(x-3)\log(5.2^{1/x}) = 0$

Thus original equation is equivalent to the collection of equations

$x - 3 = 0, \log(5.2^{1/x}) = 0$

$\therefore$ $x = 3, 5.2^{1/x} = 1$

$\Rightarrow$ $2^{1/x} = (1/5)$

$\therefore$ $x = -\log_5 2$

Hence roots of the original equation are

$x = 3$ and $x = -\log_5 2$

## Example – 2

Solve the equation $25x^2 - 30x + 11 = 0$ by using the general expression for the roots of a quadratic equation.

**Sol.** Comparing the given equation with the general form of a quadratic equation $ax^2 + bx + c = 0$, we get

$a = 25, b = -30$ and $c = 11.$

Substituting these values in

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{30 + \sqrt{900 - 1100}}{50} \text{ and } \beta = \frac{30 - \sqrt{900 - 1100}}{50}$$

$$\Rightarrow \quad \alpha = \frac{30 + \sqrt{-200}}{50} \text{ and } \beta = \frac{30 - \sqrt{-200}}{50}$$

$$\Rightarrow \quad \alpha = \frac{30 + 10i\sqrt{2}}{50} \text{ and } \beta = \frac{30 - 10i\sqrt{2}}{50}$$

$$\Rightarrow \quad \alpha = \frac{3}{5} + \frac{\sqrt{2}}{5}i \text{ and } \beta = \frac{3}{5} - \frac{\sqrt{2}}{5}i$$

Hence, the roots of the given equation are $\frac{3}{5} \pm \frac{\sqrt{2}}{5}i$

## Example – 3

Solve the following quadratic equation by factorization method :

$x^2 - 5ix - 6 = 0$

**Sol.** The given equation is

$x^2 - 5ix - 6 = 0$

$\Rightarrow \quad x^2 - 5ix + 6i^2 = 0$

$\Rightarrow \quad x^2 - 3ix - 2ix + 6i^2 = 0$

$\Rightarrow \quad x(x - 3i) - 2i(x - 3i) = 0$

$\Rightarrow \quad (x - 3i)(x - 2i) = 0$

$\Rightarrow \quad x - 3i = 0, x - 2i = 0$

$\Rightarrow \quad x = 3i, x = 2i$

Hence, the roots of the given equation are 3i and 2i.

## Example – 4

If $\alpha, \beta, \gamma$ be the roots of the equation

$$x(1 + x^2) + x^2(6 + x) + 2 = 0,$$

then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is

(a) $-3$ (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) None of these

**Ans.** (c)

**Sol.** $2x^3 + 6x^2 + x + 2 = 0$ has roots $\alpha, \beta, \gamma$.

So, $2x^3 + x^2 + 6x + 2 = 0$ has roots $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$

(writing coefficients in revers order, since roots are reciprocal)

$$\text{Hence, Sum of the roots} = -\frac{(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$$

$$\therefore \quad \alpha^{-1} + \beta^{-1} + \gamma^{-1} = -\frac{1}{2}$$

Hence, (c) is the correct answer.

## Example – 5

If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) = (.......).

**Sol.** If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$. Then, other root is $2 - i\sqrt{3}$

$$\Rightarrow -p = 2 + i\sqrt{3} + 2 - i\sqrt{3} = 4$$

$$and \ q = (2 + i\sqrt{3})(2 - i\sqrt{3}) = 7$$

$$\Rightarrow (p, q) = (-4, 7).$$

Correct Answer (–4, 7)

**Example – 6**

If the products of the roots of the equation $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ is 7, then the roots are real for $k = \ldots$.

**Ans.** (2)

**Sol.** $Since, x^2 - 3kx + 2e^{2\log_e k} - 1 = 0$ has product of roots 7.

$$\Rightarrow 2e^{2\log_e k} - 1 = 7$$

$$\Rightarrow e^{2\log_e k} = 4$$

$$\Rightarrow k^2 = 4$$

$$\Rightarrow k = 2[neglecting - 2].$$

Correct Answer $(k = 2)$

**Example – 7**

If $\alpha$ and $\beta$ are the roots of the equation $x^2 + Px + 1 = 0$; $\gamma$, $\delta$ are the roots of $x^2 + qx + 1 = 0$, then $q^2 - P^2 = (\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

**Sol.** $x^2 + Px + 1 = 0$ Roots $\alpha, \beta$

$\alpha + \beta = -P$

$\alpha\beta = 1$

$x^2 + qx + 1 = 0$ ; Roots $\gamma$ and $\delta$

$\gamma + \delta = -q$

$\gamma\delta = 1$

Now : $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

$(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

$$= \left[\alpha\beta - \gamma(\alpha+\beta) + \gamma^2\right]\left[\alpha\beta + \delta(\alpha+\beta) + \delta^2\right]$$

$$= \left[1 - \gamma(-P) + \gamma^2\right]\left[1 + \delta(-P) + \delta^2\right]$$

$$= \left[1 + P\gamma + \gamma^2\right]\left[1 - P\delta + \delta^2\right]$$

$$= \left[\left(1+\gamma^2\right) + P\gamma\right]\left[\left(1+\delta^2\right) - P\delta\right]$$

$$= \left[-q\gamma + P\gamma\right]\left[-q\delta - P\delta\right]$$

$$= \gamma[P - q]\,\delta[-P - q]$$

$$= \gamma\delta[P-q][P+q](-1)$$

$$= 1\left(P^2 - q^2\right)(-1)$$

$= q^2 - P^2$ Hence Proved.

**Example – 8**

If $\alpha$ and $\beta$ are the roots of $x^2 + px + q = 0$ and $\gamma$, $\delta$ are the roots of $x^2 + rx + s = 0$, then evaluate $(\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$ in terms of p, q, r and s.

**Sol.** $\alpha + \beta = -p, \alpha\beta = q$

$\gamma + \delta = -r,\ \gamma\delta = s$

$$= (\alpha-\gamma)(\beta-\gamma)(\alpha-\delta)(\beta-\delta)$$

$$= \left[\alpha\beta - \gamma(\alpha+\beta) + \gamma^2\right]\left[\alpha\beta - \delta(\alpha+\beta) + \delta^2\right]$$

$$= \left[q + p\gamma + \gamma^2\right]\left[q + p\delta + \delta^2\right]$$

$$= q^2 + pq\delta + q\delta^2 + pq\gamma + p^2\gamma\delta + p\gamma\delta^2 + \gamma^2 q + p\delta\gamma^2 + (\gamma\delta)^2$$

$$= q^2 + pq(\delta+\gamma) + q(\delta^2+\gamma^2) + \gamma\delta[p^2 + p(\gamma+\delta)] + (\gamma.\delta)^2$$

$$= q^2 - pqr + q\left(r^2 - 2s\right) + s\left[p^2 - rp\right] + s^2$$

$$= \left(q^2 - 2sq + s^2\right) - rpq - rsp + sp^2 + qr^2$$

$$= (q-s)^2 - rpq - rsp + sp^2 + qr^2.$$

**Example – 9**

If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then show that

$$(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$$

**Sol.** Let the roots be $\alpha$ and $\alpha^n$

$$\alpha + \alpha^n = -\frac{b}{a}$$

$$\rightarrow \alpha^{n+1} = \frac{c}{a} \rightarrow c = a\alpha^{n+1}.$$

Now

$$\left(ac^n\right)^{\frac{1}{n+1}} + \left(a^n c\right)^{\frac{1}{n+1}} + b$$

$$= \left[a.\left(\alpha^{n+1}.a\right)^n\right]^{\frac{1}{n+1}} + \left[a^n.\alpha^{n+1}.a\right]^{\frac{1}{n+1}} + b$$

$$= \left[a^{n+1}.\alpha^{n(n+1)}\right]^{\frac{1}{n+1}} + \left[a^{n+1}.\alpha^{n+1}\right]^{\frac{1}{n+1}} + b$$

$$= a\,.\,\alpha^n + a.\alpha + b$$

$$= a\left(\frac{c}{a\alpha}\right) + a\alpha + b$$

$$= \frac{c}{\alpha} + a\alpha + b$$

$$= \frac{a\alpha^2 + b\alpha + c}{\alpha} = 0$$

**Example – 10**

If $\alpha$, $\beta$ are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant $\delta$, then prove that

$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$$

**Sol.** $ax^2 + bx + c = 0 : \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$Ax^2 + Bx + C = 0 :$

$$\alpha + \delta + \beta + \delta = -\frac{B}{A}, (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$$

Now $\alpha + \beta + 2\delta = -\frac{B}{A}$

$$\Rightarrow -\frac{b}{a} + 2\delta = -\frac{B}{A}$$

Now $\frac{B^2 - 4AC}{A^2} = \left(\frac{B}{A}\right)^2 - 4\left(\frac{C}{A}\right)$

$$= \left(-\frac{b}{a} + 2\delta\right)^2 - 4\left((\alpha + \delta)(\beta + \delta)\right)$$

$$= \frac{b^2}{a^2} + 4\delta^2 - 4\delta\frac{b}{a} - 4\left(\frac{c}{a} + \delta\left(-\frac{b}{a}\right) + \delta^2\right)$$

$$= \frac{b^2}{a^2} + 4\delta^2 - 4\delta\frac{b}{a} - \frac{4c}{a} + 4\delta\frac{b}{a} - 4\delta^2$$

$$= \frac{b^2}{a^2} - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2}$$

**Example – 11**

Let a, b, c be real numbers with $a \neq 0$ and let $\alpha$, $\beta$ be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of $\alpha$, $\beta$.

**Sol.** $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$a^3x^2 + abc\,x + c^3 = 0$ Roots : $x_1, x_2$

$$x_1 + x_2 = -\frac{abc}{a^3} = -\frac{bc}{a^2} = \left(\frac{-b}{a}\right)\left(\frac{c}{a}\right) = (\alpha + \beta)(\alpha\beta)$$

$$x_1 + x_2 = \alpha^2\beta + \alpha\beta^2$$

$$x_1x_2 = \frac{c^3}{a^3} = \left(\frac{c}{a}\right)^3 = (\alpha\beta)^3 = \left(\alpha\beta^2\right)\left(\alpha^2\beta\right)$$

So, Roots are : $\alpha^2\beta$ and $\alpha\beta^2$

## Example – 12

If P (x) = $ax^2 + bx + c$ and Q (x) = $-ax^2 + bx + c$, where $ac \neq 0$, show that the equation

$P(x) \cdot Q(x) = 0$ has at least two real roots.

**Sol.** Roots of the equation P (x) Q (x) = 0

i.e., $(ax^2 + bx + c)(-ax^2 + bx + c) = 0$ will be roots of the equations

$ax^2 + bx + c = 0$ ....... (i)

and $-ax^2 + bx + c = 0$ ........ (ii)

If $D_1$ and $D_2$ be discriminants of (i) and (ii) then

$D_1 = b^2 - 4ac$ and $D_2 = b^2 + 4ac$

Now $D_1 + D_2 = 2b^2 \geq 0$

(since, b may be zero)

i.e., $D_1 + D_2 \geq 0$

Hence, at least one of $D_1$ and $D_2 \geq 0$

i.e., at least one of the equations (i) and (ii) has real roots and therefore, equation P (x) Q (x) = 0 has at least two real roots.

**Alternative Sol.**

Since, $ac \neq 0$

$\therefore$ $ac < 0$ or $ac > 0$

**Case I :**

If $ac < 0$ $\Rightarrow$ $-ac > 0$

then $D_1 = b^2 - 4ac > 0$

**Case II :**

If $ac > 0$

then $D_2 = b^2 + 4ac > 0$

So, at least one of $D_1$ and $D_2 > 0$.

Hence, at least one of the equations (i) and (ii) has real roots.

Hence, equation P (x) $\cdot$ Q (x) = 0 has at least two real roots.

## Example – 13

a, b, c $\in$ R, $a \neq 0$ and the quadratic equation $ax^2 + bx + c = 0$ has no real roots, then,

(a) $a + b + c > 0$ (b) $a(a + b + c) > 0$

(c) $b(a + b + c) > 0$ (d) $c(a + b + c) > 0$

**Ans.** (b,d)

**Sol.** Let $f(x) = ax^2 + bx + c$. It is given that $f(x) = 0$ has no real roots. So, either $f(x) > 0$ for all $x \in R$ or $f(x) < 0$ for all $x \in R$ i.e. f(x) has same sign for all values of x.

$\therefore$ $f(0)\, f(1) > 0$

$\Rightarrow$ $c(a + b + c) > 0$

Also, $af(1) > 0$

$\Rightarrow$ $a(a + b + c) > 0$.

Correct answer (b,d)

## Example – 14

If $ax^2 - bx + 5 = 0$ does not have two distinct real roots, then find the minimum value of $5a + b$.

**Ans.** (–1)

**Sol.** Let $f(x) = ax^2 - bx + 5$

Since, f (x) = 0 does not have two distinct real roots, we have either

$f(x) \geq 0\ \forall x \in R$ or $f(x) \leq 0\ \forall x \in R$

But $f(0) = 5 > 0$, so $f(x) \geq 0\ \forall x \in R$

In particular $f(-5) \geq 0 \Rightarrow 5a + b \geq -1$

Hence, the least value of $5a + b$ is –1.

## Example – 15

If $x^2 + (a - b)x + (1 - a - b) = 0$ where a, b $\in$ R, then find the values of a for which equation has unequal real roots for all values of b.

**Sol.** Let $f(x) = x^2 + (a-b)x + (1 - a - b)$

$D > 0$

$\Rightarrow$ $(a - b)^2 - 4(1-a-b) > 0$

$\Rightarrow$ $a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\Rightarrow$ $b^2 - (2a - 4)b + (a^2 + 4a - 4) > 0$

Above is a quadratic in 'b'

Whose value is +ve

So its $D < 0$

$(2a - 4)^2 - 4(a^2 + 4a - 4) < 0$

$4a^2 + 16 - 16a - 4a^2 - 16a + 16 < 0$

$32 - 32a < 0$

$a > 1$.

**Example – 16**

Find all the zeros of the polynomial $x^4+x^3-9x^2-3x+18$ if it is given that two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

**Sol.** Given polynomial $f(x)=x^4+x^3-9x^2-3x+18$ has two of its zeros $-\sqrt{3}$ and $\sqrt{3}$.

$\Rightarrow$ $(x+\sqrt{3})(x-\sqrt{3})$ is a factor of $f(x)$,

i.e., $x^2-3$ is a factor of $f(x)$.

Now, we apply the division algorithm to the given polynomial with $x^2-3$.

$$\begin{array}{r|l} & x^2+x-6 \\ \hline x^2-3 & x^4+x^3-9x^2-3x+18 \\ & x^4 \qquad -3x^2 \\ & - \qquad\quad + \\ \hline & x^3-6x^2-3x+18 \\ & x^3 \qquad\quad -3x \\ & - \qquad\quad + \\ \hline & -6x^2+18 \\ & -6x^2+18 \\ & + \qquad - \\ \hline & 0 = \text{Remainder} \end{array}$$

Thus, $x^4+x^3-9x^2-3x+18$

$=(x^2-3)(x^2+x-6)$

$=(x^2-3)\times\{x^2+3x-2x-6\}$

$=(x^2-3)\times\{x(x+3)-2(x+3)\}$

$=(x^2-3)\times(x+3)(x-2)$

Putting $x+3=0$ and $x-2=0$

we get $x=-3$ and $x=2$, i.e., $-3$ and $2$ are the other two zeros of the given polynomial.

Hence $-\sqrt{3},\sqrt{3},-3,\ 2$ are the four zeros of the given polynomial.

**Example – 17**

Find all roots of the equation $x^4+2x^3-16x^2-22x+7=0$ if one root is $2+\sqrt{3}$.

**Sol.** All coefficients are real, irrational roots will occur in conjugate pairs.

Hence another root is $2-\sqrt{3}$.

$\therefore$ Product of these roots $=(x-2-\sqrt{3})(x-2+\sqrt{3})$

$=(x-2)^2-3$

$=x^2-4x+1$

Dividing $x^4+2x^3-16x^2-22x+7$ by $x^2-4x+1$ then the other quadratic factor is $x^2+6x+7$

then the given equation reduce in the form

$(x^2-4x+1)(x^2+6x+7)=0$

$\therefore$ $x^2+6x+7=0$

then $x=\dfrac{-6\pm\sqrt{36-28}}{2}$

$=-3\pm\sqrt{2}$

Hence roots $2\pm\sqrt{3},-3\pm\sqrt{2}$

**Example – 18**

Solve for $x:-(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$

**Sol.** $\left(5+2\sqrt{6}\right)^{x^2-3}+1\left(5-2\sqrt{6}\right)^{x^2-3}=10$

Put $5+2\sqrt{6}=k$

Observe $5-2\sqrt{6}=\dfrac{\left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right)}{\left(5+2\sqrt{6}\right)}=\dfrac{25-24}{\left(5+2\sqrt{6}\right)}$

$5-2\sqrt{6}=\dfrac{1}{k}$

Now $(k)^{x^2-3}+\left(\dfrac{1}{k}\right)^{x^2-3}=10$

Let $(k)^{x^2-3}=z$

$\Rightarrow z + z^{-1} = 10$

$\Rightarrow z^2 - 10z + 1 = 0$

$\Rightarrow z = \dfrac{10 \pm \sqrt{100-4}}{2}$ $OR\ z = 5 \pm 2\sqrt{6}$

Now $k^{x^2-3} = 5 + 2\sqrt{6}$

$\Rightarrow \left(5+2\sqrt{6}\right)^{x^2-3} = 5 + 2\sqrt{6}$

$\Rightarrow x^2 - 3 = 1$

$x^2=4 \qquad x=\pm 2$

and $k^{x^2-3} = 5 - 2\sqrt{6}$

$\Rightarrow \left(5+2\sqrt{6}\right)^{x^2-3} = \left(5-2\sqrt{6}\right)$

$\Rightarrow x^2 - 3 = -1$

$x^2=2 \rightarrow x = \pm\sqrt{2}$.

**Example – 19**

For $a \le 0$, determine all real roots of the equation $x^2 - 2a\,|x-a| - 3a^2 = 0$

**Sol.** $a \le 0$, $x^2 - 2a\,|x-a| - 3a^2 = 0$

When $x < (a)$, $|x-a| = -(x-a)$

$x^2 + 2a(x-a) - 3a^2 = 0$

$x^2 + 2ax - 2a^2 - 3a^2 = 0$

$x^2 + 2ax - 5a^2 = 0$

$x = \dfrac{-2a \pm \sqrt{4a^2 + 20a^2}}{2}$

$as\ a < 0,\ So\ x. = a\left(\sqrt{6}-1\right)$

When $x \ge a, |x-a| = x-a$

$x^2 - 2a(x-a) - 3a^2 = 0$

$x^2 - 2ax + 2a^2 - 3a^2 = 0$

$x^2 - 2ax - a^2 = 0$

$x = \dfrac{2a \pm \sqrt{4a^2+4a^2}}{2} = a\left(1 \pm \sqrt{2}\right)$

as $a < 0$, So $x = a\left(1-\sqrt{2}\right)$

Correct Answer $x = \{a(1-\sqrt{2}), a(\sqrt{6}-1)\}$

**Example – 20**

The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is .... .

**Sol.** Given, $|x-2|^2 + |x-2| - 2 = 0$

**Case I:** when $x \ge 2$

$(x-2)^2 + (x-2) - 2 = 0$

$x^2 + 4 - 4x + x - 2 - 2 = 0$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$ [0 is rejected]

$x = 3$ ..(i)

**Case II:** when $x < 2$

$\Rightarrow \{-(x-2)\}^2 - (x-2) - 2 = 0$

$\Rightarrow (x-2)^2 - x + 2 - 2 = 0$

$x^2 + 4 - 4x - x = 0$

$x^2 + 4x - (x-4) = 0$

$x(x-4) - 1(x-4) = 0$

$(x-1)(x-4) = 0$

$x = 1, 4$ [4 is rejected]

$x = 1$ ...(ii)

Hence, the sum of the roots is $3 + 1 = 4$.

Alternate solution

Given $|x-2|^2 + |x-2| - 2 = 0$

$\Rightarrow \left(|x-2|+2\right)\left(|x-2|-1\right) = 0$

$\therefore |x-2| = -2, 1$ [neglecting $-2$]

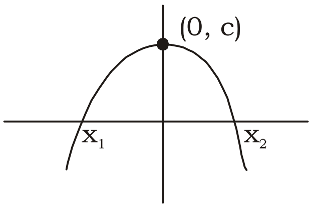
$\Rightarrow |x-2| = 1 \Rightarrow x = 3, 1$

$\Rightarrow$ Sum of the roots $= 4$

**Example – 21**

The diagram shows the graph of

$y = ax^2 + bx + c$, Then,




(a) $a > 0$ (b) $b < 0$

(c) $c > 0$ (d) $b^2 - 4ac = 0$

**Ans.** (b,c)

**Sol.** As it is clear from the figure that it is a parabola opens downwards i.e. $a < 0$.

$\Rightarrow$ It is $y = ax^2 + bx + c$ i.e. degree two polynomial

Now, if $ax^2 + bx + c = 0$

$\Rightarrow$ it has two roots $x_1$ and $x_2$ as it cuts the axis at two distinct point $x_1$ and $x_2$.

Now from the figure it is also clear that $x_1 + x_2 < 0$

(i.e. sum of roots are negative)

$\Rightarrow \frac{-b}{a} < 0 \Rightarrow \frac{b}{a} > 0$

$\Rightarrow$ $b < 0$ ($\because a < 0$) (b) is correct.

As the graph of $y = f(x)$ cuts the + y-axis at (0, c)

where $c > 0$ $\Rightarrow$ (b,c) is correct.

**Example – 22**

Let $f(x) = Ax^2 + Bx + C$ where, A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A + B and C are all integers, then $f(x)$ is an integer whenever x is an integer.

**Sol.** Supoose : $f(x) = Ax^2 + Bx + c$ is an integer wherever x is an integer

$\therefore$ f(0), f(1), f(-1) are integers.

$\Rightarrow$ C, A + B + C, A – B + C are integrs

$\Rightarrow$ C, A + B, A - B are integers.

$\Rightarrow$ C, A + B, (A + B) - (A - B) = 2A are integers.

Conversely suppose 2A, A + B and C are integers.

Let n be any integer. We have,

$f(n) = An^2 + Bn + C$

$$= 2A\left[\frac{n(n-1)}{2}\right] + (A+B)n + C$$

Since n is an integer, $\frac{n(n-1)}{2}$ is an integer.

Also, 2A, A + B and C are integers.

We get f (n) is an integer for all integer ‘n’.

**Example – 23**

Solve $2\log_x a + \log_{ax} a + 3\log_b a = 0$,

where $a > 0$, $b = a^2 x$

**Sol.** Given $2\log_x a + \log_{ax} a + 3\log_b a = 0$

$$\Rightarrow 2\frac{\log a}{\log x} + \frac{\log a}{\log ax} + 3\frac{\log a}{\log b} = 0$$

$$\Rightarrow \log a\left[\frac{2}{\log x} + \frac{1}{\log ax} + \frac{3}{\log a^2 x}\right] = 0$$

$$\left[\because b = a^2 x\right]$$

$$\Rightarrow 2\log a^2 x \log ax + \log x \log a^2 x + 3\log x \log ax = 0$$

$$\Rightarrow 2[2\log a + \log x][\log a + \log x] + \log x[2\log a + \log x]$$

$$+ 3\log x\,[\log a + \log x] = 0$$

$$\Rightarrow 2\left[2(\log a)^2 + 3\log a \log x + (\log x)^2\right]$$

$$+\left[2\log a.\log x + (\log x)^2\right] + \left[3\log x \log a + 3(\log x)^2\right]$$

$$\Rightarrow 6(\log x)^2 + 11(\log a)(\log x) + 4(\log a)^2 = 0$$

$$\Rightarrow 6(\log x)^2 + 8(\log a)(\log x) + 3(\log a)(\log x) + 4(\log a)^2 = 0$$

$$\Rightarrow 2(\log x)(3\log x + 4\log a) + \log a(3\log x + 4\log a) = 0$$

$$\Rightarrow (3\log x + 4\log a)(2\log x + \log a) = 0$$

$$\Rightarrow 3\log x = -4\log a \; OR \; 2\log x = -\log a$$

$$\Rightarrow x^3 = a^{-4} \qquad x^2 = a^{-1}$$

$$x = a^{-\frac{4}{3}} \qquad x = a^{-\frac{1}{2}}$$

**Example – 24**

Solve for x the following equation

$\log_{(2x+3)}(6x^2+23x+21)=4-\log_{(3x+7)}(4x^2+12x+9)$

**Sol.** $\log_{(2x+3)}\left(6x^2+23x+21\right)=4-\log_{(3x+7)}\left(4x^2+12x+9\right)$

$\Rightarrow \log_{(2x+3)}\left(2x+3\right)\left(3x+7\right)=4-\log_{(3x+7)}\left(2x+3\right)^2$

$\Rightarrow 1+\log_{(2x+3)}\left(3x+7\right)=4-2\log_{(3x+7)}\left(2x+3\right)$

Let $\log_{(2x+3)}\left(3x+7\right)=y$

$\Rightarrow y+\frac{2}{y}-3=0$

$\Rightarrow y^2-3y+2=0$

$\Rightarrow \left(y-2\right)\left(y-1\right)=0$

$\Rightarrow$ y = 1 OR y = 2

$\Rightarrow \log_{2x+3}(3x+7)=1$ OR $\log_{2x+3}(3x+7)=2$

$\Rightarrow (2x+3)=(3x+7)$ OR $(3x+7)=(2x+3)^2$

$\Rightarrow x=-4$ OR $3x+7=4x^2+9+12x$

$4x^2+9x+2=0$

$(4x+1)(x+2)=0$

$x=-\frac{1}{4}$ *OR* $x=-2$

So : $x=-4$, $x=-\frac{1}{4}$, $x=-2$

but log exist only when $6x^2+23x+21>0$ and $4x^2+12x+9>0$ and $2x+3>0$ and $3x+7>0$

$\therefore x=-\frac{1}{4}$ is the only solution.

**Example – 25**

If the remainder on dividing $x^3+2x^2+kx+3$ by $x-3$ is 21, find the quotient and the value of k. Hence find the zeros of the cubic polynomial $x^3+2x^2+kx-18$.

**Sol.** Let $p(x)=x^3+2x^2+kx+3$.

We are given that when p (x) is divided by the linear polynomial $x-3$, the remainder is 21.

$\Rightarrow$ $p(3)=21$ (Remainder Theorem)

$\Rightarrow$ $3^3+2\times 3^2+k\times 3+3=21$

$\Rightarrow$ $27+18+3k+3=21$

$\Rightarrow$ $3k=21-27-18-3$

$\Rightarrow$ $3k=-27$

$\Rightarrow$ $k=-9$

Hence, $p(x)=x^3+2x^2-9x+3$.

To find the quotient obtained on dividing p(x) by x–3, we perform the following division :

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x-3 & x^3+2x^2-9x+3 \\ & x^3-3x^2 \\ & -\quad + \\ & 5x^2-9x+3 \\ & 5x^2-15x \\ & -\quad + \\ & 6x+3 \\ & 6x-18 \\ & -\quad + \\ & 21 \end{array}$$

Hence, $p(x)=(x^2+5x+6)(x-3)+21$

(Divisor × Quotient + Remainder)

$\Rightarrow$ $x^3+2x^2-9x+3-21=(x^2+5x+6)(x-3)$

$\Rightarrow$ $x^3+2x^2-9x-18=(x^2+3x+2x+6)(x-3)$

$\Rightarrow$ $x^3+2x^2-9x-18=(x+3)(x+2)(x-3)$

Hence, the zeros of $x^3+2x^2-9x-18$ are given by $x+3=0, x+2=0, x-3=0$

$\Rightarrow$ $x=-3,-2,3$

$\therefore$ The zeros of $x^3+2x^2-9x-18$ are $-3, -2, 3$.

**Example – 26**

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

**Sol.** By division algorithm

$x^4 - 6x^3 + 16x^2 - 25x + 10 = (x^2 - 2x + k)\, q(x) + (x + a)$

where q (x) is the quotient.

As the degree on L.H.S. is 4; therefore, q (x) must be of degree 2.

Let $q(x) = lx^2 + mx + n,\ l \neq 0$.

Then $x^4 - 6x^3 + 16x^2 - 25x + 10 = (x^2 - 2x + k)(lx^2 + mx + n) + x + a$

$\Rightarrow\ x^4 - 6x^3 + 16x^2 - 25x + 10 = lx^4 + (m - 2l)x^3 + (n - 2m + kl)x^2 + (mk - 2n + 1)x + nk + a$

Equating coefficients of like powers of x on the two sides,

we obtain

$l = 1$ ... (1)

$m - 2l = -6$ ... (2)

$n - 2m + kl = 16$ ... (3)

$mk - 2n + l = -25$ ... (4)

and $nk + a = 10$ ... (5)

From (2), $m = -6 + 2l = -6 + 2 \times 1 = -4$ and

then from (3), $n = 16 + 2m - kl = 16 + 2 \times (-4) - k \times 1$

$\Rightarrow\ n = 8 - k$ ... (6)

From (4) and (6), we get

$(-4)k - 2(8 - k) + 1 = -25$

$\Rightarrow\ -4k - 16 + 2k + 1 = -25$

$\Rightarrow\ -2k = -25 + 16 - 1$

$\Rightarrow\ -2k = -10 \Rightarrow k = 5$

Substituting this value of k in (6), we have

$n = 8 - 5 = 3$ and then from (5),

we get

$a = 10 - nk = 10 - 3 \times 5 = -5$.

**Example – 27**

If $x^2 - ax + b = 0$ and $x^2 - px + q = 0$ have a root in common and the second equation has equal roots.

show that $b + q = \dfrac{ap}{2}$.

**Sol.** Given equations are $x^2 - ax + b = 0$ ...(i)

and $x^2 - px + q = 0$ ...(ii)

Let α be the common root. Then roots of Eq. (ii) will be α and α. Let β be the other root of Eq. (i). Thus roots of Eq. (i) are α, β and those of Eq. (ii). are α, α

$\alpha + \beta = a$ ...(iii)

$\alpha\beta = b$ ...(iv)

$2\alpha = p$ ...(v)

$\alpha^2 = q$ ...(vi)

$LHS = b + q = \alpha\beta + \alpha^2 = \alpha(\alpha + \beta)$ ...(vii)

and $RHS = \dfrac{ap}{2} = \dfrac{(\alpha + \beta)2\alpha}{2} = \alpha(\alpha + \beta)$ ...(viii)

From Eqs. (vii) and (viii), LHS = RHS

**Example – 28**

If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0\ (a \neq b)$ have a common root, then the numerical value of a + b is ... .

**Ans.** (–1)

**Sol.** Given equation are

$x^2 + ax + b = 0$ *and* $x^2 + bx + a = 0$ have common root on subtracting above equations, we get

$(a - b)x + (b - a) = 0$

$\Rightarrow x = 1$

$\therefore x = 1$ is the common root.

$\Rightarrow 1 + a + b = 0$

$\Rightarrow a + b = -1$

Correct Answer (-1)

**Example – 29**

Form an equation whose roots are cubes of the roots of equation $ax^3 + bx^2 + cx + d = 0$

**Sol.** Replacing x by $x^{1/3}$ in the given equation, we get

$a(x^{1/3})^3 + b(x^{1/3})^2 + c(x^{1/3}) + d = 0$

$\Rightarrow\ ax + d = -(bx^{2/3} + cx^{1/3})$ ....... (i)

$\Rightarrow\ (ax + d)^3 = -(bx^{2/3} + cx^{1/3})^3$

$\Rightarrow\ a^3x^3 + 3a^2dx^2 + 3ad^2x + d^3$

$= -[b^3x^2 + c^3x + 3bcx(bx^{2/3} + cx^{1/3})]$

$\Rightarrow\ a^3x^3 + 3a^2dx^2 + 3ad^2x + d^3 =$

$[-b^3x^2 - c^3x + 3bcx(ax + d)]$ [From Eq. (i)]

$\Rightarrow\ a^3x^3 + x^2(3a^2d - 3abc + b^3)$

$+ x(3ad^2 - 3bcd + c^3) + d^3 = 0$

This is the requied equation.

## Example – 30

Find the values of a for which the inequality $(x - 3a)(x - a - 3) < 0$ is satisfied for all x such that $1 \le x \le 3$.

**Sol.** Let $f(x) = (x - 3a)(x - a - 3)$

for given equality to be true for all values of $x \in [1, 3]$, 1 and 3 should lie between the roots of $f(x) = 0$.

$\Rightarrow$ $f(1) < 0$ and $f(3) < 0$

Consider $f(1) < 0$ :

$\Rightarrow$ $(1 - 3a)(1 - a - 3) < 0$

$\Rightarrow$ $(3a - 1)(a + 2) < 0$

$\Rightarrow$ $a \in (-2, 1/3)$ ...(i)

Consider $f(3) < 0$ :

$\Rightarrow$ $(3 - 3a)(3 - a - 3) < 0$

$\Rightarrow$ $(a - 1)(a) < 0$

$\Rightarrow$ $a \in (0, 1)$ ...(ii)

Combining (i) and (ii), we get :

$a \in (0, 1/3)$

## Example – 31

Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots $\alpha$ and $\beta$, where $\alpha < -1$ and $\beta > 1$, then show that

$$1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$$

**Sol.** $ax^2 + bx + c = 0$

Roots : $\alpha$ and $\beta$

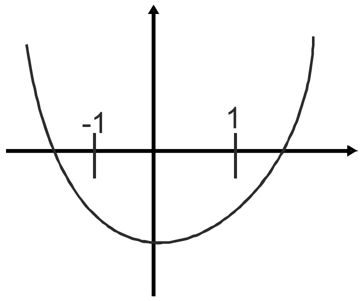
$\alpha < -1$ and $\beta > 1$

$ax^2 + bx + c = 0$

Let $f(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

upward parabola




Observe

$f(-1) < 0$ and $f(1) < 0$

Now

$$f(-1) = 1 - \frac{b}{a} + \frac{c}{a} < 0 \quad ...(1)$$

$$f(1) = 1 + \frac{b}{a} + \frac{c}{a} < 0 \quad ...(2)$$

from (1) and (2)

$$1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0$$

## Example – 32

Let $-1 \le P < 1$. Show that the equation $4x^3 - 3x - P = 0$ has a unique root in the interval [1/2, 1] and identify it.

**Sol.** Given that $-1 \le P \le 1$

Let $f(x) = 4x^3 - 3x - P = 0$

Now

$$f\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{3}{2} - P = -1 - P \le 0 \,(\because P \ge -1)$$

Also $f(1) = 4 - 3 - P = 1 - P \ge 0 \,(\because P \le 1)$

f (x) has at lest one real root between $\left[\frac{1}{2}, 1\right]$

Also, $f'(x) = 12x^2 - 3 > 0 \; on \left[\frac{1}{2}, 1\right]$

$\Rightarrow f(x)$ *is* increasing on $\left[\frac{1}{2}, 1\right]$

f has only are real root between $\left[\frac{1}{2}, 1\right]$

To find a root, we observe f(x) contains $4x^3 - 3x$ which is multiple angle formula for $\cos 3\theta$.

$\therefore$ We put $x = \cos\theta$

$\Rightarrow$ $4\cos^3\theta - 3\cos\theta = P$

$\Rightarrow$ $P = \cos 3\theta$

$\Rightarrow$ $\theta = \frac{1}{3}\cos^{-1}(P)$

$\therefore$ Root is $\cos\left(\frac{1}{3}\cos^{-1}(P)\right)$

**Example – 33**

Solve the inequality, $\frac{3x^2 - 7x + 8}{x^2 + 1} \le 2$

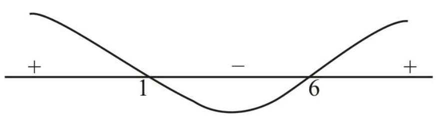
**Sol.** Domain : $x \in R$

Given inequality is equivalent to

$$\frac{3x^2 - 7x + 8}{x^2 + 1} - 2 \le 0$$

$$\Rightarrow \quad \frac{3x^2 - 7x + 8 - 2x^2 - 2}{x^2 + 1} \le 0$$

$$\Rightarrow \quad \frac{x^2 - 7x + 6}{x^2 + 1} \le 0 \quad \Rightarrow \frac{(x-1)(x-6)}{x^2 + 1} \le 0$$




$\Rightarrow$ $x \in [1, 6]$

**Example – 34**

Find the integral solutions of the following systems of inequalities

(a) $5x - 1 < (x+1)^2 < 7x - 3$

(b) $\frac{x}{2x+1} > \frac{1}{4}, \frac{6x}{4x-1} < \frac{1}{2}$

**Sol.** (a) $5x - 1 < (x+1)^2 < 7x - 3$

$5x - 1 < (x+1)^2$ and $(x+1)^2 < 7x - 3$

$\Rightarrow$ $5x - 1 < x^2 + 1 + 2x$ and $x^2 + 1 + 2x < 7x - 3$

$\Rightarrow$ $x^2 - 3x + 2 > 0$ and $x^2 - 5x + 4 < 0$

$(x-2)(x-1) > 0$

$(x-1)(x-4) < 0$

$x < 1, x > 2$ ...(i)

and $x \in (1, 4)$ ...(ii)

from (i) and (ii) $x \in (2, 4)$.

$\Rightarrow x = 3$ is the only integral solution.

(b) $\frac{x}{2x+1} > \frac{1}{4}$ *and* $\frac{6x}{4x-1} < \frac{1}{2}$

$$\frac{x}{2x+1} - \frac{1}{4} > 0 \text{ and } \frac{6x}{4x-1} - \frac{1}{2} < 0$$

$$\frac{4x - 2x - 1}{4(2x+1)} > 0 \text{ and } \frac{12x - 4x + 1}{2(4x-1)} < 0$$

$$\frac{2x-1}{2x+1} > 0 \text{ and } \frac{8x+1}{4x-1} < 0$$

$$x < -\frac{1}{2} \text{ OR } x > \frac{1}{2} \text{ and } -\frac{1}{8} < x < \frac{1}{4}$$

No common integer.

hence $x = \phi$.

**Example – 35**

For what values of m, does the system of equations

$3x + my = m$

and $2x - 5y = 20$

has solution satisfying the conditions x > 0, y > 0 ?

**Sol.** $3x + my = m$ ....(1)

$2x - 5y = 20$ ... (2)

$$\begin{array}{l} 2\left(3x + my = m\right) \\ 3\left(2x - 5y = 20\right) \\ \hline 2my + 15y = 2m + 60 \\ y\left(2m + 15\right) = 2\left(m - 30\right) \end{array}$$

$$y = \frac{2\left(m-30\right)}{2m+15} \quad as \ y > 0$$

$$\frac{2\left(m-30\right)}{2m+15} > 0$$


$$m \in \left(-\infty, -\frac{15}{2}\right) \cup \left(30, \infty\right)$$

# EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

## Polynomials

**1.** The product of the real roots of the equation $|2x+3|^2-3\,|2x+3|+2=0$, is

(a) 5/4 (b) 5/2

(c) 5 (d) 2

**2.** The roots of the equation $|x^2-x-6|=x+2$ are

(a) –2, 1, 4 (b) 0, 2, 4

(c) 0, 1, 4 (d) –2, 2, 4

**3.** If $(1-p)$ is a root of quadratic equation $x^2+px+(1-p)=0$, then its roots are

(a) 0, –1 (b) –1, 1

(c) 0, 1 (d) –1, 2

**4.** Product of real roots of the equation

$$x^2+|x|+9=0$$

(a) is always positive (b) is always negative

(c) does not exist (d) none of the above

**5.** The integral value of x satisfing $|x^2+4x+3|+2x+5=0$ is

(a) –4 (b) –3

(c) –2 (d) –1

## Nature of roots

**6.** If a, b $\in$ R & the quadratic equation $ax^2-bx+1=0$ has imaginary roots then $a+b+1$ is

(a) positive

(b) negative

(c) zero

(d) depends on the sign of b

**7.** The roots of the quadratic equation $7x^2-9x+2=0$ are

(a) Rational and different (b) Rational and equal

(c) Irrational and different (d) Imaginary and different

**8.** The roots of the equation $x^2-2\sqrt{2}\,x+1=0$ are

(a) Real and different (b) Imaginary and different

(c) Real and equal (d) Rational and different

**9.** If $l$, m, n are real, $l \neq m$, then the roots of the equation $(l-m)\,x^2-5\,(l+m)\,x-2\,(l-m)=0$ are

(a) real and equal (b) Non real

(c) real and unequal (d) none of these

**10.** If a,b,c are distinct real numbers then the equation $(b-c)\,x^2+(c-a)\,x+(a-b)=0$ has

(a) equal roots (b) irrational roots

(c) rational roots (d) none of these

**11.** If a,b,c are distinct rational numbers then roots of equation $(b+c-2a)\,x^2+(c+a-2b)x+(a+b-2c)=0$ are

(a) rational (b) irrational

(c) non-real (d) equal

**12.** If a,b,c are distinct rational numbers and $a+b+c=0$, then the roots of the equation

$(b+c-a)\,x^2+(c+a-b)\,x+(a+b-c)=0$ are

(a) imaginary (b) real and equal

(c) real and unequal (d) none of these

## Relations between roots and coefficient

**13.** If p, q are the roots of the equation $x^2+px+q=0$ where both p and q are non-zero, then (p, q) =

(a) (1, 2) (b) (1, –2)

(c) (–1, 2) (d) (–1, –2)

**14.** If the equation $(k-2)\,x^2-(k-4)\,x-2=0$ has difference of roots as 3 then the value of k is

(a) 1, 3 (b) 3, 3/2

(c) 2, 3/2 (d) 3/2, 1

**15.** The roots of the equation $x^2+px+q=0$ are tan 22° and tan 23° then

(a) $p+q=1$ (b) $p+q=-1$

(c) $p-q=1$ (d) $p-q=-1$

**16.** If $\alpha$, $\beta$ are the roots of the equation $x^2-p\,(x+1)-c=0$, then $(\alpha+1)\,(\beta+1)=$

(a) c (b) c–1

(c) 1–c (d) none of these

**17.** If the difference between the roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then

(a) $a+b+4=0$ (b) $a+b-4=0$

(c) $a-b-4=0$ (d) $a-b+4=0$

18. If roots of the equation $x^2 + ax + 25 = 0$ are in the ratio of 2 : 3 then the value of a is

(a) $\frac{\pm 5}{\sqrt{6}}$ (b) $\frac{\pm 25}{\sqrt{6}}$

(c) $\frac{\pm 5}{6}$ (d) none of these

19. If $\alpha, \beta$ are roots of $Ax^2 + Bx + C = 0$ and $\alpha^2, \beta^2$ are roots of $x^2 + px + q = 0$, then p is equal to

(a) $(B^2 - 2AC)/A^2$ (b) $(2AC - B^2)/A^2$

(c) $(B^2 - 4AC)/A^2$ (d) $(4AC - B^2)A^2$

20. If $\alpha, \beta$ are roots of the equation

$ax^2 + 3x + 2 = 0\ (a < 0)$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is greater than

(a) 0 (b) 1

(c) 2 (d) none of these

21. In a quadratic equation with leading coefficient 1, a student reads the coefficient 16 of x wrongly as 19 and obtain the roots as –15 and –4. The correct roots are

(a) 6, 10 (b) –6, –10

(c) –7, –9 (d) none of these

22. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$

(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

23. If the roots of the quadratic equations $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then the value of $2 + q - p$ is

(a) 2 (b) 3

(c) 0 (d) 1

24. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(a) $(3, \infty)$ (b) $(-\infty, -3)$

(c) $(-3, 3)$ (d) $(-3, \infty)$

25. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are

(a) –4, –3 (b) 6, 1

(c) 4, 3 (d) –6, –1

## Common roots

26. The value of a so that the equations

$(2a - 5)x^2 - 4x - 15 = 0$ and

$(3a - 8)x^2 - 5x - 21 = 0$ have a common root, is

(a) 4, 8 (b) 3, 6

(c) 1, 2 (d) None

27. If $a, b, c \in R$, the equation $ax^2 + bx + c = 0\ (a, c \neq 0)$ and $x^2 + 2x + 3 = 0$ have a common root, then a : b : c =

(a) 1 : 2 : 3 (b) 1 : 3 : 4

(c) 2 : 4 : 5 (d) None

28. If equations ax2 + bx + c = 0, ($a, b \in R$, $a \neq 0$) and $2x^2 + 3x + 4 = 0$ have a common root then a : b : c equals:

(a) 1 : 2 : 3 (b) 2 : 3 : 4

(c) 4 : 3 : 2 (d) 3 : 2 : 1

## Location of roots

29. The value of k for which the equation

$3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0$

has roots of opposite signs, lies in the interval

(a) $(-\infty, 0)$ (b) $(-\infty, -1)$

(c) $(1, 2)$ (d) $(3/2, 2)$

30. If the roots of $x^2 + x + a = 0$ exceed a, then

(a) $2 < a < 3$ (b) $a > 3$

(c) $-3 < a < 3$ (d) $a < -2$

31. The range of values of m for which the equation $(m - 5)x^2 + 2(m - 10)x + m + 10 = 0$ has real roots of the same sign, is given by

(a) $m > 10$ (b) $-5 < m < 5$

(c) $m < -10, 5 < m \leq 6$ (d) None of these

**32.** If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(a) $(6, \infty)$ (b) $(5, 6]$

(c) $[4, 5]$ (d) $(-\infty, 4)$

**33.** All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than $-2$ but less than 4, lie in the interval

(a) $-2 < m < 0$ (b) $m > 3$

(c) $-1 < m < 3$ (d) $1 < m < 4$

**34.** If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval:

(a) $(-\infty, -2) \cup (2, \infty)$ (b) $(-1, 0) \cup (0, 1)$

(c) $(1, 2)$ (d) $(-2, -1)$

## Numerical Value Type Questions

**35.** The sum of all real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$, is

**36.** The equation $x^2 - 3|x| + 2 = 0$ has how many real roots

**37.** The sum of the real roots of the equation $x^2 + |x| - 6 = 0$ is

**38.** The number of real solution of the equations $x^2 - 3|x| + 2 = 0$ is

**39.** The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is

**40.** The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has how many solutions.

**41.** The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has how many real roots

**42.** If $\alpha$ and $\beta$ are the roots of $4x^2 + 3x + 7 = 0$, then the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is

**43.** If $\alpha$ and $\beta$ are the roots of $x^2 - P(x+1) - C = 0$, then the value of $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + C} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + C}$ is

**44.** If the roots of the equations $x^2 + 3x + 2 = 0$ & $x^2 - x + \lambda = 0$ are in the same ratio then the value of $\lambda$ is given by

**45.** If $\alpha, \beta, \gamma$ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

**46.** The value of m for which the equation $x^3 - mx^2 + 3x - 2 = 0$ has two roots equal in magnitude but opposite in sign, is

**47.** The real value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assumes the least value, is

**48.** The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is

**49.** Let $\alpha$ and $\beta$ be the roots of equation $px^2 + qx + r = 0, p \ne 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

**50.** If $\alpha$ and $\beta$ are roots of the equation, $x^2 - 4\sqrt{2}\,kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$ then $\alpha^3 + \beta^3$ is equal to:

# EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

**1.** Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to: **(2015)**

(a) 3 (b) –3

(c) 6 (d) –6

**2.** The sum of all real values of x satisfying the equation $\left(x^2 - 5x + 5\right)^{x^2 + 4x - 60} = 1$ is : **(2016)**

(a) –4 (b) 6

(c) 5 (d) 3

**3.** If $b \in C$ and the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from –1, then |b| is equal to : **(2016/Online Set–1)**

(a) $\sqrt{2}$ (b) 2

(c) 3 (d) $\sqrt{3}$

**4.** If x is a solution of the equation,

$\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to : **(2016/Online Set–2)**

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) 2 (d) $2\sqrt{2}$

**5.** If, for a positive integer n, the quadratic equation,

$x(x+1) + (x+1)(x+2) + \ldots + (x+\overline{n-1})(x+n) = 10n$

has two consecutive integral solutions, then n is equal to: **(2017)**

(a) 12 (b) 9

(c) 10 (d) 11

**6.** Let p(x) be a quadratic polynomial such that p(0) = 1. If p(x) leaves remainder 4 when divided by x – 1 and it leaves remainder 6 when divided by x + 1; then : **(2017/Online Set–1)**

(a) p(2) = 11 (b) p(2) = 19

(c) p(–2) = 19 (d) p (–2) = 11

**7.** The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is : **(2017/Online Set–2)**

(a) 16 (b) 14

(c) –4 (d) –5

**8.** If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is : **(2018/Online Set–1)**

(a) $4\sqrt{2}$ (b) $2\sqrt{5}$

(c) $2\sqrt{7}$ (d) 20

**9.** If f(x) is a quadratic expression such that f(1) + f(2) = 0, and -1 is a root of f(x) = 0, then the other root of f(x) = 0 is : **(2018/Online Set–2)**

(a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$

(c) $\frac{5}{8}$ (d) $\frac{8}{5}$

**10.** Let p, q and r be real numbers $(p \neq q, r \neq 0)$, such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to : **(2018/Online Set–3)**

(a) $\frac{p^2 + q^2}{2}$ (b) $p^2 + q^2$

(c) $2(p^2 + q^2)$ (d) $p^2 + q^2 + r^2$

**11.** The number of integral values of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is: **(8-4-2019/Shift -2)**

(a) 1 (b) 2

(c) infinitely many (d) 3

**12.** Let $p, q \in R$. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then: **(9-4-2019/Shift -1)**

(a) $p^2 - 4q + 12 = 0$ (b) $q^2 - 4p - 16 = 0$

(c) $q^2 + 4p + 14 = 0$ (d) $p^2 - 4q - 12 = 0$

**13.** If m is chosen in the quadratic equation $\left(m^2+1\right)x^2-3x+\left(m^2+1\right)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: **(9-4-2019/Shift -2)**

(a) $10\sqrt{5}$ (b) $8\sqrt{3}$

(c) $8\sqrt{5}$ (d) $4\sqrt{3}$

**14.** If α and β are the roots of the quadratic equation, $x^2+x\sin\theta-2\sin\theta=0,\ \theta\in\left(0,\frac{\pi}{2}\right)$,

then $\dfrac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$ is equal to

**(10-4-2019/Shift -1)**

(a) $\dfrac{2^{12}}{(\sin\theta-4)^{12}}$ (b) $\dfrac{2^{12}}{(\sin\theta+8)^{12}}$

(c) $\dfrac{2^{12}}{(\sin\theta-8)^{6}}$ (d) $\dfrac{2^{6}}{(\sin\theta+8)^{12}}$

**15.** The number of real roots of the equation $5+\left|2^x-1\right|=2^x\left(2^x-2\right)$ is: **(10-4-2019/Shift -2)**

(a) 3 (b) 2

(c) 4 (d) 1

**16.** If α and β are the roots of the equation $375x^2-25x-2=0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r+\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to **(12-4-2019/Shift -1)**

(a) $\dfrac{21}{346}$ (b) $\dfrac{29}{358}$

(c) $\dfrac{1}{12}$ (d) $\dfrac{7}{116}$

**17.** If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $ax^2+2\beta x+\gamma=0$ and $x^2+x-1=0$ have a common root, then $\alpha(\beta+\gamma)$ is equal to ______. **(12-4-2019/Shift -2)**

(a) 0 (b) αβ

(c) αγ (d) βγ

**18.** Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to: **(9-1-2019/Shift -1)**

**19.** If both the roots of the quadratic equation $x^2-mx+4=0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval: **(9-1-2019/Shift -2)**

(a) (–5, –4) (b) (4, 5)

(c) (5, 6) (d) (3, 4)

**20.** The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+a=0$ are rational numbers is:

**(9-1-2019/Shift -2)**

(a) 3 (b) 4

(c) 2 (d) 5

**21.** Consider the quadratic equation

$(c-5)x^2-2cx+(c-4)=0, c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is:

**(10-1-2019/Shift -1)**

**22.** The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is: **(10-1-2019/Shift -2)**

(a) $\dfrac{15}{8}$ (b) 1

(c) $\dfrac{4}{9}$ (d) 2

**23.** If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is: **(11-1-2019/Shift -1)**

(a) -81 (b) 100

(c) 144 (d) -300

**24.** If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\dfrac{1}{\lambda}=1$, is **(12-1-2019/Shift -1)**

(a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$

(c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$

**25.** The number of integral values of m for which the quadratic expression, $(1+2m)x^2-2(1+3m)x+4(1+m)$, $x \in R$, is always positive, is : **(12-1-2019/Shift -2)**

(a) 3 (b) 8

(c) 7 (d) 6

**26.** Let α and β be the roots of the equation, $5x^2+6x-2=0$.

If $S_n=\alpha^n+\beta^n$, n = 1, 2, 3, ........., then : **(2-9-2020/Shift -1)**

(a) $5S_6+6S_5+2S_4=0$ (b) $6S_6+5S_5=2S_4$

(c) $6S_6+5S_5+2S_4=0$ (d) $5S_6+6S_5=2S_4$

**27.** Let f (x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f (x) = 0 is 3, then its other roots lies in : **(2-9-2020/Shift -2)**

(a) (0, 1) (b) (1, 3)

(c) (–1, 0) (d) (–3, –1)

**28.** Consider the two sets : A = $\{m \in R:$ both the roots of $x^2-(m+1)x+m+4=0$ are real$\}$ and B = [–3, 5). Which of the following is not true ? **(3-9-2020/Shift -1)**

(a) $A-B=(-\infty,-3)\cup(5,\infty)$

(b) $A\cap B=\{-3\}$

(c) $B-A=(-3,5)$

(d) $A\cup B=R$

**29.** If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then

$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to : **(3-9-2020/Shift -1)**

(a) $\frac{9}{4}(9+p^2)$ (b) $\frac{9}{4}(9+q^2)$

(c) $\frac{9}{4}(9-p^2)$ (d) $\frac{9}{4}(9-q^2)$

**30.** The set of all real values of λ for which the quadratic equations, $(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0, 1) is : **(3-9-2020/Shift -2)**

(a) (–3, –1) (b) (2, 4]

(c) (1, 3] (d) (0, 2)

**31.** Let α and β be the roots of $x^2-3x+p=0$ and λ and δ be the roots of $x^2-6x+q=0$. If α,β,λ,δ form a geometric progression. Then ratio of (2q + p) : (2q - p) is ; **(4-9-2020/Shift -1)**

(a) 33 : 31 (b) 9 : 7

(c) 3 : 1 (d) 5 : 3

**32.** Let $\lambda \neq 0$ be in R. If α are β the roots of the equation, $x^2-x+2\lambda=0$ and α and γ are the roots of the equation $3x^2-10x+27\lambda=0$, then $\frac{\beta\gamma}{\lambda}$, is equal to: **(4-9-2020/Shift -2)**

(a) 29 (b) 9

(c) 18 (d) 36

**33.** The product of the roots of the equation $9x^2-18|x|+5=0$ is : **(5-9-2020/Shift -1)**

(a) $\frac{25}{81}$ (b) $\frac{5}{9}$

(c) $\frac{5}{27}$ (d) $\frac{25}{9}$

**34.** If α and β are the roots of the equation, $7x^2-3x-2=0$, then the value of $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$ is equal to: **(5-9-2020/Shift -2)**

(a) $\frac{27}{32}$ (b) $\frac{1}{24}$

(c) $\frac{27}{16}$ (d) $\frac{3}{8}$

**35.** If $\alpha$ be $\beta$ two roots of the equation $x^2-64x+256=0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8}+\left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

**(6-9-2020/Shift -1)**

(a) 1 (b) 3

(c) 2 (d) 4

**36.** If $\alpha$ and $\beta$ are the roots of the equation $2x(2x+1)=1$, then $\beta$ is equal to : **(6-9-2020/Shift -2)**

(a) $2\alpha(\alpha-1)$ (b) $-2\alpha(\alpha+1)$

(c) $2\alpha^2$ (d) $2\alpha(\alpha+1)$

**37.** Let $\alpha$ and $\beta$ are two real roots of the equation $(k+1)\tan^2 x-\sqrt{2}\lambda\tan x=1-k$, where $(k\neq 1)$ and $\lambda$ are real numbers. If $\tan^2(\alpha+\beta)=50$, then value of $\lambda$ is

**(7-1-2020/Shift -1)**

(a) $5\sqrt{2}$ (b) $10\sqrt{2}$

(c) 10 (d) 5

**38.** :Let $\alpha$ and $\beta$ are the roots of the equation $x^2-x-1=0$. If $P_k=(\alpha)^k+(\beta)^k, k\geq 1$ then which one of the following statements is not true? **(7-1-2020/Shift -2)**

(a) $(P_1+P_2+P_3+P_4+P_5)=26$

(b) $P_5=11$

(c) $P_5=P_2\cdot P_3$

(d) $P_3=P_5-P_4$

**39.** The least positive value of 'a' for which the equation, $2x^2+(a-10)x+\frac{33}{2}=2a, a\in Z^+$ has real roots is ______. **(8-1-2020/Shift -1)**

**40.** Let S be the set of all real roots of the equation, $3^x(3^x-1)+2=|3^x-1|+|3^x-2|$. Then S:

**(8-1-2020/Shift -2)**

(a) is a singleton

(b) is an empty set

(c) contains at least four elements

(d) contains exactly two elements

**41.** The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is **(9-1-2020/Shift -1)**

(a) 3 (b) 4

(c) 1 (d) 2

**42.** Let $a,b\in R, a\neq 0$, such that the equation, $ax^2-2bx+5=0$ has a repeated root $\alpha$, which is also a root of the equation $x^2-2bx-10=0$. If $\beta$ is the root of this equation, then $\alpha^2+\beta^2$ is equal to:

**(9-1-2020/Shift -2)**

(a) 24 (b) 25

(c) 26 (d) 28

**43.** If $\alpha$ and $\beta$ are the distinct roots of the equation $x^2+(3)^{\frac{1}{4}}x+3^{\frac{1}{2}}=0$, then the value of $\alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)$ is equal to:

**(20-07-2021/Shift-1)**

(a) $56\times 3^{25}$ (b) $52\times 3^{24}$

(c) $56\times 3^{24}$ (d) $28\times 3^{25}$

**44.** If $\alpha,\beta$ are roots of the equation $x^2+5\sqrt{2}x+10=0$, $\alpha>\beta$ and $P_n=\alpha^n-\beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to ________ ?

**(25-07-2021/Shift-1)**

**45.** Let $\alpha,\beta$ be two roots of the equation $x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}}=0$. Then $\alpha^8+\beta^8$ is equal to:

**(27-07-2021/Shift-1)**

(a) 10 (b) 50

(c) 160 (d) 100

**46.** The number of real solutions of the equation, $x^2-|x|-12=0$ is : **(25-07-2021/Shift-2)**

(a) 3 (b) 1

(c) 2 (d) 4

**47.** The number of pairs $(a,b)$ of real numbers, such that whenever $\alpha$ is a root of the equation $x^2+ax+b=0$, $\alpha^2-2$ is also a root of this equation is

**(01-09-2021/Shift-2)**

(a) 6 (b) 4

(c) 8 (d) 2

**48.** Let $\lambda \neq 0$ be in R. If $\alpha$ and $\beta$ are the roots of the equation $x^2 - x + 2\lambda = 0$ and $\alpha$ and $\gamma$ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to ______.

**(26-08-2021/Shift-2)**

**49.** The sum of all integral values of $k\,(k \neq 0)$ for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____.

**(26-08-2021/Shift-1)**

**50.** The set of all value of $k > -1$, for which the equation $(3x^2+4x+3)^2 - (k+1)(3x^2+4x+3)(3x^2+4x+2)$ $+k(3x^2+4x+2)^2 = 0$ has real roots, is:

**(27-08-2021/Shift-2)**

(a) $\left[-\frac{1}{2}, 1\right)$ (b) $[2, 3)$

(c) $\left(1, \frac{5}{2}\right]$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

**51.** The sum of the roots of the equation

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$ is

**(31-08-2021/Shift-2)**

(a) $\log_2 12$ (b) $\log_2 14$

(c) $\log_2 11$ (d) $\log_2 13$

**52.** Let $\alpha$ and $\beta$ be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \geq 1$. Then, the value of $p_n^2$ is ______.

**(26-02-2021/Shift-2)**

**53.** The number of solutions of the equation

$\log_4(x-1) = \log_2(x-3)$ is **(26-02-2021/Shift-1)**

**54.** The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

**(25-02-2021/Shift-1)**

(a) 4 (b) 2

(c) 3 (d) 0

**55.** Let $\alpha$ and $\beta$ be the roots $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is :

**(25-02-2021/Shift-2)**

(a) 2 (b) 4

(c) 1 (d) 3

**56.** Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 27.2$. Then p and q are roots of the equation

**(24-02-2021/Shift-1)**

(a) $x^2 - 2x + 136 = 0$ (b) $x^2 - 2x + 8 = 0$

(c) $x^2 - 2x + 16 = 0$ (d) $x^2 - 2x + 2 = 0$

**57.** The number of roots of the equation,

$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$

In the interval $[0, \pi]$ is equal to **(16-03-2021/Shift-1)**

(a) 8 (b) 3

(c) 2 (d) 4

**58.** The value $4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{4 + \ldots\ldots\ldots \infty}}}}$ is :

**(17-03-2021/Shift-1)**

(a) $2 + \frac{4}{\sqrt{5}}\sqrt{30}$ (b) $5 + \frac{2}{5}\sqrt{30}$

(c) $4 + \frac{4}{\sqrt{5}}\sqrt{30}$ (d) $2 + \frac{2}{5}\sqrt{30}$

**59.** The number of the real roots of the equation

$y = \frac{10 + 2\sqrt{30}}{5}$ is **(24-02-2021/Shift-2)**

**60.** The number of solutions of the equation

$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0,$

is __________ ? **(20-07-2021/Shift-2)**

# EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

## Objective Questions I [Only one correct option]

**1.** If $a(p+q)^2 + 2bpq + c = 0$ and $a(p+r)^2 + 2bpr + c = 0$, $(a \neq 0)$ then

(a) $qr = p^2 + \frac{c}{a}$  (b) $qr = p^2$

(c) $qr = -p^2$  (d) None of these

**2.** If $a, b \in R$, $a \neq b$. The roots of the quadratic equation,

$x^2 - 2(a+b)x + 2(a^2+b^2) = 0$ are

(a) Rational and different  (b) Rational and equal

(c) Irrational and different  (d) Imaginary and different

**3.** If $0 \leq x \leq \pi$, then the solution of the equation

$16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(a) $\frac{\pi}{6}, \frac{\pi}{3}$  (b) $\frac{\pi}{3}, \frac{\pi}{2}$

(c) $\frac{\pi}{6}, \frac{\pi}{2}$  (d) none of these

**4.** The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is

(a) 0, 2  (b) 0, –2

(c) 2, –2  (d) none of these

**5.** If $\alpha, \beta$ are roots of the equation $ax^2 + 3x + 2 = 0$ $(a < 0)$, then $\alpha^2/\beta + \beta^2/\alpha$ is greater than

(a) 0  (b) 1

(c) 2  (d) none of these

**6.** Two real numbers $\alpha$ and $\beta$ are such that $\alpha + \beta = 3$ and $|\alpha - \beta| = 4$, then $\alpha$ and $\beta$ are the roots of the quadratic equation

(a) $4x^2 - 12x - 7 = 0$  (b) $4x^2 - 12x + 7 = 0$

(c) $4x^2 - 12x + 25 = 0$  (d) none of these

**7.** If $\alpha, \beta$ are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\,S_{n+1} + c\,S_{n-1} =$

(a) $b\,S_n$  (b) $b^2 S_n$

(c) $2bS_n$  (d) $-bS_n$

**8.** If a, b, p, q are non-zero real numbers, the two equations, $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have

(a) no common root

(b) one common root if $2a^2 + b^2 = p^2 + q^2$

(c) two common roots if $3pq = 2ab$

(d) two common roots if $3qb = 2ap$

**9.** If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then, the common factor is

(a) $(x-3)$  (b) $(x-6)$

(c) $(x-8)$  (d) none of these

**10.** If both roots of the quadratic equation $(2-x)(x+1) = p$ are distinct and positive then p must lie in the interval

(a) $p > 2$  (b) $2 < p < 9/4$

(c) $p < -2$  (d) $-\infty < p < \infty$

**11.** If $\alpha, \beta$ are the roots of the equation, $x^2 - 2mx + m^2 - 1 = 0$ then the range of values of m for which $\alpha, \beta \in (-2, 4)$ is

(a) $(-1, 3)$  (b) $(1, 3)$

(c) $(\infty, -1) \cup (3, \infty)$  (d) none

**12.** If $\alpha, \beta$ are the roots of the quadratic equation, $x^2 - 2p(x-4) - 15 = 0$ then the set of values of p for which one root is less than 1 & the other root is greater than 2 is

(a) $(7/3, \infty)$  (b) $(-\infty, 7/3)$

(c) $x \in R$  (d) none

**13.** $\frac{8x^2 + 16x - 51}{(2x-3)(x+4)} > 3$ if x is such that

(a) $x < -4$  (b) $-3 < x < 3/2$

(c) $x > 5/2$  (d) all these true

**14.** If both roots of the quadratic equation $x^2 + x + p = 0$ exceed p where $p \in R$ then p must lie in the interval

(a) $(-\infty, 1)$  (b) $(-\infty, -2)$

(c) $(-\infty, -2) \cup (0, 1/4)$  (d) $(-2, 1)$

**15.** If $a, b, c \in R$, $a > 0$ and $c \neq 0$ Let $\alpha$ and $\beta$ be the real and distinct roots of the equation $ax^2 + bx + c = |c|$ and p, q be the real and distinct roots of the equation $ax^2 + bx + c = 0$. Then

(a) p and q lie between $\alpha$ and $\beta$

(b) p and q do not lie between $\alpha$ and $\beta$

(c) Only p lies between $\alpha$ and $\beta$

(d) Only q lies between $\alpha$ and $\beta$

## Objective Questions II [One or more than one correct option]

**16.** $5^x + \left(2\sqrt{3}\right)^{2x} - 169 \le 0$ is true in the interval.

(a) $(-\infty, 2)$ (b) $(0, 2)$
(c) $(2, \infty)$ (d) $(0, 4)$

**17.** $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0, -1 < x < 0$, then the value of $\sin 2\alpha$ is

(a) 24/25 (b) –12/25
(c) –24/25 (d) 20/25

**18.** For $a > 0$, the roots of the equation

$log_{ax}\, a + log_x\, a^2 + log_{a^2x}\, a^3 = 0$, are given by

(a) $a^{-4/3}$ (b) $a^{-3/4}$
(c) $a^{-1/2}$ (d) $a^{-1}$

**19.** $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is $(a, b, c, d \in R)$

(a) $-d/a$ (b) $d/a$
(c) $(b - a)/a$ (d) $(a - b)/a$

**20.** If $a < b < c < d$, then for any positive $\lambda$, the quadratic equation $(x - a)(x - c) + \lambda(x - b)(x - d) = 0$ has

(a) non-real roots
(b) one real root between a and c
(c) one real root between b and d
(d) irrational roots

**21.** If $p, q, r, s, \in R$ and $\alpha, \beta$ are roots of the equation $x^2 + px + q = 0$ and $\alpha^4$ and $\beta^4$ are roots of $x^2 - rx + s = 0$, then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are

(a) both real (b) both positive
(c) both negative (d) none of these

**22.** If $a, b, c \in R$ and $\alpha$ is a real root of the equation $ax^2 + bx + c = 0$, and $\beta$ is the real root of the equation $-ax^2 + bx + c = 0$, then the equation $\frac{a}{2}x^2 + bx + c = 0$ has

(a) real roots
(b) none- real roots
(c) has a root lying between $\alpha$ and $\beta$
(d) None of these

**23.** If 'x' is real and satisfying the inequality, $|x| < \frac{a}{x}\ (a \in R)$, then

(a) $x \in \left(0, \sqrt{a}\right)$ for $a > 0$
(b) $x \in \left(-\sqrt{a}, 0\right)$ for $a < 0$
(c) $x \in \left(-\sqrt{-a}, 0\right)$ for $a < 0$
(d) $x \in \left(-\sqrt{a}, \sqrt{a}\right)$ for $a > 0$

**24.** The roots of the equation, $(x^2 + 1)^2 = x(3x^2 + 4x + 3)$, are given by

(a) $2 - \sqrt{3}$ (b) $\left(-1 + i\sqrt{3}\right)/2,\ i = \sqrt{-1}$
(c) $2 + \sqrt{3}$ (d) $\left(-1 - i\sqrt{3}\right)/2,\ i = \sqrt{-1}$

**25.** Equation $\frac{\pi^e}{x - e} + \frac{e^\pi}{x - \pi} + \frac{\pi^\pi + e^e}{x - \pi - e} = 0$ has

(a) one real root in $(e, \pi)$ and other in $(\pi - e, e)$
(b) one real root in $(e, \pi)$ and other in $(\pi, \pi + e)$
(c) two real roots in $(\pi - e, \pi + e)$
(d) No real root

**26.** If $0 < a < b < c$, and the roots $\alpha$, $\beta$ of the equation $ax^2 + bx + c = 0$ are non real complex roots, then

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$
(c) $|\beta| < 1$ (d) none of these

**27.** Let $a, b, c \in R$. If $ax^2 + bx + c = 0$ has two real roots A and B where $A < -1$ and $B > 1$, then

(a) $1 + \left|\frac{b}{a}\right| + \frac{c}{a} < 0$ (b) $1 - \left|\frac{b}{a}\right| + \frac{c}{a} < 0$
(c) $|c| < |a|$ (d) $|c| < |a| - |b|$

**28.** If $a < 0$, then root of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ is

(a) $a\left(-1 - \sqrt{6}\right)$ (b) $a\left(1 - \sqrt{2}\right)$
(c) $a\left(-1 + \sqrt{6}\right)$ (d) $a\left(1 + \sqrt{2}\right)$

## Numerical Value Type Questions

**29.** The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is

**30.** If the equation $(k-2)x^2 - (k-4)x - 2 = 0$ has difference of roots as 3 then the sum of all the values of k is :

**31.** If $p(x) = ax^2 + bx$ and $q(x) = lx^2 + mx + n$ with $p(1) = q(1)$; $p(2) - q(2) = 1$ and $p(3) - q(3) = 4$, then $p(4) - q(4)$ is

**32.** If $\alpha$, $\beta$ are the roots of $x^2 - p(x+1) - c = 0$ then $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$ is equal to

**33.** If $b < 0$, then the roots $x_1$ and $x_2$ of the equation $2x^2 + 6x + b = 0$, satisfy the condition $\left(\frac{x_1}{x_2}\right) + \left(\frac{x_2}{x_1}\right) < k$ where k is equal to

**34.** If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to

**35.** The maximum integral part of positive value of $a$ for which, the least value of $4x^2 - 4ax + a^2 - 2a + 2$ on $[0, 2]$ is 3, is

**36.** If $(x+1)^2$ is greater then $5x - 1$ and less than $7x - 3$ then the integral value of x is equal to

**37.** If $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \leq 4$, then the sum of the least and the highest values of $4x^2$ is

**38.** If roots $x_1$ and $x_2$ of $x^2 + 1 = \frac{x}{a}$ satisfy $|x_1^2 - x_2^2| > \frac{1}{a}$, then $a \in \left(-\frac{1}{\sqrt{k}}, 0\right) \cup \left(0, \frac{1}{\sqrt{k}}\right)$ the numerical quantity k must be equal to

**39.** If p & q are roots of the equation $x^2 - 2x + A = 0$ and r & s be roots of the equation $x^2 - 18x + B = 0$ if $p < q < r < s$ be in A.P., then $A + B$ is

**40.** If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation, $x^3 - Ax^2 + Bx - C = 0$ where A, B, C, P and Q are constants then the value of $A + B + C =$

**41.** If $\alpha, \beta, \gamma, \delta$ are the roots of the equation, $x^4 - Kx^3 + Kx^2 + Lx + M = 0$ where K, L and M are real numbers then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is

**42.** When $x^{100}$ is divided by $x^2 - 3x + 2$, the remainder is $(2^{k+1} - 1)x - 2(2^k - 1)$ where $k$ is a numerical quantity, then $k$ must be.

**43.** If the quadratic equations, $3x^2 + ax + 1 = 0$ and $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is

**44.** The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by $x_1$ and $x_2$ respectively, then $x_1 + x_2$ is

**45.** The value of a for which the equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root is

## Assertion & Reason

**(A)** **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

**(B)** **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

**(C)** **If ASSERTION is true, REASON is false.**

**(D)** **If ASSERTION is false, REASON is true.**

**46.** **Assertion:** If one roots is $\sqrt{5} - \sqrt{2}$ is then the equation of lowest degree with rational coefficient is $x^4 - 14x^2 + 9 = 0$.

**Reason:** For a polynomial equation with rational co-efficient irrational roots occurs in pairs.

(a) A (b) B

(c) C (d) D

**47.** **Assertion:** If $a > b > c$ and $a^3 + b^3 + c^3 = 3abc$, then the equation $ax^2 + bx + c = 0$ has one positive and one negative real roots.

**Reason:** If roots of opposite nature, then product of roots $< 0$ and $|\text{sums of roots}| \geq 0$.

(a) A (b) B

(c) C (d) D

## Match the Following

**Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.**

**48.**

| Column–I | Column–II |
|---|---|
| **(A)** Number of real solution of $\|x+1\| = e^x$ is | **(P)** 2 |
| **(B)** The number of non-negative real roots of $2^x - x - 1 = 0$ equal to | **(Q)** 3 |
| **(C)** If p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$, then minimum value of $p^2 + q^2$ is equal to | **(R)** 6 |
| **(D)** If $\alpha$ and $\beta$ are the roots of $2x^2 + 7x + c = 0$ & $\|\alpha^2 - \beta^2\| = \frac{7}{4}$, then c is equal to | **(S)** 5 |

**The correct matching is :**

(a) (A–P; B–Q; C–S; D–R)

(b) (A–Q; B–P; C–S; D–R)

(c) (A–S; B–P; C–Q; D–R)

(d) (A–R; B–S; C–P; D–Q)

**49.** The value of k for which the equation $x^3 - 3x + k = 0$ has

| Column–I | Column–II |
|---|---|
| **(A)** three distinct real roots | **(P)** $\|k\| > 2$ |
| **(B)** two equal roots | **(Q)** $k = -2, 2$ |
| **(C)** exactly one real root | **(R)** $\|k\| < 2$ |
| **(D)** three equal roots | **(S)** no value of k |

**The correct matching is :**

(a) (A–R; B–Q; C–P; D–S)

(b) (A–Q; B–R; C–P; D–S)

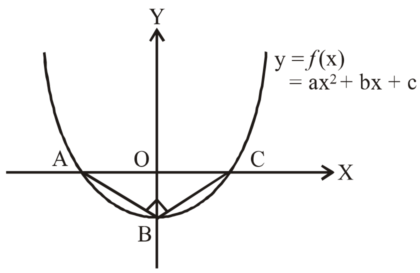
(c) (A–R; B–Q; C–S; D–P)

(d) (A–S; B–P; C–Q; D–R)

## Using the following passage, solve Q.50 to Q.52

### Passage – 1

In the given figure vertices of $\Delta$ ABC lie on $y = f(x) = ax^2 + bx + c$. The $\Delta$ ABC is right angled isosceles triangle whose hypotenuse AC = $4\sqrt{2}$ units, then




**50.** $y = f(x)$ is given by

(a) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ (b) $y = \frac{x^2}{2} - 2$

(c) $y = x^2 - 8$ (d) $y = x^2 - 2\sqrt{2}$

**51.** Minimum value of $y = f(x)$ is

(a) $2\sqrt{2}$ (b) $-2\sqrt{2}$

(c) 2 (d) –2

**52.** Number of integral value of k for which $\frac{k}{2}$ lies between the roots of $f(x) = 0$, is

(a) 9 (b) 10

(c) 11 (d) 12

## Using the following passage, solve Q.53 to Q.55

### Passage – 2

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive then

**53.** Value of b is

(a) – 54 (b) 54

(c) 27 (d) –27

**54.** Value of c is

(a) 108 (b) –108

(c) 54 (d) – 54

**55.** Root of equation $2bx + c = 0$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) –1

# EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

## Objective Questions I [Only one correct option]

**1.** For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one of the root is square of the other, then p is equal to **(2000)**

(a) 1/3 (b) 1

(c) 3 (d) 2/3

**2.** The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is **(2001)**

(a) 3 (b) 1

(c) 2 (d) 0

**3.** The set of all real numbers x for which

$x^2 - |x+2| + x > 0$ is **(2002)**

(a) $(-\infty, -2) \cup (2, \infty)$ (b) $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

(c) $(-\infty, -1) \cup (1, \infty)$ (d) $\left(\sqrt{2}, \infty\right)$

**4.** For all 'x', $x^2 + 2ax + (10 - 3a) > 0$, then the interval in which 'a' lies is **(2004)**

(a) $a < -5$ (b) $-5 < a < 2$

(c) $a > 5$ (d) $2 < a < 5$

**5.** If one root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is **(2004)**

(a) $p^3 - (3p-1)q + q^2 = 0$

(b) $p^3 - q(3p+1) + q^2 = 0$

(c) $p^3 + q(3p-1) + q^2 = 0$

(d) $p^3 + q(3p+1) + q^2 = 0$

**6.** If a, b, c are the sides of a triangle ABC such that $x^2 - 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ has real roots, then **(2006)**

(a) $\lambda < \frac{4}{3}$ (b) $\lambda > \frac{5}{3}$

(c) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

**7.** Let α, β be the roots of the equation $x^2 - px + r = 0$ and α/2, 2β be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **(2007)**

(a) $2/9\,(p-q)(2q-p)$ (b) $2/9\,(q-p)(2p-q)$

(c) $2/9\,(q-2p)(2q-p)$ (d) $2/9\,(2p-q)(2q-p)$

**8.** Let p and q be the real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **(2010)**

(a) $(p^3+q)x^2 - (p^3+2q)x + (p^3+q) = 0$

(b) $(p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$

(c) $(p^3-q)x^2 - (5p^3-2q)x + (p^3-q) = 0$

(d) $(p^3-q)x^2 - (5p^3+2q)x + (p^3-q) = 0$

**9.** Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **(2011)**

(a) 1 (b) 2

(c) 3 (d) 4

**10.** A value of b for which the equations $x^2 + bx - 1 = 0$, $x^2 + x + b = 0$ have one root in common is **(2011)**

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}$

(c) $i\sqrt{5}$ (d) $\sqrt{2}$

**11.** The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots.

Then the equation

$$p(p(x)) = 0$$

has **(2014)**

(a) only purely imaginary roots

(b) all real roots

(c) two real and two purely imaginary roots

(d) neither real nor purely imaginary roots

**12.** Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose $\alpha_1$ and $\beta_1$ are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and $\alpha_2$ and $\beta_2$ are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals. **(2016)**

(a) $2(\sec\theta - \tan\theta)$ (b) $2\sec\theta$

(c) $-2\tan\theta$ (d) 0

**13.** Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of

$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$ is **(2020)**

(a) 0 (b) 8000

(c) 8080 (d) 16000

## Objective Questions II [One or more than one correct option]

**14.** Let S be the set of all non-zero real numbers a such that the quadratic equation $ax^2 - x + a = 0$ has two distinct real roots $x_1$ and $x_2$ satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **(2015)**

(a) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{5}, 0\right)$

(c) $\left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

**15.** Let $\alpha$ and $\beta$ be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$.

For all positive integers n. define $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \ge 1$

$b_1 = 1$ *and* $b_n = a_{n-1} + a_{n+1}, n \ge 2$ then which of the following options is/ are correct? **(2019)**

(a) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(b) $b_n = \alpha^n + \beta^n$ *for all* $n \ge 1$

(c) $a_1 + a_2 + \ldots\ldots + a_n = a_{n+2} - 1$ *for all* $n \ge 1$

(d) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

## Numerical Value Type Questions

**16.** The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is.... **(2009)**

**17.** Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations $3x - y - z = 0, -3x + z = 0, -3x + 2y + z = 0$. Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is...

**(2009)**

**18.** For $x \in R$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is _____. **(2021)**

**19.** If $x^2 - 10ax - 11b = 0$ have roots c & d, $x^2 - 10cx - 11d = 0$ have roots a and b. $(a \ne c)$ Find $a + b + c + d$. **(2006)**

## Assertion & Reason

**(A)** **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

**(B)** **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

**(C)** **If ASSERTION is true, REASON is false.**

**(D)** **If ASSERTION is false, REASON is true.**

**20.** Let a, b, c, p, q be the real numbers. Suppose $\alpha, \beta$ are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$. **(2008)**

**Assertion :** $(p^2 - q)(b^2 - ac) \ge 0$

**Reason :** $b \notin pa$ or $c \notin qa$.

(a) A (b) B

(c) C (d) D

**Using the following passage, solve Q.21 to Q.23**

**Passage – 1**

If a continuous function f defined on the real line R, assumes positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x) = 0$ has a root in R.

Consider $f(x) = ke^x - x$ for all real x where k is real constant. **(2007)**

**21.** The line $y = x$ meets $y = ke^x$ for $k \le 0$ at

(a) no point (b) one point

(c) two points (d) more than two points

**22.** The positive value of k for which $ke^x - x = 0$ has only one root is

(a) $\frac{1}{e}$ (b) 1

(c) e (d) $\log_e 2$

**23.** For $k > 0$, the set of all values of k for which $ke^x - x = 0$ has two distinct roots, is

(a) $\left(0, \frac{1}{e}\right)$ (b) $\left(\frac{1}{e}, 1\right)$

(c) $\left(\frac{1}{e}, \infty\right)$ (d) (0, 1)

**Using the following passage, solve Q.24 and Q.25**

**Passage – 2**

Let P, q be integers and let $\alpha$, $\beta$ be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \ne \beta$. For $n = 0, 1, 2, \ldots\ldots,$ let $a_n = P\alpha^n + q\beta^n$.

**FACT :** If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$. **(2017)**

**24.** If $a_4 = 28$, then $P + 2q =$

(a) 12 (b) 21

(c) 14 (d) 7

**25.** $a_{12} =$

(a) $a_{11} + 2a_{10}$ (b) $a_{11} + a_{10}$

(c) $a_{11} - a_{10}$ (d) $2a_{11} + a_{10}$

# Answer Key

## CHAPTER -1 | QUADRATIC EQUATIONS

### EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

| | | | | |
|---|---|---|---|---|
| **1.** (b) | **2.** (d) | **3.** (a) | **4.** (c) | **5.** (a) |
| **6.** (a) | **7.** (a) | **8.** (a) | **9.** (c) | **10.** (c) |
| **11.** (a) | **12.** (c) | **13.** (b) | **14.** (b) | **15.** (d) |
| **16.** (c) | **17.** (a) | **18.** (b) | **19.** (b) | **20.** (d) |
| **21.** (b) | **22.** (a) | **23.** (b) | **24.** (c) | **25.** (b) |
| **26.** (a) | **27.** (a) | **28.** (b) | **29.** (c) | **30.** (d) |
| **31.** (c) | **32.** (d) | **33.** (c) | **34.** (b) | **35.** (4) |
| **36.** (4) | **37.** (0) | **38.** (4) | **39.** (1.414) | **40.** (0) |
| **41.** (0) | **42.** (0.66) | **43.** (1) | **44.** (0.22) | |
| **45.** (-3.75) | | **46.** (0.67) | **47.** (1) | **48.** (0.67) |
| **49.** (0.8) | **50.** (395.92) | | | |

### EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

| | | | | |
|---|---|---|---|---|
| **1.** (a) | **2.** (d) | **3.** (d) | **4.** (a) | **5.** (d) |
| **6.** (c) | **7.** (c) | **8.** (b) | **9.** (d) | **10.** (b) |
| **11.** (c) | **12.** (d) | **13.** (c) | **14.** (b) | **15.** (d) |
| **16.** (c) | **17.** (d) | **18.** (-256) | **19.** (b) | **20.** (a) |
| **21.** (11.00) | **22.** (d) | **23.** (d) | **24.** (b) | **25.** (c) |
| **26.** (d) | **27.** (c) | **28.** (a) | **29.** (c) | **30.** (c) |
| **31.** (b) | **32.** (c) | **33.** (a) | **34.** (c) | **35.** (c) |
| **36.** (b) | **37.** (c) | **38.** (c) | **39.** (8.00) | **40.** (a) |
| **41.** (c) | **42.** (b) | **43.** (b) | **44.** (1.00) | **45.** (b) |
| **46.** (c) | **47.** (a) | **48.** (18.00) | **49.** (66.00) | |
| **50.** (c) | **51.** (c) | **52.** (324.00) | | **53.** (1.00) |
| **54.** (c) | **55.** (a) | **56.** (c) | **57.** (d) | **58.** (d) |
| **59.** (2.00) | **60.** (1.00) | | | |

## CHAPTER -1 | QUADRATIC EQUATIONS

### EXERCISE - 3 :
### ADVANCED OBJECTIVE QUESTIONS

| | | | | |
|---|---|---|---|---|
| **1.** (a) | **2.** (d) | **3.** (a) | **4.** (b) | **5.** (d) |
| **6.** (a) | **7.** (d) | **8.** (a) | **9.** (c) | **10.** (b) |
| **11.** (a) | **12.** (b) | **13.** (d) | **14.** (b) | **15.** (a) |
| **16.** (a,b) | **17.** (a,c) | **18.** (a,c) | **19.** (a,d) | **20.** (b,c) |
| **21.** (a,d) | **22.** (a,c) | **23.** (a,c) | **24.** (a,b,c,d) | |
| **25.** (b,c) | **26.** (a,b) | **27.** (a,b) | **28.** (b,c) | **29.** (1) |
| **30.** (4.5) | **31.** (9) | **32.** (1) | **33.** (-2) | **34.** (0) |
| **35.** (8) | **36.** (3) | **37.** (81) | **38.** (5) | **39.** (74) |
| **40.** (18) | **41.** (-1) | **42.** (99) | **43.** (1) | **44.** (-12) |
| **45.** (-2) | **46.** (a) | **47.** (a) | **48.** (b) | **49.** (a) |
| **50.** (a) | **51.** (b) | **52.** (c) | **53.** (b) | **54.** (b) |
| **55.** (c) | | | | |

### EXERCISE - 4 :
### PREVIOUS YEAR JEE ADVANCED QUESTIONS

| | | | | |
|---|---|---|---|---|
| **1.** (c) | **2.** (b) | **3.** (b) | **4.** (b) | **5.** (a) |
| **6.** (a) | **7.** (d) | **8.** (b) | **9.** (c) | **10.** (b) |
| **11.** (d) | **12.** (c) | **13.** (d) | **14.** (a,d) | **15.** (a,b,c) |
| **16.** (2) | **17.** (7) | **18.** (4.00) | **19.** (1210) | **20.** (b) |
| **21.** (b) | **22.** (a) | **23.** (a) | **24.** (a) | **25.** (b) |