CHAPTER 13

LIMITS AND DERIVATIVE

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. The limit of $f(x) = x^2$ as x tends to zero equals
   (a) zero (b) one (c) two (d) three

2. Consider the function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$
   Then, left hand limit and right hand limit of f (x) at x = 0, are respectively
   (a) 1, 2 (b) 2, 1 (c) 1,1 (d) 2, 2

3. The value of $\lim\limits_{x \to -1} \left[\dfrac{x^2 - 1}{x^2 + 3x + 2}\right]$ is
   (a) 2 (b) –2 (c) 0 (d) –1

4. The value of $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$ is
   (a) 2 (b) –2 (c) 1 (d) –1

5. Evaluate $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}$
   (a) 1 (b) 2 (c) –1 (d) –2

6. Value of $\lim\limits_{x \to 5} \dfrac{1 - \sqrt{x-4}}{x-5}$ is
   (a) 0 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) does not exist

7. $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x+x^2} - 1}{x} =$
   (a) $\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) 0 (d) ∞

8. If $f(x) = \begin{cases} x^2 + 1, & x \ge 1 \\ 3x - 1, & x < 1 \end{cases}$, then the value of $\lim\limits_{x \to 1} f(x)$ is
   (a) 2 (b) –2 (c) 1 (d) –1

9. The value of $\lim\limits_{x \to 0} \dfrac{\sqrt{(1+x^2)} - \sqrt{1-x^2}}{x^2}$ is
   (a) 1 (b) –1 (c) 0 (d) does not exist

10. If $f(t) = \dfrac{1-t}{1+t}$, then the value of $f'(1/t)$ is
    (a) $\dfrac{-2t^2}{(t+1)^2}$ (b) $\dfrac{2t}{(t+1)^2}$ (c) $\dfrac{2t^2}{(t-1)^2}$ (d) $\dfrac{-2t^2}{(t-1)^2}$

11. Let f and g be two functions such that $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist. Then, which of the following is incomplete?
    (a) $\lim\limits_{x \to a} [f(x) + g(x)] = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$
    (b) $\lim\limits_{x \to a} [f(x) - g(x)] = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$
    (c) $\lim\limits_{x \to a} [f(x) \,.\, g(x)] = \lim\limits_{x \to a} f(x) \,.\, \lim\limits_{x \to a} g(x)$
    (d) $\lim\limits_{x \to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$

12. The derivative of the function $f(x) = x$ is
    (a) 0 (b) 1 (c) ∞ (d) None of these

13. The derivative of sin x at x = 0 is
    (a) 0 (b) 2 (c) 1 (d) 3

14. The derivative of the function $f(x) = 3x$ at x = 2 is
    (a) 0 (b) 1 (c) 2 (d) 3

15. The derivative of $f(x) = 3$ at x = 0 and at x = 3 are
    (a) negative (b) zero
    (c) different (d) not defined

16. Derivative of f at x = a is denoted by
    (a) $\left.\dfrac{d}{dx} f(x)\right|_a$ (b) $\left.\dfrac{df}{dx}\right|_a$
    (c) $\left(\dfrac{df}{dx}\right)_{x=a}$ (d) All of these

17. If a is a non-zero constant, then the derivative of x + a is
    (a) 1 (b) 0
    (c) a (d) None of these

18. The derivative of $\dfrac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$ is
    (a) $\dfrac{2}{(1+x)^2}$ (b) $\dfrac{-2}{(1-x)^2}$
    (c) $\dfrac{-1}{(1-x)^2}$ (d) $\dfrac{3}{(1-x)^2}$

19. The derivative of $4\sqrt{x} - 2$ is
    (a) $\dfrac{1}{\sqrt{x}}$ (b) $2\sqrt{x}$ (c) $\dfrac{2}{\sqrt{x}}$ (d) $\sqrt{x}$

20. If a and b are fixed non-zero constants, then the derivative of $(ax + b)^n$ is
(a) $n(ax + b)^{n-1}$ (b) $na(ax + b)^{n-1}$
(c) $nb(ax + b)^{n-1}$ (d) $nab(ax + b)^{n-1}$

21. The derivative of $\sin^n x$ is
(a) $n \sin^{n-1} x$ (b) $n \cos^{n-1} x$
(c) $n \sin^{n-1} x \cos x$ (d) $n \cos^{n-1} x \sin x$

22. The derivative of $(x^2 + 1) \cos x$ is
(a) $-x^2 \sin x - \sin x - 2x \cos x$
(b) $-x^2 \sin x - \sin x + 2 \cos x$
(c) $-x^2 \sin x - x \sin x + 2 \cos x$
(d) $-x^2 \sin x - \sin x + 2 x \cos x$

23. The derivative of $f(x) = \tan (ax + b)$ is
(a) $\sec^2 (ax + b)$ (b) $b \sec^2 (ax + b)$
(c) $a \sec^2 (ax + b)$ (d) $ab \sec^2 (ax + b)$

24. If $f(x) = x \sin x$, then $f'\left(\frac{\pi}{2}\right)$ is equal to
(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

25. The derivative of function $6x^{100} - x^{55} + x$ is
(a) $600x^{100} - 55x^{55} + x$ (b) $600x^{99} - 55x^{54} + 1$
(c) $99x^{99} - 54x^{54} + 1$ (d) $99x^{99} - 54x^{54}$

26. $\lim_{x \to 0} \frac{x}{\tan x}$ is
(a) 0 (b) 1 (c) 4 (d) not defined

27. Derivative of $\log_x x$ is
(a) 0 (b) 1 (c) $\frac{1}{x}$ (d) x

28. Derivative of $e^{3 \log x}$ is
(a) e^x (b) $3x^2$ (c) 3x (d) log x

29. Derivative of $x^2 + \sin x + \frac{1}{x^2}$ is
(a) $2x + \cos x$ (b) $2x + \cos x + (-2) x^{-3}$
(c) $2x - 2x^{-3}$ (d) None of these

30. Derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ is
(a) $\frac{1}{x^2}$ (b) $1 - \frac{1}{x^2}$ (c) 1 (d) $1 + \frac{1}{x^2}$

31. If $f(x) = \alpha x^n$, then $\alpha =$
(a) $f'(1)$ (b) $\frac{f'(1)}{n}$ (c) $n \cdot f'(1)$ (d) $\frac{n}{f'(1)}$

32. Derivative of x sin x
(a) x cos x (b) x sin x
(c) x cos x + sin x (d) sin x

33. Value of $\lim_{x \to 0} \frac{a^{\sin x} - 1}{\sin x}$ is
(a) log a (b) sin x (c) log (sin x) (d) cos x

34. $\lim_{x \to 0} \frac{2 \sin^2 3x}{x^2}$ is equal to :
(a) 12 (b) 18 (c) 0 (d) 6

35. $\lim_{\theta \to 0} \frac{\sin m^2 \theta}{\theta}$ is equal to :
(a) 0 (b) 1 (c) m (d) m^2

36. Derivative of the function $f(x) = 7x^{-3}$ is
(a) $21x^{-4}$ (b) $-21x^{-4}$ (c) $21x^4$ (d) $-21x^4$

37. If $f(x) = 2\sin x - 3x^4 + 8$, then $f'(x)$ is
(a) $2\sin x - 12x^3$ (b) $2\cos x - 12x^3$
(c) $2\cos x + 12x^3$ (d) $2\sin x + 12x^3$

38. Derivative of the function $f(x) = (x - 1)(x - 2)$ is
(a) $2x + 3$ (b) $3x - 2$
(c) $3x + 2$ (d) $2x - 3$

39. If $\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right]$ exists, then which one of the following correct ?
(a) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist
(b) $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ must exist
(c) Both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ need not exist
(d) $\lim_{x \to a} f(x)$ must exist but $\lim_{x \to a} g(x)$ need not exist

40. The value of $\lim_{x \to 0} \frac{1 + \frac{x}{3} - 1 + \frac{x}{3}}{x}$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

41. The value of $\lim_{x \to 0} \frac{\cos x}{\pi - x}$ is
(a) π (b) $-\pi$ (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

42. Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to
(a) $\frac{2}{7}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{7}{2}$

43. What is the derivative of
$f(x) = \frac{7x}{(2x - 1)(x + 3)}$?
(a) $-\frac{3}{(x+3)^2} - \frac{2}{(2x-1)^2}$ (b) $-\frac{3}{(x+3)^2} - \frac{1}{(2x-1)^2}$
(c) $\frac{3}{(x+3)^2} + \frac{1}{(2x-1)^2}$ (d) $\frac{3}{(x+3)^2} + \frac{2}{(2x-1)^2}$

44. As $x \to a$, $f(x) \to l$, then l is called .... of the function f(x),
(a) limit (b) value
(c) absolute value (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

45. Consider the function $g(x) = |x|, x \neq 0$.
Then
I. g (0) is not defined.
II. $\lim_{x \to 0} g(x)$ is not defined.

Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

46. Consider the function $h(x) = \frac{x^2-4}{x-2}, x \neq 2$
Then,
I. h (2) is not defined.
II. $\lim_{x\to 2} h(x) = 4$.
Which of the following is/are true?
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

47. Which of the following is/are true?
I. $\lim_{x\to 1}\left[\frac{x^{15}-1}{x^{10}-1}\right] = \frac{3}{2}$
II. $\lim_{x\to 0}\left[\frac{\sqrt{1+x}-1}{x}\right] = \frac{1}{2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

48. Which of the following is/are true?
I. $\lim_{x\to 0}\frac{\sin x}{x} = 1$
II. $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

49. Which of the following is/are true?
I. $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$ (where $a+b+c \neq 0$) is 1.
II. $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

50. $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is equal to
I. $a^2 \sin a + 2a \cos a$ II. $a^2 \cos a + 2a \sin a$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

51. Which of the following is/are true?
I. The derivative of $f(x) = \sin 2x$ is $2(\cos^2 x - \sin^2 x)$.
II. The derivative of $g(x) = \cot x$ is $-\operatorname{cosec}^2 x$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

52. Which of the following is/are true?
I. The derivative of $x^2 - 2$ at $x = 10$ is 18.
II. The derivative of $99x$ at $x = 100$ is 99.
III. The derivative of x at $x = 1$ is 1.
(a) I, II and III are true (b) I and II are true
(c) II and III are true (d) I and III are true

53. Which of the following is/are true?
I. The derivative of $y = 2x - \frac{3}{4}$ is 2.
II. The derivative of $y = (5x^3 + 3x - 1)(x - 1)$ is $20x^3 + 15x^2 + 6x - 4$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

54. Which of the following is/are true?
I. The derivative of $f(x) = x^3$ is x^2
II. The derivative of $f(x) = \frac{1}{x^3}$ is $\frac{-1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

55. Which of the following is/are true?
I. The derivative of $-x$ is -1.
II. The derivative of $(-x)^{-1}$ is $\frac{1}{x^2}$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

56. Which of the following is/are true?
I. The derivative of $\sin(x + a)$ is $\cos(x + a)$, where a is a fixed non-zero constant.
II. The derivative of cosec x cot x is $\operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x$
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Borh I and II are false

57. Which of the following is/are true?
I. The derivative of $f(x) = 1 + x + x^2 + ... + x^{50}$ at $x = 1$ is 1250.
II. The derivative of $f(x) = \frac{x+1}{x}$ is $\frac{1}{x^2}$.
(a) Both I and II are true (b) Only I is true
(c) Only II is true (d) Both I and II are false

58. Consider the following limits which holds for function f and g:
I. $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
II. $\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x). \lim_{x\to a} g(x)$
III. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$
Which of the above are true ?
(a) Only I (b) Only II
(c) Only III (d) All of the above

59. Consider the following derivatives which holds for function u and v.
I. $(u \pm v)' = u' \pm v'$ II. $(uv)' = uv' + vu'$
III. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
Which of the above holds are true ?
(a) Only I (b) Only II
(c) Only III (d) All of these

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column-I	Column-II
A. $\lim_{x\to a^-} f(x)$	1. left hand limit of f at a
B. $\lim_{x\to a^+} f(x)$	2. limit of f at a
C. $\lim_{x\to a} f(x)$	4. right hand limit of f at a

Codes

	A	B	C
(a)	3	1	2
(b)	1	3	2
(c)	1	2	3
(d)	2	3	1

61.

Column-I (Limts)	Column-II (Values)
A. $\lim_{x\to 3} x+3$	1. π
B. $\lim_{x\to\pi}\left(x-\frac{22}{7}\right)$	2. 6
C. $\lim_{r\to 1}\pi r^2$	3. $\frac{19}{2}$
D. $\lim_{x\to 4}\left(\frac{4x+3}{x-2}\right)$	4. $\frac{-1}{2}$
E. $\lim_{x\to -1}\left(\frac{x^{10}+x^5+1}{x-1}\right)$	5. $\pi-\frac{22}{7}$

Codes

	A	B	C	D	E
(a)	5	2	1	4	3
(b)	2	5	1	3	4
(c)	5	2	1	3	4
(d)	2	5	3	1	4

62.

Column-I (Limits)	Column-II (Values)
A. $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$	1. 4
B. $\lim_{x\to 0}\frac{\cos x}{\pi-x}$	2. $\frac{1}{\pi}$
C. $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$	3. $\frac{a+1}{b}$
D. $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$	4. 0
E. $\lim_{x\to 0} x\sec x$	5. 1
F. $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$ $(a, b, a+b\neq 0)$	

Codes

	A	B	C	D	E	F
(a)	2	2	1	3	5	4
(b)	2	2	3	1	4	5
(c)	2	2	1	4	3	5
(d)	2	2	1	3	4	5

63.

Column-I (Functions)	Column-II (Derivatives)
A. cosec x	1. 5 cos x + 6 sin x
B. 3 cot x + 5 cosec x	2. – 3 cosec² x – 5 cosec x cot x
C. 5 sin x – 6 cos x + 7	3. 2 sec² x – 7 sec x tan x
D. 2 tan x – 7 sec x	4. – cot x cosec x

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	4	2	3	1
(c)	2	4	1	3
(d)	4	2	1	3

64.

Column-I (Functions)	Column-II (Derivatives)
A. $f(x)=10x$	1. $2x$
B. $f(x)=x^2$	2. $-\frac{1}{x^2}$
C. f(x) = a for fixed real no. a	3. 0
D. $f(x)=\frac{1}{x}$	4. 10

Codes

	A	B	C	D
(a)	4	1	3	2
(b)	1	4	3	2
(c)	4	1	2	3
(d)	4	3	1	2

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

65. If value of $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$ is equal to $\frac{1}{a\sqrt{2}}$ then 'a' equals

(a) 1 (b) 2 (c) 3 (d) 4

66. If value of $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ is equal to $\frac{2\sqrt{3}}{m}$, where m is equal to

(a) 2 (b) 8 (c) 9 (d) 3

67. $\lim_{x\to\pi/2}(\sec x-\tan x)$ is equal to

(a) 0 (b) 2 (c) 1 (d) 3

68. Suppose $f(x)=\begin{cases}a+bx, & x<1\\ 4, & x=1\\ b-ax, & x>1\end{cases}$

and if $\lim_{x\to 1} f(x)=f(1)$ then the value of $a+b$ is

(a) 0 (b) 2 (c) 4 (d) 8

69. If $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$ is equal to p cos q, then sum of p and q is

(a) 2 (b) 1 (c) 3 (d) 4

70. If $f(x)=|x|-5$, then the value of $\lim_{x\to 5} f(x)$ is

(a) 9 (b) 1 (c) 0 (d) None of these

71. If value of $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}$ is equal to $\frac{a}{2}$ then the value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

72. Value of $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$ is

(a) 1 (b) 2 (c) 4 (d) None of these

73. If $f(x)=\begin{cases}2x+3, & x\leq 0\\ 3(x+1), & x>0\end{cases}$ then the value of $\lim_{x\to 0} f(x)$ is

(a) 0 (b) 6 (c) 2 (d) 3

74. Let $f(x)=\begin{cases} x+2\ , & x\le -1 \\ cx^2\ , & x>-1 \end{cases}$

If $\lim\limits_{x\to -1} f(x)$ exists, then c is equal to

(a) 1 (b) 0 (c) 2 (d) 3

75. If value of $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$ is $a\sqrt{2}$, then the value of 'a' is

(a) 2 (b) 3 (c) 4 (d) 5

76. If $\lim\limits_{x\to 2} \dfrac{x^n-2^n}{x-2}=80$ and $n\in N$, then the value of 'n' is

(a) 2 (b) 3 (c) 4 (d) 5

77. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$ is equal to

(a) 2 (b) 0 (c) 1 (d) 3

78. If $f(x)=x^n$ and $f'(1)=10$, then the value of 'n' is

(a) 1 (b) 5 (c) 9 (d) 10

79. If $\lim\limits_{x\to 5} \dfrac{x^k-5^k}{x-5}=500$, then k is equal to :

(a) 3 (b) 4 (c) 5 (d) 6

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

80. **Assertion:** $\lim\limits_{x\to 0} \dfrac{\sin ax}{bx}=\dfrac{a}{b}$

Reason: $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx}=\dfrac{b}{a}$ $(a, b\neq 0)$

81. **Assertion:** $\lim\limits_{x\to 0} (\operatorname{cosec} x-\cot x)=0$

Reason: $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\tan 2x}{x-\dfrac{\pi}{2}}=1$

82. **Assertion:** If a and b are non-zero constants, then the derivative of $f(x)=ax+b$ is a.

Reason: If a, b and c are non-zero constants, then the derivative of $f(x)=ax^2+bx+c$ is $ax+b$.

83. Let $a_1, a_2, a_3, ..., a_n$ be fixed real numbers and define a function $f(x)=(x-a_1)(x-a_2)...(x-a_n)$, then

Assertion: $\lim\limits_{x\to a_1} f(x)=0$.

Reason: $\lim\limits_{x\to a} f(x)=(a-a_1)(a-a_2)...(a-a_n)$, for some $a\neq a_1, a_2, ..., a_n$.

84. **Assertion:** Suppose f is real valued function, the derivative of 'f' at x is given by $f'(x)=\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

Reason: If $y=f(x)$ is the function, then derivative of 'f' at any x is denoted by $f'(x)$.

85. **Assertion.** For the function

$f(x)=\dfrac{x^{100}}{100}+\dfrac{x^{99}}{99}+...+\dfrac{x^2}{2}+x+1$, $f'(1)=100f'(0)$.

Reason: $\dfrac{d}{dx}(x^n)=n.x^{n-1}$.

86. **Assertion:** $\lim\limits_{x\to 0} (1+3x)^{1/x}=e^3$.

Reason: Since $\lim\limits_{x\to 0} (1+x)^{1/x}=e$.

87. **Assertion:** $\lim\limits_{x\to 0} \log_e\left(\dfrac{\sin x}{x}\right)=0$

Reason: $\lim\limits_{x\to 0} f(g(x))=f(\lim\limits_{x\to 0} g(x))$.

88. **Assertion:** $\lim\limits_{x\to 0} \dfrac{\tan x^0}{x^0}=1$ where x^0 means x degree.

Reason: If $\lim\limits_{x\to 0} f(x)=l$, $\lim\limits_{x\to 0} g(x)=m$, then $\lim\limits_{x\to 0}\{f(x)g(x)\}=lm$

89. **Assertion:** Derivative of $f(x)=x\,|x|$ is $2\,|x|$.

Reason: For function u and v, $(uv)'=uv'+vu'$.

90. **Assertion:** Let $\lim\limits_{x\to a} f(x)=l$ and $\lim\limits_{x\to a} g(x)=m$. If l and m both exist, then $\lim\limits_{x\to a} (fg)(x)=\lim\limits_{x\to a} f(x).\lim\limits_{x\to a} g(x)=lm$

Reason: Let f be a real valued function defined by $f(x)=x^2+1$, then $f'(2)=4$.

91. **Assertion:** Derivative of $f(x)=2$ is zero.

Reason: Differentiation of a constant function is zero.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

92. Evaluate $\lim\limits_{x\to 0} \dfrac{\sin^2 2x}{x^2}$.

(a) 4 (b) -4 (c) $\sin x$ (d) $\cos x$

93. The value of $\lim\limits_{x\to 0} \dfrac{x^3 \cot x}{1-\cos x}$ is

(a) 1 (b) -2 (c) 2 (d) 0

94. The value of $\lim\limits_{x\to 0} \dfrac{x(e^x-1)}{1-\cos x}$ is

(a) 0 (b) 2 (c) -2 (d) does not exist

95. $\lim\limits_{x\to 0} \dfrac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) -1 (c) zero (d) does not exist

96. $\lim\limits_{x\to 0} \dfrac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) -2 (c) 1/2 (d) $-1/2$

97. $\lim\limits_{x\to 0} \dfrac{\sin(\pi\cos^2 x)}{x^2}$ equals

(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

98. The value of $\lim\limits_{\theta\to -\frac{\pi}{4}} \dfrac{\cos\theta+\sin\theta}{\theta+\dfrac{\pi}{4}}$ is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{-\pi}{4}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}$

99. If $f(x)=\dfrac{x+|x|}{x}$, then the value of $\lim\limits_{x\to 0} f(x)$ is
(a) 0 (b) 2
(c) does not exist (d) None of these

100. f(x) is a function such that $f''(x)=-f(x)$ and $f'(x)=g(x)$ and h (x) is a function such that $h(x)=[f(x)]^2+[g(x)]^2$ and h (5) = 11, then the value of h (10) is
(a) 5 (b) –5 (c) –11 (d) 11

101. Let α and β be the distinct roots of $ax^2+bx=c=0$, then

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$ is equal to

(a) $\dfrac{a^2}{2}(\alpha-\beta)^2$ (b) 0
(c) $\dfrac{-a^2}{2}(\alpha-\beta)^2$ (d) $\dfrac{1}{2}(\alpha-\beta)^2$

102. If $\lim\limits_{x\to 0}\dfrac{((a-n)nx-\tan x)\sin nx}{x^2}=0$, where n is non-zero real number, then a is equal to
(a) 0 (b) $\dfrac{n+1}{n}$ (c) n (d) $n+\dfrac{1}{n}$

103. If sin y = x sin (a + y), then value of dy/dx is
(a) $\dfrac{\sin(a+y)}{\sin a}$ (b) $\dfrac{\sin^2(a+y)}{\sin a}$
(c) $\sin^2(a+y)$ (d) $\dfrac{\cos^2(a+y)}{\sin a}$

104. If $y=x\tan\dfrac{x}{2}$, then value of $(1+\cos x)\dfrac{dy}{dx}-\sin x$ is
(a) –x (b) x^2 (c) x (d) None

105. Differential coefficient of $\dfrac{x\sin x}{1+\cos x}$ is
(a) $\dfrac{-x-\sin x}{1+\cos x}$ (b) $\dfrac{x-\sin x}{1+\cos x}$ (c) $\dfrac{x+\sin x}{1-\cos x}$ (d) $\dfrac{x+\sin x}{1+\cos x}$

106. If $x^3+y^3=3xy$, then value of $\dfrac{dy}{dx}$ is
(a) $\dfrac{x-y^2}{x^2-y}$ (b) $\dfrac{x^2-y}{x-y^2}$ (c) $\dfrac{x^2-y^2}{x-y}$ (d) $\dfrac{x-y}{x^2-y^2}$

107. If $y=ax^{n+1}+bx^{-n}$, then value of $x^2\dfrac{d^2y}{dx^2}$ is
(a) n (n + 1) y (b) $n^2(n+1)$ y
(c) 2n (n – 1) y (d) None of these

108. If $y=f\left(\dfrac{2x-1}{x^2+1}\right)$ and $f'(x)=\sin x^2$, then value of $\dfrac{dy}{dx}$ is
(a) $\sin\left(\dfrac{2x+1}{x^2+1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$
(b) $\sin\left(\dfrac{2x-1}{x^2-1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$
(c) $\sin\left(\dfrac{2x-1}{x^2+1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$
(d) $\sin\left(\dfrac{2x+1}{x^2-1}\right)^2\left[\dfrac{2+2x-2x^2}{x^2+1}\right]$

109. $\lim\limits_{x\to 2}\left[\dfrac{1}{x-2}-\dfrac{2(2x-3)}{x^3-3x^2+2x}\right]$ is equal to
(a) $\dfrac{1}{2}$ (b) $\dfrac{-1}{2}$ (c) 1 (d) – 2

110. If $\lim\limits_{x\to 3}\dfrac{x^n-3^n}{x-3}=108$, the positive integer n is equal to
(a) 3 (b) 5 (c) 2 (d) 4

111. $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx}{\cos cx-1}$ is equal to $\dfrac{m}{n}$, where m and n are respectively
(a) a^2+b^2, c^2 (b) c^2, a^2+b^2
(c) a^2-b^2, c^2 (d) c^2, a^2-b^2

112. $\lim\limits_{x\to 1}[x-1]$, where [.] is greatest integer function, is equal to
(a) 1 (b) 2 (c) 0 (d) does not exist

113. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$ is equal to
(a) $\dfrac{1}{2}$ (b) 0 (c) 1 (d) Not defined

114. If a, b are fixed non-zero constant, then the derivative of $\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$ is ma + nb – p, where
(a) $m=4x^3, n=\dfrac{-2}{x^3}, p=\sin x$
(b) $m=\dfrac{-4}{x^5}, n=\dfrac{2}{x^3}, p=\sin x$
(c) $m=\dfrac{-4}{x^5}, n=\dfrac{-2}{x^3}, p=-\sin x$
(d) $m=4x^3, n=\dfrac{2}{x^3}, p=-\sin x$

115. If a is a fixed non-zero constant, then the derivative of $\dfrac{\sin(x+a)}{\cos x}$ is
(a) $\dfrac{\cos a}{\cos^2 x}$ (b) $\dfrac{-\cos a}{\cos^2 x}$ (c) $\dfrac{\sin a}{\cos^2 x}$ (d) $\dfrac{-\sin a}{\cos^2 x}$

116. $\lim\limits_{x\to 4}\dfrac{|x-4|}{x-4}$ is equal to
(a) 1 (b) 0 (c) – 1 (d) does not exist

117. Let $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{when } x\neq\dfrac{\pi}{2}\\ 3, & \text{when } x=\dfrac{\pi}{2}\end{cases}$

If $\lim\limits_{x\to\frac{\pi}{2}} f(x)=f\left(\dfrac{\pi}{2}\right)$, then k is equal to
(a) 2 (b) 4 (c) 6 (d) 8

118. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 0 (d) None of these

119. If $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R$ $(xy \neq 1)$ and $\lim\limits_{x\to 0}\frac{f(x)}{x} = 2$. Then, $f'\left(\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{6}$ (d) $\frac{3}{2}$

120. If f be a function given by $f(x) = 2x^2 + 3x - 5$. Then, $f'(0) = mf'(-1)$, where m is equal to

(a) -1 (b) -2 (c) -3 (d) -4

121. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots \frac{x^2}{2} + x + 1,$$

$f'(1) = mf'(0)$, where m is equal to

(a) 50 (b) 0 (c) 100 (d) 200

122. Evaluate: $\lim\limits_{x\to\pi/6}\frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

(a) 3 (b) -3
(c) 1 (d) -1

123. The function $u = e^x \sin x$, $v = e^x \cos x$ satisfy the equation

(a) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (b) $\frac{d^2u}{dx^2} = 2v$

(c) $\frac{d^2v}{dx^2} = -2u$ (d) All of these

124. If $f(x) = \begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>0 \end{cases}$ then $\lim\limits_{x\to a} f(x)$ exists for all

(a) $a \neq 1$ (b) $a \neq 0$ (c) $a \neq -1$ (d) $a \neq 2$

125. Evaluate : $\lim\limits_{x\to a}\frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$.

(a) $\frac{-5}{3}(a+2)^{2/3}$ (b) $\frac{5}{3}(a-2)^{2/3}$

(c) $\frac{5}{3}(a+2)^{-2/3}$ (d) $\frac{5}{3}(a+2)^{2/3}$

126. $\lim\limits_{x\to 0}\sqrt{\frac{x-\sin x}{x+\sin^2 x}}$ is equal to

(a) 1 (b) 0 (c) ∞ (d) None of these

127. What is the value of $\lim\limits_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$?

(a) 0 (b) $\frac{5}{4}$ (c) $\frac{5}{16}$ (d) $\frac{25}{4}$

128. If $\lim\limits_{x\to 0}\frac{a^x - x^a}{x^a - a^a} = -1$, then a is equal to:

(a) -1 (b) 0 (c) 1 (d) 2

129. The value of $\lim\limits_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$ (c) 0 (d) None of these

130. The value of $\lim\limits_{x\to 2a}\frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ is

(a) $\frac{1}{\sqrt{a}}$ (b) $\frac{1}{2\sqrt{a}}$ (c) $\frac{\sqrt{a}}{2}$ (d) $2\sqrt{a}$

131. $\lim\limits_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is

(a) ∞ (b) $\frac{1}{8}$ (c) 0 (d) $\frac{1}{32}$

132. $\lim\limits_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ is equal to

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

133. Let $f : R \to [0, \infty)$ be such that $\lim\limits_{x\to 5} f(x)$ exists and $\lim\limits_{x\to 5}\frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim\limits_{x\to 5} f(x)$ equals :

(a) 0 (b) 1 (c) 2 (d) 3

134. The value of $\lim\limits_{x\to 0}\frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ is at $\tan x = 3$, is

(a) 0 (b) 1 (c) 2 (d) 3

135. $\lim\limits_{x\to 0}\frac{x\sqrt[3]{z^2 - (z-x)^2}}{\left(\sqrt[3]{8xz - 4x^2} + \sqrt[3]{8xz}\right)^4}$ is equal to

(a) $\frac{z}{2^{11/3}}$ (b) $\frac{1}{2^{23/3}.z}$ (c) $2^{21/3}z$ (d) None of these

136. $\lim\limits_{h\to 0}\left(\frac{1}{h\sqrt[3]{8+h}} - \frac{1}{2h}\right)$ equals to

(a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{48}$ (d) $-\frac{1}{48}$

137. The value of $\lim\limits_{x\to 0}\frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6 (c) 1/4 (d) 1/2

138. The value of $\lim\limits_{x\to 0}\frac{1-\cos x + 2\sin x - \sin^3 x - x^2 + 3x^4}{\tan^3 x - 6\sin^2 x + x - 5x^3}$ is

(a) 1 (b) 2 (c) -1 (d) -2

139. A function f is said to be a rational function, if $f(x) = \frac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that $h(x) \neq 0$, then

(a) $h(a) \neq 0 \Rightarrow \lim\limits_{x\to a} f(x) = \frac{g(a)}{h(a)}$

(b) $h(a) = 0$ and $g(a) \neq 0 \Rightarrow \lim\limits_{x\to a} f(x)$ does not exist

(c) Both (a) and (b) are true

(d) Both (a) and (b) are false.

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** Given function $f(x) = x^2$. Observe that as x takes values very close to 0, the value of f (x) also approaches towards 0.

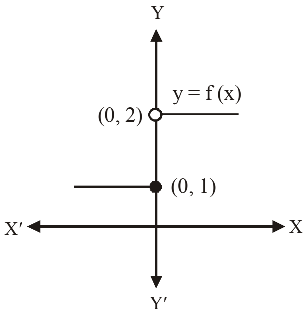
We say $\lim_{x \to 0} f(x) = 0$

(i.e, the limit of f (x) as x tends to zero equals zero).

2. **(a)** Given function $f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$

Graph of this function is shown below. It is clear that the value of f at 0 dictated by values of f (x) with $x \le 0$ equals 1, i.e. the left hand limit of f (x) at x = 0 is

$\lim_{x \to 0^-} f(x) = 1$



Similarly, the value of f at x = 0 dictated by values of f (x) with x > 0 equals 2, i.e., the right hand limit of f (x) at x = 0 is

$$\lim_{x \to 0^+} f(x) = 2$$

In this case the right and left hand limits are different, and hence we say that the limit of f (x) as x tends to zero does not exist (even though the function is defined at 0).

3. **(b)** Limit $= \lim_{x \to -1} \frac{(x-1)(x+1)}{(x+2)(x+1)} = \frac{-1-1}{-1+2} = -2$

4. **(a)** From direct substitution $\frac{\sqrt{1+0}+\sqrt{1-0}}{1+0} = \frac{2}{1} = 2$

5. **(a)** Limit $= \lim_{x \to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)} = \lim_{x \to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1$

6. **(c)** $\lim_{x \to 5} \frac{1-\sqrt{x-4}}{x-5} = \lim_{x \to 5} \frac{1-\sqrt{x-4}}{x-5} \cdot \frac{1+\sqrt{x-4}}{1+\sqrt{x-4}}$

$= \lim_{x \to 5} \frac{1-x+4}{(x-5)(1+\sqrt{x-4})} = \lim_{x \to 5} \frac{-(x-5)}{(x-5)(1+\sqrt{x-4})}$

$= \lim_{x \to 5} \frac{-1}{(1+\sqrt{x-4})} = \frac{-1}{(1+\sqrt{5-4})} = \frac{-1}{2}$

7. **(a)** By rationalisation of numerator, given expression

$= \lim_{x \to 0} \frac{\sqrt{1+x+x^2}-1}{x} \cdot \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$

$= \lim_{x \to 0} \frac{1+x+x^2-1}{x\left(\sqrt{1+x+x^2}+1\right)} = \lim_{x \to 0} \frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$

$= \lim_{x \to 0} \frac{1+x}{\sqrt{1+x+x^2}+1} = \frac{1}{2}$

8. **(a)** Left hand limit $= \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x-1) = 3.1-1 = 2$

and Right hand limit $= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2+1)$

$= 1^2 + 1 = 2$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$

So $\lim_{x \to 1} f(x) = 2$

9. **(a)** $\lim_{x \to 0} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2} \cdot \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$

$= \lim_{x \to 0} \frac{1+x^2-1+x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$

$= \lim_{x \to 0} \frac{2x^2}{x^2\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)} = \frac{2}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$

10. **(a)** $f'(t) = \frac{d}{dt}\left[\frac{1-t}{1+t}\right] = \frac{(1+t)(-1)-(1-t)\times(1)}{(1+t)^2}$

$= \frac{-1-t-1+t}{(1+t)^2} = \frac{-2}{(1+t)^2}$

$f'[1/t] = \frac{-2}{\left(1+\frac{1}{t}\right)^2} = \frac{-2t^2}{(t+1)^2}$

11. **(d)** Let f and g be two functions such that both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then,

(i) Limit of sum of two functions is sum of the limits of the functions i.e.,

$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x).$

(ii) Limit of difference of two functions is difference of the limits of the functions, i.e.,

$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x).$

(iii) Limit of product of two functions is product of the limits of the functions, i.e.,

$$\lim_{x\to a}[f(x).g(x)] = \lim_{x\to a} f(x).\lim_{x\to a} g(x).$$

(iv) Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non-zero), i.e.,

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{ if } \lim_{x\to a} g(x) \neq 0$$

12. **(b)** It is easy to see that the derivative of the function f (x) = x is the constant function 1. This is because

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{x+h-x}{h} = \lim_{h\to 0} 1 = 1$$

13. **(c)** Let f (x) = sin x. Then,

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(0+h)-\sin(0)}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$$

14. **(d)** We have,

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{3(2+h)-3(2)}{h}$$

$$= \lim_{h\to 0}\frac{6+3h-6}{h} = \lim_{h\to 0}\frac{3h}{h} = \lim_{h\to 0} 3 = 3.$$

The derivative of the function f (x) = 3x at x = 2 is 3.

15. **(b)** Since, the derivative measures the change in the function, intuitively it is clear that the derivative of the constant function must be zero at every point.

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{3-3}{h} = \lim_{h\to 0}\frac{0}{h} = 0$$

Similarly, $f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{3-3}{h} = 0$

16. **(d)** The derivative of f at x = a is denoted by

$$\left.\frac{d}{dx}f(x)\right|_a \text{ or } \left.\frac{df}{dx}\right|_a \text{ or even } \left(\frac{df}{dx}\right)_{x=a}$$

17. **(a)** Let y = x + a

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 1 + 0 = 1$$

18. **(b)** Let $y = \dfrac{1+\frac{1}{x}}{1-\frac{1}{x}} \Rightarrow y = \dfrac{x+1}{x-1}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(1+0)-(x+1)(1-0)}{(x-1)^2} = \frac{x-1-x-1}{(x-1)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{(x-1)^2} = \frac{-2}{(1-x)^2}$$

19. **(c)** Let $y = 4\sqrt{x}-2 \Rightarrow y = 4x^{1/2}-2$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 4.\frac{1}{2}x^{\frac{1}{2}-1} - 0 = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$$

20. **(b)** Let $y = (ax+b)^n$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = n(ax+b)^{n-1}\frac{d}{dx}(ax+b) = n(ax+b)^{n-1}a$$

$$\Rightarrow \frac{dy}{dx} = na(ax+b)^{n-1}$$

21. **(b)** Let $y = \sin^n x \Rightarrow y = (\sin x)^n$

Differentiating y w.r.t.x, we get

$$\frac{dy}{dx} = n(\sin x)^{n-1}\frac{d}{dx}(\sin x) \Rightarrow \frac{dy}{dx} = n(\sin x)^{n-1}\cos x$$

22. **(d)** Let $y = (x^2+1)\cos x$,

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = (x^2+1)\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x^2+1)$$

(by product rule)

$$= (x^2+1)(-\sin x) + \cos x(2x) = -x^2\sin x - \sin x + 2x\cos x$$

23. **(c)** We have, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\tan[a(x+h)+b]-\tan(ax+b)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(ax+ah+b)}{\cos(ax+ah+b)}-\frac{\sin(ax+b)}{\cos(ax+b)}}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(ax+ah+b)\cos(ax+b)-\sin(ax+b)\cos(ax+ah+b)}{}}{h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a\sin(ah)}{a.h\cos(ax+b)\cos(ax+ah+b)}$$

$$= \lim_{h\to 0}\frac{a}{\cos(ax+b)\cos(ax+ah+b)}\lim_{h\to 0}\frac{\sin ah}{ah}$$

[as $h\to 0$, $ah\to 0$]

$$= \frac{a}{\cos^2(ax+b)} = a\sec^2(ax+b)$$

24. (b) $\because f(x) = x\sin x$

$\Rightarrow \quad f'(x) = \dfrac{d}{dx}(x\sin x)$

$= \sin x\dfrac{d}{dx}x + x\dfrac{d}{dx}\sin x = \sin x + x\cos x$

$\Rightarrow f'\left(\dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} + \dfrac{\pi}{2}\cos\dfrac{\pi}{2} = 1$

25. (b) If given function is $6x^{100} - x^{55} + x$. Then, the derivative of function is $6.100.x^{99} - 55.x^{54} + 1$
or $600x^{99} - 55x^{54} + 1$

26. (b) $\lim\limits_{x\to 0}\dfrac{x}{\tan x} = 1$

27. (a) We have,

$\dfrac{d}{dx}(\log_x x) = \dfrac{d}{dx}(1) = 0 \quad [\because \log_x x = 1]$

28. (b) We have,

$\dfrac{d}{dx}(e^{3\log x}) = \dfrac{d}{dx}(e^{\log x^3}) = \dfrac{d}{dx}(x^3) = 3x^2 \quad [\because e^{\log k} = k]$

29. (b) We have,

$\dfrac{d}{dx}\left\{x^2 + \sin x + \dfrac{1}{x^2}\right\} = \dfrac{d}{dx}(x^2 + \sin x + x^{-2})$

$= \dfrac{d}{dx}(x^2) + \dfrac{d}{dx}(\sin x) + \dfrac{d}{dx}(x^{-2})$

$= 2x + \cos x + (-2)x^{-3}$

30. (b) We have,

$\dfrac{d}{dx}\left\{\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2\right\} = \dfrac{d}{dx}\left\{x + \dfrac{1}{x} + 2\right\}$

$= \dfrac{d}{dx}(x) + \dfrac{d}{dx}(x^{-1}) + \dfrac{d}{dx}(2) = 1 + (-1)x^{-2} + 0 = 1 - \dfrac{1}{x^2}$

31. (b) We have, $f(x) = \alpha x^n$
Differentiating both sides w.r.t. x, we obtain

$\dfrac{d}{dx}\{f(x)\} = \dfrac{d}{dx}(\alpha x^n)$

$\Rightarrow \quad f'(x) = \alpha\dfrac{d}{dx}(x^n) \Rightarrow f'(x) = \alpha n.x^{n-1}$

Putting x = 1 on both sides, we get

$f'(1) = \alpha.n \Rightarrow \alpha = \dfrac{f'(1)}{n}$

32. (c) We have,

$\dfrac{d}{dx}(x\sin x) = x.\dfrac{d}{dx}(\sin x) + \sin x.\dfrac{d}{dx}(x)$

$= x\cos x + \sin x \cdot 1 = x\cos x + \sin x.$

33. (a) We have,

$\lim\limits_{x\to 0}\dfrac{a^{\sin x} - 1}{\sin x} = \lim\limits_{y\to 0}\dfrac{a^y - 1}{y} = \log a$, where $y = \sin x$

$[\because x \to 0 \Rightarrow y = \sin x \to 0]$

34. (b) Consider $\lim\limits_{x\to 0}\dfrac{2\sin^2 3x}{x^2}$

$= 2.\lim\limits_{x\to 0}\left[\dfrac{\sin 3x}{x}\right]^2 = 2.\lim\limits_{x\to 0}\left[3\dfrac{\sin 3x}{3x}\right]^2$

$= 2.9.\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{3x}\right)^2 = 18\times 1 = 18$

35. (d) Consider $\lim\limits_{\theta\to 0}\dfrac{\sin m^2\theta}{\theta} = \lim\limits_{\theta\to 0}\left(\dfrac{\sin m^2\theta}{m^2\theta}\right).m^2 = 1\times m^2 = m^2$

36. (b) $f(x) = 7(-3)x^{-3-1} = -21x^{-4}$.

37. (b) $f'(x) = 2\cos x - 12x^3$

38. (d) Applying product rule,

$f'(x) = (x-1)\dfrac{d}{dx}(x-2) + (x-2)\dfrac{d}{dx}(x-1)$
$= x - 1 + x - 2 = 2x - 3$

39. (a) For $\lim\limits_{x\to a}\left[\dfrac{f(x)}{g(x)}\right]$ to exist, then both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ must exist.

40. (a) $\lim\limits_{x\to 0}\dfrac{1 + \dfrac{x}{3} - 1 + \dfrac{x}{3}}{x} = \lim\limits_{x\to 0}\dfrac{2x}{3x} = \dfrac{2}{3}$

41. (c) $\lim\limits_{x\to 0}\dfrac{\cos x}{\pi - x} = \dfrac{1}{\pi - 0} = \dfrac{1}{\pi}$

42. (b) $3f(x) - 2f\left(\dfrac{1}{x}\right) = x \quad \text{...(i)}$

Put $x = \dfrac{1}{x}$, then $3f\left(\dfrac{1}{x}\right) - 2f(x) = \dfrac{1}{x} \quad \text{...(ii)}$

Solving (i) and (ii), we get

$5f(x) = 3x + \dfrac{2}{x} \Rightarrow f'(x) = \dfrac{3}{5} - \dfrac{2}{5x^2}$

$\therefore \ f'(2) = \dfrac{3}{5} - \dfrac{2}{20} = \dfrac{1}{2}$

43. **(a)** Given function is $f(x)=\dfrac{7x}{(2x-1)(x+3)}$

Breaking into partial fraction

We get, $f(x)=\dfrac{1}{2x-1}+\dfrac{3}{x+3}$

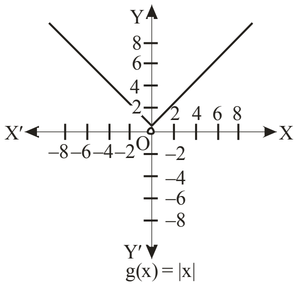
Differentiating w.r.t. x, we get

$$f'(x)=-\frac{2}{(2x-1)^2}-\frac{3}{(x+3)^2}$$

44. **(a)**

STATEMENT TYPE QUESTIONS

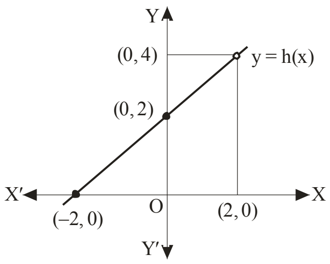
45. **(b)** Given function $g(x)=|x|, x\neq 0$. Observe that g (0) is not defined. Now, on computing the value of g (x) for values of x very near to 0, we see that the value of g(x) moves towards 0. So, $\lim_{x\to 0} g(x)=0$. This is intuitively clear from the graph of $y=|x|$ for $x\neq 0$.



46. **(a)** Given, the following function.

$h(x)=\dfrac{x^2-4}{x-2}, x\neq 2$

Now, on computing the value of h (x) for values of x very near to 2 (but not at x = 2), we get all these values are near to 4.



This is somewhat strengthened by considering the graph of the function y = h (x).

47. **(a)** I. Given,

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\div\frac{x^{10}-1}{x-1}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^{15}-1}{x-1}\right]\div\lim_{x\to 1}\left[\frac{x^{10}-1}{x-1}\right]$$

$$=15(1)^{14}\div 10(1)^9=15\div 10=\frac{3}{2}$$

II. Put $y=1+x$, so that $y\to 1$ as $x\to 0$.

$$\text{Then, }\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{y\to 1}\frac{\sqrt{y}-1}{y-1}$$

$$=\lim_{y\to 1}\frac{y^{\frac12}-1^{\frac12}}{y-1}=\frac{1}{2}(1)^{\frac12-1}=\frac{1}{2}$$

48. **(a)** I. $\lim_{x\to 0}\dfrac{\sin x}{x}=1$ (Standard Result)

II. Let us recall the trigonometric identity

$1-\cos x=2\sin^2\left(\dfrac{x}{2}\right)$.

$$\text{Then, }\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x}$$

$$=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\sin\left(\frac{x}{2}\right)=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}.\lim_{x\to 0}\sin\left(\frac{x}{2}\right)$$

$=1.0=0$

Observe that, we have implicity used the fact that $x\to 0$ is equivalent to $\dfrac{x}{2}\to 0$. This may be justified by putting $y=\dfrac{x}{2}$.

49. **(b)** I. Given, $\lim_{x\to 1}\dfrac{ax^2+bx+c}{cx^2+bx+a}=\dfrac{a\times(1)^2+b\times 1+c}{c\times(1)^2+b\times 1+a}$

$$=\frac{a+b+c}{c+b+a}=1$$

II. $$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to -2}\frac{(2+x)}{2x(x+2)}$$

$$=\lim_{x\to -2}\frac{1}{2x}=\frac{1}{2(-2)}=-\frac{1}{4}$$

50. **(c)** We have $\lim_{h\to 0}\dfrac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\frac{(a^2+h^2+2ah)\,[\sin a\cos h+\cos a\sin h]-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a(\cos h-1)}{h}+\frac{a^2\cos a\sin h}{h}+(h+2a)(\sin a\cos h+\cos a\sin h)\right]$$

$$=\lim_{h\to 0}\left[\frac{a^2\sin a\left(-2\sin^2\frac{h}{2}\right)}{\frac{h^2}{2}}.\frac{h}{2}\right]+\lim_{h\to 0}\frac{a^2\cos a\sin h}{h}+\lim_{h\to 0}(h+2a)\sin(a+h)$$

$= a^2\sin a\times 0+a^2\cos a(1)+2a\sin a=a^2\cos a+2a\sin a.$

51. **(a)** I. Recall the trigonometric rule $\sin 2x=2\sin x\cos x$. Thus,

$$\frac{df(x)}{dx}=\frac{d}{dx}(2\sin x\cos x)=2\frac{d}{dx}(\sin x\cos x)$$

$= 2[(\sin x)'\cos x+\sin x(\cos x)']$

$= 2[(\cos x)\cos x+\sin x(-\sin x)]$

$= 2(\cos^2 x-\sin^2 x)$

II. $g(x) = \cot x = \dfrac{\cos x}{\sin x}$

$$\Rightarrow \frac{d}{dx}(g(x)) = \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$
$$= \frac{(\cos x)'(\sin x) - (\cos x)(\sin x)'}{(\sin x)^2}$$
$$= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$
$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\operatorname{cosec}^2 x$$

52. (c) I. Let $f(x) = x^2 - 2$, we have

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{[(x+h)^2 - 2] - (x^2 - 2)}{h}$$
$$= \lim_{h\to 0}\frac{x^2 + h^2 + 2xh - 2 - x^2 + 2}{h}$$
$$= \lim_{h\to 0}\frac{h(h+2x)}{h} = 0 + 2x = 2x$$

At $x = 10$, $f'(10) = 2 \times 10 = 20$

II. Let $f(x) = 99x$

We have $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{99(x+h) - 99x}{h}$$
$$= \lim_{h\to 0}\frac{99x + 99h - 99x}{h} = \lim_{h\to 0}\frac{99h}{h} = 99$$

At $x = 100$, $f'(100) = 99$

III. Let $f(x) = x$

We have, $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{x+h-x}{h} \Rightarrow f'(x) = \lim_{h\to 0}\frac{h}{h} = 1$$

At $x = 1$, $f'(1) = 1$

53. (b) I. We have, $y = 2x - \dfrac{3}{4}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 2 \times 1 - 0 = 2$$

II. We have, $y = (5x^3 + 3x - 1)(x - 1)$

Differentiating y w.r.t x, we get

$$\frac{dy}{dx} = (5x^3 + 3x - 1)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(5x^3 + 3x - 1)$$
$$= (5x^3 + 3x - 1)(1 - 0) + (x - 1)(5 \times 3x^2 + 3 \times 1 - 0)$$
$$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$$
$$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$$
$$= 20x^3 - 15x^2 + 6x - 4$$

54. (d) I. $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h}$$
$$= \lim_{h\to 0}\frac{x^3 + h^3 + 3xh(x+h) - x^3}{h}$$
$$= \lim_{h\to 0}(h^2 + 3x(x+h)) = 3x^2$$

II. $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}\frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} \quad \left[\because f(x) = \frac{1}{x^3}\right]$$
$$= \lim_{h\to 0}\frac{x^3 - (x+h)^3}{(x+h)^3 x^3 h}$$
$$= \lim_{h\to 0}\frac{x^3 - [x^3 + h^3 + 3xh(x+h)]}{(x+h)^3 x^3 h}$$
$$= \lim_{h\to 0}\frac{-h^3 - 3xh(x+h)}{(x+h)^3 x^3 h}$$
$$= \lim_{h\to 0}\frac{-h[h^2 + 3x(x+h)]}{(x+h)^3 x^3 h} = \frac{-3}{x^4}$$

55. (a) I. Let $f(x) = -x$

We have, $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

(by first principle)

$$= \lim_{h\to 0}\frac{-(x+h) - (-x)}{h} = \lim_{h\to 0}\frac{-x-h+x}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{-h}{h} = -1$$

II. Let $f(x) = (-x)^{-1}$

$$\Rightarrow f(x) = -\frac{1}{x}$$

We have, $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ (by first principle)

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{-\frac{1}{x+h} + \frac{1}{x}}{h}$$
$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\frac{1}{x} - \frac{1}{x+h}}{h} = \lim_{h\to 0}\frac{x+h-x}{x(x+h)h}$$
$$= \lim_{h\to 0}\frac{h}{x(x+h)h} = \frac{1}{x(x+0)} = \frac{1}{x^2}$$

56. (b) I. Let $y = \sin(x + a)$

$y = \sin x \cos a + \cos x \sin a$

$[\because \sin(A + B) = \sin A \cos B + \cos A \sin B]$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \cos a\frac{d}{dx}(\sin x) + \sin a\frac{d}{dx}(\cos x)$$
$$= \cos a \cos x - \sin a \sin x = \cos(x + a)$$

II. Let $y = \operatorname{cosec} x \cot x$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \operatorname{cosec} x\frac{d}{dx}(\cot x) + \cot x\frac{d}{dx}(\operatorname{cosec} x)$$
$$= -\operatorname{cosec} x \operatorname{cosec}^2 x + \cot x(-\operatorname{cosec} x \cot x)$$
$$= -\operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x$$

57. **(d)** I. The derivative of the function is $1 + 2x + 3x^2 + ... + 50x^{49}$. At x = 1 the value of this function equals to

$1+2(1)+3(1)^2+...+50(1)^{49}=1+2+3+...+50$

$= \dfrac{(50)(51)}{2} = 1275$

II. Clearly, this function is defined everywhere except at x = 0. We use the quotient rule with u = x + 1 and v = x. Hence, u′ = 1 and v′ = 1. Therefore,

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{x+1}{x}\right) = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$$

$$= \frac{1(x)-(x+1)1}{x^2} = -\frac{1}{x^2}$$

58. **(d)** **59.** **(d)**

MATCHING TYPE QUESTIONS

60. **(b)** We say $\lim\limits_{x\to a^-} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the left of *a*. This value is called the left hand limit of *f* at *a*.

Now, $\lim\limits_{x\to a^+} f(x)$ is the expected value of *f* at *x* = *a* given the values of *f* near *x* to the right of *a*. This value is called the right hand limit of f (x) at *a* and if the right and left hand limits coincide, we call that common value as the limit of f(x) at *x* = *a* and denote it by $\lim\limits_{x\to a} f(x)$.

61. **(b)** A. $\lim\limits_{x\to 3} x+3 = 3+3 = 6$

B. $\lim\limits_{x\to \pi}\left(x-\dfrac{22}{7}\right) = \pi - \dfrac{22}{7}$

C. $\lim\limits_{r\to 1} \pi r^2 = \pi \times (1)^2 = \pi$

D. $\lim\limits_{x\to 4}\dfrac{4x+3}{x-2} = \dfrac{4\times4+3}{4-2} = \dfrac{19}{2}$

E. $\lim\limits_{x\to -1}\dfrac{x^{10}+x^5+1}{x-1} = \dfrac{(-1)^{10}+(-1)^5+1}{-1-1} = \dfrac{1-1+1}{-2} = \dfrac{-1}{2}$

62. **(d)** A. Given, $\lim\limits_{x\to\pi}\dfrac{\sin(\pi-x)}{\pi(\pi-x)}$

Let $\pi - x = h$, As $x \to \pi$, then $h \to 0$

$$\therefore \lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)} = \lim_{h\to0}\frac{\sin h}{\pi h} = \lim_{h\to0}\frac{1}{\pi}\times\frac{\sin h}{h}$$

$$= \frac{1}{\pi}\times 1 = \frac{1}{\pi} \quad \left(\because \lim_{h\to0}\frac{\sin h}{h} = 1\right)$$

B. Given $\lim\limits_{x\to 0}\dfrac{\cos x}{\pi - x}$

Put the limit directly, we get $\dfrac{\cos 0}{\pi - 0} = \dfrac{1}{\pi}$

C. Given, $\lim\limits_{x\to0}\dfrac{\cos 2x-1}{\cos x-1} = \lim\limits_{x\to0}\dfrac{1-\cos 2x}{1-\cos x} = \lim\limits_{x\to0}\dfrac{2\sin^2 x}{2\sin^2\frac{x}{2}}$

$$\left(\because\ 1-\cos 2x = 2\sin^2 x \text{ and } 1-\cos x = 2\sin^2\frac{x}{2}\right)$$

Multiplying and dividing by x^2 and then multiplying by $\dfrac{4}{4}$ in the numerator,

$$= \lim_{x\to0}\frac{\sin^2 x}{x^2}\times\frac{4\times\frac{x^2}{4}}{\sin^2\frac{x}{2}} = \lim_{x\to0}\left(\frac{\sin x}{x}\right)^2\times\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2\times 4$$

$= 1 \times 1 \times 4 = 4$

D. Given, $\lim\limits_{x\to0}\dfrac{ax + x\cos x}{b\sin x}$

Dividing each term by x, we get

$$= \lim_{x\to0}\frac{\frac{ax}{x}+\frac{x\cos x}{x}}{\frac{b\sin x}{x}} = \lim_{x\to0}\frac{a+\cos x}{b\left(\frac{\sin x}{x}\right)}$$

$$= \frac{a+\cos 0}{b\times 1} = \frac{a+1}{b} \quad \left(\because \lim_{x\to0}\frac{\sin x}{x} = 1\right)$$

E. $\lim\limits_{x\to0} x\sec x = 0\times\sec 0 = 0\times1 = 0$

F. $\lim\limits_{x\to0}\dfrac{\sin ax + bx}{ax + \sin bx}$

Dividing each term by x,

$$= \lim_{x\to0}\frac{\frac{\sin ax}{x}+\frac{bx}{x}}{\frac{ax}{x}+\frac{\sin bx}{x}} = \lim_{x\to0}\frac{\frac{a\sin ax}{ax}+b}{a+\frac{b\sin bx}{bx}}$$

$$= \frac{a\times1+b}{a+b\times1} = \frac{a+b}{a+b} = 1 \quad \left(\because \lim_{x\to0}\frac{\sin x}{x} = 1\right)$$

63. **(d)** A. Let $y = \operatorname{cosec} x = \dfrac{1}{\sin x}$

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sin x\frac{d}{dx}(1)-(1)\frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x\times0-1\times\cos x}{\sin^2 x}$$

$$= \frac{0-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x}\times\frac{1}{\sin x}$$

$$\Rightarrow \frac{dy}{dx} = -\cot x \operatorname{cosec} x$$

B. Let y = 3 cot x + 5 cosec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = -3\operatorname{cosec}^2 x - 5\operatorname{cosec} x\cot x$$

C. Let y = 5 sin x – 6 cos x + 7

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 5\cos x - 6(-\sin x) + 0 = 5\cos x + 6\sin x$$

D. Let y = 2 tan x – 7 sec x

Differentiating y w.r.t. x, we get

$$\frac{dy}{dx} = 2\sec^2 x - 7\sec x\tan x$$

64. (a) A. Since, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{10(x+h)-10(x)}{h}=\lim_{h\to 0}\frac{10h}{h}$

$=\lim_{h\to 0}(10)=10$

B. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$

$=\lim_{h\to 0}(h+2x)=2x$

C. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{a-a}{h}=\lim_{h\to 0}\frac{0}{h}=0$ (as $h\neq 0$)

D. We have, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$

$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]$

$=\lim_{h\to 0}\frac{-1}{x(x+h)}=\frac{-1}{x^2}$

INTEGER TYPE QUESTIONS

65. (b) $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\times\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}=\lim_{x\to 0}\frac{(2+x)-2}{x\left[\sqrt{2+x}+\sqrt{2}\right]}$

$=\lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}}=\frac{1}{2\sqrt{2}}$

66. (c) $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}$

$=\lim_{x\to a}\frac{(a+2x)-3x}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

Again rationalizing, we get

$=\lim_{x\to a}\frac{(a-x)\left[\sqrt{3a+x}+2\sqrt{x}\right]}{\left(\sqrt{a+2x}+\sqrt{3x}\right)(3a-3x)}=\frac{4\sqrt{a}}{6\sqrt{3a}}$

$=\frac{2\sqrt{3}}{9}$

67. (a) Put $y=\frac{\pi}{2}-x$

$\therefore\ \lim_{x\to\pi/2}(\sec x-\tan x)=\lim_{y\to 0}\left[\sec\left(\frac{\pi}{2}-y\right)-\tan\left(\frac{\pi}{2}-y\right)\right]$

$=\lim_{y\to 0}[\mathrm{cosec}\, y-\cot y]=\lim_{y\to 0}\left[\frac{1-\cos y}{\sin y}\right]$

$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}=\lim_{\frac{y}{2}\to 0}\tan\frac{y}{2}=0$

68. (c) $\lim_{x\to 1}f(x)=f(1)$

i.e. RHL = LHL = f (1)

$\Rightarrow\ \lim_{x\to 1^+}f(x)=\lim_{x\to 1^{-1}}f(x)=4$

$\lim_{h\to 0}f(1+h)=\lim_{h\to 0}f(1-h)=4$

$\Rightarrow\ \lim_{h\to 0}\ b-a\,(1+h)=\lim_{h\to 0}\ a+b(1-h)=4$

$\Rightarrow\ b-a(1+0)=a+b(1-0)=4$

$\Rightarrow\ b-a=4$ and $b+a=4$

On solving, we get $a=0$, $b=4$

69. (d) $\lim_{x\to 0}\frac{\sin(2+x)-\sin(2-x)}{x}$

$=\lim_{x\to 0}\frac{2\cos\frac{(2+x+2-x)}{2}\sin\frac{(2+x-2+x)}{2}}{x}$

$=\lim_{x\to 0}\frac{(2\cos 2)\sin x}{x}=2\cos 2$

$\Rightarrow\ p=2$ and $q=2$.

70. (c) At $x=5$, RHL $=\lim_{x\to 5^+}f(x)$

$=\lim_{h\to 0}f(5+h)=\lim_{h\to 0}|5+h|-5=0$

L.H.L $=\lim_{x\to 5}f(x)=\lim_{h\to 0}f(5-h)$

$=\lim_{h\to 0}|5-h|-5=0$

Hence, RHL = LHL $=\lim_{x\to 5}f(x)=0$

71. (b) $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{x\left[2\cos^2\frac{x}{2}\right]}$

$=\lim_{x\to 0}\frac{\tan x/2}{2.\frac{x}{2}}=\frac{1}{2}.\lim_{\frac{x}{2}\to 0}\frac{\tan\frac{x}{2}}{\frac{x}{2}}=\frac{1}{2}$

72. (b) $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{4x}{\sin 2x}\times\frac{2x}{2x}$

$=\lim_{x\to 0}\frac{\sin 4x}{4x}\times\frac{2x}{\sin 2x}\times\frac{4x}{2x}=\frac{4}{2}=2$

$(\because x\to 0\Rightarrow 4x\to 0 \text{ and } 2x\to 0)$

73. (d) At $x=0$,

RHL $=\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}3(0+h+1)=3$

LHL $=\lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}2(0-h)+3=3$

Hence, RHL = LHL $=\lim_{x\to 0}f(x)=3$

74. (a) At $x=-1$, limit exists.

$\therefore$ RHL = LHL

$\Rightarrow\ \lim_{x\to -1^+}f(x)=\lim_{x\to -1^-}f(x)$

$\Rightarrow\ \lim_{h\to 0}f(-1+h)=\lim_{h\to 0}f(-1-h)$

$\Rightarrow\ \lim_{h\to 0}c\,(-1+h)^2=\lim_{h\to 0}(-1-h+2)$

$\Rightarrow\ c\,(-1+0)^2=1-0\Rightarrow c=1$

75. **(d)** We have

$$\lim_{x\to\frac{\pi}{4}}\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{2^{\frac{5}{2}}-[(\cos x+\sin x)^2]^{\frac{5}{2}}}{2-(1+\sin 2x)}=\lim_{x\to\frac{\pi}{4}}\frac{(1+\sin 2x)^{\frac{5}{2}}-2^{\frac{5}{2}}}{(1+\sin 2x)-2}$$

$$=\lim_{t\to 2}\frac{t^{\frac{5}{2}}-2^{\frac{5}{2}}}{t-2}\text{, where } t=1+\sin 2x=\frac{5}{2}\times(2)^{\frac{5}{2}-1}=5\sqrt{2}$$

76. **(d)** $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80$

$\Rightarrow n.2^{n-1}=80 \Rightarrow n.2^{n-1}=5.2^{5-1} \Rightarrow n=5$

77. **(a)** $\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2=2.\lim_{x\to 0}\frac{\sin 2x}{2x}=2\times 1=2.$

78. **(d)** Let $f(x)=x^n$

$f'(x)=n.x^{n-1}$

$f'(1)=n.1^{n-1}=n$

$10=n$

79. **(b)** Let $\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$

By using $\lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}$, we have

$k.5^{k-1}=500$

Now, put $k=4$, we get

$4.5^{4-1}=500 \Rightarrow 4.5^3=500$

which is true.

$\therefore\ k=4$

ASSERTION- REASON TYPE QUESTIONS

80. **(c)** Assertion is correct but Reason is incorrect.

81. **(c)** Assertion is correct

$$\lim_{x\to 0}\left[\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right]=\lim_{x\to 0}\frac{1-\cos x}{\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}=\lim_{x\to 0}\tan\frac{x}{2}=0$$

82. **(c)** **Assertion** is correct but Reason is incorrect.

Reason: $f(x)=ax^2+bx+c$

$f'(x)=2ax+b$

83. **(b)** Both Assertion and Reason are correct but reason is not the correct explanation.

84. **(b)** Both Assertion and Reason are correct.

85. **(a)** We know that $\frac{d}{dx}(x^n)=nx^{n-1}$

$$\therefore \text{For } f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+.....+\frac{x^2}{2}+x+1$$

$$f'(x)=\frac{100x^{99}}{100}+99\frac{x^{98}}{99}+.....+\frac{2x}{2}+1$$

$=x^{99}+x^{98}+..........+x+1$

Now, $f'(1)=1+1+$..........to 100 term $=100$

$f'(0)=1$

$\therefore f'(1)=100\times 1=100f'(0)$

Hence, $f'(1)=100f'(0)$

86. **(a)** $\lim_{x\to 0}(1+3x)^{1/x}=\lim_{x\to 0}\left[\left(1+3x^{1/3x}\right)\right]^3=e^3$

because $\lim_{x\to 0}(1+x)^{1/x}=e$

87. **(c)** Obviously Assertion is true, but Reason is not always true.

Consider, $f(x)=[x]$ and $g(x)=\sin x$.

88. **(b)** $\because\ \lim_{x\to 0}\frac{\tan x^0}{x^0}=\lim_{x\to 0}\frac{\tan\left(\frac{\pi x}{180}\right)}{\left(\frac{\pi x}{180}\right)}=1$

and $\lim_{x\to 0}\{f(x)g(x)\}=\left(\lim_{x\to 0}f(x)\right)\left(\lim_{x\to 0}g(x)\right)=lm$

89. **(a)** **Assertion:** Let $u=x$, $v=|x|$

90. **(b)** Both Assertion and Reason are correct.

Reason: $f'(2)=\lim_{h\to 0}\frac{\{(2+h)^2+1\}-\{2^2+1\}}{h}$

$$=\lim_{h\to 0}\frac{h^2+4h}{h}=\lim_{h\to 0}h+4=4\Rightarrow f'(2)=4$$

91. **(a)**

CRITICALTHINKING TYPE QUESTIONS

92. **(a)** $\lim_{x\to 0}\frac{\sin^2 2x}{x^2}$

$$=\lim_{x\to 0}\frac{(2\sin x\cos x)^2}{x^2}=4\lim_{x\to 0}\frac{\sin^2 x}{x^2}.\cos^2 x=4$$

93. **(c)** $\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}=\lim_{x\to 0}\left(\frac{x^3\cot x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\right)$

$$=\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3\times\lim_{x\to 0}\cos x\times\lim_{x\to 0}(1+\cos x)=2$$

94. **(b)** $\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}=\lim_{x\to 0}\frac{2x^2(e^x-1)}{4\sin^2\frac{x}{2}}$

$$=2\lim_{x\to 0}\left[\frac{(x/2)^2}{\sin^2(x/2)}\right]\left(\frac{e^x-1}{x}\right)=2$$

95. **(d)** $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2}x}$;

$$=\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}=\lim_{x\to 0}\frac{|\sin x|}{x}$$

The limit of above does not exist as

LHS $=-1\neq$ RHL $=1$

96. **(c)** Given expression can be written as

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}$$

$$= \lim_{x\to 0} \frac{x}{4\sin^4 x}\left[\frac{2\tan x}{1-\tan^2 x} - 2\tan x\right]$$

$$= \lim_{x\to 0} \frac{2x\tan x}{4\sin^4 x}\left[\frac{1-1+\tan^2 x}{1-\tan^2 x}\right]$$

$$= \lim_{x\to 0} \frac{2x\tan^3 x}{4\sin x^4(1-\tan^2 x)}$$

$$= \frac{1}{2}\lim_{x\to 0}\frac{x}{\sin x}.\frac{1}{\cos^3 x}.\frac{1}{1-\tan^2 x} = \frac{1}{2}.1.\frac{1}{1^3}.\frac{1}{1-0} = \frac{1}{2}$$

97. (b) $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2} = \lim_{x\to 0}\frac{\sin(\pi - \pi\sin^2 x)}{x^2}$

$[\because \sin(\pi-\theta) = \sin\theta]$

$$= \lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{(\pi\sin^2 x)}{x^2} = \pi$$

98. (d) Put $\theta + \frac{\pi}{4} = h$ or $\theta = -\frac{\pi}{4} + h$

$$\text{Limit} = \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{4}-h\right) - \sin\left(\frac{\pi}{4}-h\right)}{h}$$

$$= \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{4}-h\right) - \cos\left(\frac{\pi}{4}+h\right)}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{\pi}{4}.\sin h}{h} = \sqrt{2}$$

99. (c) $\text{LHL} = \lim_{h\to 0}\frac{-h+|h|}{-h} = \lim_{h\to 0}(0) = 0$

$\text{RHL} = \lim_{h\to 0}\frac{h+|h|}{h} = 2$

LHL $\neq$ RHL $\Rightarrow$ limit does not exist

100. (d) $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$
$= 2f(x)g(x) + 2g(x)f''(x)$
$= 2f(x)g(x) - 2f(x)g(x)$
$= 0 \quad [\because f''(x) = -f(x)]$

$\Rightarrow \quad h(x) = c \Rightarrow h(10) = h(5) = 11$

101. (a) Given limit $= \lim_{x\to\alpha}\frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha}\frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2}{(x-\alpha)^2}\times\frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}}\times\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$= \frac{a^2(\alpha-\beta)^2}{2}.$$

102. (d) We are given that

$$\lim_{x\to 0}\frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0$$

where n is non zero real number

$$\Rightarrow \lim_{x\to 0} n.\frac{\sin nx}{nx}\left[\left\{(a-n)n - \frac{\tan x}{x}\right\}\right] = 0$$

$$\Rightarrow 1.n[(a-n)n - 1] = 0 \Rightarrow a = \frac{1}{n} + n$$

103. (b) $\sin y = x\sin(a+y)$

$$\therefore x = \frac{\sin y}{\sin(a+y)}$$

Differentiating the function with respect to y

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

104. (c) Let $y = x\tan\frac{x}{2} \Rightarrow \frac{dy}{dx} = 1.\tan\frac{x}{2} + x.\sec^2\frac{x}{2}.\frac{1}{2}$

$$= \tan\frac{x}{2} + \frac{x}{2}\sec^2\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} + \frac{x}{2\cos^2\frac{x}{2}}$$

$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2} + x}{2\cos^2\frac{x}{2}} = \frac{\sin x + x}{1+\cos x}$$

$$\Rightarrow (1+\cos x)\frac{dy}{dx} - \sin x = x$$

105. (d) $\frac{d}{dx}\left(\frac{x\sin x}{1+\cos x}\right)$

$$= \frac{(1+\cos x)(\sin x + x\cos x) - (x\sin x)(0-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\sin x(1+\cos x) + x\cos x + x(\cos^2 x + \sin^2 x)}{(1+\cos x)^2}$$

$$= \frac{(x+\sin x)(1+\cos x)}{(1+\cos x)^2} = \frac{x+\sin x}{1+\cos x}$$

106. (b) Differentiating w.r.t. x,

$$3x^2 + 3y^2\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

$$\Rightarrow 3(x^2 - y) = 3\frac{dy}{dx}(x - y^2) \Rightarrow \frac{dy}{dx} = \frac{x^2-y}{x-y^2}$$

107. (a) $y = ax^{n+1} + bx^{-n}$

$$\frac{dy}{dx} = (n+1)ax^n - n\,bx^{-n-1}$$

$$\frac{d^2y}{dx^2} = (n+1)n\,ax^{n-1} + n(n+1)bx^{-n-2}$$

$$\therefore x^2\frac{d^2y}{dx^2} = (n+1)\,na.\,x^{n+1} + n(n+1)\,b\,x^{-n}$$

$$= n(n+1)[ax^{n+1} + bx^{-n}] = n(n+1)y$$

108. (c) We have, $y = f\left(\frac{2x-1}{x^2+1}\right)$

$\Rightarrow \frac{dy}{dx} = f'\left(\frac{2x-1}{x^2+1}\right).\left[\frac{(x^2+1)2-(2x-1).2x}{(x^2+1)^2}\right]$

$= \sin\left(\frac{2x-1}{x^2+1}\right)^2.\left[\frac{2+2x-2x^2}{(x^2+1)}\right]$

$\left[\because f'(x) = \sin x^2, \therefore f'\left(\frac{2x-1}{x^2+1}\right) = \sin\left(\frac{2x-1}{x^2+1}\right)^2\right]$

109. (b) We have,

$\lim_{x\to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x}\right]$

$= \lim_{x\to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x(x-1)(x-2)}\right]$

$= \lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$

$= \lim_{x\to 2}\left[\frac{x^2-5x+6}{x(x-1)(x-2)}\right]$

$= \lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right] \quad (x-2\neq 0)$

$= \lim_{x\to 2}\left[\frac{x-3}{x(x-1)}\right] = \frac{-1}{2}$

110. (d) We have, $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = n(3)^{n-1}$

Therefore, $n(3)^{n-1} = 108 = 4(27) = 4(3)^{4-1}$

On comparing, we get $n = 4$

111. (c) We have, $\lim_{x\to 0}\frac{\cos ax - \cos bx}{\cos cx - 1}$

$= \lim_{x\to 0}\frac{2\sin\left[\frac{(a+b)}{2}x\right]\sin\left(\frac{(a-b)x}{2}\right)}{2\sin^2\left(\frac{cx}{2}\right)}$

$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}.\sin\frac{(a-b)x}{2}}{x^2}.\frac{x^2}{\sin^2\frac{cx}{2}}$

$= \lim_{x\to 0}\frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}.\left(\frac{2}{a+b}\right)}.\frac{\sin\frac{(a-b)x}{2}}{\frac{(a-b)x}{2}.\frac{2}{a-b}}.\frac{\left(\frac{cx}{2}\right)^2\times\frac{4}{c^2}}{\sin^2\frac{cx}{2}}$

$= \left(\frac{a+b}{2}\times\frac{a-b}{2}\times\frac{4}{c^2}\right) = \frac{a^2-b^2}{c^2}$. Hence m and n are a^2-b^2 and c^2 respectively.

112. (d) Since, RHL $= \lim_{x\to 1^+}[x-1] = 0$

and LHL $= \lim_{x\to 1^-}[x-1] = -1$

Hence, at $x = 1$ limit does not exist.

113. (a) We have, $\lim_{x\to 0}\frac{\tan x - \sin x}{\sin^3 x} = \lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{\sin^3 x}$

$= \lim_{x\to 0}\frac{1-\cos x}{\cos x\sin^2 x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\left(4\sin^2\frac{x}{2}.\cos^2\frac{x}{2}\right)} = \frac{1}{2}$

114. (b) Let $y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$

$\Rightarrow \quad y = ax^{-4} - bx^{-2} + \cos x$

Differentiating y w.r.t. x, we get

$\frac{dy}{dx} = a\frac{d}{dx}(x^{-4}) - b\frac{d}{dx}(x^{-2}) + \frac{d}{dx}(\cos x)$

$= a(-4)x^{-4-1} - b(-2)x^{-2-1}(-\sin x)$

$= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x \qquad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

115. (a) Let $y = \frac{\sin(x+a)}{\cos x} = \frac{\sin x\cos a + \cos x\sin a}{\cos x}$

$[\because \sin(A+B) = \sin A\cos B + \cos A\sin B]$

$= \frac{\sin x\cos a}{\cos x} + \frac{\cos x\sin a}{\cos x} = \cos a\tan x + \sin a$

Differentiating y w.r.t. x, we get

$\frac{dy}{dx} = \cos a\frac{d}{dx}(\tan x) + \frac{d}{dx}(\sin a)$

$= \cos a\sec^2 x + 0 = \frac{\cos a}{\cos^2 x}$

116. (d) Let $f(x) = \frac{|x-4|}{x-4}$

At $x=4$, RHL $= \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0}\frac{|4+h-4|}{(4+h-4)}$

$= \lim_{h\to 0}\left(\frac{4+h-4}{4+h-4}\right) = 1$

At $x = 4$, LHL $= \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h)$

$= \lim_{h\to 0}\frac{|4-h-4|}{(4-h-4)} = \lim_{h\to 0}\frac{-(4-h-4)}{(4-h-4)} = -1$

$\because$ RHL $\neq$ LHL

$\therefore$ Hence, at $x = 4$, limit does not exist.

117. (c) Given, $f(x) = \begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\frac{\pi}{2}\\ 3, & x = \frac{\pi}{2}\end{cases}$

Since, $\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) \Rightarrow \lim_{x\to\frac{\pi}{2}} f(x) = 3$

$\Rightarrow \lim_{h\to 0} f\left(\frac{\pi}{2}+h\right) = 3 \Rightarrow \lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)} = 3$

$\Rightarrow \quad \lim_{h\to 0}\dfrac{-k\sin h}{\pi-\pi-2h}=3 \Rightarrow \lim_{h\to 0}\dfrac{-k\sin h}{-2h}=3$

$\Rightarrow \quad \dfrac{k}{2}\times\lim_{h\to 0}\dfrac{\sin h}{h}=3 \Rightarrow \dfrac{k}{2}\times 1=3$

$\Rightarrow \quad k=6 \quad \left(\because \lim_{h\to 0}\dfrac{\sin h}{h}=1\right)$

118. (d) We have, $f(x)=|\cos x-\sin x|$

$$\Rightarrow \quad f(x)=\begin{cases}\cos x-\sin x, \text{ for } 0<x\le\dfrac{\pi}{4}\\ \sin x-\cos x, \text{ for } \dfrac{\pi}{4}<x<\dfrac{\pi}{2}\end{cases}$$

Clearly, $Lf'\left(\dfrac{\pi}{4}\right)=\left\{\dfrac{d}{dx}(\cos x-\sin x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(-\sin x-\cos x)_{x=\frac{\pi}{4}}=-\sqrt{2}$

and $Rf'\left(\dfrac{\pi}{4}\right)=\left\{\dfrac{d}{dx}(\sin x-\cos x)\right\}_{\text{at } x=\frac{\pi}{4}}$

$=(\cos x+\sin x)_{x=\frac{\pi}{4}}=\sqrt{2}$

$\because \quad Lf'\left(\dfrac{\pi}{4}\right)\neq Rf'\left(\dfrac{\pi}{4}\right)$

$\therefore \quad f'\left(\dfrac{\pi}{4}\right)$ doesn't exist.

119. (d) $f(x)+f(y)=f\left(\dfrac{x+y}{1-xy}\right)$ …(i)

Putting $x=y=0$, we get $f(0)=0$

Putting $y=-x$, we get $f(x)+f(-x)=f(0)=0$

$\Rightarrow \quad f(-x)=-f(x)$ ...(ii)

Also, $\lim_{x\to 0}\dfrac{f(x)}{x}=2$ ...(iii)

Now, $\quad f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

[using eq. (ii) $-f(x)=f(-x)$]

$$f'(x)=\lim_{h\to 0}\dfrac{f\left(\dfrac{x+h-x}{1-(x+h)(-x)}\right)}{h}$$

[using eq. (i)]

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\left[\dfrac{f\left(\dfrac{h}{1+x(x+h)}\right)}{h}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\dfrac{f\left(\dfrac{h}{1+xh+x^2}\right)}{\left(\dfrac{h}{1+xh+x^2}\right)}\times\left[\dfrac{1}{1+xh+x^2}\right]$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\dfrac{f\left(\dfrac{h}{1+xh+x^2}\right)}{\left(\dfrac{h}{1+xh+x^2}\right)}\times\lim_{h\to 0}\dfrac{1}{1+xh+x^2}$$

$\left[\text{using } \lim_{x\to 0}\dfrac{f(x)}{x}=2\right]$

$\Rightarrow \quad f'(x)=2\times\dfrac{1}{1+x^2}=\dfrac{2}{1+x^2}$

$\Rightarrow \quad f'\left(\dfrac{1}{\sqrt{3}}\right)=\dfrac{2}{1+\frac{1}{3}}=\dfrac{2}{4/3}=\dfrac{6}{4}=\dfrac{3}{2}$

120. (c) We first find the derivatives of $f(x)$ at $x=-1$ and at $x=0$.

We have,

$f'(-1)=\lim_{h\to 0}\dfrac{f(-1+h)-f(-1)}{h}$

$=\lim_{h\to 0}\dfrac{[2(-1+h)^2+3(-1+h)-5]-[2(-1)^2+3(-1)-5]}{h}$

$=\lim_{h\to 0}\dfrac{2h^2-h}{h}=\lim_{h\to 0}(2h-1)=2(0)-1=-1$

and $f'(0)=\lim_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\dfrac{[2(0+h)^2+3(0+h)-5]-[2(0)^2+3(0)-5]}{h}$

$=\lim_{h\to 0}\dfrac{2h^2+3h}{h}=\lim_{h\to 0}(2h+3)=2(0)+3=3$

Clearly, $f'(0)=-3f'(-1)$

121. (c) Given, $f(x)=\dfrac{x^{100}}{100}+\dfrac{x^{99}}{99}+\ldots+\dfrac{x^2}{2}+x+1$

$\Rightarrow \quad f'(x)=\dfrac{100x^{99}}{100}+\dfrac{99x^{98}}{99}+\ldots+\dfrac{2x}{2}+1+0$

$[\because f(x)=x^n \Rightarrow f'(x)=nx^{n-1}]$

$\Rightarrow \quad f'(x)=x^{99}+x^{98}+\ldots+x+1$ ...(i)

Putting $x=1$, we get

$f'(1)=\underbrace{(1)^{99}+1^{98}+\ldots+1+1}_{100\text{ times}}=\underbrace{1+1+1\ldots+1+1}_{100\text{ times}}$

$\Rightarrow \quad f'(1)=100$ ...(ii)

Again, putting $x=0$, we get

$f'(0)=0+0+\ldots+0+1$

$\Rightarrow \quad f'(0)=1$ ...(iii)

From eqs. (ii) and (iii), we get

$f'(1)=100f'(0)$

Hence, $m=100$

122. (b) We have,

$\lim_{x\to\pi/6}\dfrac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}=\lim_{x\to\pi/6}\dfrac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$

$=\lim_{x\to\pi/6}\dfrac{\sin x+1}{\sin x-1}=\dfrac{\frac{1}{2}+1}{\frac{1}{2}-1}=-3$

123. (d) We have, $u=e^x\sin x$

$\Rightarrow \quad \dfrac{du}{dx}=e^x\sin x+e^x\cos x=u+v$

$v=e^x\cos x$

$\Rightarrow \quad \dfrac{dv}{dx}=e^x\cos x-e^x\sin x=v-u$

$\therefore$ Consider $v\dfrac{du}{dx}-u\dfrac{dv}{dx}=v(u+v)-u(v-u)=u^2+v^2$

$\dfrac{d^2u}{dx^2}=\dfrac{du}{dx}+\dfrac{dv}{dx}=u+v+v-u=2v$

and $\dfrac{d^2v}{dx^2}=\dfrac{dv}{dx}-\dfrac{du}{dx}=(v-u)-(v+u)=-2u$

124. (b) Given, $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}=\begin{cases}-x+1, & x<0\\ 0, & x=0\\ x-1, & x>0\end{cases}$

Let us first check the existence of limit of f (x) at x = 0.
At x = 0,

$$\text{RHL}=\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}(0+h)-1$$
$$=\lim_{h\to 0} h-1=0-1=-1$$
$$\text{LHL}=\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)$$
$$=\lim_{h\to 0}-(0-h)+1$$
$$=\lim_{h\to 0} h+1=0+1=1$$

$\Rightarrow$ RHL $\neq$ LHL
$\Rightarrow$ At x = 0, limit does not exist.
Note that for any a < 0 or a > 0, $\lim_{x\to a} f(x)$ exists,
as for a < 0, $\lim_{x\to a} f(x)=\lim_{x\to a} -x+1=-a+1$ exists and for a > 0, $\lim_{x\to a} f(x)=\lim_{x\to a} x-1=a-1$ exists. Hence, $\lim_{x\to a} f(x)$ exists for all $a\neq 0$.

125. (d) $\lim_{x\to a}\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}=\lim_{x\to a}\frac{(x+2)^{5/3}-(a+2)^{5/3}}{(x+2)-(a+2)}$

$$=\lim_{y\to b}\frac{y^{5/3}-b^{5/3}}{y-b},$$

where $x+2=y$, $a+2=b$. and when $x\to a, y\to b$

$$=\frac{5}{3}b^{5/3-1}=\frac{5}{3}b^{2/3}=\frac{5}{3}(a+2)^{2/3}.$$

126. (b) $\lim_{x\to 0}\sqrt{\frac{x-\sin x}{x+\sin^2 x}}=\lim_{x\to 0}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\sin^2 x}{x}}}$

$$=\lim_{x\to 0}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\left(\frac{\sin x}{x}\right)\sin x}}=\sqrt{\frac{1-1}{1+1\times 0}}=0$$

127. (c) $\lim_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$

[multiply denominator and numerator with x]
We get,

$$\lim_{x\to 0}\frac{x^2\sin 5x}{x\sin^2 4x}=\lim_{x\to 0}\frac{\sin 5x}{x}.\frac{x^2}{\sin^2 4x}$$

Rearranging to bring a standard form, we get,

$$\lim_{x\to 0}\frac{5\sin 5x}{5x}.\frac{(4x)^2}{16\sin^2 4x}$$
$$=\frac{5}{16}\left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right).\frac{1}{\lim_{x\to 0}\left(\frac{\sin 4x}{4x}\right)^2}=\frac{5}{16}$$

128. (c) As given $\lim_{x\to 0}\frac{a^x-x^a}{x^a-a^a}=-1$

Applying limit, we have

$$\frac{1-0}{0-a^a}=-1\quad (\because\ a^0=1)$$
$$\Rightarrow\quad \frac{1}{-a^a}=-1\Rightarrow a^a=1$$

Taking log on both the sides
a log a = 0 $\Rightarrow$ a = 0 or log a = 0
a $\neq$ 0 $\Rightarrow$ log a = 0 $\Rightarrow$ a = 1

129. (a) The required limit

$$=\lim_{x\to 2}\frac{[1+\sqrt{2+x}-3]}{(x-2)\left[\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right]}\ \text{(on rationalizing)}$$
$$=\lim_{x\to 2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to 2}\frac{(x+2)-4}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{x+2}+2\right)}$$
$$=\frac{1}{2\sqrt{3}}.\frac{1}{4}=\frac{1}{8\sqrt{3}}$$

130. (b) $\lim_{x\to 2a}\frac{\sqrt{x-2a}}{\sqrt{x^2-4a^2}}+\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$

$$=\lim_{x\to 2a}\frac{1}{\sqrt{x+2a}}+\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{(x-2a)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to 2a}\frac{\sqrt{x}-\sqrt{2a}}{\sqrt{\left(\sqrt{x}-\sqrt{2a}\right)\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}$$
$$=\frac{1}{2\sqrt{a}}+\lim_{x\to 2a}\frac{\sqrt{\sqrt{x}-\sqrt{2a}}}{\sqrt{\left(\sqrt{x}+\sqrt{2a}\right)(x+2a)}}=\frac{1}{2\sqrt{a}}+0$$

131. (d) $\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x=\frac{\pi}{2}+y; y\to 0$

$$=\lim_{y\to 0}\frac{\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3}=\lim_{y\to 0}\frac{-\tan\frac{y}{2}\left(2\sin^2\frac{y}{2}\right)}{(-8).\frac{y^3}{8}.8}$$
$$=\lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2=\frac{1}{32}$$

132. (d) $\lim\limits_{x\to 2}\dfrac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}=\lim\limits_{x\to 2}\dfrac{\sqrt{2}\,|\sin(x-2)|}{x-2}$

$\underset{(\text{at }x=2)}{\text{L.H.L}}=-\lim\limits_{x\to 2}\dfrac{\sqrt{2}\sin(x-2)}{(x-2)}=-1$

$\underset{(\text{at }x=2)}{\text{R.H.L}}=\lim\limits_{x\to 2}\dfrac{\sqrt{2}\sin(x-2)}{(x-2)}=1$

Thus $\underset{(\text{at }x=2)}{\text{L.H.L}}\neq\underset{(\text{at }x=2)}{\text{R.H.L}}$

Hence, $\lim\limits_{x\to 2}\dfrac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

133. (d) $\lim\limits_{x\to 5}\dfrac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$\Rightarrow\ \lim\limits_{x\to 5}[(f(x))^2-9]=0\Rightarrow\lim\limits_{x\to 5}f(x)=3$

134. (c) Consider $\lim\limits_{x\to 0}\dfrac{\tan^2 x-2\tan x-3}{\tan^2 x-4\tan x+3}$

$=\lim\limits_{x\to 0}\dfrac{\tan^2 x-3\tan x+\tan x-3}{\tan^2 x-3\tan x-\tan x+3}$

$=\lim\limits_{x\to 0}\dfrac{(\tan x+1)(\tan x-3)}{(\tan x-1)(\tan x-3)}=\lim\limits_{x\to 0}\dfrac{\tan x+1}{\tan x-1}$

Now, at $\tan x=3$, we have

$\lim\limits_{\tan x\to 3}\dfrac{\tan x+1}{\tan x-1}=\dfrac{3+1}{3-1}=\dfrac{4}{2}=2$

135. (b) $\lim\limits_{x\to 0}\dfrac{x\sqrt[3]{z^2-(z-x)^2}}{\left(\sqrt[3]{8xz-4x^2}+\sqrt[3]{8xz}\right)^4}$

$=\lim\limits_{x\to 0}\dfrac{x\sqrt[3]{2xz-x^2}}{(\sqrt[3]{x}\sqrt[3]{8z-4x}+\sqrt[3]{8z}\sqrt[3]{x})^4}$

$=\lim\limits_{x\to 0}\dfrac{x^{4/3}\sqrt[3]{2z-x}}{x^{4/3}[\sqrt[3]{8z-4x}+\sqrt[3]{8z}]^4}$

$=\dfrac{\sqrt[3]{2z}}{\left[2\sqrt[3]{8z}\right]^4}=\dfrac{1}{2^{23/3}.z}$

136. (d) $\lim\limits_{h\to 0}\dfrac{2-\sqrt[3]{8+h}}{2h.\sqrt[3]{8+h}}$

$\lim\limits_{h\to 0}\dfrac{8-(8+h)}{2h.\sqrt[3]{8+h}\{8^{2/3}+8^{1/3}.(8+h)^{1/3}+(8+h)^{2/3}\}}=-\dfrac{1}{48}$

137. (b) $\dfrac{2\cos\left(\dfrac{\sin x+x}{2}\right).\sin\left(\dfrac{x-\sin x}{2}\right)}{x^4}$

$=\lim\limits_{x\to 0}2\left[\dfrac{\sin\left(\dfrac{\sin x+x}{2}\right)}{\left(\dfrac{\sin x+x}{2}\right)}\right]\left[\dfrac{\sin\left(\dfrac{x-\sin x}{2}\right)}{\left(\dfrac{x-\sin x}{2}\right)}\right]$

$\times\left[\dfrac{1}{2\left(\dfrac{1}{\dfrac{\sin x}{x}+1}\right).3\dfrac{x^3}{(x-\sin x)}}\right]=\dfrac{1}{6}$

138. (b)

139. (c) A function f is said to be a rational function, if

$f(x)=\dfrac{g(x)}{h(x)}$, where g (x) and h (x) are polynomials such that $h(x)\neq 0$. Then,

$\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}\dfrac{g(x)}{h(x)}=\dfrac{\lim\limits_{x\to a}g(x)}{\lim\limits_{x\to a}h(x)}=\dfrac{g(a)}{h(a)}$

If $h(a)=0$, there are two scenarios - (i) when $g(a)\neq 0$ and (ii) when $g(a)=0$. In case I, the limit does not exist. In case II, we can write $g(x)=(x-a)^k g_1(x)$, where k is the maximum of powers of $(x-a)$ in g (x). Similarly, $h(x)=(x-a)^l h_1(x)$ as $h(a)=0$. Now, if $k>l$, then

$$\lim_{x\to a}f(x)=\frac{\lim\limits_{x\to a}g(x)}{\lim\limits_{x\to a}h(x)}=\frac{\lim\limits_{x\to a}(x-a)^k g_1(x)}{\lim\limits_{x\to a}(x-a)^l h_1(x)}$$

$$=\frac{\lim\limits_{x\to a}(x-a)^{k-l}g_1(x)}{\lim\limits_{x\to a}h_1(x)}=\frac{0.g_1(a)}{h_1(a)}=0$$

If $k<l$, the limit is not defined.