CHAPTER 09

Analysis and Design of Flanged Beams by LSM

9.1 Introduction

In the previous chapters, we dealt with the analysis and design of RCC beams of rectangular section. The analysis procedure described earlier is applicable to beams of any other cross sectional shape also. In practice, the reinforced concrete floors/slabs are casted monolithically with beams. Such type of rectangular beams having slab at the top casted monolithically behaves differently from the simple reinforced concrete rectangular beam with no slab or with disconnected slabs. Such type of flanged systems where the beam and slab are monolithic, can be of either '*T*' or '*L*' type. In this chapter, we will discuss the analysis and design of such types of beams which are encountered more frequently in practice than simple rectangular beams.

9.2 Flanged Beams

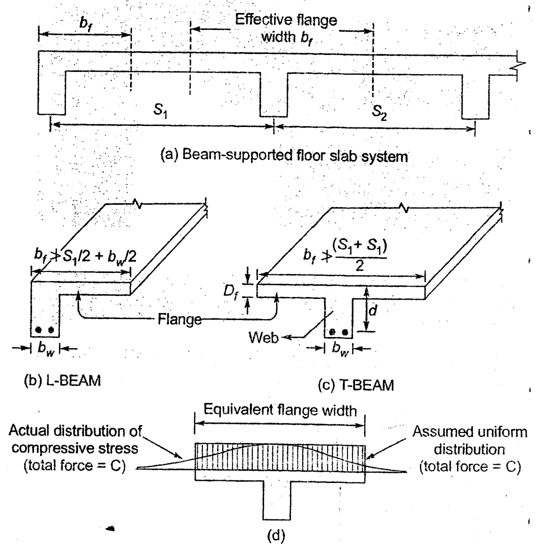


Fig.9.1 Explanation of T and L beams

In general, the floor slabs are casted monolithically with beams and hence the behavior of such beams is different from simple rectangular beams. In such type of beams, the slab acts in conjunction with beam. The slab portion is called as **flange** of '*T*' or '*L*' beam. The beam portion which is below the slab (flange) is called as **web** of '*T*' or '*L*' beam.

Actual width of the flange is the spacing of the beam which is same as the center to center distance between the adjacent spans of the slab as shown in **Fig. 9.1**. However in the flanged systems, a part of the width which is less than or equal to the actual width of the slab is considered to be effective. Thus the width of the slab is the effective width of the flange.

When the flange is too wide, then compressive stress distribution is not uniform over the entire width of web. But theory of flexure as discussed in previous chapters, assume a uniform stress distribution across the width of the section. Thus it becomes imperative to reduce the non-uniform stress distribution in the flange to a uniform stress distribution.

9.3 Effective Width of Flange

The **effective width of flange** is in fact a hypothetical concept and is defined as **the width of the flange that resists the uniform compressive stress that is equal to maximum stress in the original wide flange in such a way that resultant longitudinal compressive force is same in both the cases.** This is shown in Fig. 9.1(d).

The effective width of flange depends upon the following:

1. Span of the beam.
2. Width of the web (b_w).
3. Thickness of flange (D_f).
4. Type of loading i.e. whether the load is concentrated or uniformly distributed.
5. Type of beam supports.

As per **Cl. 23.1.1 of IS 456: 2000**, the following requirements must be satisfied for ensuring combined action of slab and rib (beam):

1. The slab shall be casted integrally with the web/rectangular beam or they shall be effectively bonded in any other manner.
2. At the mid-span of the slab, the transverse reinforcement must be atleast 60% of the main longitudinal reinforcement of the slab which is parallel to the transverse beam.

9.3.1 IS 456: 2000 Recommendations for Effective Width of Flange

Cl. 23.1.2 of IS 456: 2000 specifies the following values of effective flange width of *T*-beam and *L*-beams:

1. **For *T*-beams:** Minimum of the following is adopted as effective width of the flange:

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

or

$$b_f = b_w + \frac{l_1}{2} + \frac{l_2}{2}$$

2. **For *L*-beams:** Minimum of the following is adopted as effective width of the flange:

$$b_f = \frac{l_0}{12} + b_w + 3D_f$$

or

$$b_f = b_w + \frac{l_1}{2}$$

3. **For isolated T-beam:** $b_f = \dfrac{l_o}{\dfrac{l_o}{b}+4} + b_w$

4. **For isolated L-beam:** $b_f = \dfrac{0.5 l_o}{\dfrac{l_o}{b}+4} + b_w$

Where, l_o = Distance between points of zero moment in the beam, which is the effective span of simply supported beam and 0.7 times the effective span for continuous beams and frames.

D_f = Thickness of the flange

b_w = Width of the web

b_f = Effective width of the flange

b = Actual width of the flange

Remember: For all the above cases, in no case the effective width of flange shall exceed the actual width of the flange.

9.4 Compressive Stress Distribution in the Flange

Maximum compressive stress in the flange occurs at the location above the rib of the flanged beam as shown in Fig. 9.2. Due to this complex variation of stress distribution, **the concept of effective width of the flange was introduced.**

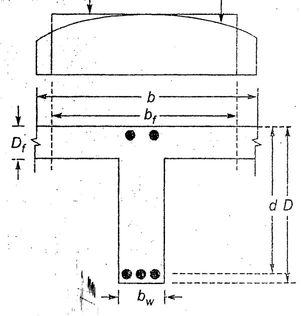


Fig. 9.2 Variation of Compressive Stress in Flanged Beams

9.5 Analysis of Flanged Beams Sections (by Limit State Method)

In a flanged beam, the neutral axis may either be in the flange or web of the section which depends upon the dimensions of effective flange width (b_f), web width (b_w), and thickness of flange (D_f) of the flanged beam.

If neutral axis lies in the flange of the T beam (i.e. $x_u \le D_f$) then flanged beam can be taken as a rectangular beam of width b_f and effective depth d because the concrete on tension side is not taken into account. Thus equations derived in the previous chapters on rectangular beams are still applicable with the only difference that width of the beam (b) will get replaced by effective width of the flange (b_f).

But if the neutral axis lies in the web of the flanged beam (i.e. $x_u > D_f$) then compression is taken by flange and part of the web of the beam. In this case, the total compressive force (C_u) comprises compressive force due to the web portion ($b_w x_u$) and compressive force due to the flange portion $[(b_f - b_w) D_f]$. The compressive force and moment of resistance in the web portion is given by:

$$C_{uw} = 0.36 f_{ck} b_w x_u$$

$$M_{uw} = C_{uw}(d - 0.42x_u) = 0.36 f_{ck} b_w x_u (d - 0.42x_u)$$

But estimation of compressive force in the flange is difficult because it depends on the shape of the compressive stress distribution curve i.e. whether only the rectangular portion of compressive stress distribution or the rectangular and part of parabolic portion of the compressive stress distribution lies in the flange portion.

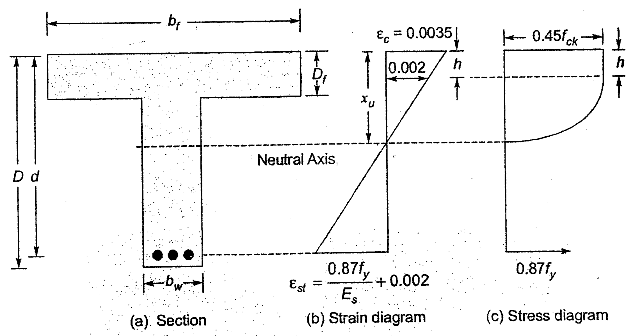


Fig.9.3 A typical T-beam section.

However it is to be noted that the compressive stress remains constant between the concrete compressive strains of 0.002 and 0.0035 which is equal to 0.45 f_{ck}.

Let h = Depth of concrete stress block where the compressive stress remains constant i.e. depth between compressive strain of 0.002 and 0.0035

If depth 'h' where the compressive strain is 0.002 lies in the web of the flange beam then whole of the flange will be under a constant compressive stress of 0.45 f_{ck}.

From Fig. 9.3(b)

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u} \quad \text{or} \quad \frac{h}{x_u} = \frac{3}{7}$$

In the limiting case when $x_u = x_{u\,lim}$,

$$h = \frac{3}{7} x_{u\,lim}$$

$= 0.227d$ for Fe 250 since $x_{u\,lim} = 0.53d$

$= 0.205d$ for Fe 415 since $x_{u\,lim} = 0.479d$

$= 0.197d$ for Fe 500 since $x_{u\,lim} = 0.46d$

From the above results, it can be inferred that,

$$\frac{h}{d} \simeq 0.2$$

Thus for all the three grades of steel i.e. Fe 250, Fe 415 and Fe 500, $h \approx 0.2d$. Let the value of 'h' reaches the depth of flange i.e., at the bottom of flange so that $h = D_f$ and thus the strain will be 0.002 if $\frac{D_f}{d} = 0.2$.

NOTE: Thus, the flange thickness can be considered as small if D_f/d is less than 0.2 and in that case the entire flange will be under a constant compressive stress of $0.45 f_{ck}$.

If $\frac{D_f}{d} > 0.2$, then only a part of flange of the beam will be under a constant compressive stress of $0.45f_{ck}$, i.e. the strain is more than 0.002.

Thus, in balanced flanged beams (when $x_u = x_{u\,lim}$), the ratio of $\frac{D_f}{d}$ is important to determine i.e. the rectangular stress block is for the full depth of flange (when $\frac{D_f}{d} \leq 0.2$) or for a part of the flange (where $\frac{D_f}{d} > 0.2$).

Similarly, for the case of under reinforced flanged beams (when $x_u < x_{u\,lim}$), replace $\frac{D_f}{d}$ by $\frac{D_f}{x_u}$.

If $\frac{D_f}{x_u} \leq 0.43\left(=\frac{3}{7}\right)$, then the constant stress block (= $0.45\,f_{ck}$) will be for the full depth of the flange.

If $\frac{D_f}{x_u} > 0.43\left(=\frac{3}{7}\right)$, then only a part of flange depth will be having a constant stress of $0.45f_{ck}$.

Thus, there are four cases of flanged beams which are explained below:

Case 1: When $x_u < D_f$ i.e. when neutral axis is in the flange

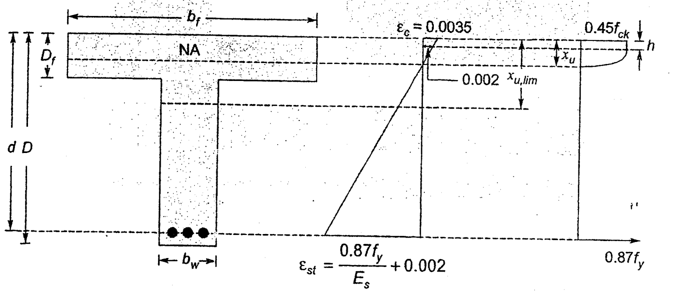


***Fig.9.4** T-beam, when $x_u < D_f$,*

Concrete below the neutral axis remains under tension and hence ignored. Thus the beam is considered as a rectangular beam of width b_f and effective depth d.

Actual depth of neutral axis is given by,

$$C = T$$

$$0.36f_{ck}b_f x_u = 0.87f_y A_{st}$$

$$x_u = \frac{0.87f_y A_{st}}{0.36f_{ck}b_f}$$

Moment of resistance is given by,

$$M_u = C \times \text{Lever Arm} = T \times \text{Lever Arm}$$

$$= 0.36f_{ck}b_f x_u(d - 0.42x_u) = 0.87f_y A_{st}(d - 0.42x_u)$$

Case 2: When $x_{u\,lim} > D_f$ i.e. when neutral axis is in the web and the section is balanced (i.e., $x_u = x_{u\,lim}$)

Sub case (i) When $\frac{D_f}{d} \leq 0.2$.

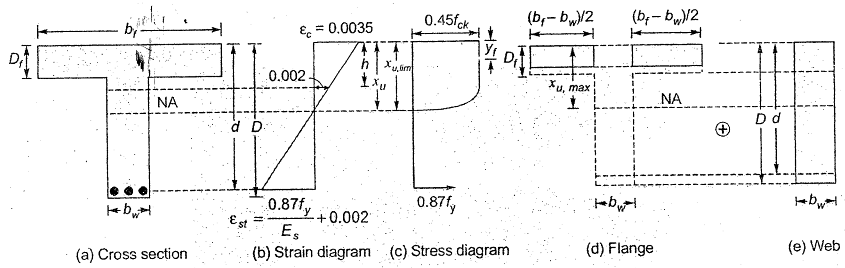


***Fig.9.5** T-beam when $D_f/d \leq 0.2$ and balanced $x_{u\,lim} \geq D_f$*

In this case, the depth of constant stress (= $0.45f_{ck}$) block is greater than D_f. The section is divided into two parts viz.

1. Rectangular beam of width b_w and effective depth d
2. Rectangular flange of width $(b_f - b_w)$ and depth D_f

Total compressive force (C) = Compressive force of rectangular beam of width b_w and depth d + Compressive force of rectangular flange of width $(b_f - b_w)$ and depth D_f

Compressive force, $\quad C = 0.36f_{ck}\,b_w x_{u\,lim} + 0.45f_{ck}(b_f - b_w)D_f$

Do you know?: In the above expression it is assumed that constant compressive stress is $0.45f_{ck}$ and NOT $0.446f_{ck}$ as per Cl. G-2 of IS 456: 2000.

Tensile force, $\quad T = 0.87f_y A_{st}$

Lever arm for rectangular web part is $(d - 0.42\,x_{u\,lim})$ and for the flange part is $(d - 0.5D_f)$.

Total moment of resistance = Moment due to rectangular web beam + Moment due to rectangular flange

$$M_u = 0.36f_{ck}b_w x_{u\,lim}(d - 0.42x_{u\,lim}) + 0.45f_{ck}(b_f - b_w)\,D_f\left(d - \frac{D_f}{2}\right)$$

or $$M_u = 0.36\left(\frac{x_{u\,lim}}{d}\right)\left[1-0.42\left(\frac{x_{u\,lim}}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\, D_f \left(d - \frac{D_f}{2}\right)$$

The above equation is given in Cl. G-2.2 of IS 456: 2000.

Sub case (ii) When $\frac{D_f}{d} > 0.2$

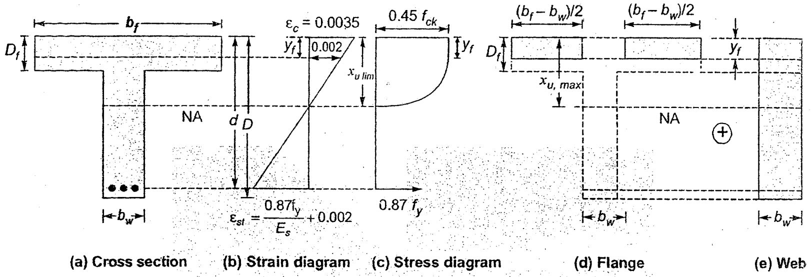


Fig.9.6 *T-beam, Case (ii-b), $D_f/d > 0.2$ and balanced $x_{u,lim} > D_f$*

In this case, the depth of rectangular portion of stress block lies in the flange of the beam only. The depth of constant stress (= $0.45 f_{ck}$) is taken as 'y_f' where 'y_f' is given by:

$$y_f = \left(0.15 x_{u\,lim} + 0.65 D_f\right) \le D_f$$

The derivation of above expression for y_f is given in the next section.

When $\frac{D_f}{d} \le 0.2$, the equations derived in sub-case(i) are applicable here also just by replacing D_f by y_f.

Thus the equations are:

$$C = 0.36 f_{ck} b_w x_{u\,lim} + 0.45 f_{ck}(b_f - b_w) y_f$$
$$T = 0.87 f_y A_{st}$$

Lever arm for rectangular web part is $(d - 0.42\, x_{u\,lim})$ and for the flange part is $(d - 0.5 y_f)$. Thus the moment equations are:

$$M_u = 0.36 f_{ck} b_w x_{u\,lim}(d - 0.42 x_{u\,lim}) + 0.45 f_{ck}(b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

or $$M_u = 0.36\left(\frac{x_{u\,lim}}{d}\right)\left[1-0.42\left(\frac{x_{u\,lim}}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

Case 3: When $x_u > D_f$ i.e. when neutral axis is in the web and the section is under reinforced

Sub case (i) When $\frac{D_f}{x_u} \le 0.43$

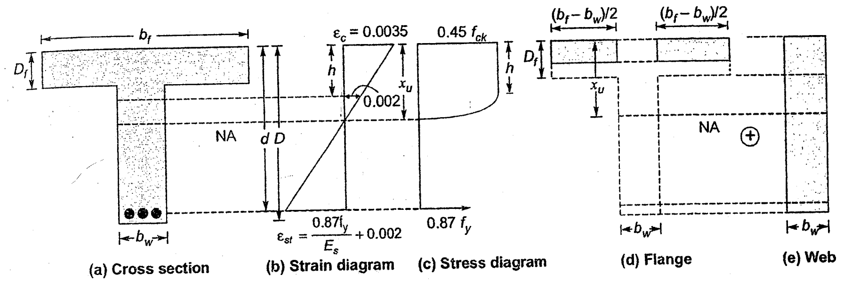


Fig.9.7 *T-beam, Case (iii-a), when $D_f/x_u \le 0.43$ and under-reinforced $x_u > D_f$*

Now since the flange depth D_f does not exceed $0.43 x_u$ and h (depth of compressive fibre of strain 0.002) is $0.43 x_u$, the whole flange is under constant compressive stress of $0.45 f_{ck}$.

Thus, $$C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) D_f$$
$$T = 0.87 f_y A_{st}$$
$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1-0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\left(d - \frac{D_f}{2}\right) D_f$$

Sub case (ii): When $\frac{D_f}{x_u} > 0.43$

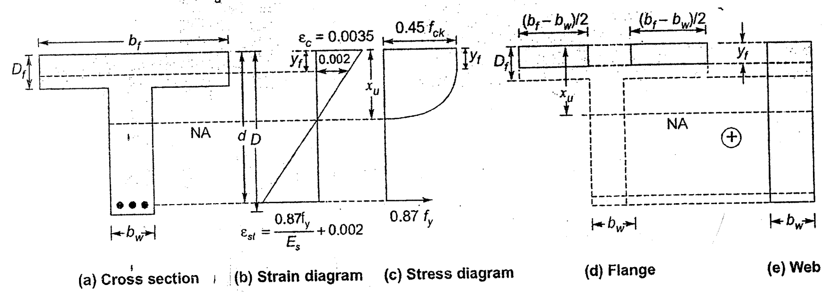


Fig.9.8 *T-beam, when $D_f/d \le 0.2$ and $x_u > D_f$*

In this case, $D_f > 0.43 x_u$ and h (depth of compressive fibre of strain 0.002) is at a depth of $0.43 x_u$, part of the flange will be under the constant compressive stress of $0.45 f_{ck}$ and this depth is denoted as y_f.

Thus

$$y_f = (0.15 x_u + 0.65 D_f) \le D_f$$
$$C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) y_f$$
$$T = 0.87 f_y A_{st}$$
$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1-0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)\, y_f \left(d - \frac{y_f}{2}\right)$$

Case 4: When $x_u > D_f$ i.e. when neutral axis is in the web and the section is over reinforced

In case of over reinforced sections, the actual depth of neutral axis (x_u) is greater than the limiting depth of neutral axis ($x_{u\,lim}$).

However x_u is restricted to $x_{u\,lim}$.

Sub case (i): When $\frac{D_f}{d} \leq 0.2$

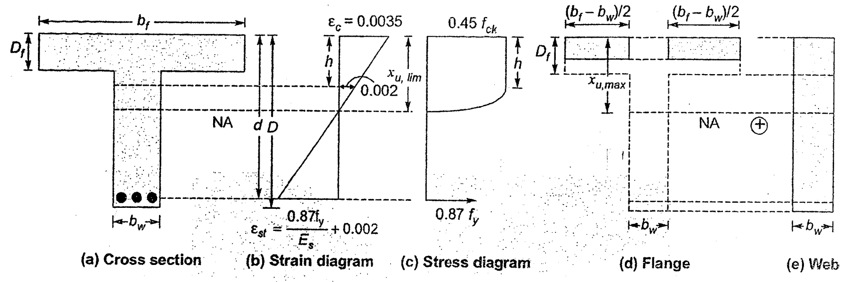


Fig.9.9 *T-beam, when $D_f/d \leq 0.2$ and $x_u > D_f$*

$$C = 0.36\, f_{ck} b_w x_{u\,lim} + 0.45\, f_{ck}(b_f - b_w)D_f$$

$$T = 0.87 f_y A_{st}$$

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)D_f\left(d - \frac{D_f}{2}\right)$$

Sub case (ii): When $D_f/d > 0.2$

$$y_f = (0.15x_u + 0.65D_f) \leq D_f$$

$$C = 0.36\, f_{ck} b_w x_{u\,lim} + 0.45\, f_{ck}(b_f - b_w)y_f$$

$$T = 0.87 f_y A_{st}$$

$$M_u = 0.36\left(\frac{x_u}{d}\right)\left[1 - 0.42\left(\frac{x_u}{d}\right)\right] f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w)y_f\left(d - \frac{y_f}{2}\right)$$

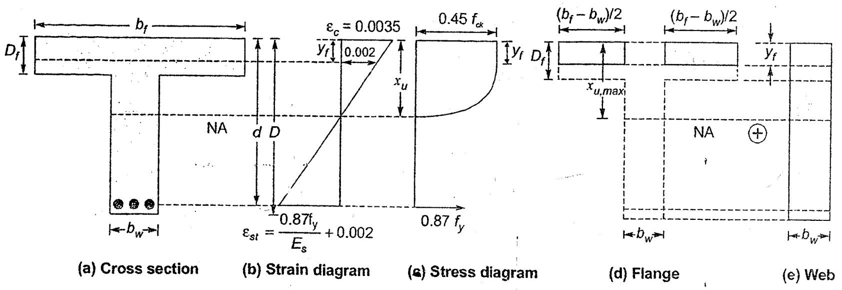


Fig.9.10 *T-beam, $D_f/d > 0.2$ and $x_u > D_f$*

From the above expressions, it is quite evident that for over reinforced beams, the additional moment of resistance beyond balanced section is not possible because it will restrict the failure/yielding of steel. Thus the section is either redesigned or we go for doubly reinforced beams.

9.6 Derivation of the Expression to Determine 'y_f'

Fig. 9.11 shows two stress blocks viz. the IS 456: 2000 stress block and the Whitney's stress block.

Now,

y_f = Depth of constant portion of stress block when $D_f/d > 0.2$.

Since y_f is a function of x_u and D_f and thus y_f can be assumed as:

$$y_f = Ax_u + BD_f$$

Here A and B are constants to be determined from the following two conditions viz.:

1. When $D_f = 0.43x_u$, $y_f = 0.43x_u$
2. When $D_f = x_u$, $y_f = 0.8x_u$

Thus solving equation of y_f for the conditions 1. and 2. gives,

$$y_f = 0.15x_u + 0.65D_f$$

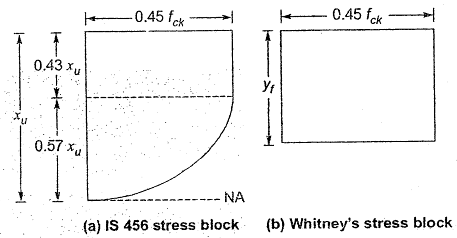
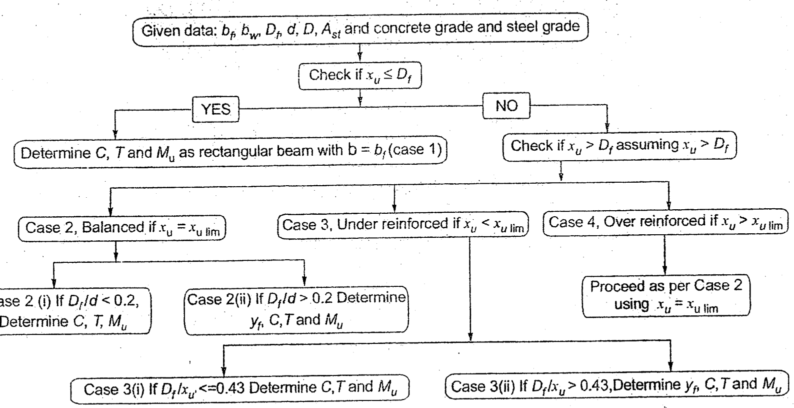


Fig.9.11 *Stress blocks*

9.7 Types of Analysis Problems



9.8 Integral Action of Slab and Beam

The combined (or the integral) action between the slab and beam can be ensured by providing stirrups. Sufficient transverse reinforcement must be provided near the top of the flange which is present in the form of negative moment reinforcement in continuous slabs which spans across and form the flange of T beams as shown in the Fig. 9.12.

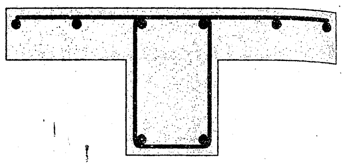


Fig.9.12 Integral action of slab and beam

In the other case (i.e. when bars run parallel to the beam in flange as shown in Fig. 9.13), Cl. 23.1.1b of IS 456: 2000 specifies to provide transverse reinforcement in the flange of the ***T or L beam.*** The area of such steel should not be less than 60% of the area of main mid-span steel reinforcement and should extend on either side of the beam for a distance not less than (span/4).

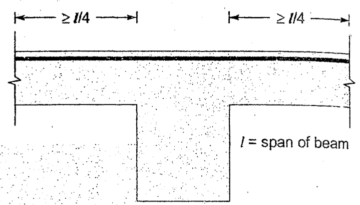


Fig.9.13 Flanged beam with top bars running parallel to flange

9.9 Design of Flanged Beam Sections (by Limit State Method)

The combined action of slab and beam occurs in *cast in-situ* constructions. If the slab and beam are not casted monolithically, flanged action on the beam cannot be considered unless special shear connectors are provided at the interface of slab and beam.

In **continuous flanged beams**, the proportioning of the section is governed by the negative moment at the support which is invariably larger than the mid span positive moment. For this case (i.e. negative moment at support in the flanged beam), the flange portion is under tension and thus the concrete in the flange portion is ignored. In other words, the flanged beam section at supports is designed as a rectangular beam section for the factored negative moment. At the mid span of the beam, the beam is actually a flanged beam with flange portion under compression. Now as the web width (b_w) and effective beam depth (d) has already been determined (hence fixed) from the factored negative moment at support, only the reinforcing steel required is to be determined. The effective width of the flange (b_f) is determined as per provisions of IS 456: 2000.

In case of **simply supported flanged beams**, the web dimension has to be determined / designed. The web width is usually fixed as 250 mm, 300 mm or 350 mm etc. with overall depth preliminary taken as 1/13th to 1/16th of span. The area of tension steel is then determined as:

$$A_{st\,required} = \frac{M_u}{0.87 f_y z}$$

Where, the lever arm (z) is approximately taken larger of $0.9d$ or $\left(d - \frac{D_f}{2}\right)$.

Determination of reinforcing steel in flanged beams depends on the location of neutral axis (x_u), which in any case should not exceed $x_{u\,lim}$. If factored moment (M_u) exceeds the limiting moment of resistance of the flanged beam section ($M_{u\,lim}$) then either the beam depth should be increased or the section has to be designed as doubly reinforced.

Case 1: Neutral Axis Lies Within the Flange ($x_u \le D_f$)

In practical situations, the compressive force contributed by the flange of T or L beams is usually very large and thus the depth of neutral axis (x_u) is less than the flange depth (D_f). Here, the section behaves as a rectangular section of width b_f and effective depth d.

Now the question arises,

"How to check whether $x_u \le D_f$ without actually calculating the neutral axis depth x_u?"

The answer to the above question is that first calculate moment of resistance of the section at $x_u = D_f$ i.e. $(M_{uR})_{x_u=D_f}$ as:

$$(M_{uR})_{x_u=D_f} = 0.362 f_{ck} b_f D_f (d - 0.42 D_f)$$

Compare the above calculated moment $(M_{uR})_{x_u=D_f}$ with the factored design moment (M_u). If $M_u \le (M_{uR})_{x_u=D_f}$ then it implies that $x_u \le D_f$. But all this is valid only if $x_{u\,lim} > D_f$. If $x_{u\,lim} < D_f$ then $M_{u\,lim}$ is calculated from the above expression by replacing D_f by $x_{u\,lim}$.

Case 2: Neutral Axis Lies within the Web ($x_u > D_f$)

In this case, $M_u > (M_{uR})_{x_u=D_f}$ and thus $x_u > D_f$. This leads to the accurate determination of the depth of neutral axis (x_u) which is quite tedious. Now as discussed above, the compressive force in concrete consists of compressive force due to the web (C_{uw}) and due to the flange (C_{uf}) as:

$$C_{uw} = 0.362 f_{ck} b_w x_u$$

$$C_{uf} = 0.45 f_{ck} (b_f - b_w) y_f$$

Here the equivalent flange thickness (y_f) is less than or more than D_f depends on whether x_u is greater than or less than, $\frac{7D_f}{3}$.

If $x_{u\,lim} > \frac{7D_f}{3}$, then the ultimate moment of resistance $(M_{uR})_{x_u=7D_f/3}$ (i.e. at $x_u = \frac{7D_f}{3}$) and $y_f = D_f$ is computed first. If $M_u \ge (M_{uR})_{x_u=7D_f/3}$ (i.e. at $x_u = \frac{7D_f}{3}$) then it implies that $x_u > \frac{7D_f}{3}$ and $y_f = D_f$

Else $D_f < x_u < \frac{7D_f}{3}$ for $(M_u)_{x_u=D_f} < M_u < (M_{uR})_{x_u=7D_f/3}$ and

$$y_f = 0.15 x_u + 0.65 D_f$$

x_u is then calculated from the following equation,

$$M_u = C_{uw}(d - 0.42 x_u) + C_{uf}\left(d - \frac{y_f}{2}\right)$$

Once x_u has been determined, C_{uw} and C_{uf} can be determined from the above equations. Area of reinforcing steel required is computed as:

$$A_{st\,required} = \frac{C_{uw} + C_{uf}}{0.87 f_y}$$

Example 9.1 Determine the moment of resistance of a flanged beam as shown below. The area of steel in the tension zone is 2945 mm² (6-25 ϕ). Use M 20 concrete and Fe 415 steel.

Solution:

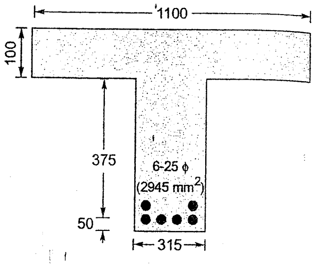
Calculating the depth of natural axis

From statical equilibrium, $C = T$

$$\Rightarrow \quad 0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow \quad x_u = \frac{0.87 \times 415 \times 2945}{0.36 \times 20 \times 1100} = 133.51 \text{ mm} > D_f (= 100 \text{ mm})$$

$$\frac{D_f}{d} = \frac{100}{475} = 0.211 > 0.2$$



$$\therefore \quad y_f = 0.15 x_u + 0.65 D_f$$

$$= 0.15 x_u + 0.65(100)$$

$$= 0.15 x_u + 65$$

and from $C = T$

$$0.36 f_{ck} b_w x_u + 0.445 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36(20)(315) x_u + 0.445(20)(1100 - 315)(0.15 x_u + 65)$$

$$= 0.87(415)(2945)$$

$$\Rightarrow \quad 2280.6 x_u + 1050.33 x_u + 455143 = 1063292.25$$

$$\Rightarrow \quad x_u = 182.58 \text{ mm}$$

$$x_{u\,lim} = 0.479 d = 0.479(475) = 227.525 \text{ mm} > x_u$$

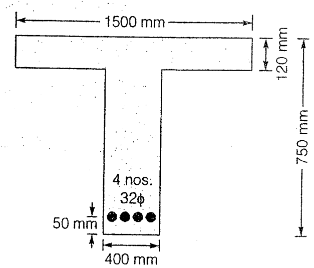
Thus beam is under-reinforced

Moment of resistance $\quad (MR) = 0.87 f_y A_{st}(d - 0.42 x_u)$

$$= 0.87 \times 415 \times 2945 (475 - 0.42 \times 182.58) \text{ Nmm}$$

$$= 423.53 \text{ kNm}$$

Example 9.2 Calculate the moment of resistance of a continuous isolated T-beam of clear span 9 m and cross-section as shown in the figure. Use M 25 concrete and Fe 500 steel. Use WSM.



Solution:

Given:

Flange width (b) = 1500 mm

Effective depth (d) = 750 − 50 = 700 mm

Flange depth (d_f) = 120 mm

Area of tension steel (A_{st}) = $4 \times \frac{\pi}{4} \times 32^2$ mm² = 3216.9 mm²

Design coefficients

$$m = 11,\ c = 8.5,\ t = 275 \text{ N/mm}^2$$

$$k = \frac{mc}{mc + t} = \frac{11 \times 8.5}{11 \times 8.5 + 275} = 0.253$$

Critical depth of neutral axis

$$x_c = kd = 0.253 \times 750 \text{ mm} = 189.75 \text{ mm}$$

Effective width of flange

$$b_f = \frac{l_0}{\frac{l_0}{b} + 4} + b_w$$

$$\because \quad l_0 = 0.7l = 0.7 \times 9 \text{ m} = 6.3 \text{ m} = 6300 \text{ mm}$$

$$b_f = \frac{6300}{\frac{6300}{1500} + 4} + 400 = 1168.3 \text{ mm} < 1500 \text{ mm}$$

Actual depth of neutral axis

(a) If neutral axis lies in the flange of the beam $(x_a \le d_f)$

$$b_f \frac{x_a^2}{2} = m A_{st}(d - x_a)$$

$$1168.3 \times \frac{x_a^2}{2} = 11 \times 3216.9(750 - x_a)$$

$$389.43 x_a^2 = 35385.9(750 - x_a)$$

$$x_a = 185 \text{ mm} > 120 \text{ mm}$$

∴ Neutral axis does not lies in the flange of the beam

(b) If neutral axis lies in the web of the beam $(x_a > d_f)$

$$b_f d_f \left(x_a - \frac{d_f}{2}\right) + b_w \frac{(x_a - d_f)^2}{2} = m A_{st}(d - x_a)$$

$$1168.3 \times 120\left(x_a - \frac{120}{2}\right) + \frac{400}{2}(x_a - 120)^2 = 11 \times 3216.9(750 - x_a)$$

$$140196(x_a - 60) + 200(x_a^2 + 120^2 - 240 x_a) = 35385.9(750 - x_a)$$

$$x_a = 193 \text{ mm}$$

$x_a = 193$ mm > 120 mm $\quad$ ∴ NA lies in the web of the beam

Now, $\quad x_a > x_c$

∴ Section is over reinforced $\quad c_a = \sigma_{cbc} = 8.5$ N/mm²

Stress at the junction of flange and web

$$c_1 = \frac{x_a - d_f}{x_a} \times \sigma_{cbc} = \frac{193 - 120}{193} \times 8.5 = 3.215 \text{ N/mm}^2$$

Moment of resistance

$$M_R = b_f d_f \left(\frac{c_a + c_1}{2}\right)\left(d - \frac{c_a + 2c_1}{c_a + c_1} \times \frac{d_f}{3}\right)$$

$$+ b_w (x_a - d_f)\frac{c_1}{2}\left(d - d_f - \frac{x_a - d_f}{3}\right)$$

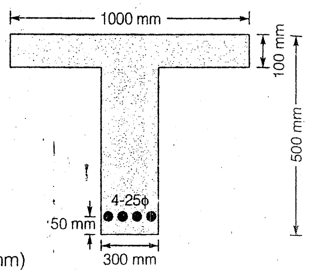
$$= 1168.3 \times 120\left(\frac{8.5 + 3.215}{2}\right)\left(700 - \frac{8.5 + 2 \times 3.215}{8.5 + 3.215} \times \frac{120}{3}\right)$$

$$+ 400(193 - 120)\frac{3.215}{2}\left(700 - 120 - \frac{193 - 120}{3}\right)$$

$$= 559.05 \text{ kNm}$$

Example 9.3 Find the moment of resistance of the T-beam of cross-section as shown in the figure below. Use M 20 concrete and Fe 415 steel.

Solution:



Step-1: Determine the depth of N.A.

Let NA lies in the flange i.e., $x_u \le D_f$

$C = T$

$0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$$x_u = \frac{0.87(415)4\times\frac{\pi}{4}(25)^2}{0.36\times20\times1000}$$

$= 97.92 \text{ mm} < D_f (= 100 \text{ mm})$

Thus assumption of $x_u \le D_f$ is correct.

Also $x_{u\,lim} = 0.48d$ (for Fe 415) $= 0.48\,(450) = 216 \text{ mm} > x_u$

So section is under reinforced

Step-2: Determine moment of resistance of section

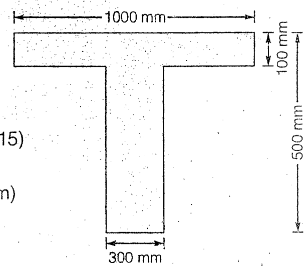
∵ Section is under reinforced

∴ Steel will yield first

∴ Moment of resistance $= 0.87 f_y A_{st}(d - 0.42x_u)$

$= 0.87 \times 415 \times 1963\,(450 - 0.42 \times 97.92) = 289.78 \text{ kNm}$

Example 9.4 A T-beam of cross-section as shown in the figure below. Using M20 concrete and Fe 415 steel, determine $A_{st\,lim}$ and $M_{u\,lim}$.



Solution:

Step-1: Determine (D_f/d) ratio

Let Effective cover = 50 mm

In the limiting case, $x_u = x_{u\,lim} = 0.48d$ (for Fe 415)

$= 0.48\,(450)$

$= 216 \text{ mm} > D_f (= 100 \text{ mm})$

∴ $\frac{D_f}{d} = \frac{100}{450} = 0.222 > 0.2$

Let N.A. lies in the web.

Step-2: Calculate y_f, C and T.

$y_f = 0.15\,x_{u\,lim} + 0.65\,D_f \not> D_f$

$= 0.15\,(0.48 \times 450) + 0.65\,(100)$

$= 97.4 \text{ mm} < D_f (= 100 \text{ mm})$ (OK)

∴ $C = 0.362\,f_{ck}\,b_w x_{u\,lim} + 0.45 f_{ck}\,(b_f - b_w) y_f$

$= 0.362\,(20)\,(300)\,(216) + 0.45\,(20)(1000 - 300)97.4$

$= 1082.772 \text{ kN}$

In the limiting case, $f_{st} = 0.87 f_y$

∴ $T = 0.87 f_y A_{st}$

from $C = T$

$$A_{st} = \frac{1082.772\times10^3}{0.87 f_y} = \frac{1082.772\times10^3}{0.87(415)} = 2998.95 \text{ mm}^2$$

∴ $A_{st\,lim} = 2998.95 \text{ mm}^2 \approx 2999 \text{ mm}^2$

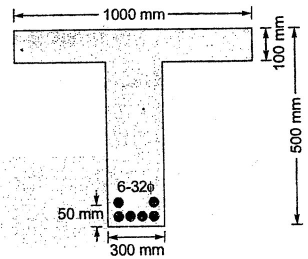
Step-3: Calculate $M_{u\,lim}$

$M_{u\,lim} = 0.362\,f_{ck}\,b_w x_{u\,lim}\,(d - 0.42x_{u\,lim}) + 0.45 f_{ck}\,(b_f - b_w) y_f (d - y_f/2)$

$= 0.362 \times 20 \times 300 \times 216\,(450 - 0.42 \times 216) + 0.45 \times 20(1000 - 300)$ $97.4\,(450 - 97.4/2)$

$= 414.81 \text{ kNm}$

Example 9.5 In the previous question, determine the moment of resistance of the T-beam when it is reinforced with 6-32ϕ bars on tension side. Use M20 concrete and Fe415 steel.



Solution:

Area of tension steel $A_{st} = 6\times\frac{\pi}{4}\times32^2 = 4825.5 \text{ mm}^2$

Step-1: Determine depth of N.A. (x_u)

Let N.A. lies in the flange i.e., $x_u \le D_f$

∴ $$x_u = \frac{0.87 f_y A_{st}}{0.362 f_{ck} b_f} = \frac{0.87(415)4825.5}{0.362(20)1000}$$

$= 240.64 \text{ mm}$

Limiting depth of N.A. $x_{u\,lim} = 0.48d = 0.48(450) = 216 \text{ mm}$

∴ $x_u > x_{u\,lim}$

So section is over reinforced

∴ Limit x_u to $x_{u\,lim}$ i.e., $x_u = x_{u\,lim} = 216 \text{ mm}$

Also $\frac{D_f}{d} = \frac{100}{450} = 0.222 > 0.2$

∴ Depth of constant stress block ($= 0.45 f_{ck}$) $< D_f$

∴ $y_f = 0.15\,x_u + 0.65\,D_f \not> D_f$

Here $x_u = x_{u\,lim} = 216 \text{ mm}$

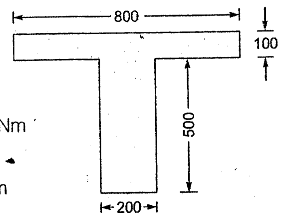
∴ $y_f = 0.15\,(216) + 0.65\,(100) = 97.4 \text{ mm}$

Step-2: Calculate moment of resistance of beam

$M_u = 0.362\,f_{ck}\,b_w x_{u\,lim}\,(d - 0.42x_{u\,lim}) + 0.45 f_{ck}\,(b_f - b_w) y_f (d - y_f/2)$

$= 0.362\,(20)\,(300)\,(216)\,(450 - 0.42 \times 216) + 0.45(20)(1000 - 300)97.4$ $(450 - 97.4/2) = 414.81 \text{ kNm}$

Example 9.6 A T-beam section as shown below is supposed to resist a moment of 300 kNm at working loads. Using M20 concrete and Fe415 steel, design the reinforcement required in the beam.



Solution:

Factored BM (M_u) $= 1.5 \times 300 \text{ kNm} = 450 \text{ kNm}$

Let effective cover to reinforcement is 50 mm.

So, effective depth of the section (d) $= 600 - 50 \text{ mm} = 550 \text{ mm}$

Depth of the flange (D_f) = 100 mm

So, $\frac{D_f}{d} = \frac{100}{550} = 0.1818 \approx 0.2$

Limiting depth of neutral axis ($x_{u\,lim}$) = $0.479d = 0.479 \times 550$ mm = 263.45 mm

Let neutral axis lies in the web of the T-beam section.

Ultimate moment of resistance of the T-beam section:

$$M_{uR} = 0.138 f_{ck} b_w d^2 + 0.362 f_{ck} (b_f - b_w) D_f (d - 0.5D_f)$$

$$= 0.138(20)(200)(550)^2 + 0.362(20)(800-200)(100)(550 - 0.5 \times 100)$$

$$= 166.98 \times 10^6 + 217.2 \times 10^6 \text{ Nmm}$$

$$= 384.18 \text{ kNm} < 450 \text{ kNm}$$

Thus a doubly reinforced beam section is required.

Area of tension steel corresponding to limiting moment of resistance is $p_{t\,lim}$ which is given by:

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right) \cdot \frac{x_{ulim}}{d}$$

$$p_{t\,lim} = 41.61\left(\frac{20}{415}\right)(0.479) = 0.9605\,\%$$

$$A_{st\,lim} = \frac{0.9605}{100}(200)(550) = 1056.55 \text{ mm}^2$$

Balance moment to be resisted by compression steel (ΔM_u) = 450 − 384.18 kNm = 65.82 kNm

Let effective cover to compression steel (d') = 50 mm

So $\frac{d'}{d} = \frac{50}{550} = 0.0909 \approx 0.1$

For $\frac{d'}{d} = 0.1$ and Fe415, $f_{sc} = 351.9$ N/mm²

A_{sc} Required in the compression side:

$$\Delta M_u = f_{sc} A_{sc} (d - d')$$

$$65.82 \times 10^6 = (351.9) A_{sc} (550 - 50)$$

$$A_{sc} = 374.08 \text{ mm}^2$$

Provide 2-16 mm diameter bars in the compression side.

Thus, $A_{sc\,provided} = 402.13$ mm²

A_{st} Required in the tension side:

$$f_{sc} A_{sc} = 0.87 f_y \Delta A_{st}$$

$$\Delta A_{st} = \frac{(351.9)(402.13)}{(0.87 \times 415)} = 391.94 \text{ mm}^2$$

Thus total area of tension steel = $A_{st\,lim} + \Delta A_{st}$ = 1056.55 + 391.94 mm² = 1448.49 mm²

Provide 5-20 mm diameter bars in the tension side.

Thus, $A_{st\,provided}$ = 1570.8 mm² > 1448.49 mm²

Maximum percentage of steel in a beam section ≤ 4%

= 0.04 × 200 × 600 mm²

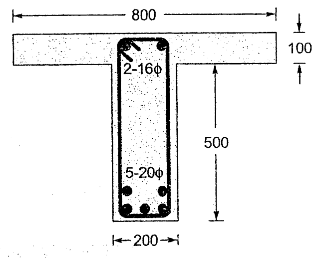
= 4800 mm²

Thus, $A_{sc\,provided}$ = 402.13 mm² < 4800 mm² (OK)

and $A_{st\,provided}$ = 1570.8 mm² < 4800 mm² (OK)

Minimum tension steel $A_{st\,min} \geq 0.85 \times 200 \times \frac{550}{415} = 225.3$ mm²

< $A_{st\,provided}$ (= 1570.8 mm²) (OK)



Reinforcement Detailing

Objective Brain Teasers

Q.1 A slab casted monolithically on a cantilever beam under the influence of gravity loads will act as a

(a) Flanged beam
(b) Curved beam
(c) Deep beam
(d) Rectangular beam

Q.2 A shed is casted with a central row of columns with a central beam on the columns. Cantilevered slabs are casted on either side of the central beam to cover the shed area. The mid span transverse section will be designed as a

(a) Flanged beam
(b) Continuous beam
(c) Spandrel beam
(d) None of the above

Q.3 When $\frac{D_f}{d} < 0.2$ is the limiting case then the flange of *T* beam will have:

(a) A constant compressive stress of $0.45 f_{ck}$
(b) A varying compressive stress
(c) A constant shear stress
(d) All of the above

Q.4 In order to have a constant compressive stress in the flange (D_f) of a *T* beam, D_f/x_u should be

(a) Greater than 3/7 (b) Greater than 4/7
(c) Lesser than 4/7 (d) Lesser than 3/7

Q.5 The expression for equivalent depth of flange 'y_f' for a constant compressive stress is based on

(a) Marcus stress block
(b) Grashoff's stress block
(c) Whitney's stress block
(d) Rankine stress block

Q.6 State true or false

An inverted T-beam subjected to gravity loads act as T beam.

Q.7 The location of neutral axis in a RC T-beam

(a) depends on flange depth and total depth
(b) Lies in the flange always
(c) Lies in the web always
(d) Lies at the junction of flange and web

Answers

1. (d) 2. (a) 3. (a) 4. (d) 5. (c)
6. False 7. (a)

Conventional Practice Questions

Q.1 Find the steel reinforcement required for a simply supported flanged beam of flange width 1000 mm, flange depth 100 mm, web width 300 mm and overall depth of 700 mm. Take effective cover to reinforcement as 50 mm. The span of beam is 12 m and it carries an imposed load of 10 kN/m^2. Use M20 and Fe415.

Q.2 Find the moment of resistance of a flanged beam with b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, effective cover = 50 mm and overall depth of 500 mm. (a) using M20 and Fe250, (b) using M20 and Fe415.

Q.3 Find the area of steel required for a simply supported flanged beam of flange width 1500 mm, flange depth 120 mm, web width 350 mm and overall depth of 700 mm. Take effective cover to reinforcement as 60 mm. The span of beam is 10 m and it carries a live load of 5 kN/m^2. Use M25 and Fe500.

Q.4 Design a flanged beam with b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, subjected to a live load of 4 kN/m^2. Use M30 and Fe500.