METHODS OF DIFFERENTIATION

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$ then $\frac{dy}{dx}$ equals -

(A) 2 sec x (sec x − tan x)

(B) $-2\sec x (\sec x - \tan x)^2$

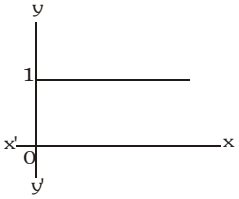
(C) $2 \sec x (\sec x + \tan x)^2$

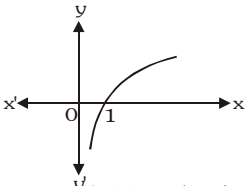
(D) $-2 \sec x (\sec x + \tan x)^2$

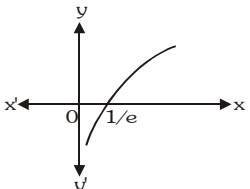
2. If $y = \frac{1+x^2+x^4}{1+x+x^2}$ and $\frac{dy}{dx} = ax + b$, then values of a & b are -

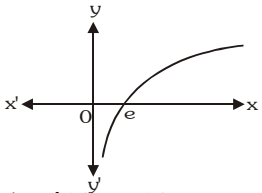
(A) a = 2 , b = 1 (B) a = −2 , b = 1 (C) a = 2 , b = −1 (D) a = −2 , b = −1

3. Which of the following could be the sketch graph of $y = \frac{d}{dx}(x \ell n x)$?

(A) 

(B) 

(C) 

(D) 

4. Let f(x) = x +3 *l*n(x − 2) & g(x) = x +5 *l*n(x - 1), then the set of x satisfying the inequality f'(x) < g'(x) is -

(A) $\left(2, \frac{7}{2}\right)$ (B) $(1, 2) \cup \left(\frac{7}{2}, \infty\right)$ (C) $(2, \infty)$ (D) $\left(\frac{7}{2}, \infty\right)$

5. Differential coefficient of $\left(x^{\frac{\ell+m}{m-n}}\right)^{\frac{1}{n-\ell}} \cdot \left(x^{\frac{m+n}{n-\ell}}\right)^{\frac{1}{\ell-m}} \cdot \left(x^{\frac{n+\ell}{\ell-m}}\right)^{\frac{1}{m-n}}$ w.r.t. x is -

(A) 1 (B) 0 (C) −1 (D) x^{lmn}

6. If $y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$ then $\frac{dy}{dx}$ at $x = e^{m^{np}}$ is equal to -

(A) e^{mnp} (B) $e^{mn/p}$ (C) $e^{np/m}$ (D) none of these

7. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$ then $\frac{dy}{dx}$ =

(A) $-\frac{x}{y}$ (B) $-\frac{y}{x}$ (C) $\frac{y}{x}$ (D) $\frac{x}{y}$

8. If $f(x) = \prod_{n=1}^{100}(x-n)^{n(101-n)}$; then $\frac{f(101)}{f'(101)}$ =

(A) 5050 (B) $\frac{1}{5050}$ (C) 10010 (D) $\frac{1}{10010}$

9. If $f(x) = (|x|)^{|\sin x|}$, then $f'\left(-\frac{\pi}{4}\right)$ is -

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

(B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$

(D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

10. If $y = \cfrac{x}{a+\cfrac{b}{x+\cfrac{a}{b+\cfrac{x}{a+\cfrac{b}{x+}}}}}\cdots\cdots\infty$ then $\frac{dy}{dx}$ -

(A) $\frac{a}{ab+2ay}$ (B) $\frac{b}{ab+2by}$ (C) $\frac{a}{ab+2by}$ (D) $\frac{b}{ab+2ay}$

11. If $y = x^{x^2}$ then $\frac{dy}{dx} =$

(A) $2\,\ell n\, x.x^{x^2}$ (B) $(2\,\ell n\, x+1).x^{x^2}$ (C) $(\ell n\, x+1).x^{x^2+1}$ (D) $x^{x^2+1}.\ell n\,(ex^2)$

12. If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is -

(A) xy (B) $\frac{x}{y}$ (C) $\frac{y}{x}$ (D) $\frac{x+y}{xy}$

13. If $x\sqrt{(1+y)}+y\sqrt{(1+x)}=0$, then $\frac{dy}{dx}$ equals -

(A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$ (C) $-\frac{1}{(1+x)}+\frac{1}{(1+x)^2}$ (D) none of these

14. If $x^2\, e^y + 2xye^x + 13 = 0$, then $\frac{dy}{dx}$ equals -

(A) $-\frac{2xe^{y-x}+2y(x+1)}{x(xe^{y-x}+2)}$ (B) $\frac{2xe^{x-y}+2y(x+1)}{x(xe^{y-x}+2)}$ (C) $-\frac{2xe^{x-y}+2y(x+1)}{x(xe^{y-x}+2)}$ (D) none of these

15. If $x = e^{y+e^{y+\cdots\cdots\text{to }\infty}}$, $x > 0$ then $\frac{dy}{dx}$ is -

(A) $\frac{x}{1+x}$ (B) $\frac{1+x}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{1}{x}$

16. If $x = \theta - \frac{1}{\theta}$ and $y = \theta + \frac{1}{\theta}$, then $\frac{dy}{dx}$ =

(A) $\frac{x}{y}$ (B) $\frac{y}{x}$ (C) $\frac{-x}{y}$ (D) $\frac{-y}{x}$

17. The derivative of $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $(x > 0)$ is -

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

18. Let g is the inverse function of f & $f'(x) = \frac{x^{10}}{\left(1+x^2\right)}$. If g (2) = a then g'(2) is equal to -

(A) $\frac{5}{2^{10}}$ (B) $\frac{1+a^2}{a^{10}}$ (C) $\frac{a^{10}}{1+a^2}$ (D) $\frac{1+a^{10}}{a^2}$

19. Let f(x) = sinx ; $g(x) = x^2$ & $h(x)=\log_e x$ & F(x) = h[g(f(x))] then $\frac{d^2F}{dx^2}$ is equal to -

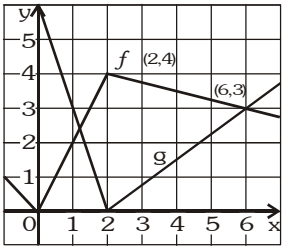
(A) $2\ \text{cosec}^3 x$ (B) $2\ \cot(x^2) - 4x^2\ \text{cosec}^2(x^2)$

(C) $2x\ \cot x^2$ (D) $-2\ \text{cosec}^2 x$

20. If $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$, then $f'(h'(g'(x))) =$

(A) 0 (B) $\frac{1}{\sqrt{x^2+1}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{x}{\sqrt{x^2+1}}$

21. If f & g are the functions whose graphs are as shown, let u(x) = f (g(x)); w(x) = g(g(x)), then the value of u'(1) + w'(1) is -



(A) $-\frac{1}{2}$ (B) $-\frac{3}{2}$

(C) $-\frac{5}{4}$ (D) does not exist

22. f'(x) = g(x) and g'(x) = - f(x) for all real x and f(5) = 2 = f'(5) then $f^2(10) + g^2(10)$ is -

(A) 2 (B) 4 (C) 8 (D) none of these

23. If f(x) = x^n, then the value of $f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots\ldots+\frac{(-1)^n f^{\text{''''''''.....(n times)}}(1)}{n!}$ -

(A) 2^{n-1} (B) 0 (C) 1 (D) 2^n

24. A function y = f(x) has second order derivative f''(x) = 6(x – 1). If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is -

(A) $(x + 1)^3$ (B) $(x + 1)^2$ (C) $(x - 1)^2$ (D) $(x - 1)^3$

25. If f (x) = $x + \frac{x^2}{1!}+\frac{x^3}{2!}+\ldots\ldots\frac{x^n}{(n-1)!}$, then f (0) + f '(0) + f ''(0) + ... + $f^{\text{''''..... n times}}$(0) is equal to -

(A) $\frac{n(n+1)}{2}$ (B) $\frac{(n^2+1)}{2}$ (C) $\left(\frac{n(n+1)}{2}\right)^2$ (D) $\frac{n(n+1)(2n+1)}{6}$

26. Let f (x) = $\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$. Then $\lim_{x\to 0}\frac{f'(x)}{x}$ =

(A) 2 (B) –2 (C) –1 (D) 1

27. If f is differentiable in (0, 6) & f'(4) =5 then $\lim_{x\to 2}\frac{f(4)-f(x^2)}{2-x}$ =

(A) 5 (B) 5/4 (C) 10 (D) 20

28. If f(4) = g(4) = 2 ; f' (4) = 9 ; g' (4) = 6 then $\lim_{x\to 4}\frac{\sqrt{f(x)}-\sqrt{g(x)}}{\sqrt{x}-2}$ is equal to -

(A) $3\sqrt{2}$ (B) $\frac{3}{\sqrt{2}}$ (C) 0 (D) none of these

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

29. The slope(s) of common tangent(s) to the curves $y = e^{-x}$ & $y = e^{-x}$ sinx can be -

(A) $-e^{-\pi/2}$ (B) $-e^{-\pi}$ (C) $\frac{\pi}{2}$ (D) 1

30. If y + ℓn(1 + x) = 0, which of the following is true ?

(A) $e^y = xy' + 1$ (B) $y' = -\frac{1}{(x+1)}$ (C) $y' + e^y = 0$ (D) $y' = e^y$

31. If $y = 2^{3^x}$, then y' equals -

(A) $3^x \ell n3 \ell n2$ (B) $y(\log_2 y) \ell n3 \ell n2$ (C) $2^{3^x} . 3^x \ell n6$ (D) $2^{3^x} . 3^x \ell n3 \ell n2$

32. If $y = 3t^2$ & $x = 2t$ then $\frac{d^2y}{dx^2}$ equals-

(A) 3t (B) 3 (C) $\frac{3}{2}$ (D) None of these

33. If g is inverse of f and $f(x) = x^2 + 3x - 3$ $(x > 0)$ then $g'(1)$ equals-

(A) $\frac{1}{2g(1)+3}$ (B) -1 (C) $\frac{1}{5}$ (D) $\frac{-f'(1)}{(f(1))^2}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	C	D	B	D	C	B	A	D	D	C	B	A	C
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	C	B	D	C	B	C	B	D	A	B	D	A	A,B	A,B,C
Que.	31	32	33												
Ans.	B,D	C	A,C												

EXTRA PRACTICE QUESTIONS ON METHODS OF DIFFERENTIATION

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If $y = fofof(x)$ and $f(0) = 0$, $f'(0) = 2$, then find $y'(0)$ -

(A) 6 (B) 7 (C) 8 (D) 9

2. If $y^2 = p(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ is equal to -

(A) $p'''(x) \cdot p'(x)$ (B) $p''(x) \cdot p'''(x)$ (C) $p(x) \cdot p'''(x)$ (D) none of these

3. If y is a function of x then $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$. If x is a function of y then the equation becomes -

(A) $\frac{d^2x}{dy^2} + x\frac{dx}{dy} = 0$ (B) $\frac{d^2x}{dy^2} + y\left(\frac{dx}{dy}\right)^3 = 0$ (C) $\frac{d^2x}{dy^2} - y\left(\frac{dx}{dy}\right)^2 = 0$ (D) $\frac{d^2x}{dy^2} - x\left(\frac{dx}{dy}\right)^2 = 0$

4. If $y = \tan x \tan 2x \tan 3x$ then $\frac{dy}{dx}$ is equal to-

(A) $3\sec^2 3x \tan x \tan 2x + \sec^2 x \tan 2x \tan 3x + 2\sec^2 2x \tan 3x \tan x$

(B) $2y(\operatorname{cosec} 2x + 2\operatorname{cosec} 4x + 3\operatorname{cosec} 6x)$

(C) $3\sec^2 3x - 2\sec^2 2x - \sec^2 x$

(D) $\sec^2 x + 2\sec^2 2x + 3\sec^2 3x$

5. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ then $\frac{dy}{dx}$ equals -

(A) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$ (B) $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$ (C) $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$ (D) $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

6. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\infty}}}$ then $\frac{dy}{dx}$ -

(A) $\frac{1}{2y-1}$ (B) $\frac{x}{x-2y}$ (C) $\frac{1}{\sqrt{1+4x}}$ (D) $\frac{y}{2x+y}$

7. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ has the value equal to -

(A) $-\frac{2^y}{2^x}$ (B) $\frac{1}{1-2^x}$ (C) $1-2^y$ (D) $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

8. The functions $u = e^x \sin x$; $v = e^x \cos x$ satisfy the equation -

(A) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ (B) $\frac{d^2u}{dx^2} = 2v$ (C) $\frac{d^2v}{dx^2} = -2u$ (D) none of these

9. Two functions f & g have first & second derivatives at x = 0 & satisfy the relations,

$f(0) = \frac{2}{g(0)}$, $f'(0) = 2$ $g'(0) = 4g(0)$, $g''(0) = 5$ $f''(0) = 6$ $f(0) = 3$ then -

(A) if $h(x) = \frac{f(x)}{g(x)}$ then $h'(0) = \frac{15}{4}$ (B) if $k(x) = f(x) \cdot g(x) \sin x$ then $k'(0) = 2$

(C) $\underset{x\to 0}{\text{Limit}}\frac{g'(x)}{f'(x)} = \frac{1}{2}$ (D) none of these

10. If $y^2 + b^2 = 2xy$, then $\frac{dy}{dx}$ equals -

(A) $\frac{1}{xy - b^2}$ (B) $\frac{y}{y - x}$ (C) $\frac{xy - b^2}{(y - x)^2}$ (D) $\frac{xy - b^2}{y}$

11. If $\sqrt{y + x} + \sqrt{y - x} = c$, then $\frac{dy}{dx}$ is equal to -

(A) $\frac{2x}{c^2}$ (B) $\frac{x}{y + \sqrt{y^2 - x^2}}$ (C) $\frac{y - \sqrt{y^2 - x^2}}{x}$ (D) $\frac{c^2}{2y}$

12. $\lim\limits_{x \to 0^+} \left(\left(x^{x^x} \right) - x^x \right)$ is -

(A) equal to 0 (B) equal to 1 (C) equal to –1 (D) non existent

13. Select the correct statements -

(A) The function f defined by f(x) = $\begin{cases} 2x^2 + 3 & \text{for } x \le 1 \\ 3x + 2 & \text{for } x > 1 \end{cases}$ is neither differentiable nor continuous at $x = 1$.

(B) The function $f(x) = x^2|x|$ is twice differentiable at x = 0

(C) If f is continuous at x = 5 and f(5) = 2 then $\lim\limits_{x \to 2} f(4x^2 - 11)$ exists.

(D) If $\lim\limits_{x \to a} (f(x)+g(x)) = 2$ and $\lim\limits_{x \to a} (f(x) - g(x)) = 1$ then $\lim\limits_{x \to a} f(x).\ g(x)$ may not exist.

14. Let $\ell = \lim\limits_{x \to 0} x^m \left(\ell n x \right)^n$ where m, n $\in$ N then -

(A) ℓ is independent of m and n (B) ℓ is independent of m and depends on m

(C) ℓ is independent of n and depends on m (D) ℓ is dependent on both m and n

15. $\lim\limits_{x \to 0} \frac{\log_{\sin^2 x} \cos x}{\log_{\sin^2 \frac{x}{2}} \cos \frac{x}{2}}$ has the value equal to -

(A) 1 (B) 2 (C) 4 (D) none of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	C	A,B,C	A,C	A,C,D	A,B,C,D	A,B,C	A,B,C	B,C
Que.	11	12	13	14	15					
Ans.	A,B,C	C	B,C	A	C					