DPP No: 19		MATHS
Maximum Time 50 Min		TARGET JEE-MAIN

SYLLABUS : DEFINITE INTEGRATION

1. Evaluate : $\int_0^{\pi/2} \sin^9 x \cos^4 x\, dx$ -

(A) $\frac{128}{1515}$ (B) $\frac{128}{15015}$ (C) $\frac{64}{15015}$ (D) None of these

2. If $f(x) = \int_{\frac{1}{x}}^{\sqrt{x}} \cos t^2\, dt$ $(x > 0)$ then $\frac{df(x)}{dx}$ is

(A) $\frac{\sqrt{x}\cos x + 2\cos(x^{-2})}{2x\sqrt{x}}$ (B) $\frac{x\sqrt{x}\cos x + 2\cos(x^{-2})}{2x^2}$

(C) $2\sqrt{x}\cos x - \frac{2}{x}\cos\left(\frac{1}{x}\right)$ (D) none of these.

3. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then, $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals-

(A) 18 (B) 12 (C) 36 (D) 24

4. $f(x) = \int_0^x (t-1)(t-2)^2(t-3)^3(t-4)^5\, dt$ $(x > 0)$ then number of points of extremum of f(x) is

(A) 4 (B) 3 (C) 2 (D) 1

5. $\underset{h\to 0}{\text{Limit}} \frac{\int_a^{x+h} \ell n^2 t\,dt - \int_a^x \ell n^2 t\,dt}{h}$ equals to :

(A) 0 (B) $\ell n^2 x$ (C) $\frac{2\ell nx}{x}$ (D) does not exist

6. The value of the function $f(x) = 1 + x + \int_1^x (\ell n^2 t + 2\,\ell nt)\, dt$, where $f'(x)$ vanishes is:

(A) e^{-1} (B) 0 (C) $2e^{-1}$ (D) $1 + 2e^{-1}$

7. If $\int_a^y \cos t^2 dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{2\sin^2 x}{x\cos^2 y}$ (B) $\frac{2\sin x^2}{x\cos y^2}$ (C) $\frac{2\sin x^2}{x\left(1-2\sin\frac{y^2}{2}\right)}$ (D) $\frac{\sin x^2}{2y}$

8. If $\int_{\sin x}^{1} t^2 (f(t))\, dt = (1 - \sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

(A) 1/3 (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

9. The value of $\lim_{a\to\infty} \frac{1}{a^2} \int_0^a \ln(1+e^x)\, dx$ equals

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) non-existent

10. $f(x) = \int_1^x \frac{\sin x \cos y}{y^2 + y^2 + 1} dy$, then

(A) $f'(x) = 0 \ \forall x = \frac{n\pi}{2}, n \in Z$ (B) $f'(x) = 0 \ \forall x = (2n+1)\frac{\pi}{2}, n \in Z$

(C) $f'(x) = 0 \ \forall x = n\pi, n \in Z$ (D) $f'(x) \neq 0 \ \forall x \in R$

11. $\int_0^{\pi/2} \sin^4 x \cos^3 x\, dx$ is equal to

(A) $\frac{6}{35}$ (B) $\frac{2}{21}$ (C) $\frac{2}{15}$ (D) $\frac{2}{35}$

12. $\int_0^1 x^2(1-x)^3\, dx$ is equal to :

(A) $\frac{1}{60}$ (B) $\frac{1}{30}$ (C) $\frac{2}{15}$ (D) $\frac{\pi}{120}$

13. $f(x) = \int_x^{x^2} \frac{e^t}{t} dt$, then $f'(1)$ is equal to :

14. (i) If $f(x) = 5^{g(x)}$ and $g(x) = \int_{2}^{x^2} \frac{t}{\ell n\,(1+t^2)}\,dt$, then find the value of $f'(\sqrt{2})$.

(ii) The value of $\lim_{x\to 0} \dfrac{\frac{d}{dx}\int_{0}^{x^3}\sqrt{\cos t}\,dt}{1-\sqrt{\cos x}}$

(iii) Find the slope of the tangent to the curve $y = \int_{x}^{x^2} \cos^{-1} t^2$ at $x = \frac{1}{\sqrt[4]{2}}$

15. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to -

(A) 2 (B) –2 (C) 1/2 (D) –1/2

16. If $I_1 = \int_{0}^{1} 2^{x^2} dx$, $I_2 = \int_{0}^{1} 2^{x^3} dx$, $I_3 = \int_{1}^{2} 2^{x^2} dx$ and $I_4 = \int_{1}^{2} 2^{x^3} dx$ then

(A) $I_3 > I_4$ (B) $I_3 = I_4$ (C) $I_1 > I_2$ (D) $I_2 > I_1$

17. Let $I = \int_{0}^{1} \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_{0}^{1} \frac{\cos x}{\sqrt{x}}\,dx$. Then, which one of the following is true ?

(A) $I > \frac{2}{3}$ and $J > 2$ (B) $I < \frac{2}{3}$ and $J < 2$ (C) $I < \frac{2}{3}$ and $J > 2$ (D) $I > \frac{2}{3}$ and $J < 2$

18. Let $I = \int_{1}^{3} \sqrt{x^4 + x^2}\,dx$, then

(A) $I > 6\sqrt{10}$ (B) $I < 2\sqrt{2}$ (C) $2\sqrt{2} < I < 6\sqrt{10}$ (D) $I < 1$

19. $I = \int_{0}^{2\pi} e^{\sin^2 x + \sin x + 1}\,dx$, then

(A) $\pi e^3 < I < 2\pi e^5$ (B) $2\pi e^{3/4} < I < 2\pi e^3$ (C) $2\pi e^3 < I < 2\pi e^4$ (D) $0 < I < 2\pi$

20. Let $f''(x) \geq 0$, $f'(x) > 0$, $f(0) = 3$ & $f(x)$ is defined in [–2, 2]. If $f(x)$ is non-negative, then

(A) $\int_{-1}^{0} f(x)dx > 6$ (B) $\int_{-2}^{2} f(x)dx > 12$ (C) $\int_{-2}^{2} f(x)dx \geq 12$ (D) $\int_{-1}^{1} f(x)dx > 12$

21. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x\, dx}{1+e^x}$ is-

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

22. The value of $\int_{-2n}^{2n+\frac{1}{2}} (\sin \pi x)\left\{\frac{x}{2}\right\}dx$ is (where {x} denotes the fractional part of x) -

(A) $\frac{-2n\pi+1}{\pi^2}$ (B) $\frac{n}{\pi}$ (C) $\frac{(n+1)}{\pi}$ (D) $\frac{2n\pi-1}{\pi^2}$

23. Let f,g and h be continuous function on [0,a] such that f(x) = f(a–x), g(x) = – g(a–x) and 3h(x) – 4h(a – x) = 5. Then $\int_0^a f(x)\, g(x)\, h(x)\, dx =$

(A) 5/4 (B) 3/4 (C) 1 (D) 0

24. The inflection points on the graph of function y = $\int_0^x (t-1)(t-2)^2 dt$ are

(A) x = –1 (B) x = 3/2 (C) x = 4/3 (D) x = 1

25. Let $I_n = \int_0^1 (1-x^3)^n dx,\ (n \in N)$ then

(A) $3n\, I_n = (3n-1)\, I_{n-1}\ \forall n \geq 2$ (B) $(3n-1)I_n = 3n\, I_{n-1}\ \forall n \geq 2$

(C) $(3n-1)I_n = (3n+1)\, I_{n-1}\ \forall n \geq 2$ (D) $(3n+1)I_n = 3n\, I_{n-1}\ \forall n \geq 2$

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ANSWER KEY OF DPP NO. : 19

1.	(B)	2.	(B)	3.	(A)	4.	(B)	5.	(B)	6.	(D)	7.	(B)
8.	(C)	9.	(C)	10.	(B)	11.	(D)	12.	(A)	13.	(e)		
14.	(i) $4\sqrt{2}$		(ii) 12		(iii) $\left(\frac{\sqrt[4]{8}}{3}-\frac{1}{4}\right)\pi$			15.	(A)	16.	(C)	17.	(B)
18.	(C)	19.	(B)	20.	(C)	21.	(B)	22.	(A)	23.	(D)	24.	(C)
25.	(D)												