22

Electromagnetic Induction

The phenomenon of generating current/emf in a conducting circuit by change in strength, position or orientation of an associated external magnetic field is called *electromagnetic induction*. The emf, so developed is called *induced emf*.

IN THIS CHAPTER ....

- Magnetic Flux
- Faraday's Laws of EMI
- Lenz's Law and Conservation of Energy
- Motional Electromotive Force
- Fleming's Right Hand Rule
- Self-inductance
- Mutual Inductance
- Eddy Currents
- AC Generator or Dynamo
- DC Motor

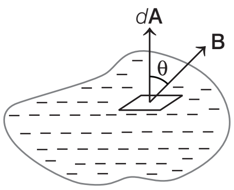
Magnetic Flux

Magnetic flux is the volume flow rate of magnetic field linked with an area.

It is also equals to total number of field lines passing through any surface normally when it is placed in magnetic field.

For an element of area $d\mathbf{A}$ on an arbitrary shaped surface as shown in below figure. If the magnetic field at this element is $\mathbf{B}$, then the magnetic flux through the element is

$$d\phi_B = \mathbf{B} \cdot d\mathbf{A} = BdA \cos\theta$$



Here, $d\mathbf{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA and θ is the angle between $\mathbf{B}$ and dA at that element.

In general, total magnetic flux through the surface is given as

$$\phi_B = \int BdA\cos\theta = \int \mathbf{B} \cdot d\mathbf{A}$$

It is a scalar quantity. The SI unit of magnetic flux is tesla-metre^2 (1 T-m^2). This unit is called *weber* (1Wb).

$$1 \text{ Wb} = 1 \text{ T-m}^2 = 1 \text{ N-m/A}$$

The CGS unit of flux is maxwell (Mx). 1 Wb = 10^8 Mx

The dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.

Specific conditions

For $\theta = 0°$	For $\theta = 90°$	For $\theta = 180°$
B is parallel to surface vector, *i.e.* **A**. $\Rightarrow \phi_B = BA\cos 0° = BA$ $\Rightarrow \phi_B =$ maximum	**B** is perpendicular to surface vector, *i.e.* **A**. $\Rightarrow \phi_B = BA\cos 90° = 0$	**B** is anti-parallel to surface vector, *i.e.* **A**. $\Rightarrow \phi_B = BA\cos 180° = BA(-1) = -BA$

Faraday's Laws of EMI

Experiments revealed that when magnetic flux linked with a conductor varies, an electromotive force (emf) is produced in the conductor. If the conductor forms part of a closed circuit, then emf produced causes an electric current in the circuit. This phenomena is known as **electromagnetic induction** (EMI).

Magnitude and direction of emf and current induced are found by Faraday's and Lenz's laws.

According to Faraday's law, "the induced emf in a conductor is proportional to the time rate of change of magnetic flux linked with the conductor."

$$e \propto \frac{d\phi_B}{dt} \quad \text{or} \quad e = -\frac{d\phi_B}{dt}$$

In SI system, proportionality constant is unity.

If a circuit is a coil consisting of N loops all of the same area and if ϕ_B is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by the expression,

$$e = -N\frac{d\phi_B}{dt}$$

Induced current produced in the circuit is given by

$$i = \frac{e}{R} = -\frac{N}{R}\cdot\frac{d\phi_B}{dt}$$

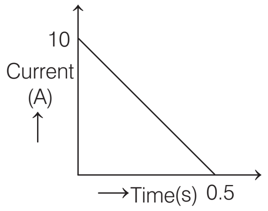
where, R = resistance of loop.

This induced emf and current exists only when flux linked with the conductor is changing.

Induced charge is given as $dq = \frac{1}{R}(d\phi_B)$

Note Integral form of Faraday's law of electromagnetic induction is $\oint \mathbf{E}\cdot d\mathbf{l} = \frac{-d\phi}{dt}$.

Example 1. *In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is* **[JEE Main 2017]**



(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

Sol. (b) Induced current, $i = \frac{e}{R}$

Here, e = induced emf $= -\frac{d\phi_B}{dt}$

$$i = -\left(\frac{d\phi_B}{dt}\right)\cdot\frac{1}{R}$$

$$d\phi_B = -iRdt$$

$$\phi_B = -\int iRdt$$

Here, R is constant.

$\therefore \quad |\phi_B| = R\int idt$

$$\int i\cdot dt = \text{Area under } i\text{-}t \text{ graph}$$

$$= \frac{1}{2}\times 10\times 0.5 = 2.5$$

$\therefore \quad \phi_B = R\times 2.5 = 100\times 2.5 = 250 \text{ Wb}$

Example 2. *A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate dB / dt = 0.032 Ts^{-1}. The induced current in the loop is close to (Take, resistivity of the metal wire $= 1.23\times 10^{-8}$ Ωm)* **[JEE Main 2020]**

(a) 0.61 A (b) 0.43 A (c) 0.53 A (d) 0.34 A

Sol. (a) Induced emf in square loop,

$$E = A\left(\frac{dB}{dt}\right) = l^2\left(\frac{dB}{dt}\right) = \left(\frac{L}{4}\right)^2\left(\frac{dB}{dt}\right) \qquad (\because \phi_B = BA)$$

Here, $L = 30 \text{ cm} = 30\times 10^{-2}$ m

$\frac{dB}{dt} = 0.032 \text{ Ts}^{-1}$

$\therefore \quad E = 1.8\times 10^{-4}$ V

Induced current, $I = \frac{E}{R} = \frac{E}{\rho\frac{L}{A}} = \frac{EA}{\rho L}$

Here, $\rho = 1.23\times 10^{-8}$ Ω-m

$L = 30\times 10^{-2}$ m

$A = \pi r^2 = \pi(2\times 10^{-3})^2 = 12.56\times 10^{-6} \text{ m}^2$

So, $I = \frac{1.8\times 10^{-4}\times 1.256\times 10^{-5}}{1.23\times 10^{-8}\times 30\times 10^{-2}} = 0.61 \text{ A}$

Lenz's Law and Conservation of Energy

The negative sign in Faraday's equation of electromagnetic induction describes the direction in which induced emf drives the current. This direction is easily determined with the help of Lenz's law, an associated principle with Faraday's law.

Lenz's law states that, "the direction of any magnetic induction effect is such as to oppose the cause of effect."

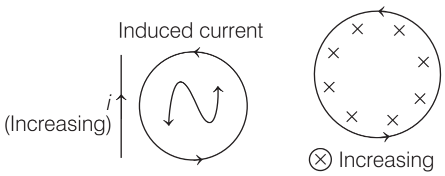
This law is based upon law of conservation of energy.

Direction of induced current with the help of Lenz's law

Direction of induced current can be determined by checking whether the flux through a conducting loop or circuit is increasing or decreasing.

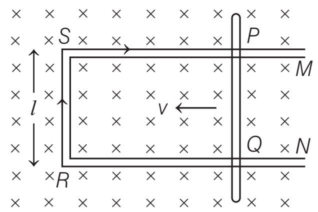
- If flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.
- If flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

> **Note** Also to apply Lenz's law, you can remember RIN or ⊗ IN (when the loop lies on the plane of paper), where
>
> (i) **RIN** In RIN, R stands for right, I stands for increasing and N stands for north pole (anti-clockwise). It means, if a loop is placed on the right side of a straight current-carrying conductor and the current in the conductor is increasing, then induced current in the loop is anti-clockwise ( ).
>
> (ii) ⊗ **IN** In ⊗ IN, suppose the magnetic field in the loop is perpendicular to paper inwards ⊗ and this field is increasing, then induced current in the loop is anti-clockwise ( ).



Motional Electromotive Force

Consider a straight conductor of length (as shown below) which is moving through a uniform magnetic field directed into the page.



Due to the motion of conductor in **B** a potential difference is maintained between the ends of the conductor as long as the conductor continuous to move through the uniform magnetic field.

This potential difference is called *motional electromagnetic force*, which is denoted by e and is given by

$$e = Bvl$$

If R is the resistance of the closed circuit (as shown above), then current will be written as

$$i = \frac{e}{R} = \frac{Bvl}{R}$$

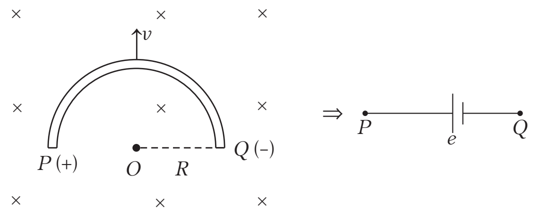
For a conductor of any shape, moving in a uniform or non-uniform magnetic field, motional emf is given as

$$e = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \text{ (for closed conducting loop)}$$

Some important points related to motional emf are given below

- For a semi-circular conducting loop of radius R with the centre at O and moving with velocity v, then the emf is given by

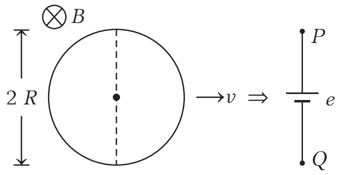
$$e = Bv(2R)$$



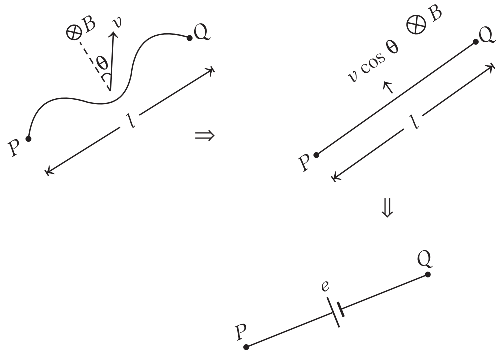
$$V_P - V_Q = 2BvR$$

- For a circular loop of radius R moving with velocity v, the emf is given by

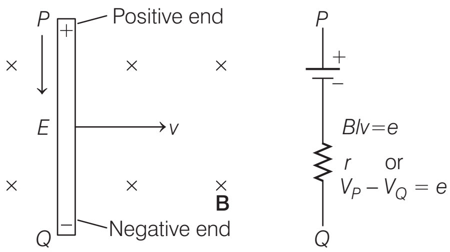
$$e = Bv(2R) = 2BvR$$



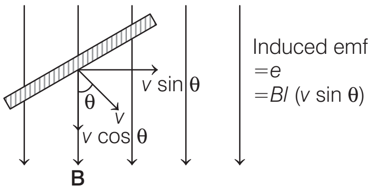
- For a irregular shape, a conducting body of length l. Then, $e = B(v\cos\theta)l$



- A moving conductor (PQ) is equivalent to a cell or battery as shown below



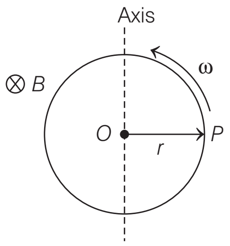
- For a conductor moving at some angle 'θ' with direction of magnetic field, velocity component perpendicular to magnetic field is taken,



- An emf is induced between the ends of rod, when a conducting rod of length l rotates about an axis passing through one of its ends (that end may be fixed) with an angular velocity ω in a plane perpendicular to the magnetic field B and is given by

$$|\varepsilon| = \frac{1}{2} Bl^2\omega$$

- If a conducting disc of radius r rotates with constant angular velocity ω about its axis in a uniform magnetic field parallel to its axis of rotation



The induced emf is given by

$$e = \frac{1}{2} B\omega r^2$$

That means, disc is equivalent to rod.

- Due to presence of current and magnetic field , arm PQ (as shown in the above figure) experiences a magnetic force given by

$$F = BIl = \frac{B^2l^2v}{R}$$

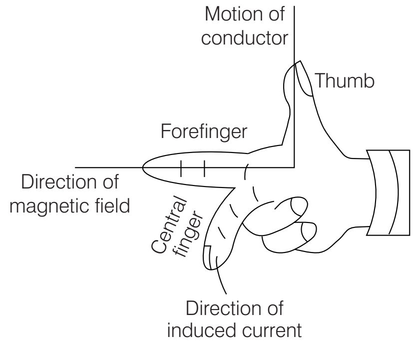
- The force given above is directed opposite to the velocity of the rod and hence the power required to maintain constant speed v is given as

$$P = Fv = \frac{B^2l^2v^2}{R}$$

- The direction of induced current is given by Fleming's right hand rule.

Fleming's Right Hand Rule

If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all three are perpendicular to each other, then if thumb represent the direction of motion, the forefinger represent the direction of magnetic field, then central finger will represent the direction of induced current.



Example 3. *A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of $5.0\ ms^{-1}$, at right angles to the horizontal component of the earth's magnetic field of $0.3 \times 10^{-4}\ Wb/m^2$. The value of the induced emf in wire is* **[JEE Main 2019]**

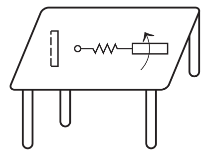
(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 0.3×10^{-3} V (d) 2.5×10^{-3} V

Sol. (a) Wire falls perpendicularly to horizontal component of earth's magnetic field, so induced emf $(e) = Blv$

Substituting the given values, we get

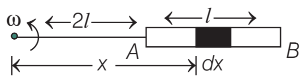
$$e = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3}\ \text{V}$$

Example 4. *A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is* **[JEE Main 2013]**



(a) $\frac{2B\omega l^2}{2}$ (b) $\frac{3B\omega l^2}{2}$
(c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

Sol. (d) ∵ Induced emf is rate of change of magnetic flux.



∴ $$e = \int Bv\,dx$$

⇒ $$e = \int_{2l}^{3l} (\omega x) \cdot B \cdot dx \qquad [\because v = \omega x]$$

Using $$\int x^n dx = \frac{x^{n+1}}{n+1}$$

⇒ $$e = B\omega \left[\frac{x^2}{2}\right]_{2l}^{3l}$$

$$e = B\omega \frac{[(3l)^2 - (2l)^2]}{2} = \frac{5Bl^2\omega}{2}$$

Example 5. *In a uniform magnetic field of induction B, a wire in the form of a semi-circle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is*

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$ (c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

Sol. (*b*) The flux associated with coil of area *A* and magnetic induction *B* is

$$\phi = BA\cos\theta = \frac{1}{2}B\pi r^2\cos\omega t$$

$$\left[\because A = \frac{1}{2}\pi r^2, \text{ for semi-circle}\right]$$

$$\therefore \quad e_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2\cos\omega t\right) = \frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\therefore \text{ Power, } P = \frac{e^2_{\text{induced}}}{R} \qquad [\because P = V^2/R]$$

$$= \frac{B^2\pi^2r^4\omega^2\sin^2\omega t}{4R}$$

Hence, $P_{\text{mean}} = \langle P \rangle$

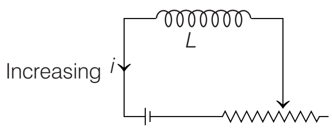
$$= \frac{B^2\pi^2r^4\omega^2}{4R}\cdot\frac{1}{2} \qquad \left[\because \langle\sin^2\omega t\rangle = \frac{1}{2}\right]$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

Self-inductance

It is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.

This induced emf is also called *back emf*. When the current in a coil is switched on, then the self-induction opposes the growth of the current and when it is switched off, then the self-induction opposes the decay of the current. Hence, self-inductance is also known as *inertia of electricity*.



For a coil of N turns, the total flux ($N\phi_B$) linked with the coil is directly proportional to the current (i) in the coil. *i.e.*

$$N\phi_B \propto i \quad \text{or} \quad N\phi_B = Li \quad \text{or} \quad L = \frac{N\phi_B}{i}$$

L is coefficient of self-inductance of coil.

The induced emf e produced in the coil is directly proportional to the rate of change of current. Thus,

$$e = -L\frac{di}{dt}$$

The minus sign here is a reflection of Lenz's law. It says that the self induced emf in a circuit opposes any change in the current in that circuit. From the above equation,

$$L = \left|\frac{-e}{di/dt}\right|$$

Following are few important results for self-inductance

- Self-inductance of a circular coil of radius r,

$$L = \frac{\mu_0\pi N^2 r}{2}$$

$$\Rightarrow \quad L \propto N^2$$

$$\Rightarrow \quad \frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right) \text{ (For constant } r\text{)}$$

- Self-inductance for a square coil of side a,

$$L = \frac{2\sqrt{2}\mu_0 N^2 a}{\pi}$$

$$\Rightarrow \quad L \propto N^2$$

- Self-inductance of a solenoid $L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al$

  where, N = total number of turns in the solenoid,
  l = length of the coil,
  n = number of turns in the coil
  and A = area of cross-section of the coil.

- If core of the solenoid is of any other magnetic material, then

$$L = \frac{\mu_0\mu_r N^2 A}{l}$$

- Self-inductance of a toroid, $L = \frac{\mu_0 N^2 A}{2\pi r}$

  where, r = radius of the toroid.

- Self-inductance of a toroid with radius r,

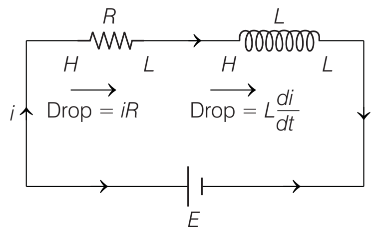
$$L = \frac{\mu_0 N^2 r}{2}$$

Kirchhoff's Second Law with an Inductor

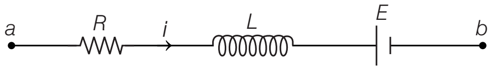
According to Kirchhoff's second law (loop rule), when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L\,di/dt$, where di/dt is to be substituted with sign.

e.g. In the loop shown in figure, Kirchhoff's second law gives the equation,

$$\Rightarrow \quad E - iR - L\frac{di}{dt} = 0$$



Example 6. *In the circuit diagram shown in figure, $R = 10\ \Omega$, $L = 5H$, $E = 20\ V$, $i = 2\ A$. This current is decreasing at a rate of $-1.0\ A/s$. Find V_{ab} at this instant.*



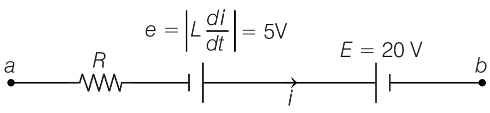
(a) 20 V (b) 35 V
(c) 40 V (d) 25 V

Sol. (b) PD across inductor, $V_L = L\dfrac{di}{dt} = (5)\,(-1.0) = -5\ \text{V}$

Now, using Kirchhoff's law, $V_a - iR - V_L - E = V_b$

$\therefore \quad V_{ab} = V_a - V_b = E + iR + V_L$
$= 20 + (2)(10) - 5 = 35\ \text{V}$

Note As the current is decreasing the inductor can be replaced by a source of emf, $e = \left|L \cdot \dfrac{di}{dt}\right| = 5$ V in such a manner that this emf supports the decreasing current or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards b. Thus, the given circuit can be drawn as



Now, we can find V_{ab}.

Example 7. *A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil*

(a) increases by a factor of 3 ***[JEE Main 2019]***
(b) decreases by a factor of $9\sqrt{3}$
(c) increases by a factor of 27
(d) decreases by a factor of 9

Sol. (c) Self-inductance of a coil is given by the relation

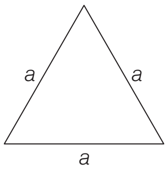
$$L = \mu_0 n^2 A \cdot l$$

where, n is number of turns per unit length. Shape of the wooden frame is equilateral triangle.

$\therefore$ Area of equilateral triangle, $A = \dfrac{\sqrt{3}}{4}a^2$

(where, a is side of equilateral triangle)

$\therefore$ Self-inductance, $L = \mu_0 n^2 \left[\dfrac{\sqrt{3}}{4}a^2\right] \cdot l$



Here, $l = 3a \times N$ (where, N is total turns)

$\therefore \quad L = \mu_0 n^2 \left[\dfrac{\sqrt{3}}{4}a^2\right] \times 3aN$ or $L \propto a^3$

When each side of frame is increased by a factor 3 keeping the number of turns per unit length of the frame constant,

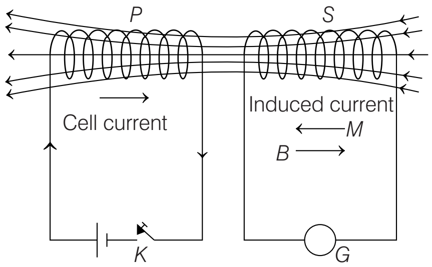
then $\quad a' = 3a$

$\therefore \quad L' \propto (a')^3$ or $L' \propto (3a)^3$ or $L' \propto 27a^3$ or $L' = 27L$

So, self-inductance will becomes 27 times.

Mutual Inductance

The phenomenon according to which an opposing emf is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called *mutual inductance*.



Suppose the circuit 1 has a current i_1 flowing in it. Then total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in circuit 1. Thus,

$$N_2\,\phi_{B_2} \propto i_1$$

or $\quad N_2\phi_{B_2} = Mi_1$

Here, the proportionality constant M is known as the mutual inductance M of the two circuits.

Thus, $\quad M = \dfrac{N_2\phi_{B_2}}{i_1}$

From the above expression, M can be defined as the total flux linked with circuit 2 per unit current in circuit 1.

If we change the current in circuit 1 at a rate di_1/dt, an induced emf e_2 is developed in circuit 1, which is proportional to the rate di_1/dt.

Thus, $\quad e_2 \propto di_1/dt$

or $\quad e_2 = -M di_1/dt$

or $\quad M = \left|\dfrac{-e_2}{di_1/dt}\right|$

So, the mutual inductance of two circuits is the magnitude of induced emf e_2 per unit rate of change of current di_1/dt.

The SI unit of mutual inductance is henry (H).

M depends upon closeness of the two circuits, their orientations and sizes and the number of turns, etc.

Reciprocity theorem $M_{21} = M_{12} = M$

As, $\quad e_2 = -M(di_1/dt)$

and $\quad e_1 = -M(di_2/dt)$

$$M_{12} = \frac{N_2 \phi_{B_2}}{i_1} \quad \text{and} \quad M_{21} = \frac{N_1 \phi_{B_1}}{i_2}$$

Following are few important results for mutual inductance

- Mutual inductance for two concentric coils having N_1 and N_2 turns with radius r and R ($R >> r$), respectively is

$$M = \frac{\pi \mu_0 N_1 N_2 r^2}{2R} \Rightarrow \quad M \propto \frac{r^2}{R}$$

- Mutual inductance for two long coaxial solenoids is

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 \, Al$$

where, N_1 and N_2 are total number of turns in both coils, n_1 and n_2 are number of turns per unit length in coils, A is area of cross-section of coils and l is length of the coils.

Example 8. *Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross- sectional area $A = 10\ cm^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$)*

(a) $2.4\pi \times 10^{-5}$ *H* *(b)* $4.8\pi \times 10^{-4}$ *H*

(c) $4.8\pi \times 10^{-5}$ *H* *(d)* $2.4\pi \times 10^{-4}$ *H*

Sol. (*d*) $M = \frac{\mu_0 N_1 \times N_2 \times A}{l}$

where, $N_1 = 300$ turns, $N_2 = 400$ turns,

$A = 10\ \text{cm}^2 = 10 \times 10^{-4} \text{m}^2$ and $l = 20\ \text{cm} = 20 \times 10^{-2}$ m

Substituting the values in the given formula, we get

$M = 2.4\,\pi \times 10^{-4}$ H

Coefficient of Coupling

The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

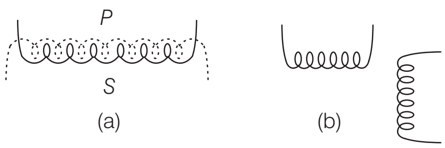
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

It is also defined as

$$K = \frac{\text{magnetic flux linked in secondary coil}}{\text{magnetic flux linked in primary coil}}$$

where, $0 \le K \le 1$.

Coefficient of coupling is maximum ($K = 1$) in case (a), when coils are coaxial and minimum in case (b), when coils are placed a right angles.

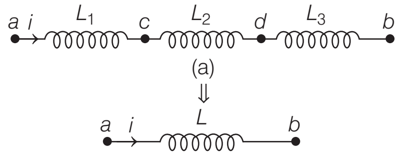


(a) (b)

Combination of Inductances

If coefficient of coupling $k = 0$, then

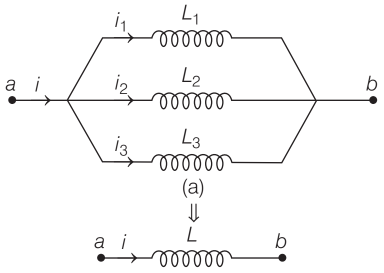
(i) When three coils of inductances L_1, L_2 and L_3 are connected in series and the coefficient of coupling $K = 0$ as in series, then $L = L_1 + L_2 + L_3$



(a) ⇓ (b)

(ii) When three coils of inductances L_1, L_2 and L_3 are connected in parallel and the coefficient of coupling $K = 0$ as in parallel, then

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



(a) ⇓ (b)

If coefficient of coupling $K = 1$, then

(i) **In series**

(a) If current in two coils are in the same direction, then

$$L = L_1 + L_2 + 2M$$

(b) If current in two coils are in opposite directions, then

$$L = L_1 + L_2 - 2M$$

(ii) **In parallel**

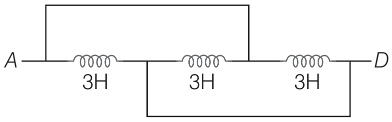
(a) If current in two coils are in same direction, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

(b) If current in two coils are in opposite directions, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

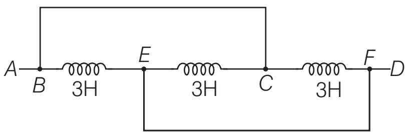
Example 9. *The inductance between A and D is*

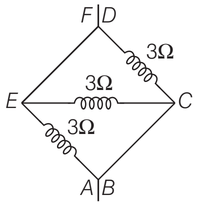


(a) 3.66 *H* *(b)* 9 *H*

(c) 0.66 *H* *(d)* 1 *H*

Sol. (d)





Here, inductors are in parallel.

$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ or $L = 1$ H

Energy Stored in an Inductor

The total energy stored by the inductor when the current in the circuit increases from zero to final value i is given as

$$U = \int_0^i Lidi = \frac{1}{2}Li^2; \text{ Also } U = \frac{1}{2}(Li)i = \frac{N\phi i}{2}$$

Also, energy density is given by $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

Energy density is property of magnetic field.

Example 10. *The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is* **[JEE Main 2019]**

(a) 437.5 J (b) 740 J (c) 637.5 J (d) 540 J

Sol. (a) Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2}LI^2$$

When current is being changed from I_1 to I_2, change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2 \quad \text{...(i)}$$

As I_1 and I_2 are given, we need to find value of L.

Now, induced emf in a coil is

$$\varepsilon = L\frac{dI}{dt}$$

Here, $\varepsilon = 25$ V,

$dI = I_2 - I_1 = (25 - 10) = 15$ A and $dt = 1$s

$\Rightarrow \quad 25 = L \times \frac{15}{1}$ or $L = \frac{25}{15} = (5/3)$ H

Putting values of L, I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525$$

$$\Delta E = 437.5 \text{ J}$$

Example 11. *A 10 H inductor is used in an induction cooker plate operating on a current of 20 A. How much ice at 0°C could be melted by the cooker if whole of energy is used to heat the ice? (Take, latent heat of ice = 22.6×10^3 J/kg)*

(a) 1 kg (b) 2.2 kg
(c) 0.88 kg (d) 0.9 kg

Sol. (c) Energy stored by a inductor $= \frac{1}{2}Li^2$

This energy is completely used in melting the ice.

Hence, $\frac{1}{2}Li^2 = mL_f$

where, L_f = latent heat of fusion.

Hence, mass of ice melted, $m = \frac{Li^2}{2L_f}$

Substituting the values, we have

$$m = \frac{(10)(20)^2}{2(2.26 \times 10^3)} = 0.88 \text{ kg}$$

Eddy Currents

The currents induced in bulk pieces of conductors when the magnetic flux linked with the conductor changes are known as eddy currents. These currents are always produced in a plane which is perpendicular to the direction of magnetic field. Eddy current shows both heating and magnetic effects.

Eddy currents can be reduced by

(i) Reducing the area of metallic core of transformers, electric motor and other such devices.

(ii) Using laminations of metal to make a metal core. The plane of the laminations must be arranged parallel to the magnetic field.

Various applications of eddy currents are as follows

(i) **Magnetic Breaking in Trains** Strong electromagnets are situated above the rails in some electrically powered trains. When electromagnets situated above rails are activated, the eddy currents induced in the rails opposes the motion of train.

(ii) **Electromagnetic Damping** Certain galvanometer have a fixed core made of non-magnetic metallic material. When the coil oscillates, the eddy currents generated in the core opposes the motion and bring the coil to rest quickly.

(iii) **Induction Furnace** It can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals.

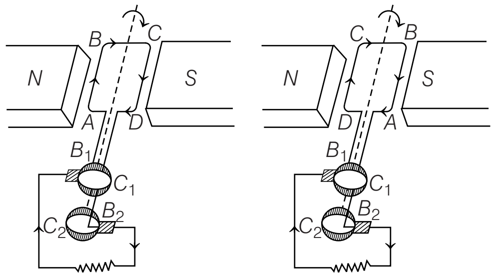
(iv) **Electric Power Meters** Electric currents are induced in the shiny metal disc by magnetic fields produced by sinusoidally varying currents in a coil.

AC Generator or Dynamo

It is a device which converts mechanical energy into alternating current energy. Its working is based on electromagnetic induction.

The induced emf produced by the AC generator is given by

$$e = NBA\omega \sin \omega t = e_0 \sin \omega t$$



Working of AC dynamo

There are four main parts of an AC generator

(i) **Armature** It is rectangular coil of insulated copper wire having a large number of turns.

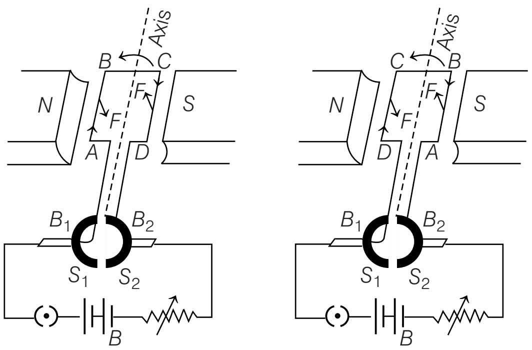
(ii) **Field Magnets** These are two pole pieces of a strong electromagnet.

(iii) **Slip Rings** These are two hollow metallic rings.

(iv) **Brushes** These are two flexible metals or carbon rods, which remains slightly in contact with slip rings.

DC Motor

It is a device which converts electrical energy into mechanical energy. Its working is based on the fact that when a current carrying coil is placed in uniform magnetic field a torque acts on it.



Torque acting on a current carrying coil placed in uniform magnetic field, $\tau = NBIA \sin \theta$

When armature coil rotates a back emf is produced in the coil,

$$\text{Efficiency of a motor, } \eta = \frac{\text{Back emf}}{\text{Applied emf}} = \frac{E}{V}$$

Example 12. *A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But, if an aluminium plate is placed near to the coil, it stops. This is due to*

(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

Sol. (*d*) According to Lenz's law, electromagnetic induction takes place in the aluminium plate due to which eddy current is developed which oppose the motion or vibrations of coil.

This causes loss in energy which results in damping of oscillatory motion of the coil.

Example 13. *A planar loop of wire rotates in a uniform magnetic field. Initially at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10s about an axis in its plane, then the magnitude of induced emf will be maximum and minimum respectively at* **[JEE Main 2020]**

(a) 2.5 *s and* 7.5 *s* *(b)* 2.5 *s and* 5.0 *s*
(c) 5.0 *s and* 10.0 *s* *(d)* 5.0 *s and* 7.5 *s*

Sol. (*b*) Induced emf in the wire loop, $e = BA\omega \sin \omega t$

Induced emf is maximum, when $\sin \omega t = \pm 1$

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{2}$$

Here, $T = 10$ s

So, $t = \frac{10}{4} = 2.5$ s

and induced emf is minimum, when $\sin \omega t = 0$

$$\Rightarrow \quad \omega t = \pi \Rightarrow \frac{2\pi}{T} \cdot t = \pi \Rightarrow t = \frac{10}{2} = 5 \text{ s}$$

Practice Exercise

ROUND I Topically Divided Problems

Magnetic Flux, Faraday's Law and Lenz's Law

1. A square of side L metres lies in the XY-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0 (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$T, where B_0 is constant. The magnitude of flux passing through the square is **[NCERT Exemplar]**

(a) $2 B_0 L^2$ Wb (b) $3 B_0 L^2$ Wb
(c) $4 B_0 L^2$ Wb (d) $\sqrt{29} B_0 L^2$ Wb

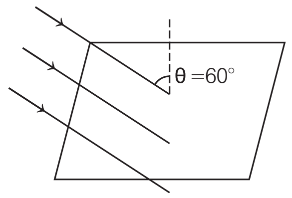
2. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_o. Which of the following option is correct? **[JEE Main 2020]**

(a) $\phi_i > \phi_o$ (b) $\phi_i < \phi_o$
(c) $\phi_i = \phi_o$ (d) $\phi_i = -\phi_o$

3. The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t) \times 10^{-3}$ Wb, where t is in second. The induced emf at $t = 2$s is

(a) -4 V (b) -1 V
(c) $+1$ V (d) $+4$ V

4. A square loop of wire of side 5 cm is lying on a horizontal table. An electromagnet above and to one side of the loop is turned on, causing a uniform magnetic field downwards at an angle of 60° to the vertical as shown in figure. The magnetic induction is 0.50 T. The average induced emf in the loop, if the field increases from zero to its final value in 0.2 s is

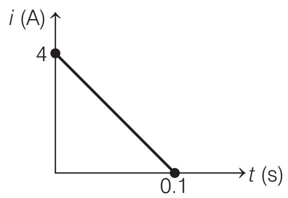


(a) 5.4×10^{-3} V (b) 3.12×10^{-3} V
(c) 0 (d) 0.25×10^{-3} V

5. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved in time t second from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is

(a) $\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

6. Some magnetic flux is changed from a coil of resistance 10Ω. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil (in Wb) is



(a) 2 (b) 4 (c) 6 (d) 0

7. The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in a time Δt, then the total quantity of electric charge Q that passes any point in the circuit during the time Δt is represented by

(a) $\frac{\Delta\phi}{R}$ (b) $\frac{\Delta\phi}{\Delta t}$ (c) $R \cdot \frac{\Delta\phi}{\Delta t}$ (d) $\frac{1}{R} \cdot \frac{\Delta\phi}{\Delta t}$

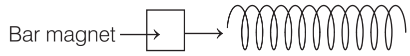
8. A circular ring of diameter 20 cm has a resistance of 0.01 Ω. The charge that will flow through the ring, if it is turned from a position perpendicular to a uniform magnetic field of 2.0 T to a position parallel to the field is about

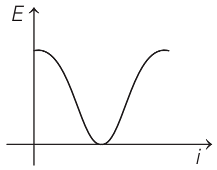
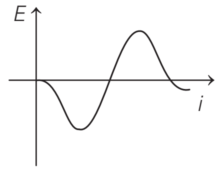
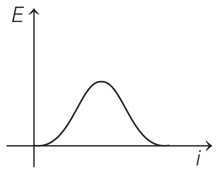
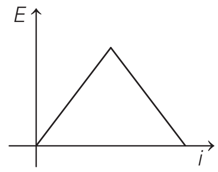
(a) 63 C (b) 0.63 C (c) 6.3 C (d) 0.063 C

9. The charge which will flow through a 200 Ω galvanometer connected to a 400 Ω circular coil of 1000 turns wound on a wooden stick 20 mm in diameter, if a magnetic field $B = 0.012$ T parallel to the axis of the stick decreased suddenly to zero is

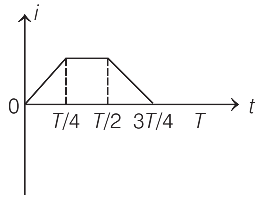
(a) 6.3 μC (b) 63 μC (c) 0.63 μC (d) 630 μC

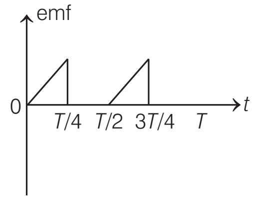
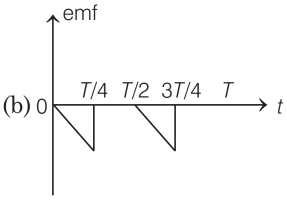
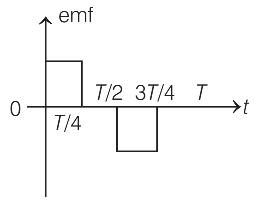
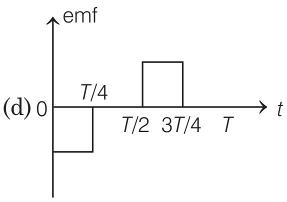
10. The variation of induced emf (E) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as



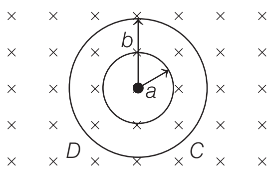
(a) 
(b) 
(c) 
(d) 

11. The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be



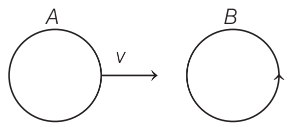
(a) 
(b) 
(c) 
(d) 

12. Plane figure made of thin wires of resistance $R = 50$ milli Ω/m are located in a uniform magnetic field perpendicular into the plane of the figures and which decrease at the rate $dB/dt = 0.1$ m T/s. Then currents in the inner and outer boundaries are (The inner radius a = 10 cm and outer radius b = 20 cm)



(a) 10^{-4} A (clockwise), 2×10^{-4} A (clockwise)
(b) 10^{-4} A (anti-clockwise), 2×10^{-4} A (clockwise)
(c) 2×10^{-4} A (clockwise), 10^{-4} A (anti-clockwise)
(d) 2×10^{-4} A (anti-clockwise), 10^{-4} A (anti-clockwise)

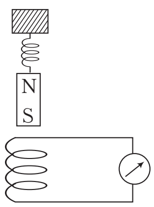
13. There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that



[NCERT Exemplar]

(a) there is a constant current in the clockwise direction in A
(b) there is a varying curent in A
(c) there is no current in A
(d) there is a constant current in the counter clockwise direction in A

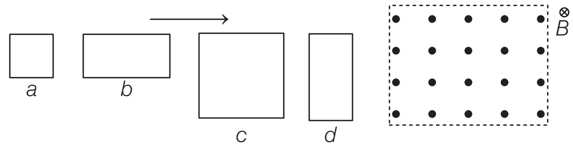
14. A magnet N-S is suspended from a spring and while it oscillates, the magnet moves in and out of the coil C. The coil is connected to a galvanometer G, then as the magnet oscillates,



(a) G shows deflection to the left and right with constant amplitude
(b) G shows deflection on one side
(c) G shows no deflection
(d) G shows deflection to the left and right but the amplitude steadily decreases

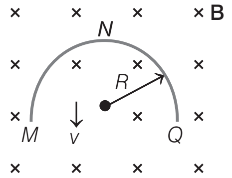
Motional EMF

15. The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field B (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the emf induced as they move through the field, greatest first



(a) $(e_c = e_d) < (e_a = e_b)$
(b) $(e_c = e_d) > (e_a = e_b)$
(c) $e_c > e_d > e_b > e_a$
(d) $e_c < e_d < e_b < e_a$

16. A thin semi-circular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\mathbf{B}$. At the position MNQ, the speed of the ring is v. The potential difference developed across the ring is



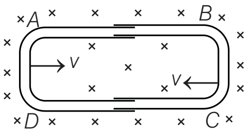
(a) zero

(b) $\frac{1}{2}Bv\pi R^2$ and M is at a higher potential

(c) πRBv and Q is at a higher potential

(d) $2RBv$ and Q is at a higher potential

17. One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be

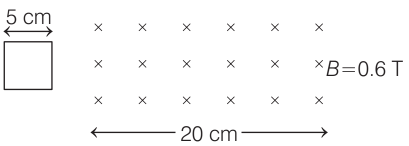


(a) Blv (b) $-Blv$ (c) zero (d) $2\,Blv$

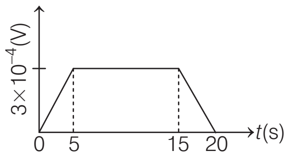
18. An airplane with 20 m wing spread is flying at $250\ \text{ms}^{-1}$ straight south parallel to the earth's surface. The earth's magnetic field has a horizontal component of $2 \times 10^{-5}\ \text{Wbm}^{-2}$ and the dip angle is 60°. Calculate the induced emf between the plane tips.

(a) 0.174 V (b) 0.173 V (c) 1.173 V (d) 0.163 V

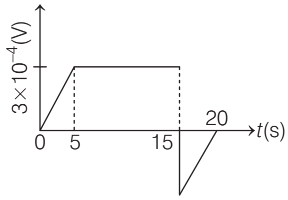
19. A square loop of side 5 cm enters a magnetic field with $1\ \text{cms}^{-1}$. The front edge enters the magnetic field at $t = 0$, then which graph best depicts emf variation correctly?



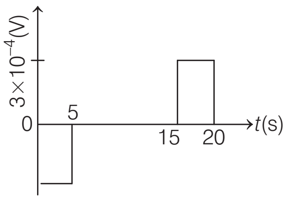
(a)



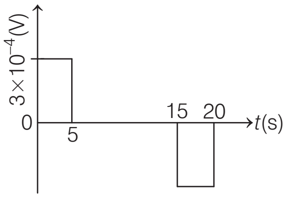
(b)



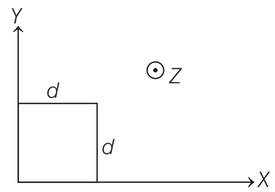
(c)



(d)



20. The magnetic field in a region is given by $B = B_0\left(\frac{x}{a}\right)\hat{\mathbf{k}}$. A square loop of side d is placed with its edges along the X and Y-axes. The loop is moved with a constant velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$, then emf induced in the loop is **[JEE Main 2021]**



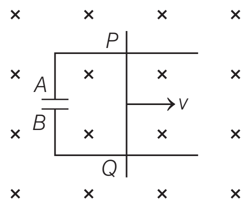
(a) $\frac{B_0 v_0^2 d}{2a}$ (b) $\frac{B_0 v_0 d}{2a}$

(c) $\frac{B_0 v_0 d^2}{a}$ (d) $\frac{B_0 v_0 d^2}{2a}$

21. The rails of a railway track insulated from each other and the ground are connected to a millivoltmeter. Find the reading of voltmeter, when a train travels with a speed of 180 km/h along the track. Given that, the vertical component of earth magnetic field is $0.2 \times 10^{-4}\ \text{Wb/m}^2$ and the rails are separated by 1m.

(a) 10^{-4} V (b) 10^{-2} V (c) 10^{-3} V (d) 1 V

22. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2.0\ \text{ms}^{-1}$ in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\ \mu\text{F}$ is connected as shown in figure, then



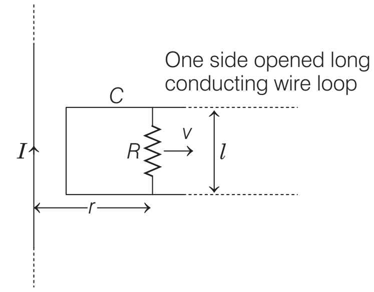
(a) $q_A = -80\ \mu\text{C}$ and $q_B = +80\ \mu\text{C}$

(b) $q_A = +80\ \mu\text{C}$ and $q_B = -80\ \mu\text{C}$

(c) $q_A = 0 = q_c$

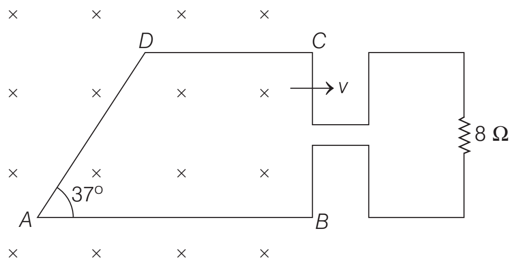
(d) charge stored in the capacitor increases exponentially with time

23. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self-inductance of the loop are negligible. The induced current in the loop, as a function of separation r between the connector and the straight wire is **[JEE Main 2020]**



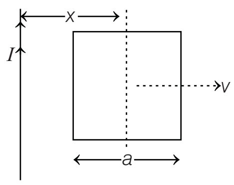
(a) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$ (b) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (c) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (d) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

24. The loop $ABCD$ is moving with velocity v towards right. The magnetic field is 4 T. The loop is connected to a resistance of 8 Ω. If steady current of 2 A flows in the loop, then value of v, if loop has resistance of 4 Ω, is (Given, $AB = 30$ cm, $AD = 30$ cm, $\sin 37° = \frac{3}{5}$)



(a) $\frac{50}{3}$ ms^{-1} (b) 20 ms^{-1} (c) 10 ms^{-1} (d) $\frac{100}{3}$ ms^{-1}

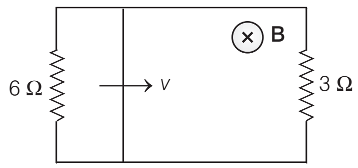
25. A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity v. The induced emf in the frame will be proportional to



(a) $\frac{1}{x^2}$ (b) $\frac{1}{(2x-a)^2}$

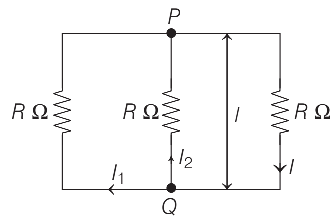
(c) $\frac{1}{(2x+a)^2}$ (d) $\frac{1}{(2x-a)(2x+a)}$

26. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$ T. Perpendicular to the plane of loop. Resistance of connector is $R = 2\ \Omega$. Two resistance of $6\ \Omega$ and $3\ \Omega$ are connected as shown in figure. The external force required to keep the connector moving with a constant velocity $v = 2\ \text{ms}^{-1}$ is



(a) 2 N (b) 1 N (c) 4 N (d) 6 N

27. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \frac{Blv}{R}$

28. A very small circular loop of radius a is initially (at $t = 0$) co-planar and concentric with a much larger fixed circular loop of radius b. A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is

(a) $\frac{\pi a^2 \mu_0 I}{2b}\omega\cos\omega t$ (b) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin\omega^2 t^2$

(c) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin\omega t$ (d) $\frac{\pi a^2 \mu_0 I}{2b}\omega\sin^2\omega t$

Self and Mutual Inductances

29. When current in a coil changes from 2 A to −2 A in 0.05s, an emf of 8 V is induced in the coil. The coefficient of self-inductance of the coil is

(a) 0.1 H (b) 0.2 H (c) 0.4 H (d) 0.8 H

30. An air cored coil has a self-inductance of 0.1 H. A soft iron core of relative permeability 1000 has number of coil as (1/10)th of previous. The value of self-inductance now becomes

(a) 1 mH (b) 10 mH (c) 1 H (d) 10 H

31. A coil is wound on a core of rectangular cross-section. If all the linear dimensions of core are increased by a factor 2 and number of turns per unit length of coil remains same, the self-inductance increases by a factor of
(a) 16 (b) 8 (c) 4 (d) 2

32. The self-inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as **[NCERT Exemplar]**
(a) Both l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) Both l and A decrease

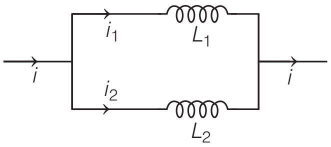
33. A coil of wire of certain radius has 100 turns and a self-inductance of 15 mH. The self-inductance of a second similar coil of 500 turns will be
(a) 75 mH (b) 375 mH (c) 15 mH (d) 400 mH

34. Two coils have mutual inductance 0.005 H. The current changes in the first coil according to equation $i = i_0 \sin \omega t$, where $i_0 = 10$ A and $\omega = 100\pi$ rads^{-1}. The maximum value of emf in second coil is
(a) 2π (b) 5π (c) π (d) 4π

35. A current of 10 A in the primary coil of a circuit is reduced to zero. If the coefficient of mutual inductance is 3H and emf induced in secondary coil is 30 kV, time taken for the change of current is
(a) 10^3 s (b) 10^2 s (c) 10^{-3} s (d) 10^{-2} s

36. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?
(a) 30 Wb (b) 33 Wb (c) 23 Wb (d) 42 Wb

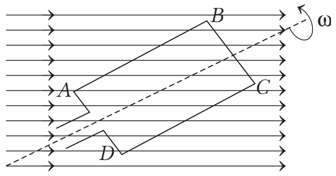
37. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of current, $\frac{i_1}{i_2}$ at any time t is



(a) $\frac{L_2}{L_1}$ (b) $\frac{L_1}{L_2}$

(c) $\frac{L_2^2}{(L_1 + L_2)^2}$ (d) $\frac{L_1^2}{(L_1 + L_2)^2}$

38. A rectangular coil $ABCD$ which is rotated at a constant angular velocity about an horizontal as shown in the figure. The axis of rotation of the coil as well as the magnetic field B are horizontal. Maximum current will flow in the circuit when the plane of the coil is

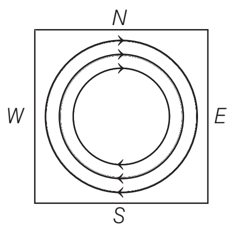


(a) inclined at 30° to the magnetic field
(b) perpendicular to the magnetic field
(c) inclined at 45° to the magnetic field
(d) parallel to the magnetic field

ROUND II Mixed Bag

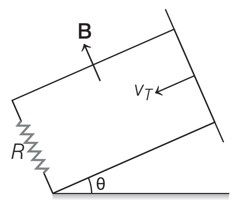
Only One Correct Option

1. When a sheet of metal is placed in a magnetic field, which changes from zero to a maximum value, the induced currents are set up in the direction shown in figure. What is the direction of magnetic field?



(a) Into the plane of the paper
(b) Out of the plane of the paper
(c) West to East
(d) South to North

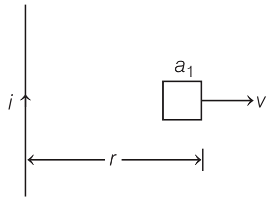
2. A copper rod of mass m slides under gravity on two smooth parallel rails l distance apart and set at an angle θ to the horizontal. At the bottom, the rails are joined by a resistance R (figure). There is a uniform magnetic field B perpendicular to the plane of the rails. The terminal velocity of the rod is



(a) $\frac{mgR \tan\theta}{B^2 l^2}$ (b) $\frac{mgR \cot\theta}{B^2 l^2}$

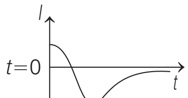
(c) $\frac{mgR \sin\theta}{B^2 l^2}$ (d) $\frac{mgR \cos\theta}{B^2 l^2}$

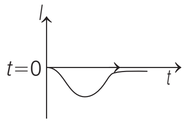
3. A square loop of side a placed in the same plane as a long straight wire carrying a current i. The centre of the loop is at a distance r from the wire, where $r >> a_1$ (see the figure). The loop is moved away from the wire with a constant velocity v. The induced emf in the loop is

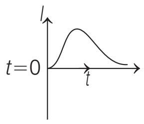


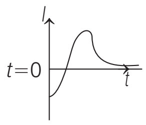
(a) $\frac{\mu_0 \, iav}{2\pi r}$
(b) $\mu_0 \, ia^3 v$
(c) $\frac{\mu_0 \, iv}{2\pi}$
(d) $\frac{\mu_0 \, ia^2 v}{2\pi r^2}$

4. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by **[JEE Main 2019]**

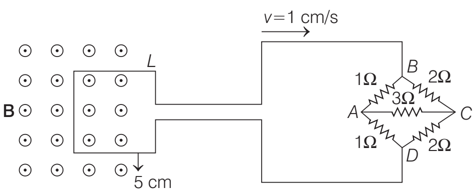
(a) 

(b) 

(c) 

(d) 

5. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to **[JEE Main 2019]**



(a) 60 μA
(b) 170 μA
(c) 150 μA
(d) 115 μA

6. A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0 t(1 - t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \leq t \leq 1$, the induced current (I_R) and the induced emf (V_R) in the ring change as **[JEE Main 2020]**

(a) direction of I_R remains unchanged and V_R is maximum at $t = 0.5$s
(b) at $t = 0.25$s, direction of I_R reverses and V_R is maximum
(c) direction of I_R remains unchanged and V_R is zero at $t = 0.25$s
(d) at $t = 0.5$s, direction of I_R reverses and V_R is zero

7. Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P, a magnetic flux of 10^{-3} Wb passes through Q and no current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is **[JEE Main 2019]**

(a) 6.67×10^{-3} Wb
(b) 6.67×10^{-4} Wb
(c) 3.67×10^{-3} Wb
(d) 3.67×10^{-4} Wb

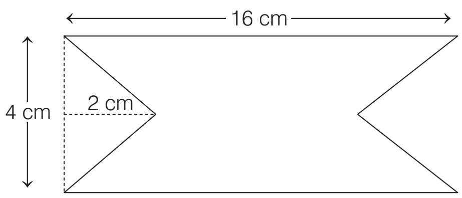
8. There are two long coaxial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is **[JEE Main 2019]**

(a) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$
(b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$
(c) $\frac{n_2}{n_1}$
(d) $\frac{n_1}{n_2}$

9. The total number of turns and cross-section area in a solenoid is fixed, however its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to **[JEE Main 2019]**

(a) $1/L$
(b) L^2
(c) L
(d) $1/L^2$

10. At time $t = 0$, magnetic field of 1000 G is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 G, in the next 5 s, then induced emf in the loop is **[JEE Main 2020]**



(a) 48 μV
(b) 28 μV
(c) 56 μV
(d) 36 μV

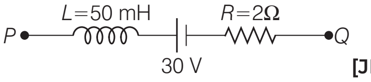
11. A conducting circular loop is made of a thin wire has area $3.5 \times 10^{-3} m^2$ and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T) \sin(0.5 \pi t)$. The field is uniform in space, then the net charge flowing through the loop during $t = 0$ s to $t = 10$ ms is close to **[JEE Main 2019]**

(a) 6 mC
(b) 21 mC
(c) 7 mC
(d) 14 mC

Numerical Value Questions

12. In a fluorescent lamp choke (a small transformer), 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25A to 0 in a duration of 0.025 ms.The self-inductance of the choke (in mH) is estimated to be .... **[JEE Main 2020]**

13. A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at the rate of $10^2 As^{-1}$. The value of the potential difference $V_P - V_Q$ (in volt) at that instant, is ....



[JEE Main 2020]

14. Two concentric circular coils C_1 and C_2 are placed in the XY-plane. C_1 has 500 turns and radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$A, where t is in second. The emf induced in C_1 (in mV), at the instant $t = 1$ s is $\frac{4}{x}$. The value of x is .... **[JEE Main 2020]**

15. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field, so that it undergoes half of rotation in 0.2s.The maximum value of emf induced (in μV) in the coil will be close to the integer .... **[JEE Main 2020]**

Answers

Round I

1. (c)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)
11. (d)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (c)
21. (c)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (a)	**38.** (d)		

Round II

1. (b)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** 10	**13.** 33	**14.** 5	**15.** 15					

Solutions

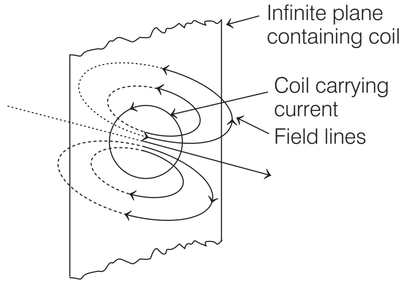
Round I

1. Here, $\mathbf{A} = L^2\,\hat{\mathbf{k}}$ and $\mathbf{B} = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, $\phi = ?$

As, $\phi = \mathbf{B}\cdot\mathbf{A} = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\cdot L^2\hat{\mathbf{k}}$

$\therefore\quad \phi = 4\,B_0 L^2$ Wb

2. We are given with following situation



From the diagram of field lines, we can observe that whatever be the number of field lines emitted from coil, all of them goes back into the infinite plane only.

So, magnetic flux emanating from coil is equal and opposite to the flux linked with infinite plane.

So, $\qquad \phi_i = -\phi_o$

3. Magnetic flux, $\phi = (t^3 - 2t)\times 10^{-3}$

$$\frac{d\phi}{dt} = (3t^2 - 2)\times 10^{-3}$$

$$\left.\frac{d\phi}{dt}\right|_{t=2} = (3\times 4 - 2)\times 10^{-3}\ \text{Wbs}^{-1}$$

$$= 10^{-2}\ \text{Wbs}^{-1}$$

$$e = -N\frac{d\phi}{dt} = -100\times 10^{-2} = -1\ \text{V}$$

4. $e = \dfrac{d\phi}{dt} = \dfrac{(NBA\cos\theta - 0)}{t}$

$$= \frac{1\times 0.5\times 25\times 10^{-4}\times\cos 60^\circ - 0}{0.2}$$

$e = 3.12\times 10^{-3}$ V

5. The rate of change of flux or induced emf in the coil is

$$e = -n\frac{d\phi}{dt}$$

$\therefore$ Induced current, $I = \dfrac{e}{R'} = -\dfrac{n}{R'}\dfrac{d\phi}{dt}$ $\qquad$ ...(i)

Given, $\quad R' = R + 4R = 5R,\ d\phi = W_2 - W_1,\ dt = t$

[here, W_1 and W_2 are flux associated with one turn]

Putting the given values in Eq. (i), we get

$$I = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

6. $|dq| = \dfrac{d\phi}{R} = i\ dt =$ Area under i-t graph

$\therefore d\phi =$ (Area under i-t graph) R

$$= \frac{1}{2}\times 4\times 0.1\times(10) = 2\ \text{Wb}$$

7. $|e| = \dfrac{\Delta\phi}{\Delta t}$

$$i = \frac{|e|}{R} = \frac{1}{R}\frac{\Delta\phi}{\Delta t}$$

$$Q = i\cdot\Delta t = \frac{\Delta\phi}{R\Delta t}\cdot\Delta t = \frac{\Delta\phi}{R}$$

8. As, $q = \dfrac{-d\phi}{R} = \dfrac{BA(\cos 0^\circ - \cos 90^\circ)}{R}$

$$= \frac{B\pi r^2(1-0)}{R} = \frac{B\pi r^2}{R} = \frac{2\times 3.143\times(10^{-1})^2}{0.01}$$

$= 6.286$ C $\simeq 6.3$ C

9. $q = \dfrac{d\phi}{R} = \dfrac{NA(B_2 - B_1)}{R} = \dfrac{N\pi r^2(B_2 - B_1)}{R}$

$$= \frac{1000\times\pi\times 10^{-4}\times(0.012 - 0)}{(200 + 400)}$$

$= 6.3\times 10^{-6}\text{C} = 6.3\ \mu\text{C}$

10. As the magnet moves towards the coil, the magnetic flux increases (non-linearly). Also, there is a change in polarity of induced emf when the magnet passes on to the other side of the coil. It is shown in option (b).

11. $e = -L\dfrac{di}{dt}$

During 0 to $\dfrac{T}{4}$, $\dfrac{di}{dt} =$ constant

So, $e = -$ ve (negative)

For $\dfrac{T}{4}$ to $\dfrac{T}{2}$, $\dfrac{di}{dt} = 0$

$e = 0$

For $\dfrac{T}{2}$ to $\dfrac{3T}{4}$, $\dfrac{di}{dt} =$ constant

$e = +$ ve (positive)

12. Current in the inner coil, $i = \dfrac{e}{R} = \dfrac{A_1}{R_1}\dfrac{dB}{dt}$

Length of the inner coil $= 2\pi a$

So, its resistance, $R_1 = 50\times 10^{-3}\times 2\pi(a)$

$$\therefore i_1 = \frac{\pi a^2}{50\times 10^{-3}\times 2\pi(a)}\times 0.1\times 10^{-3}\ \text{A} = 10^{-4}\ \text{A}$$

According to Lenz's law, direction of i_1 is clockwise.

Induced current in outer coil, $i_2 = \dfrac{e_2}{R_2} = \dfrac{A_2}{R_2}\dfrac{dB}{dt}$

$$\Rightarrow \quad i_2 = \frac{\pi b^2}{50 \times 10^{-3} \times (2\pi b)} \times 0.1 \times 10^{-3}$$
$$= 2 \times 10^{-4} \text{ A (clockwise)}$$

13. Coil A must be carrying a constant current in counter clockwise direction. That is why when A moves towards B, current induced in B is in counter clockwise direction, as per Lenz's law. The current in B would stop when A stops moving.

14. A magnet oscillates the magnetic flux through the coil changes and thus current or emf is produced in it, but in opposite direction, whose magnitude and direction changes with time. So, G shows deflection to the left and right but the amplitude decreases.

15. Emf induces across the length of the wire which cuts the magnetic field. (Length of c = Length of d) > (Length of a = Length of b).

So, $(e_c = e_d) > (e_a = e_b)$.

16. The emf induced in a conductor does not depend on its shape, but only on its end points, M and Q in this case. Thus, the conductor is equivalent to an imaginary straight conductor of $l = MQ = 2R$. Therefore, potential difference developed across the ring $= Blv = B(2R)v$ and the direction of induced current is from Q to M. Therefore, Q is at higher potential.

17. Relative velocity $= v - (-v) = 2v = \dfrac{dl}{dt}$

Now, $\quad e = \dfrac{d\phi}{dt}$

$\Rightarrow \quad e = \dfrac{Bl\,dl}{dt} \qquad [\because \phi = BA]$

Induced emf, $e = 2Blv \qquad \left[\because \dfrac{dl}{dt} = 2v\right]$

18. As the plane is flying horizontally it will cut the vertical component of earth's field B_V, so the emf induced between its tips

$$e = B_V vl$$

But as by definition of angle of dip,

$$\tan\theta = \frac{B_V}{B_H}$$

i.e. $\quad B_V = B_H \tan\theta$

So, $e = (B_H \tan\theta)vl = 2 \times 10^{-5} \times \sqrt{3} \times 250 \times 20$

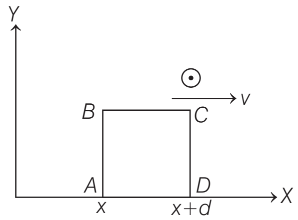
i.e. $\quad e = (\sqrt{3}) \times 10^{-1} \text{ V} = 0.173 \text{ V}$

19. When loop is entering in the field, magnetic flux (*i.e.* ×) linked with the loop increases so induced emf in it $e = Bvl = 0.6 \times 1 \times 10^{-2} \times 5 \times 10^{-2} = 3 \times 10^{-4}$ V (negative).

When loop completely entered in the field (after 5 s) flux linked with the loop remains constant, so $e = 0$.

After 15 s, loop begins to exit out, linked magnetic flux decreases, so induced emf, $e = 3 \times 10^{-4}$ V (positive).

20. Let at time t, the loop is at places shown in figure



The emf induced in AB, $\varepsilon_{AB} = \dfrac{B_0(x)}{a} v_0 d$

and the emf induced in CD, $\varepsilon_{CD} = \dfrac{B_0(x+d)}{a} v_0 d$

$$\varepsilon_{\text{net}} = \varepsilon_{CD} - \varepsilon_{AB} = \frac{B_0 v_0 d^2}{a}$$

21. Given, $B_V = 0.2 \times 10^{-4}$ Wb/m^2

We know that, $v = 180$ km/h

$$e = B_V vl$$
$$= 0.2 \times 10^{-4} \times 180 \times \frac{5}{18} \times 1$$
$$= 2 \times 10^{-5} \times 50$$
$$= 1 \times 10^{-3} \text{ V} = 10^{-3} \text{ V}$$

22. Motional emf across PQ,

$$V = Blv = 4(1)(2) = 8 \text{ V}$$

This is the potential to which the capacitor is charged.

As, $\quad q = CV$

$\therefore \quad q = (10 \times 10^{-6}) \times 8$

$= 8 \times 10^{-5}$ C

$= 80 \mu$C

As magnetic force on electron in the conducting rod PQ is towards Q, therefore A is positively charged and B is negatively charged.

i.e. $\quad q_A = +80 \mu$C

and $\quad q_B = -80 \mu$C

23. Magnetic field due to wire at a distance r is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Induced emf, $e_{\text{induced}} = Bvl = \dfrac{\mu_0 I vl}{2\pi r}$

Now, induced current in the loop,

$$I_{\text{induced}} = \frac{e_{\text{induced}}}{R} = \frac{\mu_0 I}{2\pi} \frac{vl}{Rr}$$

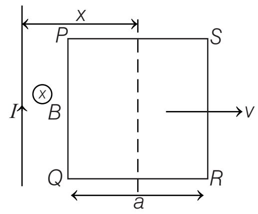
24. Current, $I = \dfrac{\text{Potential difference}}{\text{Resistance}}$

$\therefore \quad$ Potential difference $= 2 \times 12 = 24 \text{ V} = Bvl$

Here, $\quad l = AD \sin 37° = 0.3 \times \dfrac{3}{5} = 0.18$ m

$\therefore \quad$ Velocity, $v = \dfrac{24}{Bl} = \dfrac{24}{4 \times 0.18} = \dfrac{100}{3}$ ms^{-1}

25.



Potential difference (induced emf) across PQ,

$$V_P - V_Q = B_1 av$$
$$= \frac{\mu_0 I}{2\pi\left(x - \frac{a}{2}\right)} av$$

Potential difference (induced emf) across side RS of frame,

$$V_S - V_R = B_2 av$$
$$= \frac{\mu_0 I}{2\pi\left(x + \frac{a}{2}\right)} av$$

Hence, the net potential difference (induced emf) in the loop will be

$$V_{\text{net}} = (V_P - V_Q) - (V_S - V_R)$$
$$= \frac{\mu_0 Iav}{2\pi}\left[\frac{1}{\left(x - \frac{a}{2}\right)} - \frac{1}{\left(x + \frac{a}{2}\right)}\right]$$
$$= \frac{\mu_0 Iav}{2\pi}\left[\frac{a}{\left(x - \frac{a}{2}\right)\left(x + \frac{a}{2}\right)}\right]$$

Thus, $V_{\text{net}} \propto \dfrac{1}{(2x - a)(2x + a)}$

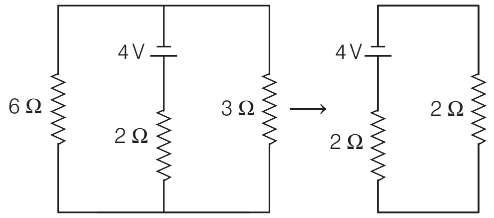
26. Motional emf induced in the connector,

$$e = Blv = 2(1)(2) = 4\text{ V}$$

This acts as a cell of emf 4 V and internal resistance 2Ω, 6Ω and 3Ω resistors are in parallel.

$$\therefore \quad \frac{1}{R_p} = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$R_p = 2\,\Omega$$



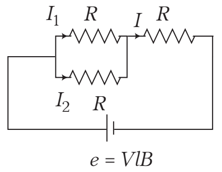
$\therefore$ Current through the connector, i

$$= \frac{E}{R_p + r} = \frac{4}{2+2} = 1\text{ A}$$

Magnetic force on the connector $= Bil = 2(1)(1) = 2$ N

Therefore, to keep the connector moving with a constant velocity, a force of 2 N has to be applied to the right side.

27. Circuit can be reduced as



$$I = \frac{e}{3R/2} = \frac{2\,vlB}{3R}$$

$$\Rightarrow \quad I_1 = I_2 = \frac{I}{2} = \frac{vlB}{3R}$$

28. We know that, $e = NBA\,\omega \sin \omega t$

where, N = number of loops = 1,

$$B = \frac{\mu_0 I}{2b}\text{ N/A-m}$$

and $\quad A = \pi a^2\text{ m}^2$.

Putting the values in Eq. (i), we get

$\therefore$ Induced emf, $e = \dfrac{\mu_0 I}{2b}(\pi a^2)\,\omega \sin \omega t$

$$= \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

29. From $e = -L\,di/dt$, $L = \left|\dfrac{-e\,dt}{di}\right| = \dfrac{8 \times 0.05}{2} = 0.2$ H

30. From $L = \dfrac{\mu_0 N^2 A}{l} = \dfrac{\mu_0 \mu_r N^2 A}{l}$

When $\mu_r = 1000$ and N becomes $\dfrac{1}{10}$,

L becomes $1000 \times \left(\dfrac{1}{10}\right)^2 = 10$ times

i.e. $\quad L = 10 \times 0.1 = 1$ H

31. As, $\quad L = \mu_0 n^2 A l$

$\therefore \quad L \to 2 \times 2 \times 2$ times = 8 times

32. The self-inductance L of a solenoid of length l and area of cross-section A with fixed number of turns N is

$$L = \frac{\mu_0 N^2 A}{l}$$

Obviously, L increases, when l decreases and A increases.

33. $\dfrac{L_2}{L_1} = \dfrac{N_2^2}{N_1^2}$

$$\therefore \quad L_2 = L_1 \frac{N_2^2}{N_1^2} = 1.5\left(\frac{500}{100}\right)^2 = 375\text{ mH}$$

34. As, $e = -M\dfrac{di}{dt} = -M\dfrac{d}{dt}(i_0 \sin \omega t)$

$\therefore \quad e = Mi_0 \cos \omega t (\omega)$

$e_{max} = Mi_0 \times 1 \times \omega$

$= 0.005 \times 10 \times 100\pi = 5\pi$

35. As, $e = -M\dfrac{di}{dt}$

$\therefore \quad 30 \times 10^3 = -3 \times \dfrac{(0-10)}{dt}$

$dt = \dfrac{30}{30 \times 10^3} = 10^{-3}$ s

36. Given, mutual inductance of coil, $M = 1.5$ H

Current change in coil, $dI = (0 - 20) = -20$ A

Time taken in change, $dt = 0.5$ s

Induced emf in the coil, $e = -M\dfrac{dI}{dt} = \dfrac{d\phi}{dt}$

or $\quad d\phi = -M \cdot dI = 1.5 \times 20$

$d\phi = 30$ Wb

Thus, the change of flux linkage is 30 Wb.

37. As the inductors are in parallel, therefore induced emf across the two inductors is the same, *i.e.* $e_1 = e_2$.

$\therefore \quad L_1\left(\dfrac{di_1}{dt}\right) = L_2\left(\dfrac{di_2}{dt}\right)$

Integrating both sides w.r.t. t, we get

$L_1 i_1 = L_2 i_2$

$\therefore \quad \dfrac{i_1}{i_2} = \dfrac{L_2}{L_1}$

38. As the coil is rotated, angle θ (angle which is normal to the coil makes with **B** at any instant t) changes, therefore magnetic flux ϕ linked with the coil changes and hence an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NSB \cos \omega t)$$

where, N is number of turns in the coil.

or $\quad e = -NSB\dfrac{d}{dt}(\cos \omega t) = -NSB(-\sin \omega t)\,\omega$

or $\quad e = NSB\omega \sin \omega t \quad \ldots$(i)

The induced emf will be maximum,

when $\sin \omega t$, *i.e.* maximum

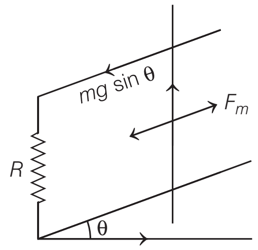
$\therefore \quad e_{max} = e_0 = NSB\omega \times 1 \quad$ or $\quad e = e_0 \sin \omega t$

Therefore, e would be maximum, hence current is maximum (as $i_0 = e_0 / R$), when $\theta = 90°$, i.e. normal to plane of coil is perpendicular to the field or plane of coil is parallel to magnetic field.

Round II

1. Induced current are clockwise, therefore induced magnetic field is into the plane of the paper. As it opposes the increasing inducing field, the inducing field must be out of the plane of the paper.

2. Terminal velocity of the rod is attained when magnetic force on the rod (Bil) balances the component of weight of the rod ($mg \sin \theta$) as shown in figure.



i.e. $\quad Bil = mg \sin \theta$

$B\left(\dfrac{e}{R}\right) l = mg \sin \theta$

$\dfrac{Bl}{R}(e) = mg \sin \theta$

$\dfrac{Bl}{R}(Blv_T) = mg \sin \theta$

$v_T = \dfrac{mg R \sin \theta}{B^2 l^2}$

3. Magnetic field intensity at a distance r from the straight wire carrying current is

$$B = \frac{\mu_0 i}{2\pi r}$$

As area of loop, $A = a^2$

and magnetic flux, $\phi = BA$

$\therefore \quad \phi = \dfrac{\mu_0 i a^2}{2\pi r}$

The induced emf in the loop is

$$e = \left|\frac{d\phi}{dt}\right| = \left|\frac{d}{dt}\frac{\mu_0 i a^2}{2\pi r}\right|$$

$$e = \frac{\mu_0 i a^2}{2\pi r^2}\frac{dr}{dt} = \frac{\mu_0 i a^2 v}{2\pi r^2}$$

where, $v = \dfrac{dr}{dt}$ = velocity.

4. Magnetic flux associated with the outer coil is

$\phi_{outer} = \mu_0 \pi N R \cdot I$

$= \mu_0 N\pi R(kte^{-\alpha t}) = Cte^{-\alpha t}$

where, $C = \mu_0 N\pi R k$ = constant.

Induced emf,

$e = \dfrac{-d\phi_{outer}}{dt}$

$= Ce^{-\alpha t} + (-\alpha t C e^{-\alpha t})$

$= Ce^{-\alpha t}(1 - \alpha t)$

$\therefore$ Induced current, $I = \dfrac{e}{\text{Resistance}}$

At $t = 0$, $I = -$ ve

$\therefore$ The correct graph representing this condition is given in option (d).

5. Induced emf in the conductor of length L moving with velocity of 1 cm/s in the magnetic field of 1T is given by

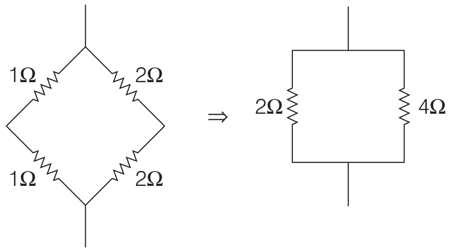
$$V = BLv \qquad \text{...(i)}$$

If equivalent resistance of the circuit is R_{eq}, then current in the loop will be

$$i = \frac{V}{R_{eq}} = \frac{BLv}{R_{eq}} \qquad \text{...(ii)}$$

Now, given network is a balanced Wheatstone bridge $\left(\frac{P}{Q} = \frac{R}{S}\right)$.

So, equivalent resistance of the Wheatstone bridge is



$$R_W = \frac{2 \times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\,\Omega$$

Again, resistance of conductor is $1.7\,\Omega$.

So, effective resistance will be

$$R_{eq} = \frac{4}{3} + 1.7 = \frac{4}{3} + \frac{17}{10}$$

$$= \frac{40+51}{30} = \frac{91}{30} \simeq 3\Omega$$

By putting given values of R_{eq} B and v in Eq. (ii), we have

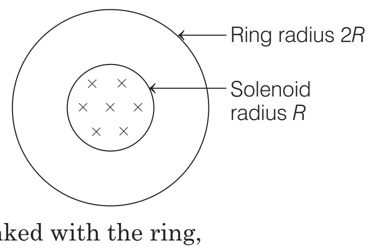
$$i = \frac{(1)\,(5 \times 10^{-2}) \times 10^{-2}}{3}$$

[Here, $L = 5 \times 10^{-2}$ m, $v = 1$ cm/s $= 10^{-2}$ m/s]

$$i = \frac{5 \times 10^{-4}}{3} = 1.67 \times 10^{-4}\,\text{A}$$

$$i = 167\,\mu\text{A} \approx 170\,\mu\text{A}$$

6. We are given following situation



Flux linked with the ring,

$$\phi = (B_{\text{solenoid}})(A_{\text{solenoid}}) = \mu_0 n I_R \times \pi R^2$$

Here, $I_R = I_0 t(1-t)$

So, $\phi = \mu_0 \pi R^2 \times n I_0 \cdot t \cdot (1-t)$

Magnitude of induced emf in the ring is

$$V_R = |E| = \frac{d\phi}{dt}$$

$$= \frac{d}{dt} \mu_0 \pi R^2 \times n I_0 t(1-t)$$

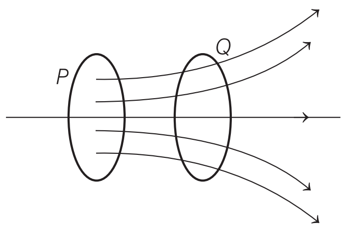
$$= \mu_0 \pi R^2\, n I_0 (1 - 2t)$$

Clearly, induced emf is zero, when $1 - 2t = 0$

or $t = \frac{1}{2}\,\text{s} = 0.5\,\text{s}$

So, induced emf and induced current are zero at $t = 0.5\,\text{s}$ and their direction also reverses.

7. As, coefficient of mutual induction is same for both coils. i.e.



$\therefore \quad M_{PQ} = M_{QP}$

$$\Rightarrow \frac{N_P \phi_{PQ}}{I_Q} = \frac{N_Q \phi_{QP}}{I_P} \qquad \text{... (i)}$$

Here, $N_P = N_Q = 1$,

$\phi_{PQ} = ?$, $\phi_{QP} = 10^{-3}$ Wb

$I_Q = 2\text{A}$, $I_P = 3\text{A}$

Substituting values in Eq (i), we get

$$\phi_{PQ} = \frac{N_Q \phi_{QP} \cdot I_Q}{N_p \cdot I_p} = \frac{1 \times 10^{-3} \times 2}{1 \times 3} = \frac{2}{3} \times 10^{-3}$$

$$= 0.667 \times 10^{-3} = 6.67 \times 10^{-4}\ \text{Wb}$$

8. Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2, respectively is given as,

$M = \mu_0 n_1 n_2\, \pi\, r_1^2\, l$ (for internal coil of radius r_1)

Self-inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi\, r_1^2 l$$

$$\therefore \quad \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

9. Self-inductance L_{sol} of a solenoid is given by

$$L_{sol} = \mu_0 n^2 \pi r^2 L$$

(Here, $n = N/L$ and L = length of solenoid)

or $L_{sol} = \dfrac{\mu_0 N^2 \pi r^2}{L}$

Clearly, $L_{sol} \propto \dfrac{1}{L}$

($\because$ all other parameters are fixed)

10. Induced emf in the loop,

$$e = -\frac{\Delta\phi}{\Delta t} = -\left(\frac{\Delta B}{\Delta t}\right)(A) \qquad [\because \phi = BA]$$

$$= -\left(\frac{B_2 - B_1}{\Delta t}\right) A \qquad \text{...(i)}$$

Here, $B_2 = 500\ \text{G} = 500 \times 10^{-4}$ T,

$B_1 = 1000\text{ G} = 1000 \times 10^{-4}$ T,

$\Delta t = 5$s

$\therefore \quad A$ = area of loop

= Area of rectangle − Area of two triangles

$= \left(16 \times 4 - 2 \times \frac{1}{2} \times 4 \times 2\right) \text{cm}^2$

$= 56 \times 10^{-4}\text{ m}^2$

Using Eq. (i), we get

$$e = \frac{(1000 - 500) \times 10^{-4} \times 56 \times 10^{-4}}{5}$$

$= 56 \times 10^{-6}\text{ V} = 56\,\mu\text{V}$

11. Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\quad \Delta\phi_B = B\,A = BA\cos\theta$

where, A is the surface area of the loop and θ is an angle between B and A.

Here, $\quad \theta = 0 \Rightarrow \Delta\phi_B = BA$

$\therefore$ For the time interval, $t = 0$ s to $t = 10$ ms,

$$Q = \frac{\Delta\phi_B}{R} = \frac{A}{R}(B_{f\text{ at }0.01\text{s}} - B_{i\text{ at }0\text{s}})$$

Substituting the given values, we get

$$= \frac{3.5 \times 10^{-3}}{10}[0.4\sin(0.5\pi) - 0.4\sin 0]$$

$= 3.5 \times 10^{-4}\,(0.4\sin\pi/2)$

$= 1.4 \times 10^{-4}$C

$= 14$ mC

12. From an inductor (choke), $E_{\text{induced}} = -L\frac{dI}{dt}$

$\Rightarrow \quad L = \left|\frac{-E \times \Delta t}{\Delta I}\right|$

Here, $\quad E = 100\text{ V},\ \Delta I = 0.25\text{ A}$

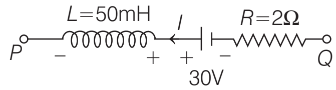
and $\quad \Delta t = 0.025\text{ ms} = 0.025 \times 10^{-3}$s

So, $\quad L = \left|\frac{-100 \times 0.025 \times 10^{-3}}{0.25}\right|$

$= 100 \times 10^{-4} = 10$ mH

13. Given that, at any instant, $I = 1$ A

$\frac{dI}{dt} = -10^2$ A/s $\qquad$ (as, current is decreasing)



Applying KVL from point P to point Q,

$$V_P + \frac{LdI}{dt} - 30 + IR = V_Q$$

$$V_P - V_Q = 30 - 1 \times 2 - 50 \times 10^{-3} \times (-10^2) = 33\text{ V}$$

14. Given, $I(t) = 5t^2 - 2t + 3$

$$\frac{dI}{dt} = 10t - 2$$

At $t = 1$ s, $\frac{dI}{dt} = 8$ A/s

Magnetic flux, $\phi = \left(\frac{\mu_0 I n_2}{2R}\right)(\pi r^2 n_1)$

Induced emf, $e = \left|\frac{d\phi}{dt}\right| = \left(\frac{\mu_0 n_2}{2R}\right)\pi r^2 n_1 \frac{dI}{dt}$

Given, $\quad n_1 = 200,\ r = 1\text{ cm} = 10^{-2}\text{ m},\ n_2 = 500,$

$R = 20\text{ cm} = 20 \times 10^{-2}$ m

and $\quad \mu_0 = 4\pi \times 10^{-7}$

$$\therefore \quad e = \frac{4\pi \times 10^{-7} \times 200 \times \pi \times 10^{-4} \times 500}{2 \times 20 \times 10^{-2}} \times 8$$

$= 8 \times 10^{-4}\text{ V} = 0.8$ mV

According to question, 0.8 mV $= \frac{4}{x} \Rightarrow x = \frac{4}{0.8} = 5$

15. Flux linked with the coil is

$$\phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\omega t$$

Magnitude of emf induced in coil

$$= \left|\frac{d}{dt}\phi\right| = |BA\omega\sin\omega t|$$

Maximum value of induced emf $= BA\omega = \frac{BA2\pi}{T}$

$$= \frac{3 \times 10^{-5} \times \pi \times (0.1)^2 \times 2\pi}{0.4} = 14.7 \times 10^{-6}\text{ V}$$

$\approx 15 \times 10^{-6}\text{ V} = 15\,\mu\text{V}$