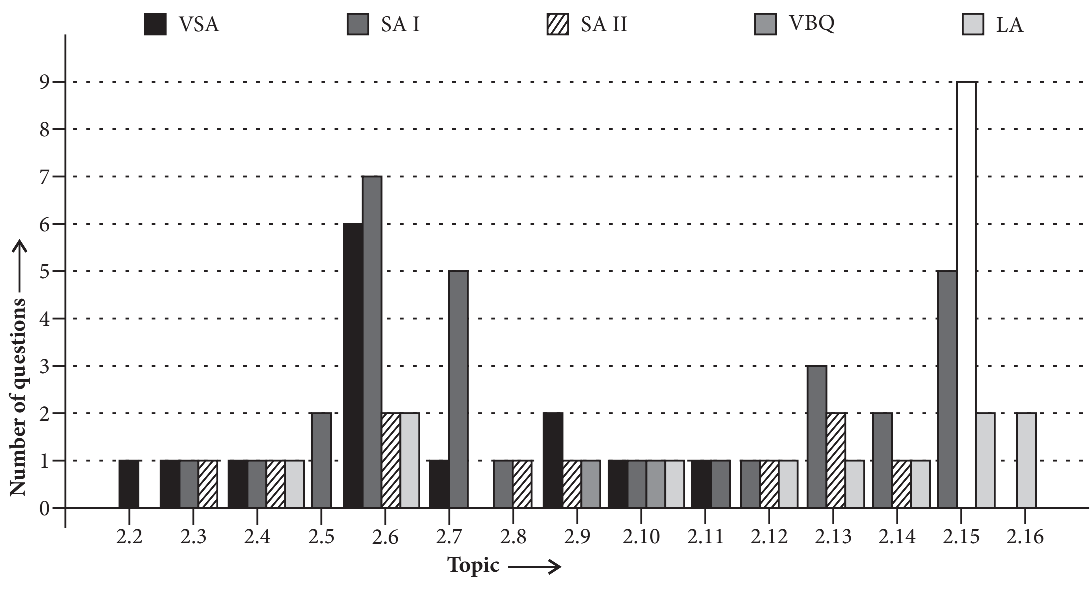
02 Electrostatic Potential and Capacitance

Topicwise Analysis of Last 10 Years' CBSE Board Questions



- Maximum weightage is of *Energy Stored in a Capacitor.*
- Maximum VSA and SA I type questions were asked from *Equipotential surfaces.*
- Maximum SA II type questions were asked from *Energy Stored in a Capacitor.*

QUICK RECAP

Electric potential : Electric potential at a point is defined as amount of work done in bringing a unit positive charge from infinity to that point. It is denoted by symbol V.

$$V = \frac{W}{q}$$

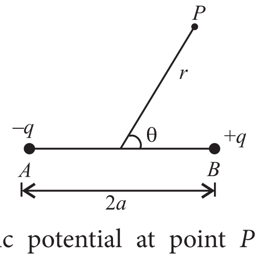
- Electric potential is a scalar quantity. The SI unit of potential is volt and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.
- Electric potential at a point distant r from a point charge q is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- Electric potential due to group of charges : The electric potential at a point due to a group of charges is equal to the algebraic sum of the electric potentials due to individual charges at that point.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... \frac{q_n}{r_n}\right)$$
$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

- Electric potential at any point due to an electric dipole



The electric potential at point P due to an electric dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

– At axial point : When the point P lies on the axial line of dipole *i.e.*, $\theta = 0°$.

$$V = \frac{p}{4\pi\varepsilon_0 r^2}.$$

– At equatorial point : When the point P lies on the equatorial line of the dipole, *i.e.*, $\theta = 90°$ $\therefore$ $V = 0$.

- Electric potential due to a uniformly charged spherical shell of uniform surface charge density σ and radius R at a distance r from the centre the shell is given as follows :

– At a point outside the shell *i.e.*, $r > R$

$$V = \frac{\sigma R^2}{\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

– At a point on the shell *i.e.*, $r = R$

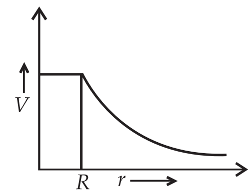
$$V = \frac{\sigma R}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

– At a point inside the shell *i.e.*, $r < R$

$$V = \frac{\sigma R}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Here, $q = 4\pi R^2\sigma$

The variation of V with r for a uniformly charged thin spherical shell is shown in the figure.



- Electric potential due to a non-conducting solid sphere of uniform volume charge density ρ and radius R at distant r from the sphere is given as follows :
- At a point outside the sphere *i.e.*, $r > R$

$$V = \frac{\rho R^3}{3\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

- At a point on the sphere *i.e.*, $r = R$

$$V = \frac{\rho R^2}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

- At a point inside the sphere *i.e.*, $r < R$

$$V = \frac{\rho}{3\varepsilon_0}\frac{(3R^2 - r^2)}{2} = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$$

Here $q = \frac{4}{3}\pi R^3\rho$

Equipotential surface : A surface on which the electric potential is constant is known as equipotential surface.

- **Properties of an equipotential surface :**
  - Electric field lines are always perpendicular to an equipotential surface.
  - Work done in moving an electric charge from one point to another on an equipotential surface is zero.
  - Two equipotential surfaces can never intersect one another.

Relationship between $\vec{E}$ and $\vec{V}$

$$\vec{E} = -\vec{\nabla} V$$

where $\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$

–ve sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

Electric potential energy

- Electric potential energy of a system of two point charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

- Electric potential energy of a system of n point charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}; \quad >$$

- The SI unit of electric potential energy is joule.
- **Conductors :** Those substances which can easily allow electricity to pass through them are known as conductors. They have a large number of free charge carriers that are free to move inside the material. *e.g.*, metals, human beings, earth etc.
- Basic electrostatics properties of a conductor are as follows :
  - Inside a conductor, electric field is zero.
  - At the surface of a charged conductor, electric field must be normal to the surface at every point.
  - The interior of a conductor can have no excess charge in the static situation.
  - Electric potential is constant throughtout the volume of the conductor and has the same value (as inside) on its surface.
  - Electric field at the surface of a charged conductor, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

- **Electrostatic shielding :** It is the phenomenon of protecting a certain region of space from external electric field.

Polar and non-polar molecule

- Polar molecule : A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). A polar molecule has a permanent dipole moment *e.g.*, water (H_2O) and HCl.
- Non-polar molecule : A non-polar molecule is one in which the centres of positive and negative charges coincide. A non polar molecule has no permanent dipole moment. *e.g.*, oxygen (O_2) and hydrogen (H_2).

Capacitance : Capacitance (C) of a capacitor is the ratio of charge(Q) given and the potential (V) to which it is raised. *i.e.*, $C = Q/V$.

- The SI unit of capacitance is farad (F).
  - 1 millifarad (mF) = 10^{-3} farad
  - 1 microfarad (μF) = 10^{-6} farad
  - 1 picofarad (pF) = 10^{-12} farad.
- The dimensional formula of capacitance is $[M^{-1}L^{-2}T^4A^2]$.
- Capacitance of a spherical conductor of radius R is $C = 4\pi\varepsilon_0 R$

Taking earth to be a conducting sphere of radius 6400 km, its capacity will be

$$C = 4\pi\varepsilon_0 R = \frac{6.4\times10^6}{9\times10^9} = 711\ \mu\text{F}$$

Capacitor : A condenser or a capacitor is a device that stores electric charge. It consists of two conductors separated by an insulator or dielectric. The two conductors carry equal and opposite charges $\pm Q$.

- Capacitance of an air filled parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

where A is area of each plate and d is separation between the two plates.

- Capacitance of an air filled spherical capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

where a and b are the inner and outer radii.

- Capacitance of an air filled cylindrical capacitor

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

where a and b are the inner and outer radii and L is the length.

- Capacitance of a parallel plate capacitor with a dielectric slab of dielectric constant K, completely filled between the plates of the capacitor, is given by

$$C = \frac{K\varepsilon_0 A}{d} = \frac{\varepsilon_0\varepsilon_r A}{d}$$

- When a dielectric slab of thickness t and dielectric constant K is introduced between the plates, then the capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

- When a metallic conductor of thickness t is introduced between the plates, then capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

Combination of capacitors in series and parallel

- Capacitors in series : For n capacitors connected in series the equivalent capacitance C_S is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

- Capacitors in parallel : For n capacitors connected in parallel, the equivalent capacitance C_P is given by

$$C_P = C_1 + C_2 + ... + C_n$$

- When capacitors are connected in series, the charge through each capacitor is same. When capacitors are connected in parallel, the potential difference across each capacitor is same.
- When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential till their potentials become equal. The equal potential is called common potential (V), where

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

It should be clearly understood that in sharing charges, there is absolutely no loss of charge. Some energy is, however, lost in the process in the form of heat etc. which is given by

$$U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}.$$

Energy stored in a capacitor : Work done in charging a capacitor gets stored in the capacitor in the form of its electric potential energy and it is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

- Energy density : The energy stored per unit volume in the electric field between the plates is known as energy density (u). It is given by

$$U = \frac{1}{2}\varepsilon_0 E^2$$

- When a dielectric slab of dielectric constant K is introduced between the plates of a charged parallel plate capacitor and the charging battery remains connected, then
  - Potential difference between the plates remains constant *i.e.*, $V = V_0$
  - Capacitance C increases *i.e.*, $C = KC_0$
  - Charge on a capacitor increases *i.e.*, $Q = KQ_0$
  - Electric field between the plates remains unchanged *i.e.*, $E = E_0$
  - Energy stored in a capacitor increases *i.e.*, $U = KU_0$

- When a dielectric slab of dielectric constant K is introduced in between the plates of a charged parallel plate capacitor and the charging battery is disconnected, then
  - Charge remains unchanged *i.e.*, $Q = Q_0$
  - Capacitance increases *i.e.*, $C = KC_0$
  - Potential difference between the plates decreases *i.e.*, $V = \frac{V_0}{K}$
  - Electric field between the plates decreases *i.e.*, $E = \frac{E_0}{K}$
  - Energy stored in the capacitor decreases *i.e.*, $U = \frac{U_0}{K}$

where Q_0, C_0, V_0, E_0 and U_0 represents the charge, capacitance, potential difference, electric field and energy stored in the capacitor of a charged air filled parallel plate capacitor.

Van de Graaff generator

- A Van de Graaff generator consists of a large spherical conducting shell (a few metres in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell, and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

Previous Years' CBSE Board Questions

2.2 Electrostatic Potential

VSA (1 mark)

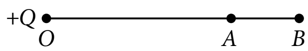
1. Name the physical quantity whose S.I. unit is J C^{-1}. Is it a scalar or a vector quantity?

(AI 2010)

2.3 Potential due to a Point Charge

VSA (1 mark)

2. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?



(Delhi 2016, Foreign 2016, Delhi 2011)

SA I (2 marks)

3. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.

(Delhi 2012)

SA II (3 marks)

4. Plot a graph comparing the variation of potential 'V' and electric field 'E' due to a point charge 'Q' as a function of distance 'R' from the point charge.

(1/3, Foreign 2010)

2.4 Potential due to an Electric Dipole

VSA (1 mark)

5. What is the electrostatic potential due to an electric dipole at an equatorial point?

(AI 2009)

SA I (2 marks)

6. Derive the expression for the electric potential at any point along the axial line of an electric dipole?

(Delhi 2008)

SA II (3 marks)

7. Deduce an expression for the electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electric potential of a dipole at a point as compared to that due to a single charge.

(Delhi 2007)

LA (5 marks)

8. Obtain the expression for the potential due to an electric dipole of dipole moment p at a point 'x' on the axial line.

(2/5, AI 2013C)

2.5 Potential due to a System of Charges

SA I (2 marks)

9. Two point charges q and $-2q$ are kept 'd' distance apart. Find the location of point relative to charge 'q' at which potential due to this system of charges is zero.

(AI 2014C)

10. Two point charges 4 μC and -2 μC are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero.

(AI 2007)

2.6 Equipotential Surfaces

VSA (1 mark)

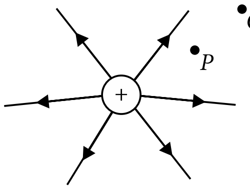
11. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor?

(AI 2015C)

12. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify.

(Delhi 2014)

13. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reasion.



(Foreign 2014)

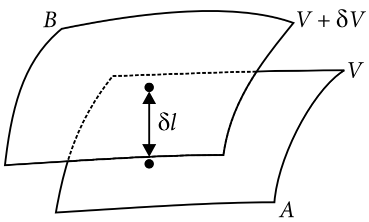
14. What is the geometrical shape of equipotential surfaces due to a single isolated charge? *(Delhi 2013, AI 2010C)*

15. Two charges 2 μC and -2 μC are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. *(Delhi 2013C)*

16. What is the amount of work done in moving a point charge around a circular arc of radius r at the centre of which another point charge is located ? *(AI 2013C)*

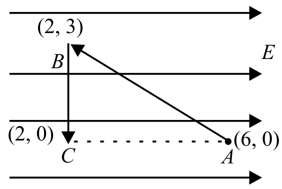
SA I (2 marks)

17. Two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, (where δV is the change in V), are kept δl distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potentials.



(Delhi 2014C)

18. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure. (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why?



(AI 2012)

19. Two uniformly large parallel thin plates having densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates, what is the magnitude and direction of this field? *(Delhi 2011)*

20. (a) Draw equipotential surfaces due to point $Q > 0$.
(b) Are these surfaces equidistant from each other? If no, explain why? *(Delhi 2011C)*

21. Can two equipotential surfaces intersect each other? Give reasons. *(Delhi 2011C)*

22. Two point charges 2 μC and -2μC are placed at points A and B, 6 cm apart.
(i) Draw equpotential surfaces of the system.
(ii) Why do the equipotential surfaces get closer to each other near the point charges? *(AI 2011C)*

23. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along Z-direction. How are these surfaces different from that of a constant electric field along Z-direction? *(AI 2009)*

SA II (3 marks)

24. Define an equipotential surface. Draw equipotential surfaces:
(i) in the case of a single point charge and
(ii) in a constant electric field in Z-direction. Why the equipotential surface about a single charge are not equidistant ?
(iii) Can electric field exist tangential to an equipotential surface? Give reason. *(AI 2016)*

25. Depict the equipotential surfaces for a system of two identical positive point charges placed a distance 'd' apart. *(1/3, Delhi 2010)*

LA (5 marks)

26. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero. *(2/5, AI 2013)*

27. Write two properties of equipotential surfaces. Depict equipotential surfaces due to an isolated point charge. Why do the equipotential surfaces get closer as the distance between the equipotential surface and the source charge decreases? *(AI 2012C)*

2.7 Potential Energy of a System of Charges

VSA (1 mark)

28. A 500 μC charge at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 mC between two diagonally opposite points on the square. *(Delhi 2008)*

SA I (2 marks)

29. Calculate the amount of work done to dissociate a system of three charges 1μC, 1μC and –4μC placed on the vertices of an equilateral triangle of side 10 cm. *(AI 2013C)*

30. Two charges $-q$ and $+q$ are located at point $A(0, 0, -a)$ and $B(0, 0, +a)$ respectively. How much work is done in moving a test charge from point $P(7, 0, 0)$ to $Q(-3, 0, 0)$? *(1/2, Delhi 2011C)*

31. Find out the expression for the potential energy of a system of three charges q_1, q_2 and q_3 located respectively at $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ with respect to the common origin O. *(Delhi 2010C)*

32. Two point charges, $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air.

(i) Find at what distance from the 1st charge, q_1, would the electric potential be zero.

(ii) Also calculate the electrostatic potential energy of the system.

33. Two points charges $4Q$, Q are separated by 1 m in air. At what point on the line joining the charges is the electric field intensity zero?

Also calculate the electrostatic potential energy of the system of charges, $Q = 2 \times 10^{-7}$ C. *(AI 2008)*

2.8 Potential Energy in an External Field

SA I (2 marks)

34. A dipole, with its charges, $-q$ and $+q$, located at the points $(0, -b, 0)$ and $(0, +b, 0)$, is present in a uniform electric field $\vec{E}$. The equipotential surfaces of this field, are planes parallel to the y-z planes.

(i) What is the direction of the electric field $\vec{E}$?

(ii) How much torque would the dipole experience in this field? *(Delhi 2010C)*

SA II (3 marks)

35. Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the presence of external electric field $\vec{E}$. *(2/3, Delhi 2010)*

2.9 Electrostatics of Conductors

VSA (1 mark)

36. Why is the potential inside a hollow spherical charged conductor constant and has the same value as on its surface? *(Foreign 2012)*

37. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere? *(AI 2011)*

SA II (3 marks)

38. Show that the capacitance of a spherical conductor is $4\pi\varepsilon_0$ times the radius of the spherical conductor. *(Delhi 2010C)*

VBQ (4 marks)

39. While travelling back to his residence in the car, Dr. Pathak was caught up in a thunderstorm. It became very dark. He stopped driving the car and waited for thunderstorm to stop. Suddenly he noticed a child walking alone on the road. He asked the boy to come inside the car till the thunderstorm stopped. Dr. Pathak dropped the boy at his residence. The boy insisted that Dr. Pathak should meet his parents. The parents expressed their gratitude to Dr. Pathak for his concern for safety of the child.

Answer the following questions based on the above information:

(a) Why is it safer to sit inside a car during a thunderstorm?

(b) Which two values are displayed by Dr. Pathak in his action?

(c) Which values are reflected in parents' response to Dr. Pathak?

(d) Give an example of similar action on your part in the past from everyday life.

(Delhi 2013)

2.10 Dielectrics and Polarisation

VSA (1 mark)

40. Distinguish between a dielectric and a conductor? *(Delhi 2012C)*

SA I (2 marks)

41. Distinguish between polar and non-polar dielectric. *(AI 2010C)*

VBQ (4 marks)

42. Immediatly after school hour, as Bimla with her friends came out, they noticed that there was a sudden thunderstorm accompanied by the lightening. They could not find any suitable place for shelter. Dr. Kapoor who was passing thereby in his car noticed these children and offered them to come in their car. He even took care to drop them to the locality where they were staying. Bimla's parents, who were waiting, saw this and expressed their gratitude to Dr. Kapoor.

(i) What values did Dr. Kapoor and Bimla's parents displayed?

(ii) Why is it considered safe to be inside a car especially during lightening and thunderstorm?

(iii) Define the term 'dielectric strength'. What does this term signify? *(AI 2015C)*

LA (5 marks)

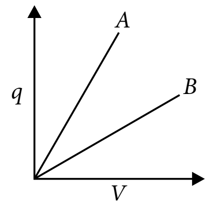
43. Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.

(Delhi 2012C)

2.11 Capacitors and Capacitance

VSA (1 mark)

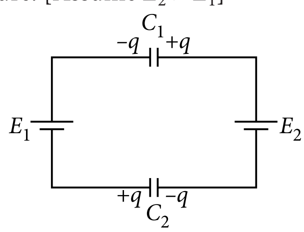
44. The given graph shows variation of charge 'q' versus potential difference 'V' for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1. Which line (A or B) corresponds to C_1 and why?



(AI 2014C)

SA I (2 marks)

45. Determine the potential difference across the plates of the capacitor 'C_1' of the network shown in the figure. [Assume $E_2 > E_1$]



(Delhi 2012C)

2.12 The Parallel Plate Capacitor

SA I (2 marks)

46. What is the area of the plates of 2 F parallel plate capacitor having separation between the plates is 0.5 cm? *(AI 2011)*

SA II (3 marks)

47. Explain the underlying principle of working of a parallel plate capacitor.

If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for

(i) the electric field at points between the two plates.

(ii) the potential difference between the plates.

(iii) the capacitance of the capacitor so formed.

(AI 2007)

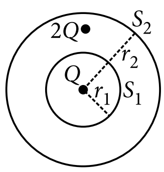
LA (5 marks)

48. If two similar large plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, find the expressions for
(a) field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.
(b) the potential difference between the plates.
(c) the capacitance of the capacitor so formed.
(3/5, AI 2016)

2.13 Effect of Dielectric on Capacitance

SA I (2 marks)

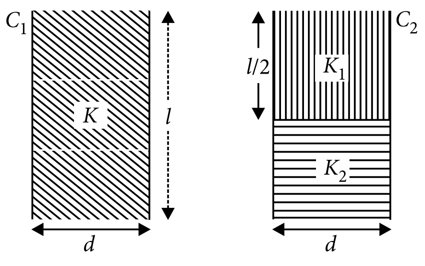
49. A sphere S_1 of radius r_1 encloses a net charge Q. If there is another concentric sphere S_2 of radius $r_2 (r_2 > r_1)$ enclosing charge $2Q$, find the ratio of the electric flux through S_1 and S_2. How will the electric flux through sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air?



(AI 2014C)

50. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.
(AI 2013)

51. Two identical parallel plate (air) capacitor C_1 and C_2 have capacitances C each. The area between their plates in now filled with dielectrics as shown.



If the two capacitors still have equal capacitance, obtained the relation between dielectric constants K, K_1 and K_2. *(Foreign 2011)*

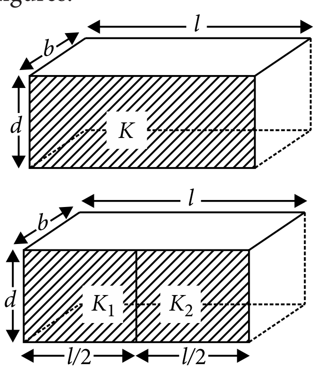
SA II (3 marks)

52. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the separation between the plates is 3 mm.
(i) Calculate the capacitance of the capacitor.
(ii) If this capacitor is connected to 100V supply, what would be the charge on each plate?
(iii) How would charge on the plates be affected, if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected? *(Foreign 2014)*

53. (a) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?
(b) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.
(Foreign 2010)

LA (5 marks)

54. Two identical capacitors of plate dimensions $l \times b$ and plate separation d have dielectric slabs filled in between the space of the plates as shown in the figures.

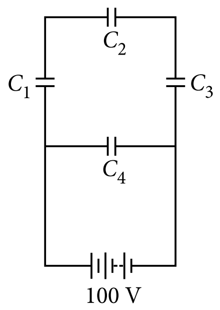


Obtain the relation between the dielectric constants K, K_1 and K_2.
(AI 2013C)

2.14 Combination of Capacitors

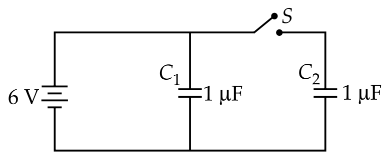
SA I (2 marks)

55. A network of four capacitors, each of capacitance 15 μF, is connected across a battery of 100 V, as shown in the figure. Find the net capacitance and the charge on the capacitor C_4.



(AI 2012C)

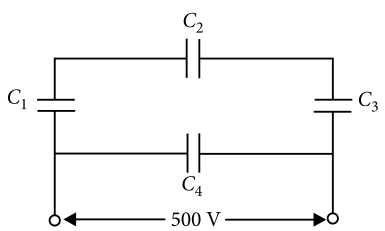
56. 1 mF capacitance connected to a battery of 6 V. Initially switch *S* is closed. After sometime *S* is left open and dielectric slabs of dielectric constant K = 3 are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



(Delhi 2011)

SA II (3 marks)

57. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor.



(AI 2010)

LA (5 marks)

58. Show that the effective capacitance, C, of a series combination, of three capacitors, C_1, C_2 and C_3 is given by

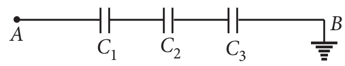
$$C = \frac{C_1C_2C_3}{(C_1C_2 + C_2C_3 + C_3C_1)}$$

(AI 2010C)

2.15 Energy Stored in a Capacitor

SA I (2 marks)

59. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given potential at A is 90 V, C_1 = 20 μF, C_2 = 30 μF and C_3 = 15 μF.



(AI 2015)

60. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor.

(AI 2014)

61. A parallel plate capacitor, each of plate area A and separation 'd' between the two plates, is charged with charges $+Q$ and $-Q$ on the two plates. Deduce the expression for the energy stored in capacitor.

(Foreign 2013)

62. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C' and having charge 'Q'.

How will the (i) energy stored and (ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'K'?

(AI 2012)

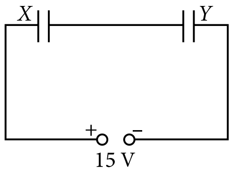
63. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

(AI 2011)

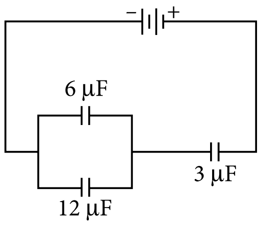
SA II (3 marks)

64. Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric of $\varepsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.
(ii) Calculate the potential difference between the plates of X and Y.
(iii) Estimate the ratio of electrostatic energy stored in X and Y. *(Delhi 2016)*

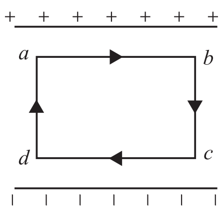
65. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is E. Find the value of the following
(i) Energy stored in 12 μF capacitor
(ii) Energy stored in 3 μF capacitor
(iii) Total energy drawn from the battery



(Foreign 2016)

66. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination. *(Delhi 2015)*

67. (a) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.



(b) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop $a\,b\,c\,d\,a$. *(Delhi 2014)*

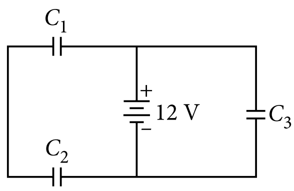
68. A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC.
Calculate:
(i) The potential V and the unknown capacitance C.
(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V? *(Delhi 2013)*

69. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. *(Foreign 2012)*

70. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will (i) the capacitance of the capacitor, (ii) potential difference between the plates and (iii) the energy stored in the capacitor be affected? Justify your answer in each case.
(Delhi 2011C, 2010, AI 2009, Delhi 2007)

71. Find the ratio of the the potential difference that must be applied across the parallel and the series combination of two identical capacitors so that the energy stored in the two cases, becomes the same. *(2/3, Foreign 2010)*

72. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown.



Find
(i) charge on the capacitor

(ii) equivalent capacitance of the network
(iii) energy stored in the network of capacitors *(Delhi 2009)*

LA (5 marks)

73. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same. *(3/5, AI 2016)*

74. (a) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.

(b) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor. *(AI 2015)*

75. Derive an expression for the energy stored in a parallel plate capacitor.
On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved, and a dielectric medium of $\varepsilon_r = 10$ is introduced between the plates, without disconnecting the d.c. source. Explain, using suitable expressions, how the (i) capacitance, (ii) electric field and (iii) energy density of the capacitor change. *(AI 2008)*

2.16 Van de Graaff Generator

LA (5 marks)

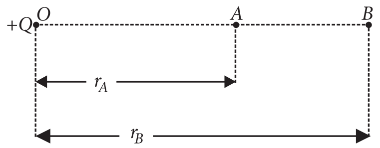
76. Draw a labelled diagram of Van de Graaff generator. State its working principle to show how by introducing a small charged sphere into a larger sphere, a large amount of charge can be transferred to the outer sphere. State the use of this machine and also point out its limitations. *(AI 2014)*

77. Explain the principle of a device that can build up high voltages of the order of a few million volts. Draw a schematic diagram and explain the working of this device.
Is there any restriction on the upper limit of the high voltages set up in this machine? Explain. *(Delhi 2012)*

Detailed Solutions

1. J C^{-1} is the S.I. unit of electrostatic potential. It is a scalar quantity.

2.



Potential difference due to a point charge Q at a distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$\therefore$ From the given figure

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A}, \quad V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B}$$

$$\therefore \quad V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

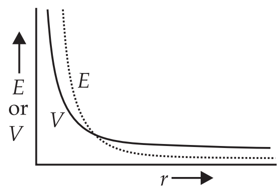
$$\because \quad r_B > r_A \Rightarrow \frac{1}{r_B} < \frac{1}{r_A} \Rightarrow \left(\frac{1}{r_A} - \frac{1}{r_B}\right) > 0$$

Hence, $(V_A - V_B) > 0$

i.e., potential difference $(V_A - V_B)$ is positive.

3. Electric field due to a point charge,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, \quad E \propto \frac{1}{r^2}$$



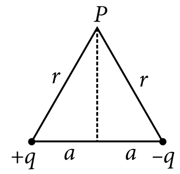
Potential due to a point charge,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}; \; V \propto \frac{1}{r}$$

The variation of electric field E with distance r and also the variation of potential v with r as shown in the figure.

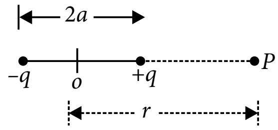
4. *Refer to answer 3.*

5. Required potential at point P



$$V_P = \frac{q}{4\pi\,\varepsilon_0 r} + \frac{-q}{4\pi\varepsilon_0 r} = 0$$

6.



Let P be an axial point at distance r from the centre of the dipole. Electric potential at point P will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2 - a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2 - a^2} \qquad [\because p = q\,(2a)]$$

For a far away point, $r >> a$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \quad \text{or} \quad V \propto \frac{1}{r^2}$$

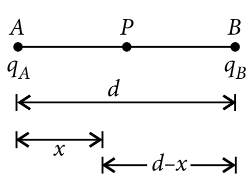
Thus, due to a dipole potential at a point is $V \propto \frac{1}{r^2}$.

7. *Refer to answer 6.*

Whereas, due to a single charge potential at a point is $V \propto \frac{1}{r}$.

8. *Refer to answer 6.*

9. $q_A = q$ and $q_B = -2q$



$$V_{PA} = \frac{kq_A}{x}$$

$$V_{PB} = \frac{kq_B}{(d-x)}$$

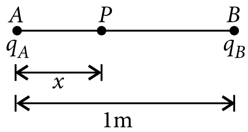
$$V_{PA} + V_{PB} = 0$$

$$\frac{kq}{x} = \frac{2kq}{(d-x)}; \; d - x = 2x$$

$$3x = d; \; x = \frac{d}{3}$$

10. $q_A = 4 \times 10^{-6}$C; $q_B = -2 \times 10^{-6}$C

$$V_{PA} = \frac{kq_A}{x}$$



$$V_{PB} = \frac{kq_B}{(1-x)}$$

$V_{PA} + V_{PB} = 0$

$$\frac{9\times10^9\times4\times10^{-6}}{x} = \frac{9\times10^9\times2\times10^{-6}}{1-x}$$

$2 = \frac{x}{1-x}$; $2 - 2x = x$; $x = \frac{2}{3}$m

11. If the field were not normal to the equipotential surface, it would have a non zero component along the surface. So to move a test charge against this component, a work would have to be done. But there is no potential difference between any two points on an equipotential surface and consequently no work is required to move a test charge on the surface. Hence, the electric field must be normal to the equipotential surface at every point.

12. *Refer to answer 11.*

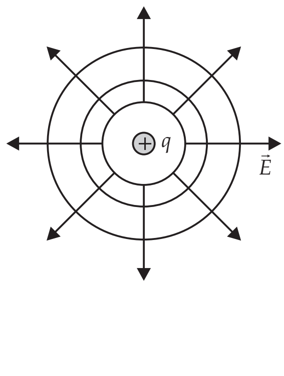
13. Work done = q (Potential at Q – Potential at P), where q is the small positive charge.
The electric potential at a point distance r due to the field created by a positive charge Q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

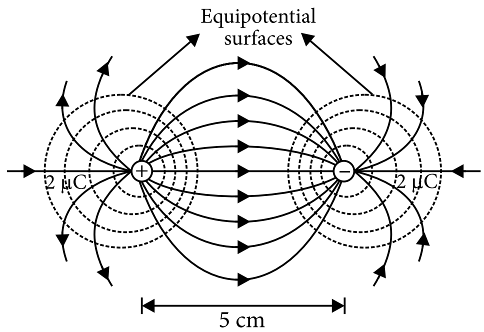
$\because\ r_P < r_Q\ \therefore\ V_P > V_Q$

Hence, work done will be negative.

14. For an isolated charge the equipotential surfaces are concentric spherical shells and the separation between consecutive equipotential surfaces increases in the weaker electric field.



15.



16. Work done in carrying a charge on equipotential surface is always zero.

17. Electric field as gradient of potential consider a point charge $+q$ placed at point O. Suppose that V and $V+\delta V$ are electrostatic potential at points A and B, where distance from the charge $+q$ are r and $r - \delta r$ respectively.

$$(V + \delta V) = V + \frac{\delta W}{q_0}$$

$$\delta V = \frac{\delta W}{q_0} \qquad \text{...(i)}$$

If $\vec{E}$ is electric field at point P due to charge $+q$ placed at point O, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge against the electric field $\vec{E}$ is given by

$\vec{F} = -q_0\vec{E}$

Therefore, work done to move the test charge through an infinitesimally small displacement $\overrightarrow{PQ} = \vec{\delta l}$ is given by

$\Delta W = \vec{F}\cdot\vec{\delta l} = (-q\vec{E})\cdot\vec{\delta l} = -q_0 E\delta l\cos180° = q_0 E\delta l$

As the distance r decreases in the direction of δl, then

$\delta W = -q_0 E\delta r$

$$\frac{\delta W}{q_0} = -E\delta r \qquad \text{...(ii)}$$

From equations (i) and (ii), we get

$\delta V = -E\delta r$; $E = -\frac{\delta V}{\delta r}$

Therefore, electric field at a point is equal to the negative gradient of the electrostatic potential at that point.

Important conclusions :

(i) No work is done in moving a test charge over an equipotential surface.

(ii) The electric field is always at right angles to the equipotential surface.

(iii) The equipotential surfaces tell the direction of the electric field.

18. (i) In the relation

$$\boxed{E = \frac{-dV}{dr}} \Rightarrow E = -\left[\frac{V_C - V_A}{(2-6)}\right]$$

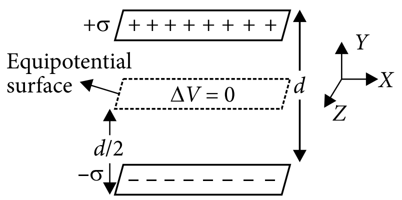
$V_C - V_A = 4E$

(ii) As $V_C - V_A = 4E$ is positive

$\therefore V_C > V_A$

Potential is greater at point C than point A, as potential decreases along the direction of electric field.

19.

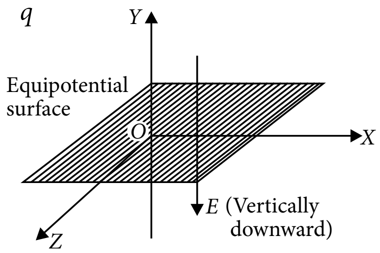


The equipotential surface is at a distance $d/2$ from either plate in XZ-plane. $-q$ charge experiences a force in a direction opposite to the direction of electric field.

$\therefore$ $-q$ charge balances when

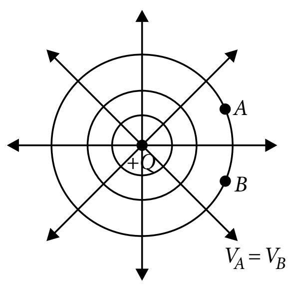
$qE = mg$

$E = \dfrac{mg}{q}$



The direction of electric field along vertically downward direction. The XZ-plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charge particle could not be balanced.

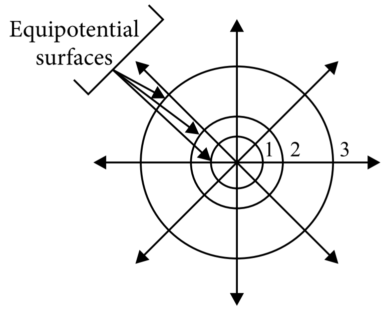
20. (a)



(b) These surfaces are not equidistant from each other because electric field at a point, distance r from point charge, is given by $E = +\dfrac{Q}{4\pi\varepsilon_0 r^2}$

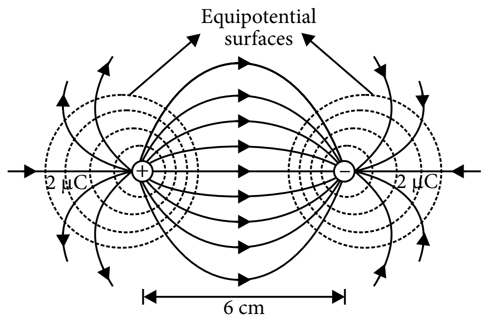
As electric field $E \propto \dfrac{1}{r^2}$, the field is non uniform.

So, distance between adjacent equipotential surfaces goes on increasing as shown in figure.



21. No, if two equipotential surfaces intersect then at the point of intersection, there will be two directions of electric field intensity which is not possible.

22. (i) Equipotential surface



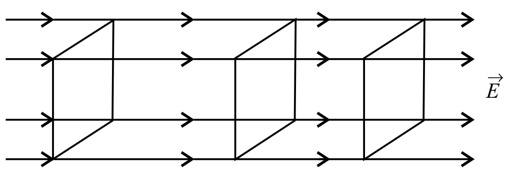
(ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

$\therefore \quad E = -\dfrac{\Delta V}{\Delta r}$

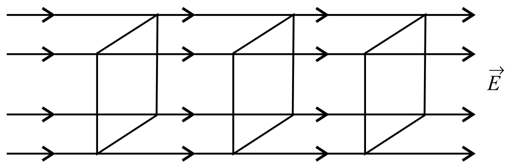
$E \propto -\dfrac{1}{\Delta r}$

For given equipotential surfaces, small Δr represents strong electric field and vice versa.

23.



For increasing electric field



For constant electric field

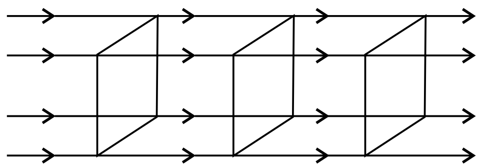
For increasing electric field, separation between equipotential surfaces decreases, in the direction of increasing field, for the same potential difference between them.

For constant electric field, equipotential surfaces are equidistant for same potential difference between these surfaces.

24. Equipotential surface is the surface with a constant value of potential at all points on the surface.

(i) *Refer to answer 14.*

(ii) Equipotential surfaces in a constant electric field in Z-direction.



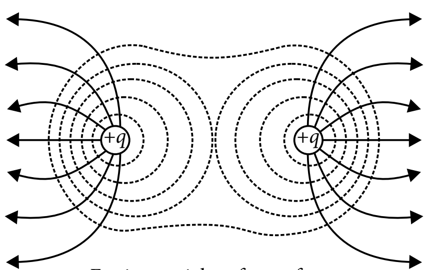
For constant electric field

Equipotential surfaces about a single charge are not equidistant because electric potential, $V \propto \dfrac{1}{r}$.

(iii) Electric field tangential to an equipotential surface cannot exist.

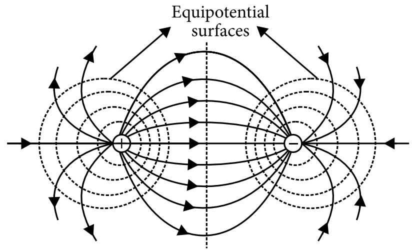
If the field lines are tangential, work will be done in moving a charge on the surface whereas on equipotential surface $W_{AB} = q_0(V_B - V_A) = 0$

25. The figure is shown as below



Equipotential surfaces of two identical positive charges

26. Equipotential surface of an electric dipole is :



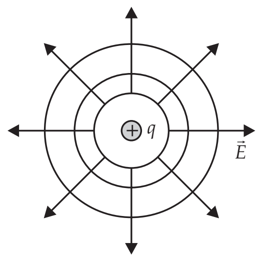
Potential is zero on the points located on the line passing through the centre of dipole and perpendicular to the dipole axis.

27. (a) Properties of equipotential surface are:

(i) Work done in moving a test charge over an equipotential surface is zero.

(ii) Electric field is always directed normal to equipotential surface.

Equipotential surface due to an isolated point charge:



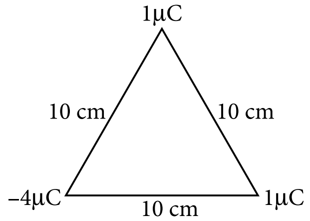
$\because \quad E = -\dfrac{dV}{dr}$, *i.e.*, $dr = -\dfrac{dV}{E}$

for given dV, $dr \propto \dfrac{1}{E}$

Hence, dr is small, then E is large. Hence, for small dr, equipotential surfaces are crowded.

28. Zero, as the diagonally opposite corners of square with point charge at its centre are at same electric potential or potential difference between them is zero.

29. $U = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{1\times10^{-6}(-4\times10^{-6})}{0.10} + \dfrac{1\times10^{-6}(1\times10^{-6})}{0.10} + \dfrac{-4\times10^{-6}(1\times10^{-6})}{0.10}\right]$



$$U = \frac{1}{4\pi\varepsilon_0} \times 10^{-12}[-4\times10+10-4\times10]$$

$U = -9 \times 10^9 \times 10^{-12} \times 70$

$U = -0.630$ J.

Work done to dissociate the system of charges

$W = -V = 0.630$ J

30. Potential at $P(7, 0, 0)$ is

$$V_1 = \frac{-q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(7-0)^2+0+(-a-0)^2}} + \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(7-0)^2+0+(a-0)^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{49+a^2}} + \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{49+a^2}} = 0$$

Potential at $Q(-3, 0, 0)$ is

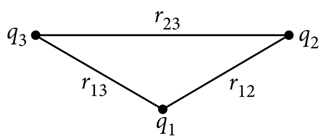
$$V_2 = \frac{-q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(-3-0)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(-3-0)^2+(-a)^2}}$$

$$= \frac{-q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{9+a^2}} + \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{9+a^2}} = 0$$

$\therefore$ Work done $= q(V_2 - V_1) = q(0 - 0) = 0$

Hence, $W = 0$.

31. Potential energy of a system of three charges : A system of three charges q_1, q_2 and q_3 are located at $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ respectively with respect to the common origin O.



To bring q_1 from infinity to $\vec{r}_1$, no work is required.

Work done is bringing charge q_2 from infinity to $\vec{r}_2$ is

$$q_2 V_1(\vec{r}_2) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} \quad \text{...(i)}$$

The charges q_1 and q_2 produce a potential, which at any point p is given by

$$V_{12} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}}\right)$$

Work done next in bringing q_3 from infinity to the point $\vec{r}_3$ is

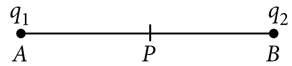
$$q_3 V_{1,2}(\vec{r}_3) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \quad \text{...(ii)}$$

The total work done in assembling the charges at the given location is obtained by adding the work done in steps (i) and (ii) is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

32.



(i) Here, $q_1 = 10 \times 10^{-8}$ C, $q_2 = -2 \times 10^{-8}$ C and $AB = 60$ cm $= 0.6$ m

Let $AP = x$ then $PB = 0.6 - x$

Potential P due to charge $q_1 = \dfrac{Kq_1}{AP}$

Potential P due to charge $q_2 = \dfrac{Kq_2}{AP}$

$\therefore$ Potential at $P = 0$

$$\Rightarrow \frac{Kq_1}{AP} + \frac{Kq_2}{PB} = 0$$

$$\frac{q_1}{AP} = \frac{-q_2}{PB}$$

$$\therefore \quad \frac{10\times10^{-8}}{x} = \frac{-(-2\times10^{-8})}{0.6-x} \Rightarrow \frac{10}{x} = \frac{2}{0.6-x}$$

$2x = 6.0 - 10x \Rightarrow 2x + 10x = 6$

$\therefore$ $12x = 6 \Rightarrow x = \dfrac{6}{12} = 0.5$ m

$\therefore$ Distance from first charge $= 0.5$ m $= 50$ cm.

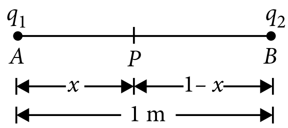
(ii) Electrostatic potential energy of the system

$$U = K\frac{q_1 q_2}{r}$$

$$U = 9\times10^9 \times \frac{10\times10^{-8}\times(-2\times10^{-8})}{0.6}$$

$$U = \frac{-18\times10^{-6}}{0.6} \Rightarrow U = -30\times10^{-6} = -3\times10^{-5}\text{ J}$$

33.



Let the point be at a distance x from $4Q$ charge. Electric field at P due to $4Q$ = Electric field at P due to Q

$$\therefore \quad K \times \frac{4Q}{x^2} = K \times \frac{Q}{(1-x)^2}$$

$$\frac{4}{x^2} = \frac{1}{(1-x)^2} \Rightarrow \frac{2}{x} = \pm\frac{1}{1-x}$$

$$\frac{2}{x} = \frac{1}{1-x} \text{ or } \frac{2}{x} = \frac{-1}{1-x}$$

$x = 2 - 2x$ or $-x = 2 - 2x$

$x + 2x = 2$ or $-x + 2x = 2$

$3x = 2$ or $x = 2$

$$x = \frac{2}{3} \text{ or } x = 2$$

$\because \quad x = 2$ m is not possible.

$$\therefore \quad x = \frac{2}{3} \text{ m}$$

Electrostatic potential energy of the system is

$$U = K\frac{q_1 q_2}{r}$$

$$\Rightarrow \quad U = K \cdot \frac{4Q \cdot Q}{r} = K \cdot \frac{4Q^2}{r}$$

$$U = 9 \times 10^9 \times \frac{4 \times (2 \times 10^{-7})^2}{1} \qquad [\because Q = 2 \times 10^{-7} \text{ C}]$$

$$= 9 \times 10^9 \times \frac{4 \times 4 \times 10^{-14}}{1}$$

$= 144 \times 10^{-5} = 1.44 \times 10^{-3}$ J

34. The direction of electric field is perpendicular to the equipotential surface.

(i) The direction of electric field is along x-axis as it should be perpendicular to equipotential surface lying in yz-plane.

Lengh of the dipole = $2b$

As dipole's axis is along the y-axis.

$\therefore$ Electric dipole moment

$$\vec{p} = q(2b)\hat{j} \qquad \text{...(i)}$$

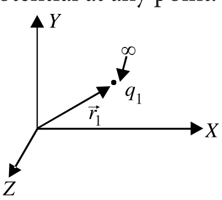
(ii) Electric field $E = E\hat{i}$

$$\because \quad \vec{\tau} = \vec{p} \times \vec{E} = q(2b)\hat{j} \times E\hat{i}$$

$$= +2qbE(\hat{j} \times \hat{i}) = 2qbE(-\hat{k})$$

$\because$ Torque $|\tau| = 2qbE$ ...(i)

35. Potential energy of a system of two point charges : Let no source charge be present in the system initially and hence no potential at any point.



Now the charge q_1 is brought at point A from infinite

work done to bring charge q_1 at A

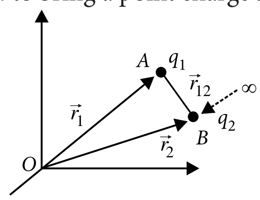
$$W_1 = q_1 V_A$$

or $\quad W_1 = 0 \qquad \text{...(i)} \qquad [\because V_A = 0]$

Due to presence of q_1 a potential develops at point B *i.e.*,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}}$$

work required to bring a point charge q_2 from ∞ to B



$$W_2 = q_2 V_B$$

$$W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} \qquad \text{...(ii)}$$

Total work done to form the system of two point charges or the potential energy of the system of charges is then given by

$$U = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

36. Electric field intensity is zero inside the hollow spherical charge conductor. So, no work is done in moving a test charge inside the conductor and on its surface. Therefore, there is no potential difference between any two points inside or on the surface of the conductor.

37. Potential inside the charged sphere is constant and equal to potential on the surface of the conductor. Therefore, potential at the centre of the sphere is 10 V.

38. The potential at any point on the surface of the conductor having radius r and charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2}

The capacitance of the spherical conductor situated in vaccum is given by

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}}$$

$C = 4\pi\varepsilon_0 r$.

Hence, the capacitance of an isolated spherical conductor situated in vaccum is $4\pi\varepsilon_0$ times its radius.

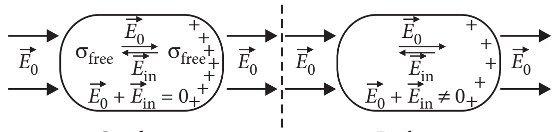
39. (a) It is safer to be inside a car during thunderstorm because the car is a hollow conductor acts like a Faraday cage for electrostatic shielding. The metal in the car will shield you from any external electric fields and thus prevent the lightning from traveling within the car.
(b) Scientific awareness in practical life and humanity
(c) Gratitude and obliged
(d) For example : Once, I came across to a situation where a puppy was stuck in the middle of a busy road during rain and was not able to cross due to heavy flow, so, I quickly rushed and helped him.

40. Dielectrics are non-conductors and do not have free electrons at all. While conductors has free electrons which makes it able to pass the electricity through it.

41. A dielectric whose molecules possess electric moment even when electric field is not applied is called polar dielectric. On the other hand a dielectric, whose molecules do not possess parmanent dipole moment, is called non-polar dielectric.

42. (i) Carefullness, awareness and helping towards the social cause.
(ii) Car is the safest place during thunder storm, because car is made up of metal which is good conductor of electricity when lighting strikes the car, it will pass it to the ground without harming any one inside the car.
(iii) The maximum voltage that a dielectric material can withstand, under specific conditions, without rupturing is called dielectric strength. It is usually expressed as volts/unit thickness.

43. When a conductor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved *i.e.*, when the two fields cancel each other and the net electrostatic field in the conductor becomes zero.
Dielectrics are non-conducting substances *i.e.*, they have no charge carriers. Thus, in a dielectric, free movement of charges is not possible. It turns out that the external field induces dipole moment by reorienting molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produce a field that opposes the external field, unlike a conductor in an external electric field. However, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric.
The effect of electric field on a conductor and a dielectric is shown in the figure.



The dipole moment per unit volume is called polarisation and is denoted by P. For linear isotropic dielectrics, $P = \chi E$
where χ is electric susceptibility of the dielectric medium.

44. The plate area of C_2 is greater than that of C_1. Since capacitance of a capacitor is directly proportional to the area of the plates,

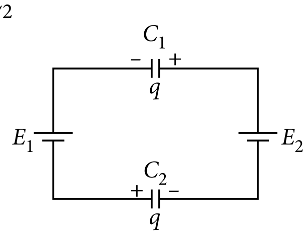
$\therefore \quad C_2 > C_1$

Now, $C = \dfrac{q}{V}$

Therefore, slope of a line $(=q/V)$ is directly proportional to the capacitance of the capacitor, it represents. Since the slope of line A is more than that of B, line A represents C_2 and the line B represents C_1.

45. $\dfrac{-q}{C_1} - E_1 - \dfrac{q}{C_2} + E_2 = 0$

or $\dfrac{q}{C_1} + \dfrac{q}{C_2} = E_2 - E_1$



Now,

$V_1 = \dfrac{q}{C_1}, V_2 = \dfrac{q}{C_2}$

46. Here $C = 2F$

$d = 0.5$ cm $= 0.5 \times 10^{-2}$ m

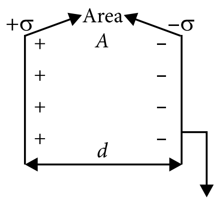
$\varepsilon_0 = 8.854 \times 10^{-12}\, C^2 N^{-1} m^{-2}$

$\therefore \quad C = \dfrac{\varepsilon_0 A}{d}$

$$A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

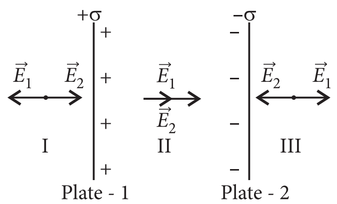
$A = 1.13 \times 10^9$ m^2

47. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel plate capacitor.



(i) (a) Magnitude of electric field intensities

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$



(i) In region I (outside)

$$E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(ii) In region II (inside)

$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

(iii) In region III (outside)

$$E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the region II *i.e.*, in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charge plate.

(b) The potential difference between the plates is

$$V = E_{II}.d = \frac{\sigma}{\varepsilon_0} d \quad \text{or} \quad V = \frac{Q}{A\varepsilon_0} d$$

(c) Capacitance of the capacitor so formed is

$$C = \frac{Q}{V} = \frac{Q}{Qd / A\varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

48. *Refer to answer - 47.*

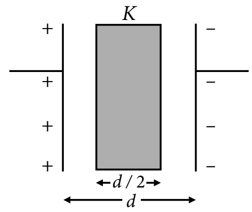
49. (i) $\phi_1 = \dfrac{Q}{\varepsilon_0}$, $\phi_2 = \dfrac{3Q}{\varepsilon_0}$

$$\frac{\phi_1}{\phi_2} = \frac{1}{3}$$

(ii) If a medium of dielectric constant 5 is filled in the space inside S_1, the flux inside S_1

$$\phi_1' = \frac{Q}{5\varepsilon_0} = \frac{\phi_1}{5}$$

50.



Capacitance of a capacitor partially filled with a dielectric

$$C = \frac{\varepsilon_0 A}{d - t + \dfrac{t}{K}} = \frac{\varepsilon_0 A}{d - \dfrac{d}{2} + \dfrac{d}{2K}} = \frac{2\varepsilon_0 AK}{d(K+1)}$$

51. Let $A \rightarrow$ area of each plate and C_1 and C_2 are capacitance of each slab.

Let initially $C_1 = C = \dfrac{\varepsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs:

$C_1' = KC$ ...(i)

and $C_2' = K_1 \dfrac{\varepsilon_0 (A/2)}{d} + K_2 \dfrac{\varepsilon_0 (A/2)}{d}$

$= \dfrac{\varepsilon_0 A}{2d}(K_1 + K_2)$; $C_2' = \dfrac{C}{2}(K_1 + K_2)$ ...(ii)

From (i) and (ii)

$C_1' = C_2'$

$$KC = \frac{C}{2}(K_1 + K_2)$$

$$K = \frac{1}{2}(K_1 + K_2)$$

52. (i) Capacitance

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-4}} = 17.7 \times 10^{-11}\,\text{F}$$

(ii) Charge $Q = CV = 17.7 \times 10^{-11} \times 100$
$= 17.7 \times 10^{-9}$ C

(iii) $C' = KC$
$\therefore$ $Q' = KQ = 10.62 \times 10^{-8}$ C

53. (a) Initial electric field between the plates of parallel plate capacitor $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{q/A}{\varepsilon_0} = \dfrac{q}{A\varepsilon_0}$

After introduction of dielectric; the permittivity of medium becomes $K\varepsilon_0$.
so, final electric field between the plates of parallel plate capacitor $E = \dfrac{q}{AK\varepsilon_0} = \dfrac{E_0}{K}$

i.e., electric field reduces to $\dfrac{1}{K}$ times.

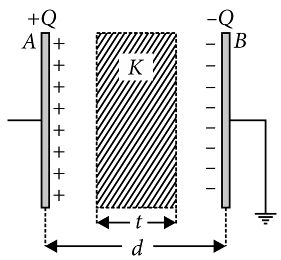
(b) Consider a parallel plate capacitor, area of each plate being A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness $t < d$ be placed between the plates. The thickness of air between the plates is $(d - t)$. If charges on plates are $+Q$ and $-Q$, then surface charge density

$$\sigma = \frac{Q}{A}$$

The electric field between the plates in air,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The electric field between the plates in the slab,



$$E_2 = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}$$

$\therefore$ The potential difference beetween the plates
V_{AB} = work done in carrying unit positive charge from one plate to another
$= \sum Ex$ (as field between the plates is not constant).

$$= E_1(d-t) + E_2 t = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{K\varepsilon_0 A}t$$

$$\therefore \quad V_{AB} = \frac{Q}{\varepsilon_0 A}\left[d - t + \frac{t}{K}\right]$$

$\therefore$ Capacitance of capacitor,

$$C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)}$$

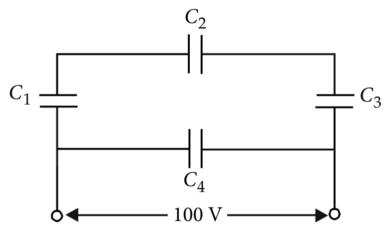
$$\text{or,} \quad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Here, $t = \dfrac{d}{2}$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

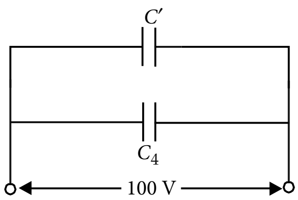
54. *Refer to answer 51.*

55. (a)



Here C_1, C_2 and C_3 are in series, hence their equivalent capacitance is C' given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



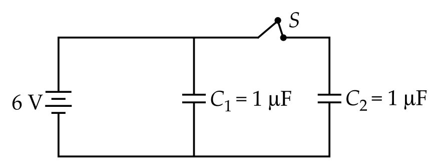
$$C' = \frac{15}{3}\ \mu\text{F}$$

$C' = 5\ \mu$F

The circuit can be redrawn as shown, in the figure.
Since C' and C_4 are in parallel
$\therefore$ $C_{net} = C' + C_4 = 5\ \mu\text{F} + 15\ \mu\text{F} = 20\ \mu\text{F}$

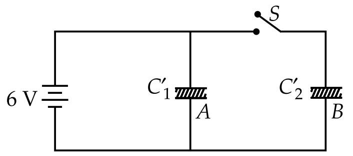
(b) Since C' and C_4 are in parallel, potential difference across both of them is 100 V.
$\therefore$ Charge across C_4 is $Q_4 = C_4 \times 100$ C
$= 15 \times 10^{-6} \times 100$ C $= 1.5$ mC

56. When the switch S is closed, the two capacitors in parallel will be charged by the same potential difference V.



So, charge on capacitor C_1

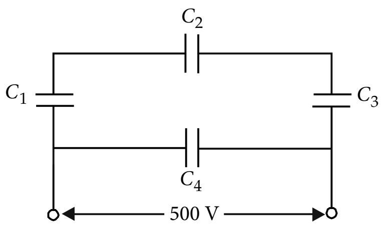
$q_1 = C_1 V$
$q_1 = 1 \times 6 = 6\ \mu C$
and charge on capacitor C_2
$q_2 = C_2 V = 1 \times 6 = 6\ \mu C$
$\therefore \quad q = q_1 + q_2 = 6 + 6 = 12\ \mu C.$
When switch S is opened and dielectric is introduced. Then



Capacity of both the capacitors becomes K times
i.e., $C'_1 = C'_2 = KC = 3 \times 1 = 3\ \mu F$
Capacitor A remains connected to battery
$\therefore \quad V'_1 = V = 6\ V$
$q'_1 = Kq_1 = 3 \times 6\ \mu C = 18\ \mu C$
Capacitor B becomes isolated
$\therefore \quad q'_2 = q_2$ or $C'_2 V'_2 = C_2 V_2$ or $(KC)V'_2 = CV$

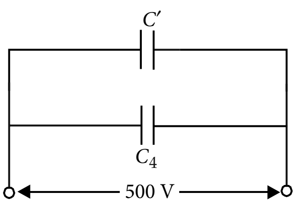
or $\quad V'_2 = \left(\frac{V}{K}\right) = \frac{6}{3} = 2\ V$

57. (a)



Here C_1, C_2 and C_3 are in series, hence their equivalent capacitance is C' given by

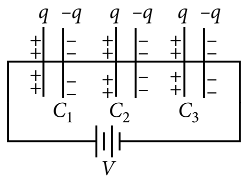
$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



$$C' = \frac{12}{3}\ \mu F$$

$C' = 4\ \mu F$
The circuit can be redrawn as shown in the figure.
Since C' and C_4 are in parallel
$\therefore \quad C_{net} = C' + C_4 = 4\ \mu F + 12\ \mu F = 16\ \mu F$
(b) Since C' and C_4 are in parallel, potential difference across both of them is 500 V.
$\therefore \quad$ Charge across C_4 is $Q_4 = C_4 \times 500$
$= 12 \times 10^{-6} \times 500 = 6$ mC
Charge across C', $Q' = C' \times 500$
$= 4 \times 10^{-6} \times 500 = 2$ mC
$\because \quad C_1, C_2, C_3$ are in series, charge across them is same, which is $Q' = 2$ mC

58. Capacitors is series : Consider three capacitors C_1, C_2 and C_3 are connected in series. The left plate of C_1 and the right plate of C_3 are connected to two terminals of a battery and have charges q and $-q$ respectively.
The total potential drop V across the combination is the sum of the potential drops V_1, V_2 and V_3 across C_1, C_2 and C_3 respectively.



$$\therefore \quad V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\therefore \quad \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \text{...(i)}$$

The effective capacitance C of the combination is

$$C = \frac{q}{V} \Rightarrow \frac{1}{C} = \frac{V}{q} \qquad \text{...(ii)}$$

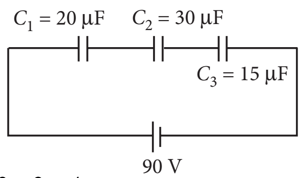
On comparing Eq (i) and (ii), we get

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2C_3 + C_3C_1 + C_1C_2}{C_1C_2C_3}$$

$$\therefore \quad C = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

59. The equivalent capacitance (C_{eq}) of the circuit is given by

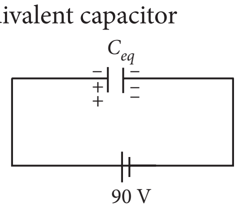
$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$



$$\frac{1}{C_{eq}} = \frac{3+2+4}{60}$$

$$C_{eq} = \frac{60}{9}\ \mu F$$

Charge on equivalent capacitor



$$Q = C_{eq}V = \frac{60}{9}\times 10^{-6}\times 90$$

$Q = 600\ \mu C$

Charge on each capacitor is same as they are in series.

Now, potential drop across C_2

$$V_2 = \frac{Q}{C_2} = \frac{600\times 10^{-6}}{30\times 10^{-6}} = 20 \text{ volt}$$

Energy, $U = \frac{1}{2}C_2V_2^2$

$$U = \frac{1}{2}\times 30\times 10^{-6}\times (20)^2 = 6\times 10^{-3} \text{ joule}$$

60. Energy stored in a capacitor

$$= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

Capacitance of the (parallel) combination

$= C + C = 2C$

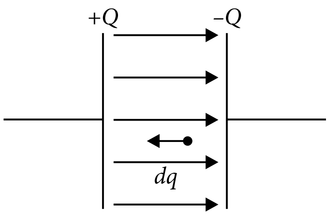
Here, total charge Q, remains the same

$\therefore$ Initial energy (Single capacitor) $= \frac{1}{2}\frac{Q^2}{C}$

and final energy (Combined capacitor) $= \frac{1}{2}\frac{Q^2}{2C}$

$$\therefore \quad \frac{\text{Final energy}}{\text{Initial energy}} = \frac{1}{2}$$

61. Q and $-Q$ are charges on the plates and produces a uniform electric field $E = \frac{\sigma}{\varepsilon_0}$ between the plates and a potential difference $V = \frac{q}{C}$ ...(i)



If a charge dq is transported in steps from negative charged plate to positive charged plate, till charges rises to $+Q$ and $-Q$, then

Work done $dW = dq.\ V$ ...(ii)

From equations (i) and (ii)

$$dW = dq\left(\frac{q}{C}\right)$$

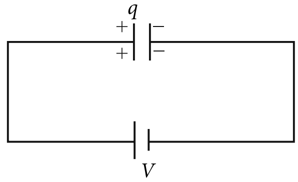
Total electrostatic potential energy stored can be given as

$$U = W = \int_0^Q \frac{q}{C}.dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C}$$

62. Potential difference between the plates of capacitor

$$V = \frac{q}{C}$$



Work done to add additional charge dq on the capacitor

$dW = V \times dq = (q/C) \times dq$

$\therefore$ Total energy stored in the capacitor

$$U = \int dW = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}$$

When battery is disconnected

(i) Energy stored will be decreased or energy stored $= \frac{1}{K}$ times the initial energy.

(ii) Electric field would decrease

or $E' = \frac{E}{K}$

63. Net capacitance in series, $C_s = 1\ \mu F = 10^{-6}$ F

if $C_1 = C_2 = C_3 = C$

Let C be the capacitance of each of three capacitors and C_S and C_R be the capacitance of series and parallel combination respectively.

then, $\frac{1}{C_S} = \frac{1}{C}+\frac{1}{C}+\frac{1}{C} = \frac{3}{C}$

$C_s = \frac{C}{3}$ $[C_s = 1\ \mu F]$

$\therefore$ $1\ \mu F = \frac{C}{3}; C = 3\ \mu F$

Also $C_P = C + C + C$

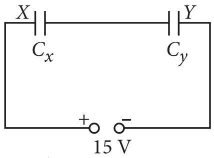
$= 3 + 3 + 3 = 9\ \mu F$

Energy stored in capacitor

$$E = \frac{1}{2}CV^2$$

$$\frac{E_S}{E_P} = \frac{\frac{1}{2}C_SV^2}{\frac{1}{2}C_PV^2} = \frac{C_S}{C_P} = \frac{1}{9}$$

64. Here, $C_x = \dfrac{\varepsilon_0 A}{d}$



$C_y = \dfrac{\varepsilon_0 \varepsilon_r A}{d} = \varepsilon_r C_x = 4\, C_x$

(i) C_x and C_y are in series, so equivalent capacitance is given by

$$C = \frac{C_x \times C_y}{C_x + C_y}$$

$$\Rightarrow \quad 4 = \frac{C_x \times 4\,C_x}{C_x + 4\,C_x} \qquad (\because\ C = 4\ \mu\text{F})$$

$$\Rightarrow \quad 4 = \frac{4\,C_x}{5} \quad \therefore\ C_x = 5\ \mu\text{F}$$

and $C_y = 4\, C_x = 20\ \mu$F

(ii) Charge on each capacitor, $Q = CV$

$Q = 4 \times 10^{-6} \times 15 = 60 \times 10^{-6}$ C

Potential difference between the plates of X,

$$V_x = \frac{Q}{C_x} = \frac{60 \times 10^{-6}}{5 \times 10^{-6}} = 12\ \text{V}$$

Potential difference between the plates of Y,

$V_y = V - V_x = 15 - 12 = 3$ V

(iii) Ratio of electrostatic energy stored,

$$\frac{U_x}{U_y} = \frac{\dfrac{Q^2}{2C_x}}{\dfrac{Q^2}{2C_y}} = \frac{C_y}{C_x} = \frac{4C_x}{C_x} = 4$$

65. (i) Given that energy of the 6 μF capacitor is E

Let V be the potential difference along the capacitor of capacitance 6 μF.

Since $\dfrac{1}{2}CV^2 = E$

$$\therefore \quad \frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E$$

$$\Rightarrow \quad V^2 = \frac{E}{3} \times 10^6 \qquad \text{...(i)}$$

Since potential is same for parallel connection, the potential through 12 μF capacitor is also V. Hence, energy of 12 μF capacitor is

$$E_{12} = \frac{1}{2} \times 12 \times 10^{-6} \times V^2 = \frac{1}{2} \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$$

(ii) Since charge remains constant in series, the charge on 6 μF and 12 μF capacitors combined will be equal to the charge on 3 μF capacitor. Using the formula, $Q = CV$, we can write

$(6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$

$V' = 6$ V

Using (i) and squaring both sides, we get

$V'^2 = 12E \times 10^6$

$$\therefore \quad E_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 12E \times 10^6 = 18E$$

(iii) Total energy drawn from battery is

$E_{total} = E + E_{12} + E_3 = E + 2E + 18E = 21E$

66. When two capacitors C_1 and C_2 are in parallel,

Equivalent capacitance, $C_p = C_1 + C_2$

Energy stored, $U_p = \dfrac{1}{2} C_p V^2 = \dfrac{1}{2}(C_1 + C_2)V^2$

Here, $U_p = 0.25$ J, $V = 100$ V

$$C_1 + C_2 = \frac{2U_p}{V^2} = \frac{2 \times 0.25}{(100)^2}$$

$$\therefore \quad C_1 + C_2 = 5 \times 10^{-5} \qquad \text{...(i)}$$

When C_1 and C_2 are connected in series

Equivalent capacitance, $C_s = \dfrac{C_1 C_2}{C_1 + C_2}$

Energy stored, $U_s = \dfrac{1}{2} C_s V^2 = \dfrac{1}{2}\left(\dfrac{C_1 C_2}{C_1 + C_2}\right) V^2$

Here, $U_s = 0.045$ J

$$\therefore \quad C_1 C_2 = \frac{2U_s (C_1 + C_2)}{V^2}$$

$$= \frac{2 \times 0.045 \times 5 \times 10^{-5}}{10^4} = 4.5 \times 10^{-10}$$

$$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1 C_2}$$

$$= \sqrt{(5 \times 10^{-5})^2 - 4 \times 4.5 \times 10^{-10}}$$

$$C_1 - C_2 = 2.64 \times 10^{-5} \qquad \text{...(ii)}$$

Solving eqn. (i) and (ii), we get

$C_1 = 38.2$ μF, $C_2 = 11.8$ μF

When capacitors are connected in parallel they have different amount of charge and given by

$Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100 = 38.2 \times 10^{-4}$ C

$Q_2 = C_2 V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4}$ C.

67. (a) Energy stored in a charged capacitor :

If q is the charge and V is the potential difference across a capacitor at any instant during its charging, then small work done is storing an additional small charge dq against the repulsion of charge q already stored on it is

$dW = V.dq = (q/C)dq$

So, the total amount of work done in storing the maximum charge Q on capacitor is

$$W = \int_0^Q \frac{q}{C}\cdot dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

which gets stored in the capacitor in the form of electrostatic energy. So the energy stored in capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

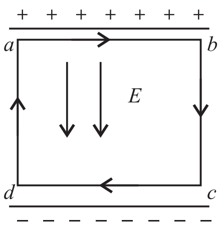
whereas the energy density *i.e.*, energy stored per unit volume in a charged parallel plate capacitor is given by

$$\text{Energy density} = \frac{\text{Total energy within plates}}{\text{Volume within plates}}$$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\varepsilon_0 A}{d}\cdot E^2 d^2}{A\cdot d}$$

$$\boxed{\text{Energy density} = \frac{1}{2}\varepsilon_0 E^2}$$

(b) Electric field inside a parallel plate capacitor = E



Here, electric field is conservative. Work done by the conservative force in closed loop is zero.
So, required work done = 0.

68. (i) Let the capacity of given capacitor is C and initial voltage $V_1 = V$

$Q_1 = 360\ \mu C$

$\therefore\ Q_1 = CV_1$ ..(i)

Changed potential, $V_2 = V - 120$

$Q_2 = 120\ \mu\text{C}$

$Q_2 = CV_2$ ...(ii)

Dividing equation (i) by (ii), we get $\dfrac{Q_1}{Q_2} = \dfrac{CV_1}{CV_2}$

$$\Rightarrow \frac{360}{120} = \frac{V}{V-120}$$

$$\Rightarrow V = 180$$

$$\therefore\ C = \frac{Q_1}{V_1} = \frac{360\times10^{-6}}{180} = 2\times10^{-6}\,\text{F} = 2\,\mu\text{F}$$

(ii) If the voltage applied had increased by 120 V, then $V_3 = 180 + 120 = 300$ V. Hence, charge stored in the capacitor,

$Q_3 = CV_3 = 2 \times 10^{-6} \times 300 = 600\ \mu\text{C}$

69. Initial energy of capacitor (U_i) = $\frac{1}{2}CV^2$

$$U_i = \frac{1}{2}\times200\times10^{-12}\times(300)^2 = 9\times10^{-6}\ \text{J}$$

Charge on capacitor

$$Q = CV = 200 \times 10^{-12} \times 300 = 6 \times 10^{-8}\text{C}$$

When both capacitors are connected then let V be common potential difference across the two capactiros.
The charge would be shared between them.
Hence, $Q = q + q'$,

$q \rightarrow$ charge on capacitor (first)

$q' \rightarrow$ charge on capacitor (second)

$C = 200$ pF, $C' = 100$ pF

$$\frac{q}{200\times10^{-12}} = \frac{q'}{100\times10^{-12}} \quad \Rightarrow q = 2q'$$

Then $Q = 2q' + q' = 3q'$

$$\Rightarrow \quad q' = \frac{Q}{3} = \frac{60\ \text{nC}}{3} = 20\ \text{nC}$$

and $q = 2q' = 40$ nC

Hence, total final energy $U_f = \dfrac{q^2}{2C} + \dfrac{q'^2}{2C'}$

$$U_f = \frac{1}{2}\times\frac{(40\times10^{-9})^2}{200\times10^{-12}} + \frac{1}{2}\times\frac{(20\times10^{-9})^2}{100\times10^{-12}}$$

$U_f = 6 \times 10^{-6}$ J

Energy difference (ΔU) = $U_f - U_i$

$= 6 \times 10^{-6} - 9 \times 10^{-6}\ \text{J} = -3 \times 10^{-6}$ J

$\Rightarrow\ \Delta U = 3 \times 10^{-6}$ J (in magnitude)

70. (i) On filling the dielectric of constant K in the space between the plates, capacitance of parallel plate capacitor becomes K times *i.e.*

$C = KC_0$

(ii) As the battery was disconnected, so the charge on the capacitor remains the same *i.e.*

$Q = Q_0$

So, the electric field in the space between the plates becomes

$$E = \frac{Q_0}{KA\varepsilon_0} \text{ or } E = \frac{E_0}{K}$$

i.e. electric field becomes $\dfrac{1}{K}$ times.

(iii) Energy stored in capacitor becomes

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{KC} \text{ or } U = \frac{1}{K}U_0$$

i.e. becomes $\frac{1}{K}$ times

71. Let C be capacitance of each capacitor.

In series arrangement net capacitance $C_S = \frac{C}{2}$

In parallel arrangement net capacitance $C_P = 2C$

Energy stored $U = \frac{1}{2}CV^2$

If V_S and V_P are potential difference applied across series and parallel arrangement, then given

$U_S = U_P$

$$\Rightarrow \frac{1}{2}C_S V_S^2 = \frac{1}{2}C_P V_P^2$$

$$\Rightarrow \frac{V_P}{V_S} = \sqrt{\frac{C_S}{C_P}} = \sqrt{\frac{C/2}{2C}} = \frac{1}{2}$$

72. (i) Here $V = 12$ V

and $C_1 = C_2 = C_3 = 6\ \mu\text{F} = 6 \times 10^{-6}$ F

charge on capacitor C_3 is

$q_3 = C_3V = 6 \times 10^{-6} \times 12 = 72 \times 10^{-6} = 72\ \mu\text{C}$

Since capacitor C_1 and C_3 are in series

$\therefore$ Equivalent capacitance $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_S} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ $C_S = 3\ \mu\text{F}$

Charge on capacitor C_1 and C_2 is

$q = C_SV = 3 \times 10^{-6} \times 12 = 36 \times 10^{-6} = 36\ \mu\text{C}$

$\therefore$ Charge on each capacitor C_1 and C_2 is 36 μC.

(ii) Since C_1 and C_2 are in series

$\therefore$ Equivalent capacitance $C_S = 3\ \mu\text{F}$

Now, C_3 and C_S are in parallel

$\therefore$ Equivalent capacitance $C = C_3 + C_S = 6 + 3 = 9\ \mu\text{F}$

(iii) Energy stored

$$= \frac{1}{2}CV^2 = \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2$$

$$= \frac{1}{2} \times 9 \times 10^{-6} \times 144 = 648 \times 10^{-6} = 6.48 \times 10^{-4} \text{ J}$$

73. Given $\frac{C_1}{C_2} = \frac{1}{2}$ or $C_2 = 2C_1$

In parallel, $C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1$

In series, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{2C_1} = \frac{2+1}{2C_1} = \frac{3}{2C_1}$

or $C_S = \frac{2}{3}C_1$

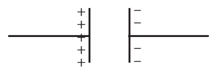
Given $U_S = U_P$

$$\frac{1}{2}C_S V_S^2 = \frac{1}{2}C_P V_P^2 \text{ or } \frac{2}{3}C_1 V_S^2 = 3C_1 V_P^2$$

$$\text{or } \frac{V_S^2}{V_P^2} = \frac{9}{2} \text{ or } \frac{V_S}{V_P} = \frac{3}{\sqrt{2}}$$

74. (a) *Refer to answer 67(a)*

(b) Let fully charge capacitor C has charge Q.



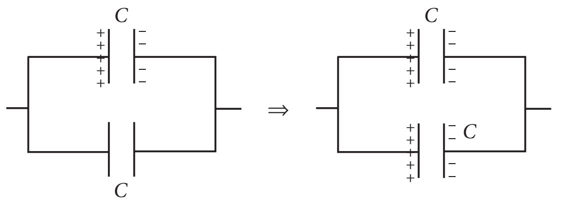
C, Q, V

$V = Q/C$

Energy stored in the capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Now, the charged capacitor is connected to identical uncharged capacitor.



The two capacitor will have same potential.

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q + 0}{2C} = \frac{Q}{2C}$$

Now, total energy

$$U' = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$U' = \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 + \frac{1}{2}C\left(\frac{Q}{2C}\right)^2 = \frac{Q^2}{4C}$$

So, $U > U'$

Energy lost as heat during charging the another capacitor.

$$U - U' = \frac{Q^2}{2C} - \frac{Q^2}{4C} = \frac{Q^2}{4C}$$

75. Energy stored in a parallel plate capacitor is equal to work done in charging a capacitor. This work done is stored as its electrical potential energy. Suppose a capacitor is charged with charge q so that potential difference between its plates is

$V = \frac{q}{C}$

Work done to increase the charge by dq is

$dW = Vdq = \frac{q}{C}dq$

Total work done to charge the capacitor from 0 to Q is

$$W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

$\therefore$ Energy of the capacitor, $U = \frac{Q^2}{2C}$

$= \frac{1}{2}QV \qquad \left[\because C = \frac{Q}{V}\right]$

(i) Here $K = E_r = 10$

$\because$ Capacitance $C = \frac{\varepsilon_0 A}{d}$

$\therefore \quad C' = K\frac{\varepsilon_0 A}{d/2} = 10 \times 2 \times \frac{\varepsilon_0 A}{d} = 20C$

$\therefore$ Capacitance becomes 20 times.

(ii) Initial electric field, $E = \frac{V}{d}$

Final, $E' = \frac{V}{d/2} = 2\frac{V}{d} = 2E$

Hence electric field is doubled.

(iii) Energy density of the capacitor, $U = \frac{1}{2}\varepsilon_0 E^2$

Finally, $U' = \frac{1}{2}\varepsilon_0 E'^2 = \frac{1}{2}\varepsilon_0 (2E)^2$

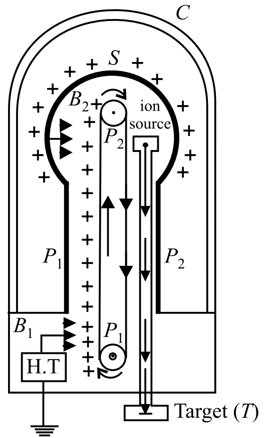
$= 4\ U'$

Energy density becomes four times.

76. Van de Graaff generator is a device used for building up high potential differences of the order of a few million volts.

Principle **:** It is based on the principle that charge given to a hollow conductor is transferred to the outer surface and is distributed uniformly over it.

Construction : It consists of a large spherical conducting shell (S) supported over the insulating pillars. A long narrow belt of insulating material is wound around two pulleys P_1 and P_2. B_1 and B_2 are two sharply pointed metal combs. B_1 is called the spray comb and B_2 is called the collecting comb.



Working : The spray comb is given a positive potential by a high tension source. The positive charge gets sprayed on the belt. As the belt moves and reaches the sphere, a negative charge is induced on the sharp ends of the collecting comb B_2 and equal positive charge is induced on the farther end of B_2.

This positive charge shifts immediately to the outer surface of S. Due to discharging action of sharp points of B_2, the positive charge on the belt is neutralised. The uncharged belt returns downwards and collects the positive charge from B_1, which in turn is collected by B_2. This procss is repeated and the positive charge on S goes on accumulating. In this way, voltage differences of as much as 6 or 8 million volts (with respect to the ground) can be built up.

Uses : Van de Graaff generator generates high potential differences that are used to accelerate charged particles such as electrons, protons, ions, etc. used for nuclear disintegration.

Limitations :

1. It's a series combination that allows only one route for the movement of charge.
2. It can accelerate only the charged particles not the uncharged particles.

77. *Refer to answer 76.*

Yes, the Van de Graaff generator can only be charged upto a limit when the electric field around it is less than breakdown field of the surrounding air.