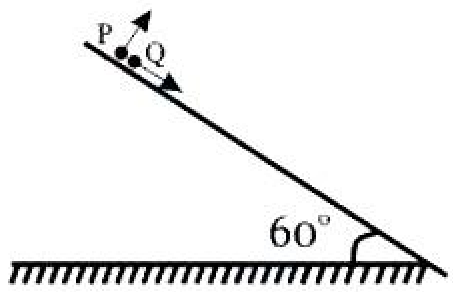
Q1: NTA Test 01 (Numerical)

A particle P is projected from a point on the surface of a smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide on the inclined plane after t = 4 seconds. Find the speed of projection in m/s.



Q2: NTA Test 02 (Single Choice)

Rain droplets are falling in vertically downward direction with velocity $5\ \mathrm{m/s}$. A cyclist is moving in northward direction with velocity $10\ \mathrm{m/s}$. The rain droplets will appear to the cyclist to be coming from

(A) $\tan^{-1}(2)$ above south horizon

(B) $\tan^{-1}\left(\frac{1}{2}\right)$ above north horizon

(C) Vertically downward

(D) $\tan^{-1}(2)$ above north horizon

Q3: NTA Test 02 (Numerical)

The position vector of a particle is determined by the expression $\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 9\hat{k}$. The magnitude of its displacement (in m) in first two seconds is

Q4: NTA Test 03 (Single Choice)

A ball is thrown with a speed u, at an angle θ with the horizontal. At the highest point of its motion, the strength of gravity is somehow doubled. Taking this change into account, the total time of flight of the projectile is

(A) $\frac{2u\sin\theta}{g}$

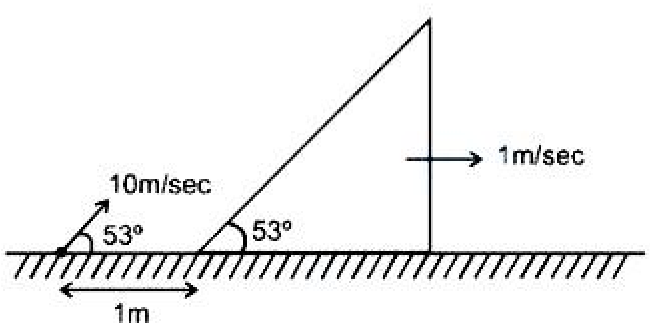
(B) $\frac{3}{2}\frac{u\sin\theta}{g}$

(C) $\frac{3}{4}\frac{u\sin\theta}{g}$

(D) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\frac{u\sin\theta}{g}$

Q5: NTA Test 08 (Single Choice)

A particle is projected from the ground and simultaneously a wedge starts moving towards the right, as shown in the figure. The maximum height of the wedge for which the particle will not hit the wedge is $\left[g = 10\ \mathrm{ms^{-2}}\right]$



(A) $\frac{28}{9}$ m

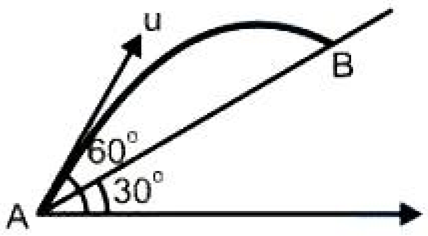
(B) $\frac{28}{3}$ m

(C) $\frac{7}{3}$ m

(D) $\frac{7}{4}$ m

Q6: NTA Test 18 (Single Choice)

Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to



(A) $\frac{ut}{\sqrt{3}}$

(B) $\frac{\sqrt{3}ut}{2}$

(C) $\sqrt{3}$ ut

(D) 2 ut

Q7: NTA Test 23 (Numerical)

A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m s^{-1} over the hill. The cannon is located at a distance of 800 m from the foot of the hill and can be moved on the ground at a speed of 2 m s^{-1} so that its distance from the hill can be adjusted. What is the shortest time (in seconds) in which a packet can reach on the ground across the hill? (Take $g = 10 \text{ m s}^{-2}$)

Q8: NTA Test 34 (Single Choice)

Two trains are moving in the same direction along parallel tracks. One of them is 200 m long travelling with a velocity of 20 m s^{-1}. The second one is 800 m long travelling with a velocity of 7.5 m s^{-1}. The time taken for the first train to overtake the second is

(A) 20 s

(B) 40 s

(C) 60 s

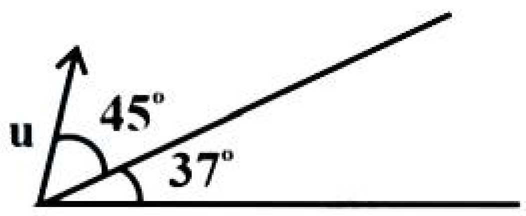
(D) 80 s

Q9: NTA Test 34 (Numerical)

Two second after projection a projectile is travelling in a direction inclined at 30° with horizontal, after one more second it is travelling horizontally. Angle of projection (in degree) with horizontal divided by 10 is.

Q10: NTA Test 35 (Numerical)

A particle is projected on a frictionless inclined plane of inclination 37° at an angle of projection 45° from the inclined plane as shown in the figure. If after the first collision from the plane, the particle returns to its point of projection, then what is the value of the reciprocal of the coefficient of restitution between the particle and the plane?



Q11: NTA Test 36 (Single Choice)

A car is travelling at a velocity 10 km h^{-1} on a straight road. The driver of the car throws a parcel with a velocity $10\sqrt{2} \text{ km h}^{-1}$ with respect to the car, when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with the direction of motion of the car

(A) 135°

(B) 45°

(C) $\tan^{-1}\sqrt{2}$

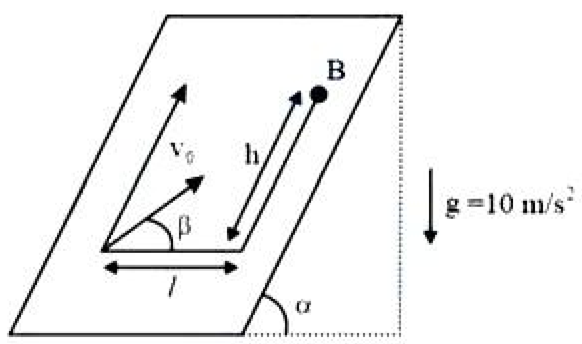
(D) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Q12: NTA Test 39 (Numerical)

Rain drops fall vertically at a speed of 20 m s^{-1}. At what angle (in degree) do they fall on the wind screen of a car moving with a velocity 15 m s^{-1}, if the wind screen is inclined at an angle of 23° to the vertical? $[\tan^{-1}(0.75) \approx 37^{\circ}]$

Q13: NTA Test 40 (Single Choice)

An inclined plane is located at angle $\alpha = 53^{\circ}$ to the horizontal. There is a hole at point B in the inclined plane as shown in the figure. A particle is projected along the plane with speed v_0 at an angle $\beta = 37^{\circ}$ to the horizontal in such a way so that it gets into the hole. Neglect any type of friction. Find the speed v_0 (in m s^{-1}) if $h = 1$ m and $l = 8m$.



(A) 9

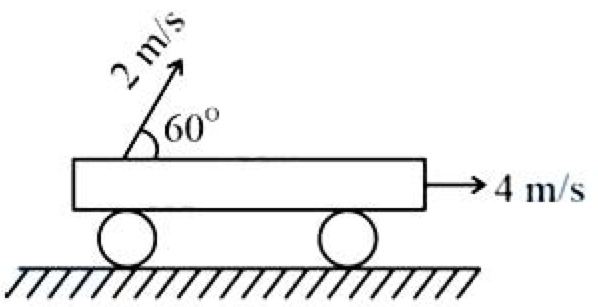
(B) 3

(C) 6

(D) 12

Q14: NTA Test 42 (Single Choice)

A ball is projected with a speed of $2\ \mathrm{m\ s^{-1}}$ with respect to a trolley which is moving with $4\ \mathrm{m\ s^{-1}}$, as shown in the figure. The angle at which ball is projected with respect to the horizontal in ground frame is



(A) $\tan^{-1}\left(\sqrt{\frac{3}{28}}\right)$

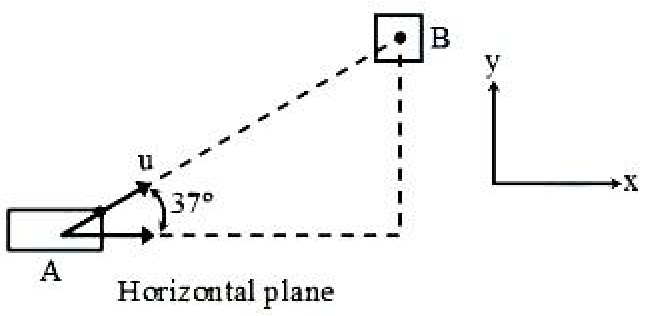
(B) $\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

(C) $\sin^{-1}\left(\sqrt{\frac{3}{28}}\right)$

(D) $\sin^{-1}\left(\frac{5}{\sqrt{28}}\right)$

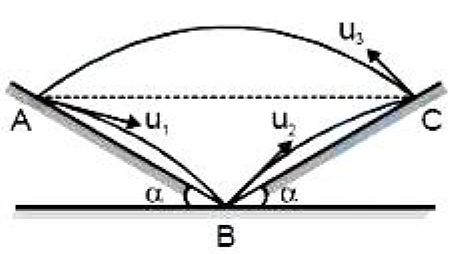
Q15: NTA Test 43 (Numerical)

A man throws a ball of mass m from a moving plank (A). The ball has a horizontal component of velocity $u = 3\sqrt{34}\ \mathrm{m\ s^{-1}}$ with respect to ground towards another man standing on stationary plank (B), as shown in the figure. The combined mass of plank A with man as well as that of plank B with the other man is $2m$ each. the x-y plane represents the horizontal plane which is frictionless. If initially, plank A was moving with speed $u\hat{i}$ with respect to ground, then the relative velocity of plank B (in $\mathrm{m\ s^{-1}}$) with respect to plank A (after the man on plank B catches the ball) minus $10\ \mathrm{m\ s^{-1}}$ is



Q16: NTA Test 45 (Single Choice)

Three particles are projected in the air with the minimum possible speeds, such that the first goes from A to B, the second goes from B to C and the third goes from C to A. Points A and C are at the same horizontal level. The two inclines make the same angle α with the horizontal, as shown. The relation among the projection speeds of the three particles is



(A) $u_3 = u_1 + u_2$

(B) $u_3^2 = 2u_1u_2$

(C) $\frac{1}{u_3} = \frac{1}{u_1} + \frac{1}{u_2}$

(D) $u_3^2 = u_1^2 + u_2^2$

Q17: NTA Test 46 (Single Choice)

An artillery piece which consistently shoots its shells with the same muzzle speed has a maximum range R. To hit a target which is $\frac{R}{2}$ from the gun and on the same level, the angle of elevation of the gun should be

(A) 15°

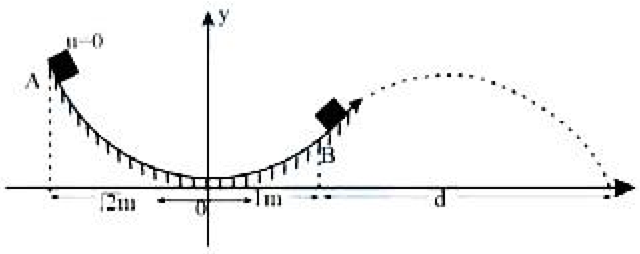
(B) 45°

(C) 30°

(D) 60°

Q18: NTA Test 47 (Numerical)

A small body is released from point A of smooth parabolic path $y = x^2$, where y is vertical axis and x is horizontal axis at ground, as shown. The body leaves the surface from point B. If $g = 10\ \mathrm{m\ s^{-2}}$ then what is the value of d (in m)?



Q19: NTA Test 48 (Numerical)

A particle is projected towards the north with speed 20 $^{m}/_{s}$ at an angle 45^{o} with horizontal. Ball gets horizontal acceleration of 7.5 $^{m}/_{s^2}$ towards east due to wind. Range of ball (in meter) minus 42 m will be

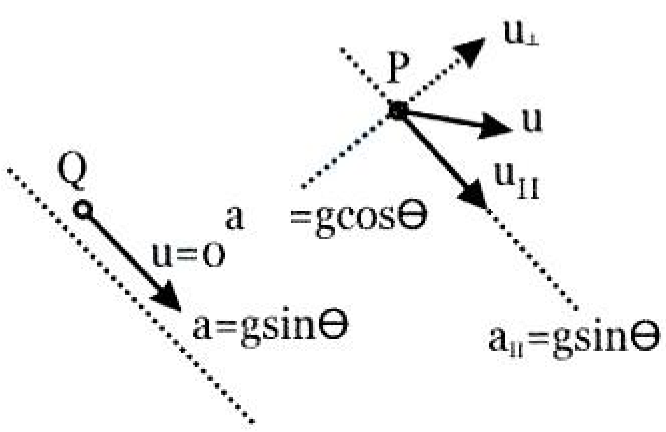
Answer Keys

Q1: 10	**Q2:** (B)	**Q3:** 10
Q4: (D)	**Q5:** (A)	**Q6:** (A)
Q7: 45	**Q8:** (D)	**Q9:** 6
Q10: 3	**Q11:** (A)	**Q12:** 60
Q13: (A)	**Q14:** (C)	**Q15:** 7
Q16: (B)	**Q17:** (A)	**Q18:** 2
Q19: 8		

Solutions

Q1: 10

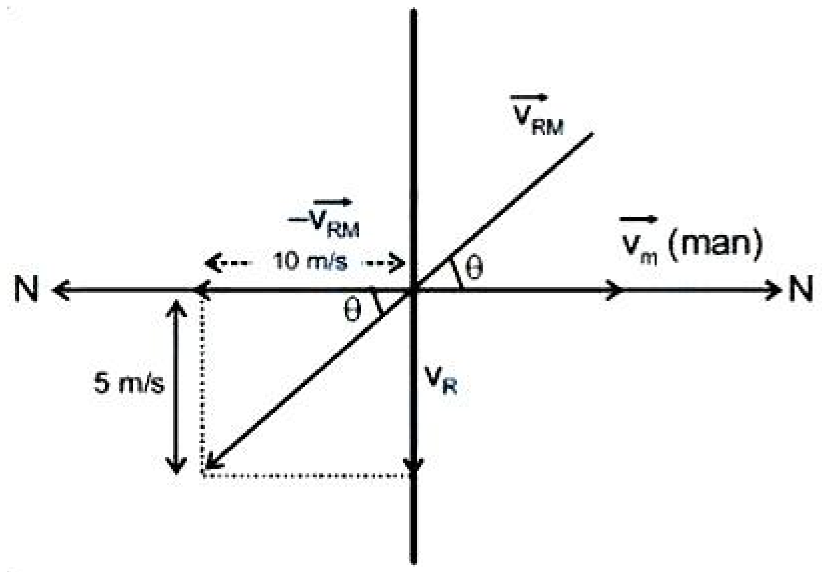
It can be observed from figure that P and Q shall collide if the initial component of velocity of P and Q on inclined plane i.e along incline should be equal. That is particle is projected perpendicular to incline.



$\therefore$ Time of flight $T = \frac{2u_{\perp}}{g\cos\theta} = \frac{2u}{g\cos\theta}$

$\therefore u = \frac{gT\cos\theta}{2} = 10\ \mathrm{m/s}$

Q2: (B) $\tan^{-1}\left(\frac{1}{2}\right)$ **above north horizon**



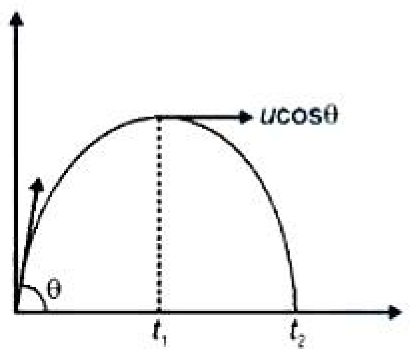
$\tan\theta = \frac{v_R}{v_M}$
$\tan\theta = \frac{5}{10}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$ above north horizon.

Q3: 10

$\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 9\hat{k}$
At $t = 0, \vec{r_1} = 9\hat{k}$
At $t = 2s, \vec{r_2} = 8\hat{i} + 6\hat{j} + 9\hat{k}$
$\therefore \quad \Delta\vec{r} = \vec{r_2} - \vec{r_1} = 8\hat{i} + 6\hat{j}$
$\therefore \left|\Delta\vec{r}\right| = \left|\vec{r_2} - \vec{r_1}\right| = \sqrt{(8)^2 + (6)^2} = 10\text{m}$

Q4: (D) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\frac{u\sin\theta}{g}$



Time of ascent

$t_1 = \frac{u\sin\theta}{g}$

Time of descent

$t_2 = \sqrt{\frac{2H}{2g}}$

$t_2 = \sqrt{\frac{u^2\sin^2\theta}{2g^2}} = \frac{u\sin\theta}{\sqrt{2}g}$

$t = t_1 + t_2 = \frac{u\sin\theta}{g} + \frac{u\sin\theta}{\sqrt{2}g}$

$\Rightarrow \frac{u\sin\theta}{g}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$

Q5: (A) $\frac{28}{9}$ m

Initial velocities of the particle

$u_x = 10\cos 53 = 6\ \text{m/s}$

$u_y = 10\sin 53 = 8\ \text{m/s}$

$6t = 1 + t + x.\frac{3}{5}$ [where x is the length of the hypotenuse of the wedge]

$5t = 1 + \frac{3x}{5}$ (in x - direction)

$8t - 5t^2 = 4.\frac{x}{5}$ (in y - direction)

$x = \frac{35}{9}\text{m}$

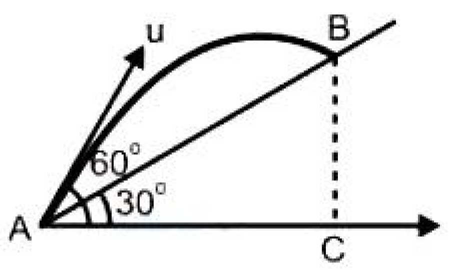
$\text{Height} = \frac{35}{9} \times \frac{4}{5} = \frac{28}{9}\text{m}$

Q6: (A) $\frac{ut}{\sqrt{3}}$

Horizontal component of velocity

$$u_H = u\cos 60^\circ = \frac{u}{2}$$

$$\therefore \quad AC = (u_H)t = \frac{ut}{2}$$



and $AB = AC\ \sec\ 30^\circ$

$$= \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = \frac{ut}{\sqrt{3}}$$

Q7: 45

Given, height of the hill (h) = 500 m

u = 125 m/s

To cross the hill, the vertical component of the velocity should be sufficient to cross such height.

$$\therefore \quad u_y \geq \sqrt{2gh}$$

$$\geq \sqrt{2 \times 10 \times 500}$$

$$\geq 100\ \text{m/s}$$

But $u^2 = u_x^2 + u_y^2$

$\therefore$ Horizontal component of initial velocity

$$u_x = \sqrt{u^2 - u_y^2}$$

$$= \sqrt{(125)^2 - (100)^2}$$

$$= 75 \text{ m/s}$$

Time taken to reach the top of the hill

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 500}{10}} = 10 \text{ s}$$

Time taken to reach the ground from the top of the hill t' = t = 10 s

Horizontal distance travelled in 10 s

$$x = u_x \times t$$

$$= 75 \times 10$$

$$= 750 \text{ m}$$

$\therefore$ Distance through which canon has to be moved

$$= 800 - 750$$

$$= 50 \text{ m}$$

Speed with which canon can move = 2 m/s

$\therefore$ Time taken by canon $= \frac{50}{2}$

$$t'' = 25 \text{ s}$$

$\therefore$ Total time taken by a packet to reach on the ground

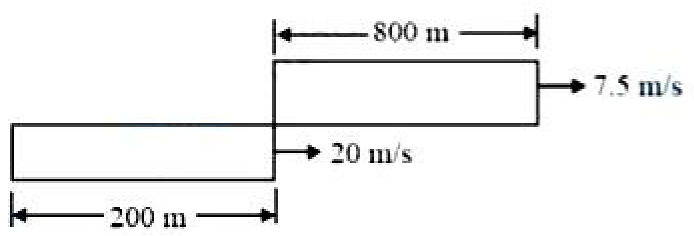
$$= t'' + t + t'$$

$$= 25 + 10 + 10$$

$$= 45 \text{ s} .$$

Q8: (D) 80 s

From the figure, the relative displacement is



$s_{rel} = (800 + 200)\text{m} = 1000 \text{ m}$

$v_{rel} = v_1 - v_2 = (20 - 7.5) \text{ m s}^{-1}$

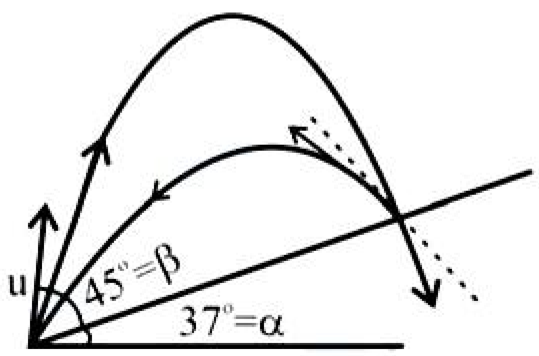
$= 12.5 \text{ m s}^{-1}$

$\therefore t = \frac{s_{rel}}{v_{rel}} = \frac{1000}{12.5} = 80 \text{ s}$

Q9: 6

Let angle made by $\overrightarrow{v}$ initially and after time t be θ and α respectively.
$\tan 30^o = \frac{u \sin \theta - g \times 2}{u \cos \theta}$..........(i)
$\tan 0^o = \frac{u \sin \theta - g \times 3}{u \cos \theta}$ ....(ii)
$\therefore \theta = 60^o$

Q10: 3



While going upward the time of flight is

$T = \frac{2u\sin\alpha}{g\cos\beta}$

So the range on the inclined plane is

$R = (u\cos\alpha)T - \frac{1}{2}(g\sin\beta)T^2$

Just after the collision of the projectile with the inclined plane, the components of its velocity down the inclined plane and perpendicular to the inclined plane are

$v_{\|} = (g\sin\beta)T - u\cos\alpha, v_{\perp} = eu\sin\alpha$

The time of flight while going down the inclined plane is

$T' = \frac{2eu\sin\alpha}{g\cos\beta} = eT$

The range of the projectile while going down is

$R = [(g\sin\beta)T - u\cos\alpha]eT + \frac{1}{2}(g\sin\beta)e^2T^2$

$\Rightarrow [(g\sin\beta)T - u\cos\alpha]eT + \frac{1}{2}(g\sin\beta)e^2T^2 = (u\cos\alpha)T - \frac{1}{2}(g\sin\beta)T^2$

$\Rightarrow u\cos\alpha\,(1 + e) = (g\sin\beta)T\left[\frac{1}{2} + e + \frac{e^2}{2}\right]$

Substituting the value as $T = \frac{2u\sin\alpha}{g\cos\beta}$ we get

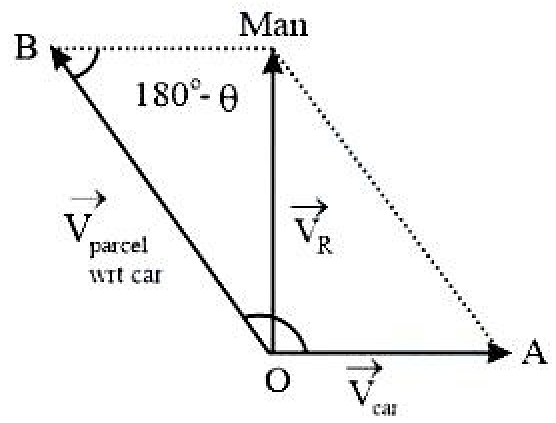
$\Rightarrow \cot\alpha\,(1 + e) = 2\tan\beta\left[\frac{1}{2} + e + \frac{e^2}{2}\right]$

$\Rightarrow 3e^2 + 2e - 1 = 0$

$\Rightarrow e = \frac{1}{3}$ or $\frac{1}{e} = 3$

Q11: (A) 135°

In the vector diagram, $\overline{OA}$ and $\overline{OB}$ represent the velocity vectors for the car and the parcel, respectively. $\overrightarrow{V}_R$ is the resultant velocity vector. Let θ be the angle between $\overrightarrow{V}_{car}$ and $\overrightarrow{V}_{Parcel}$. Then,

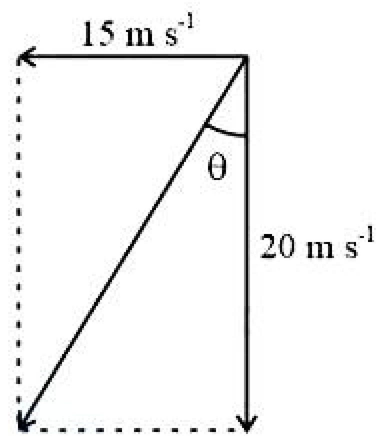


$\cos(180^\circ - \theta) = \frac{\left|\vec{V}_{car}\right|}{\left|\vec{V}_{parcel}\right|} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}}$

$180^\circ - \theta = 45^\circ$

$\therefore \theta = 135^\circ$

Q12: 60



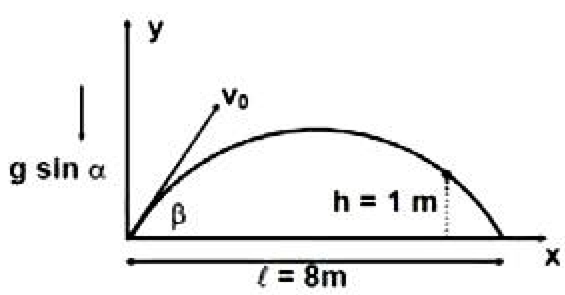
$\tan\left(90^\circ - \theta\right) = \frac{20}{15}$

$\therefore \cot\theta = \frac{20}{15} = \frac{4}{3}$

$\Rightarrow \theta = 37^\circ$

$\therefore \phi = 37^\circ + 23^\circ = 60^\circ$ (Angle made with the windscreen)

Q13: (A) 9



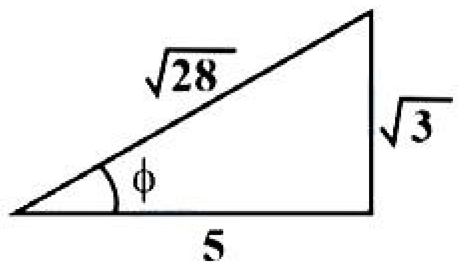
$h = l\tan\beta - \frac{1}{2}\frac{g\sin\alpha l^2}{v_0^2\cos^2\beta}$

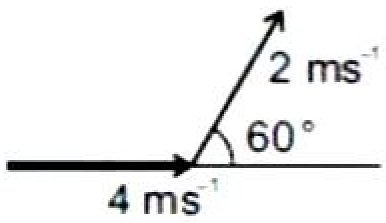
$v_0 = \sqrt{\frac{g\sin\alpha l^2}{2\cos^2\beta(l\tan\beta - h)}}$

$= \sqrt{\frac{10\times\frac{4}{5}\times8\times8}{2\times\frac{4}{5}\times\frac{4}{5}\left(8\times\frac{3}{4}-1\right)}} = 9\text{ m s}^{-1}$

Q14: (C) $\sin^{-1}\left(\sqrt{\frac{3}{28}}\right)$

$\vec{v}_b = \vec{v}_{bT} + \vec{v}_T$





$\tan\phi = \frac{2\sin 60^\circ}{4+2\cos 60^\circ} = \frac{\sqrt{3}}{5}$

$\sin\phi = \sqrt{\frac{3}{28}}$

$\phi = \sin^{-1}\left(\sqrt{\frac{3}{28}}\right)$

Q15: 7

By conservation of momentum,

$3mu\hat{i} = m\left(u\cos 37^o\hat{i} + u\sin 37^o\hat{j}\right) + 2m\overrightarrow{V_A}$

$= \overrightarrow{V_A} = \frac{11u}{10}\hat{i} - \frac{3u}{10}\hat{j}$

Similarly $= \overrightarrow{V_B} = \frac{4u}{15}\hat{i} + \frac{3u}{10}\hat{j}$

$\therefore |V_{BA}| = 17 \text{ m s}^{-1}$

Q16: (B) $u_3^2 = 2u_1u_2$

For a given speed,

the maximum range of a projectile on a horizontal level is

$R_{\text{max}} = \frac{u^2}{g}$

the maximum range up and down the incline are

$(R_{\text{up}})_{\text{max}} = \frac{u^2}{g(1+\sin\alpha)}$

$(R_{\text{down}})_{\text{max}} = \frac{u^2}{g(1-\sin\alpha)}$

by substituting the values of the velocities given in the problem, we get

$R = \frac{u_2^2}{g(1+\sin\alpha)} \Rightarrow u_2^2 = Rg\,(1+\sin\alpha)\ \ldots(1)$

$2R\cos\alpha = \frac{u_3^2}{g} \Rightarrow u_3^2 = 2Rg\cos\alpha\ \ldots(2)$

Now, $u_1u_2 = Rg\sqrt{1-\sin^2\alpha} = Rg\cos\alpha = \frac{u_3^2}{2}$

$\therefore u_3^2 = 2u_1u_2$

Q17: (A) 15°

$\text{R}_{\text{max}} = \text{R} = \frac{\text{u}^2}{\text{g}}$ (at an angle of 45°

$\text{u}^2 = \text{Rg}$

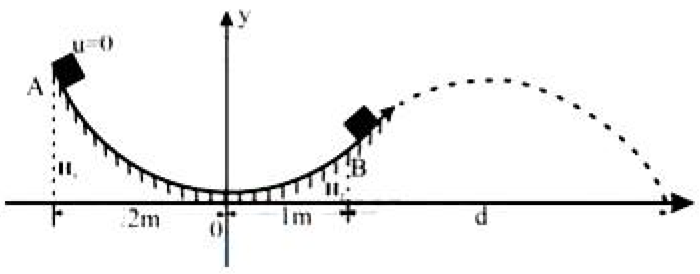
Using Range $= \frac{u^2 \sin 2\theta}{g}$

then $\frac{R}{2} = (Rg)\frac{\sin 2\theta}{g}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = 30^{\circ} \Rightarrow \theta = 15^{\circ}$

Q18: 2



$H_1 = (\sqrt{2})^2 = 2\ \mathrm{m}$
$H_2 = (1)^2 = 1\ \mathrm{m}$
$\therefore v = \sqrt{2 \times g \times (H_2 - H_1)}$
$v = \sqrt{20}\ \mathrm{m\ s^{-1}}$
$\tan\theta = \frac{dy}{dx}\Big|_{x=1} = 2x|_{x=1} = 2$
$-H_2 = d\tan\theta - \frac{g d^2}{2u^2\cos^2\theta}$
$-1 = 2d - \frac{10d^2}{2(\sqrt{20})^2\left({}^{1}/_{\sqrt{5}}\right)^2}$

$\frac{5}{4}d^2 - 2d - 1 = 0$

On solving we get $d = 2\ \mathrm{m}$

Q19: 8

Time of flight : $T = \frac{2u\sin\theta}{g} = 2\sqrt{2}\ \mathrm{s}$
Range (along north) $= \frac{u^2\sin 2\theta}{g} = 40\ \mathrm{m}$
Range (along east) $= \frac{1}{2}aT^2 = 30\ \mathrm{m}$
$\therefore$ Range $= \sqrt{30^2 + 40^2} = 50\ \mathrm{m}$