CHAPTER 7

Straight Lines and Pair of Straight Lines

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The area enclosed within the curve $|x|+|y|=1$ is .... *(1981 - 2 Marks)*
2. $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is .... *(1982 - 2 Marks)*
3. The set of lines $ax+by+c = 0$, where $3a + 2b + 4c = 0$ is concurrent at the point .... *(1982 - 2 Marks)*
4. Given the points A (0, 4) and B (0, –4), the equation of the locus of the point $P(x, y)$ such that $|AP-BP|=6$ is .... *(1983 - 1 Mark)*
5. If a, b and c are in A.P., then the straight line $ax+by+c=0$ will always pass through a fixed point whose coordinates are .... *(1984 - 2 Marks)*
6. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in quadrant number .... *(1985 - 2 Marks)*
7. Let the algebraic sum of the perpendicular distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero; then the line passes through a fixed point whose cordinates are .... *(1991 - 2 Marks)*
8. The vertices of a triangle are A (–1, –7), B (5, 1) and C (1, 4). The equation of the bisector of the angle $\angle ABC$ is .... *(1993 - 2 Marks)*

B True / False

1. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x+y+5=0$. *(1983 - 1 Mark)*
2. The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes in concyclic points. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. The points $(-a, -b)$, (0, 0), (a, b) and (a^2, ab) are : *(1979)*
   (a) Collinear
   (b) Vertices of a parallelogram
   (c) Vertices of a rectangle
   (d) None of these
2. The point (4, 1) undergoes the following three transformations successively. *(1980)*
   (i) Reflection about the line $y=x$.
   (ii) Translation through a distance 2 units along the positive direction of x-axis.
   (iii) Rotation through an angle p/4 about the origin in the counter clockwise direction.

   Then the final position of the point is given by the coordinates.
   (a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-\sqrt{2}, 7\sqrt{2})$
   (c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$
3. The straight lines $x+y=0$, $3x+y-4=0$, $x+3y-4=0$ form a triangle which is *(1983 - 1 Mark)*
   (a) isosceles (b) equilateral
   (c) right angled (d) none of these
4. If $P=(1, 0)$, $Q=(-1, 0)$ and $R=(2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2+SR^2=2SP^2$, is *(1988 - 2 Marks)*
   (a) a straight line parallel to x-axis
   (b) a circle passing through the origin
   (c) a circle with the centre at the origin
   (d) a straigth line parallel to y-axis.
5. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then *(1990 - 2 Marks)*
   (a) $a^2+b^2=p^2+q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$
   (c) $a^2+p^2=b^2+q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$
6. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is *(1992 - 2 Marks)*
   (a) square (b) circle
   (c) straight line (d) two intersecting lines
7. The locus of a variable point whose distance from (–2, 0) is 2/3 times its distance from the line $x=-\frac{9}{2}$ is *(1994)*
   (a) ellipse (b) parabola
   (c) hyperbola (d) none of these
8. The equations to a pair of opposite sides of parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$, the equations to its diagonals are *(1994)*
   (a) $x+4y=13, y=4x-7$ (b) $4x+y=13, 4y=x-7$
   (c) $4x+y=13, y=4x-7$ (d) $y-4x=13, y+4x=7$
9. The orthocentre of the triangle formed by the lines $xy = 0$ and $x+y=1$ is *(1995S)*
   (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (c) (0, 0) (d) $\left(\frac{1}{4}, \frac{1}{4}\right)$

10. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is *(1999 - 2 Marks)*

(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
(d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

11. If x_1, x_2, x_3 as well as y_1, y_2, y_3, are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). *(1999 - 2 Marks)*

(a) lie on a straight line (b) lie on an ellipse
(c) lie on a circle (d) are vertices of a triangle

12. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is *(2000S)*

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

13. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is *(2000S)*

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

14. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is *(2001S)*

(a) 2 (b) 0 (c) 4 (d) 1

15. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals *(2001S)*

(a) $|m + n|/(m - n)^2$ (b) $2/|m + n|$
(c) $1/(|m + n|)$ (d) $1/(|m - n|)$

16. Let $0 < \alpha < \frac{\pi}{2}$ be fixed angle. If

$P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha - \theta), \sin(\alpha - \theta))$,

then Q is obtained from P by *(2002S)*

(a) clockwise rotation around origin through an angle α
(b) anticlockwise rotation around origin through an angle α
(c) reflection in the line through origin with slope $\tan\alpha$
(d) reflection in the line through origin with slope $\tan(\alpha/2)$

17. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is *(2002S)*

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

18. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segemnt PQ in the ratio *(2002S)*

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

19. The number of intergral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0,0), (0,21) and (21,0), is *(2003S)*

(a) 133 (b) 190 (c) 233 (d) 105

20. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is *(2003S)*

(a) $\left(3, \frac{5}{4}\right)$ (b) (3, 12) (c) $\left(3, \frac{3}{4}\right)$ (d) (3, 9)

21. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is *(2004S)*

(a) 2 sq. units (b) 4 sq. units
(c) 6 sq. units (d) 8 sq. units

22. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangles OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are *(2007 -3 marks)*

(a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$ (c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

23. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is *(2011)*

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

D MCQs with One or More than One Correct

1. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent if *(1985 - 2 Marks)*

(a) $p + q + r = 0$
(b) $p^2 + q^2 + r^2 = qr + rp + pq$
(c) $p^3 + q^3 + r^3 = 3pqr$
(d) none of these.

2. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of

(a) an obtuse angled triangle *(1986 - 2 Marks)*
(b) an acute angled triangle
(c) a right angled triangle
(d) an isosceles triangle
(e) none of these.

3. All points lying inside the triangle formed by the points (1, 3), (5, 0) and (–1, 2) satisfy *(1986 - 2 Marks)*

(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \geq 0$
(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$
(e) none of these.

4. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components $p + 1$ and 1, then *(1986 - 2 Marks)*

(a) $p = 0$ (b) $p = 1$ or $p = -\frac{1}{3}$

(c) $p=-1$ or $p=\frac{1}{3}$ (d) $p=1$ or $p=-1$

(e) none of these .

5. If $(P(1, 2), Q(4, 6), R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then *(1998 - 2 Marks)*
(a) $a=2, b=4$ (b) $a=3, b=4$
(c) $a=2, b=3$ (d) $a=3, b=5$

6. The diagonals of a parallelogram $PQRS$ are along the lines $x+3y=4$ and $6x-2y=7$. Then $PQRS$ must be a. *(1998 - 2 Marks)*
(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus.

7. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational point(s)? *(1998 - 2 Marks)*
(a) centroid (b) incentre
(c) circumcentre (d) orthocentre
(A rational point is a point both of whose co-ordinates are rational numbers.)

8. Let L_1 be a strainght line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1? *(1999 - 3 Marks)*
(a) $x+y=0$ (b) $x-y=0$
(c) $x+7y=0$ (d) $x-7y=0$

9. For $a>b>c>0$, the distance between (1, 1) and the point of intersection of the lines $ax+by+c=0$ and $bx+ay+c=0$ is less than $2\sqrt{2}$. Then *(JEE Adv. 2013)*
(a) $a+b-c>0$ (b) $a-b+c<0$
(c) $a-b+c>0$ (d) $a+b-c<0$

E Subjective Problems

1. A straight line segment of length ℓ moves with its ends on two mutually perpendicular lines. Find the locus of the point which divides the line segment in the ratio 1 : 2. *(1978)*

2. The area of a triangle is 5. Two of its vertices are A (2, 1) and B (3, –2). The third vertex C lies on $y=x+3$. Find C. *(1978)*

3. One side of a rectangle lies along the line $4x+7y+5=0$. Two of its vertices are (–3, 1) and (1, 1). Find the equations of the other three sides. *(1978)*

4. (a) Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocentre of the triangle is the origin, find the coordinates of the third point.
(b) Find the equation of the line which bisects the obtuse angle between the lines $x-2y+4=0$ and $4x-3y+2=0$. *(1979)*

5. A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L. *(1980)*

6. The end A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ *(1983 - 2 Marks)*

7. The vertices of a triangle are $[at_1t_2, a(t_1+t_2)]$, $[at_2t_3, a(t_2+t_3)]$, $[at_3t_1, a(t_3+t_1)]$. Find the orthocentre of the triangle. *(1983 - 3 Marks)*

8. The coordinates of A, B, C are (6, 3), (–3, 5), (4, – 2) respectively, and P is any point (x, y). Show that the ratio of the area of the triangles ΔPBC and ΔABC is $\left|\frac{x+y-2}{7}\right|$ *(1983 - 2 Marks)*

9. Two equal sides of an isosceles triangle are given by the equations $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point (1, –10). Determine the equation of the third side. *(1984 - 4 Marks)*

10. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. *(1985 - 3 Marks)*

11. Two sides of a rhombus $ABCD$ are parallel to the lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find possible co-ordinates of A. *(1985 - 5 Marks)*

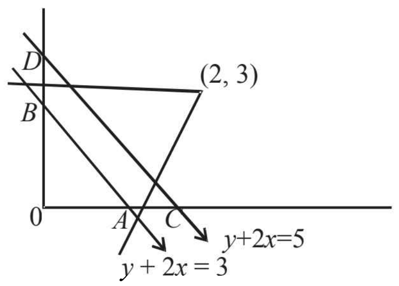
12. Lines $L_1 \equiv ax+by+c=0$ and $L_2 \equiv lx+my+n=0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. *(1988 - 5 Marks)*

13. Let ABC be a triangle with $AB=AC$. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE. *(1989 - 5 Marks)*

14. Straight lines $3x+4y=5$ and $4x-3y=15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB=AC$. Determine the possible equations of the line BC passing through the point (1, 2). *(1990 - 4 Marks)*

15. A line cuts the x-axis at A (7, 0) and the y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. *(1990 - 4 Marks)*

16. Find the equation of the line passing through the point (2, 3) and making intercept of length 2 units between the lines $y+2x=3$ and $y+2x=5$. *(1991 - 4 Marks)*



17. Show that all chords of the curve $3x^2-y^2-2x+4y=0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. *(1991 - 4 Marks)*

18. Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines

$2x + 3y - 1 = 0$ *(1992 - 6 Marks)*

$x + 2y - 3 = 0$

$5x - 6y - 1 = 0$

19. Tagent at a point P_1 {other than (0, 0)} on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3, and so on. Show that the abscissae of $P_1, P_2, P_3 ....P_n$, form a G.P. Also find the ratio.

[area $(\Delta P_1, P_2, P_3)$]/[area$(P_2 P_3, P_4)$] *(1993 - 5 Marks)*

20. A line through A (–5, –4) meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at the points B, C and D respectively. If $(15/AB)^2 + (10/AC)^2 = (6/AD)^2$, find the equation of the line. *(1993 - 5 Marks)*

21. A rectangle $PQRS$ has its side PQ parallel to the line $y = mx$ and vertices P, Q and S on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R. *(1996 - 2 Marks)*

22. Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent. *(1998 - 8 Marks)*

23. For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram.

(2000 - 10 Marks)

24. Let ABC and PQR be any two triangles in the same plane. Assume that the prependiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the prependiculars from P, Q, R to BC, CA, AB respectively are also concurrent.

(2000 - 10 Marks)

25. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that

the equation $\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$

represents a straight line. *(2001 - 6 Marks)*

26. A straight line L through the origin meets the lines $x + y = 1$ and $x + y = 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line.

(2002 - 5 Marks)

27. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of $OP + OQ$, as L varies, where O is the origin. *(2002 - 5 Marks)*

28. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P.

(2005 - 2 Marks)

H Assertion & Reason Type Questions

1. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT-1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles. *(2007 - 3 marks)*

(a) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

I Integer Value Correct Type

1. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is *(JEE Adv. 2014)*

Section-B JEE Main / AIEEE

1. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **[2002]**

(a) isosceles and right angled

(b) isosceles but not right angled

(c) right angled but not isosceles

(d) neither right angled nor isoceles

2. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ whre p is constant is **[2002]**

(a) $x^2 + y^2 = \dfrac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$

(c) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{2}{p^2}$ (d) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$

3. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect on the y-axis then **[2002]**

(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \ne ch^2$

(c) $abc = 2fgh$ (d) none of these

4. The pair of lines represented by

$3ax^2 + 5xy + (a^2 - 2)y^2 = 0$

are perpendicular to each other for **[2002]**

(a) two values of a (b) $\forall\, a$

(c) for one value of a (d) for no values of a

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **[2003]**
(a) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$
(b) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$
(c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$
(d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$.

6. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then **[2003]**
(a) $pq=-1$ (b) $p=q$ (c) $p=-q$ (d) $pq=1$.

7. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0), where t is a parameter, is **[2003]**
(a) $(3x+1)^2+(3y)^2=a^2-b^2$
(b) $(3x-1)^2+(3y)^2=a^2-b^2$
(c) $(3x-1)^2+(3y)^2=a^2+b^2$
(d) $(3x+1)^2+(3y)^2=a^2+b^2$.

8. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) **[2003]**
(a) are vertices of a triangle
(b) lie on a straight line
(c) lie on an ellipse
(d) lie on a circle.

9. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1-b_2)x+(a_1-b_2)y+c=0$, then the value of `c` is **[2003]**
(a) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$
(b) $\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$
(c) $a_1^2-a_2^2+b_1^2-b_2^2$
(d) $\frac{1}{2}(a_1^2+a_2^2+b_1^2+b_2^2)$.

10. Let $A(2,-3)$ and $B(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x+3y=1$, then the locus of the vertex C is the line **[2004]**
(a) $3x-2y=3$ (b) $2x-3y=7$
(c) $3x+2y=5$ (d) $2x+3y=9$

11. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is **[2004]**
(a) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$
(b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(c) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$
(d) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

12. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product c has the value **[2004]**
(a) –2 (b) –1 (c) 2 (d) 1

13. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals **[2004]**
(a) –3 (b) –1 (c) 3 (d) 1

14. The line parallel to the x- axis and passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$, where $(a, b)\neq(0, 0)$ is **[2005]**
(a) below the x - axis at a distance of $\frac{3}{2}$ from it
(b) below the x - axis at a distance of $\frac{2}{3}$ from it
(c) above the x - axis at a distance of $\frac{3}{2}$ from it
(d) above the x - axis at a distance of $\frac{2}{3}$ from it

15. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is **[2005]**
(a) $\left(-1,\frac{7}{3}\right)$ (b) $\left(\frac{-1}{3},\frac{7}{3}\right)$
(c) $\left(1,\frac{7}{3}\right)$ (d) $\left(\frac{1}{3},\frac{7}{3}\right)$

16. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[2006]**
(a) $x+y=7$ (b) $3x-4y+7=0$
(c) $4x+3y=24$ (d) $3x+4y=25$

17. If (a, a^2) falls inside the angle made by the lines $y=\frac{x}{2}$, $x>0$ and $y=3x$, $x>0$, then a belong to **[2006]**

(a) $\left(0,\frac{1}{2}\right)$ (b) $(3,\infty)$

(c) $\left(\frac{1}{2},3\right)$ (d) $\left(-3,-\frac{1}{2}\right)$

18. Let A (h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1square unit, then the set of values which 'k' can take is given by **[2007]**

(a) $\{-1,3\}$ (b) $\{-3,-2\}$ (c) $\{1,3\}$ (d) $\{0,2\}$

19. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[2007]**

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3}y=0$

(c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$.

20. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the lines xy = 0, then m is **[2007]**

(a) 1 (b) 2 (c) –1/2 (d) –2

21. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is **[2008]**

(a) 1 (b) 2 (c) –2 (d) – 4

22. The shortest distance between the line y – x = 1 and the curve $x = y^2$ is : **[2009]**

(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

23. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for : **[2009]**

(a) exactly one values of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

24. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: **[2009]**

(a) $\left(\frac{5}{4},0\right)$ (b) $\left(\frac{5}{2},0\right)$ (c) $\left(\frac{5}{3},0\right)$ (d) $(0,0)$

25. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is **[2010]**

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$ (c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

26. The lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2}:\sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

27. If the line $2x+y=k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : **[2012]**

(a) $\frac{29}{5}$ (b) 5 (c) 6 (d) $\frac{11}{5}$

28. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[JEE M 2013]**

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$

(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

29. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[JEE M 2013]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

30. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6,-1)$ and $R(7, 3)$. The equation of the line passing through (1, –1) and parallel to PS is: **[JEE M 2014]**

(a) $4x+7y+3=0$ (b) $2x-9y-11=0$
(c) $4x-7y-11=0$ (d) $2x+9y+7=0$

31. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then **[JEE M 2014]**

(a) $3bc-2ad=0$ (b) $3bc+2ad=0$
(c) $2bc-3ad=0$ (d) $2bc+3ad=0$

32. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : **[JEE M 2015]**

(a) 820 (b) 780 (c) 901 (d) 861

33. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? **[JEE M 2016]**

(a) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (b) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

(c) (–3, –9) (d) (–3, –8)

7 Straight Lines and Pair of Straight Lines

Section-A : JEE Advanced/ IIT-JEE

A
1. 2 sq. units **2.** $y = x$ **3.** $\left(\frac{3}{4}, \frac{1}{2}\right)$ **4.** $\frac{y^2}{9} - \frac{x^2}{7} = 1$ **5.** $(1, -2)$ **6.** first quadrant
7. (1, 1) **8.** $x - 7y + 2 = 0$

B
1. T **2.** T

C
1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (b) **6.** (a)
7. (a) **8.** (c) **9.** (c) **10.** (b) **11.** (a) **12.** (d)
13. (d) **14.** (a) **15.** (d) **16.** (d) **17.** (c) **18.** (b)
19. (b) **20.** (c) **21.** (a) **22.** (c) **23.** (b)

D
1. (a, b, c) **2.** (e) **3.** (a, c) **4.** (b) **5.** (c) **6.** (d)
7. (a,c, d) **8.** (b, c) **9.** (a)

E
1. $9x^2 + 36y^2 = 4\ell^2$
2. $\left(\frac{-3}{2}, \frac{3}{2}\right)$ or $\left(\frac{7}{2}, \frac{13}{2}\right)$
3. $4x + 7y - 11 = 0,\ 7x - 4y - 3 = 0;\ 7x - 4y + 25 = 0$
4. **(a)** $(-4, -7)$ **(b)** $(4 - \sqrt{5})x + (2\sqrt{5} - 3)y - (4\sqrt{5} - 2) = 0$
5. $x + 5y - 5\sqrt{2} = 0$ or $x + 5y + 5\sqrt{2} = 0$
7. $(-a,\ a(t_1 + t_2 + t_3) + at_1t_2t_3)$
9. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$
10. 32 sq. units
11. (0, 0) or (0, 5/2)
12. $(a^2 + b^2)(lx + my + n) - 2(al + bm)(ax + by + c) = 0$
14. $x - 7y + 13 = 0$ or $7x + y - 9 = 0$
15. $x^2 + y^2 - 7x + 5y = 0$
16. $3x + 4y - 18 = 0$ or $x - 2 = 0$
17. $(1, -2)$
18. $\alpha \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{1}{2}, 1\right)$
19. $\frac{1}{64}$ sq units
20. $2x + 3y + 22 = 0$
21. $x(m^2 - 1) - ym + (m^2 + 1)b + am = 0$
27. 18
28. $y = 2x + 1$ or $y = -2x + 1$

H
1. (c)

I
1. 6

Section-B : JEE Main/ AIEEE

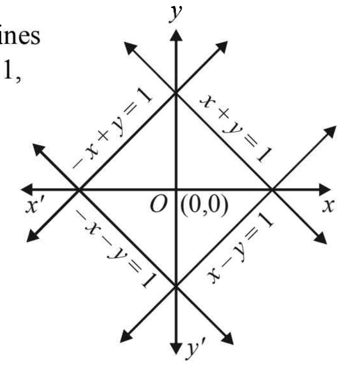
1. (a) **2.** (d) **3.** (a) **4.** (a) **5.** (a) **6.** (a) **7.** (c) **8.** (b) **9.** (b) **10.** (d) **11.** (a) **12.** (c)
13. (a) **14.** (a) **15.** (c) **16.** (c) **17.** (c) **18.** (a) **19.** (c) **20.** (a) **21.** (d) **22.** (d) **23.** (a) **24.** (a)
25. (c) **26.** (b) **27.** (c) **28.** (b) **29.** (b) **30.** (d) **31.** (a) **32.** (b) **33.** (a)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $|x|+|y|=1$

The curve represents four lines $x+y=1, x-y=1, -x+y=1, -x-y=1$ which enclose a square of side = distance between opp. sides $x+y=1$ and $x+y=-1$



Side $=\dfrac{1+1}{\sqrt{1+1}}=\sqrt{2}$

$\therefore$ Req. area = (side)^2 = 2 sq. units.

2. As $y=\log_{10}x$ can be obtained by replacing x by y and y by x in $y=10^x$

$\therefore$ The line of reflection is $y=x$.

3. Given that $3a+2b+4c=0 \Rightarrow \dfrac{3}{4}a+\dfrac{1}{2}b+c=0$

$\Rightarrow$ The set of lines $ax+by+c=0$ passes through the point (3/4, 1/2).

4. $|AP-BP|=6$

We know that locus of a point, difference of whose distances from two fixed points is constant, is hyperbola with the fixed points as focii and the difference of distances as length of transverse axis.

Thus, $ae=4$ and $2a=6 \Rightarrow a=3, e=4/3$

$\Rightarrow b^2=9\left(\dfrac{16}{9}-1\right)=7 \quad \therefore$ Equation is $\dfrac{y^2}{9}-\dfrac{x^2}{7}=1$

(foci being on y-axis, it is vertical hyperbola)

5. If a, b, c are in A.P. then

$a+c=2b \Rightarrow a-2b+c=0$

$\Rightarrow ax+by+c=0$ passes through (1,– 2).

6. **First quadrant.**

The equations of sides of triangle ABC are

$AB \;: x+y=1$

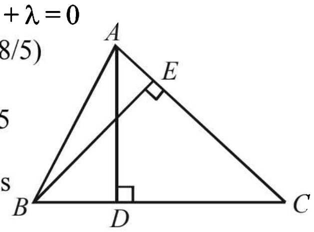
$BC \;: 2x+3y=6$

$CA \;: 4x-y=-4$

Solving these pairwise we get the vertices of Δ as follows $A(-3/5, 8/5)$ $B(-3, 4)$ $C(-3/7, 16/7)$

Now AD is line $\perp^{lar}$ to BC and passes through A. Any line perpendicular to BC is $3x-2y+\lambda=0$

As it passes through $A(-3/5, 8/5)$



$\therefore \quad \dfrac{-9}{5}-\dfrac{16}{5}+\lambda=0 \Rightarrow \lambda=5$

$\therefore$ Equation of altitude AD is $3x-2y+5=0$ ...(1)

Any line perpendicular to side AC is $x+4y+\mu=0$

As it passes through point B (– 3, 4)

$\therefore \quad -3+16+\mu=0 \Rightarrow \mu=-13$

$\therefore$ Equation of altitude BE is $x+4y-13=0$ ...(2)

Now orthocentre is the point of intersection of equations (1) and (2) (AD and BE)

Solving (1) and (2), we get $x=3/7, y=22/7$

As both the co-ordinates are positive, orthocentre lies in first quadrant.

7. Let the variable line be $ax+by+c=0$ .... (1)

Then $\perp^{lar}$ distance of line from $(0, 2)=\dfrac{2a+c}{\sqrt{a^2+b^2}}=p_1$

$\perp^{lar}$ distance of line from $(0, 2)=\dfrac{2b+c}{\sqrt{a^2+b^2}}=p_2$

$\perp^{lar}$ distance of line from $(1, 1)=\dfrac{a+b+c}{\sqrt{a^2+b^2}}=p_3$

ATQ $p_1+p_2+p_3=0$

$\Rightarrow \dfrac{2a+c+2b+c+a+b+c}{\sqrt{a^2+b^2}}=0$

$\Rightarrow 3a+3b+3c=0$

$\Rightarrow a+b+c=0$ .... (2)

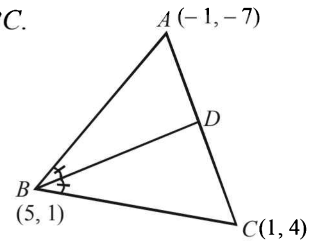
From (1) and (2), we can say variable line (1) passes through the fixed point (1, 1).

8. Let BD be the bisector of $\angle ABC$.

NOTE THIS STEP:

Then $AD : DC = AB : BC$

And



$AB=\sqrt{(5+1)^2+(1+7)^2}=10$

$BC=\sqrt{(5-1)^2+(1-4)^2}=5$

$\therefore \quad AD : DC = 2 : 1$

$\therefore$ By section formula $D\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$

Therefore equation of BD is

$y-1=\dfrac{1/3-1}{1/3-5}(x-5) \Rightarrow y-1=\dfrac{-2/3}{-14/3}(x-5)$

$\Rightarrow 7y-7=x-5 \Rightarrow x-7y+2=0$

B. True / False

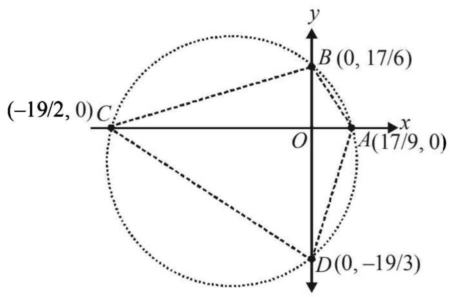
1. Intersection point of $x+2y-10=0$ and $2x+y+5=0$ is $\left(\dfrac{-20}{3}, \dfrac{25}{3}\right)$ which clearly satisfies the line $5x+4y=0$. Hence the given statement is true.

2. The given lines cut x-axis at

$$A\left(\frac{17}{9},0\right),C\left(\frac{-19}{2},0\right)$$

and y-axis at $B\left(0,\frac{17}{6}\right)$ and $D\left(0,\frac{-19}{3}\right)$.

Now A, B, C, D are concyclic if for AC and BD intersecting at O we have $AO \times OC = BO \times OD$



or, $\frac{AO}{BO}=\frac{OD}{OC}$ if $\frac{17/9}{17/6}=\frac{-19/3}{-19/2}$ i.e. $\frac{2}{3}=\frac{2}{3}$ which is true.

$\therefore$ The given statement is true.

C. MCQs with ONE Correct Answer

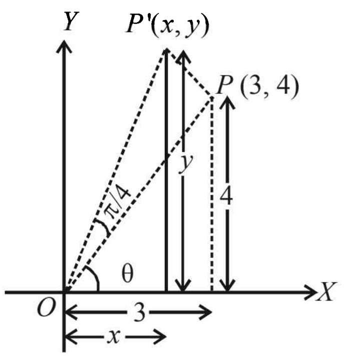
1. (a) The given points are $A(-a,-b)$, $B(0,0)$, $C(a,b)$ and $D(a^2, ab)$.

Slope of $AB=\frac{b}{a}$ = slope of BC = slope of BD

$\therefore$ A, B, C, D are collinear.

2. (c) Reflection about the line $y=x$, changes the point (4, 1) to (1, 4).

On translation of (1, 4) through a distance of 2 units along +ve direction of x-axis the point becomes $(1+2, 4)$, i.e., (3, 4).



On rotation about origin through an angle $\pi/4$ the point P takes the position P' such that

$OP = OP'$

Also $OP = 5 = OP'$ and $\cos\theta=\frac{3}{5}$, $\sin\theta=\frac{4}{5}$

Now, $x = OP'\cos\left(\frac{\pi}{4}+\theta\right)$

$$=5\left(\cos\frac{\pi}{4}\cos\theta-\sin\frac{\pi}{4}\sin\theta\right)=5\left(\frac{3}{5\sqrt2}-\frac{4}{5\sqrt2}\right)=-\frac{1}{\sqrt2}$$

$$y=OP'\sin\left(\frac{\pi}{4}+\theta\right)=5\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)$$

$$=5\left(\frac{3}{5\sqrt2}+\frac{4}{5\sqrt2}\right)=\frac{7}{\sqrt2} \quad \therefore \quad P'=\left(-\frac{1}{\sqrt2},\frac{7}{\sqrt2}\right)$$

3. (a) Solving the given equations of lines pairwise, we get the vertices of Δ as

$A(-2,2)\ B(2,-2),\ C(1,1)$

Then $AB=\sqrt{16+16}=4\sqrt2$

$BC=\sqrt{1+9}=\sqrt{10}$

$CA=\sqrt{9+1}=\sqrt{10}$ $\quad\therefore\ \Delta$ is isosceles.

4. (a) We have

$P=(1,0),\ Q=(-1,0),\ R=(2,0)$

Let $S=(x,y)$

ATQ $SQ^2+SR^2=2SP^2$

$\Rightarrow (x+1)^2+y^2+(x-2)^2+y^2=2[(x-1)^2+y^2]$

$\Rightarrow 2x^2+2y^2-2x+5=2x^2+2y^2-4x+2$

$\Rightarrow 2x+3=0 \Rightarrow x=-3/2$

Which is a straight line parallel to y-axis.

5. (b) As L has intercepts a and b on axes, equation of L is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{........ (1)}$$

Let x and y axes be rotated through an angle θ in anticlockwise direction.

In new system intercepts are p and q, therefore equation of L becomes

$$\frac{x}{p}+\frac{y}{q}=1 \qquad \text{........ (2)}$$

KEY CONCEPT : As the origin is fixed in rotation, the distance of line from origin in both the cases should be same.

$$\therefore \text{ We get } d=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\left|\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}\right|$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$$

$\therefore$ (b) is the correct answer.

6. (a) Let the two perpendicular lines be the co-ordinate axes. Let (x, y) be the point sum of whose distances from two axes is 1 then we must have

$|x|+|y|=1$ or $\pm x\pm y=1$

These are the four lines

$x+y=1, x-y=1, \quad -x+y=1, -x-y=1$

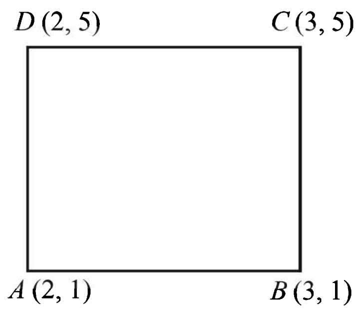
Any two adjacent sides are perpendicular to each other. Also each line is equidistant from origin. Therefore figure formed is a square.

7. **(a)** If variable point is P and $S(-2, 0)$ then $PS = \frac{2}{3}PM$

where PM is the perpendicular distance of point P from given line $x = -9/2$

$\therefore$ By definition P describes an ellipse. $\left(e = \frac{2}{3} < 1\right)$

8. **(c)** The sides of parallelogram are $x = 2, x = 3, y = 1, y = 5$.



$\therefore$ Diagonal AC is $\frac{y-1}{5-1} = \frac{x-2}{3-2}$ or $y = 4x - 7$

Equation diagonal BD is $\frac{x-2}{3-2} = \frac{y-5}{1-5}$ or $4x + y = 13$

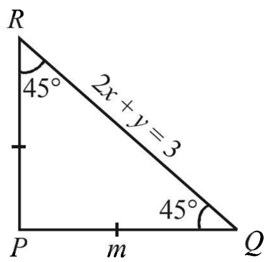
9. **(c)** The lines by which Δ is formed are $x = 0, y = 0$ and $x + y = 1$.
Clearly, it is right Δ and we know that in a rightΔ orthocentre coincides with the vertex at which right $\angle$ is formed.
$\therefore$ Orthocentre is (0, 0).

10. **(b)** Let m be the slope of PQ then

$$\tan 45° = \left|\frac{m-(-2)}{1+m(-2)}\right|$$

$$\Rightarrow 1 = \left|\frac{m+2}{1-2m}\right| \Rightarrow \pm 1 = \frac{m+2}{1-2m}$$

$\Rightarrow m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$
$\Rightarrow m = -1/3$ or $m = 3$



As PR also makes $\angle 45°$ with RQ.
$\therefore$ The above two values of m are for PQ and PR.

$\therefore$ Equation of PQ, $y - 1 = -\frac{1}{3}(x-2)$

$\Rightarrow 3y - 3 = -x + 2 \quad \Rightarrow x + 3y - 5 = 0$

and equation of PR is $\Rightarrow 3x - y - 5 = 0$
$\therefore$ Combined equation of PQ and PR is
$(x - 3y - 5)(3x - y - 5) = 0$
$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

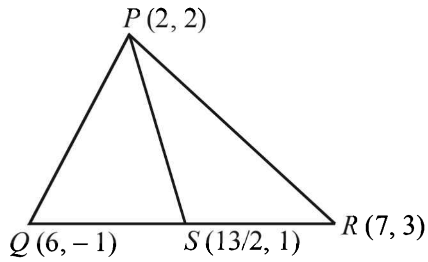
11. **(a)** $x_2 = x_1 r, x_3 = x_1 r^2$ and so is $y_2 = y_1 r, y_3 = y_1 r^2$

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = r.r^2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 & y_1 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0$$

Hence the points lie on a line, i.e., they are collinear.

12. **(d)** S is the midpoint of Q and R

Therefore, $S \equiv \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$



Now slope of $PS = m = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Now equation of the line passing through (1, – 1) and parallel to PS is

$$y + 1 = -\frac{2}{9}(x-1) \text{ or } 2x + 9y + 7 = 0$$

13. **(d)** Here AB = BC = CA = 2. So, it is an equilaterial triangle and the incentre coincides with centroid.
Therefore,

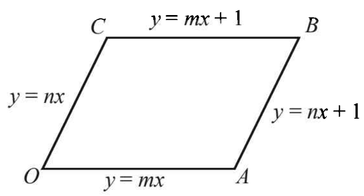
$$\text{Incentre} = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

14. **(a)** Intersection of $3x + 4y = 9$ and $y = mx + 1$.
For x co-ordinate
$3x + 4(mx + 1) = 9 \Rightarrow (3 + 4m)x = 5$

$$x = \frac{5}{3+4m}$$

For x to be an integer $3 + 4m$ should be a divisor of 5 i.e., $1, -1, 5$ or -5.
$3 + 4m = 1 \Rightarrow m = -1/2$ (not integer)
$3 + 4m = -1 \Rightarrow m = -1$ (integer)
$3 + 4m = 5 \Rightarrow m = 1/2$ (not an integer)
$3 + 4m = -5 \Rightarrow m = -2$ (integer)
$\therefore$ There are 2 integral values of m.
$\therefore$ (a) is the correct alternative.

15. **(d)**



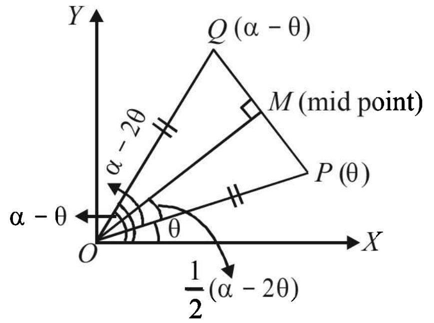
The vertices, $O(0,0), A\left(\frac{1}{m-n}, \frac{m}{m-n}\right), B(0,1)$

$Ar(\|^{gm} OABC = 2\,Ar(\Delta OAB)$

$$= 2\frac{1}{2}\left|\left[0\left(\frac{m}{m-n}-1\right)+\frac{1}{m-n}(1-0)+0\left(0-\frac{m}{m-n}\right)\right]\right|$$

$$= \frac{1}{|m-n|}$$

16. (d) Clearly $OP = OQ = 1$ and $\angle QOP = \alpha - \theta - \theta = \alpha - 2\theta$.



The bisector of $\angle QOP$ will be a perpendicular to PQ and also bisect it. Hence Q is reflection of P in the line OM which makes an angle $\angle MOP + \angle POX$ with x- axis,

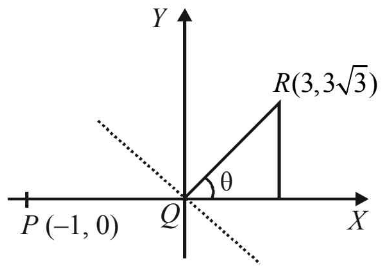
i.e., $\frac{1}{2}(\alpha - 2\theta) + \theta = \alpha/2$.

So that slope of OM is $\tan \alpha/2$.

17. (c) $\tan\theta = \sqrt{3} \Rightarrow \theta = 60° \Rightarrow \angle PQR = 120°$

$\Rightarrow$ bisector will have slope tan 120°

$\Rightarrow$ equation of bisector is $\sqrt{3}x + y = 0$

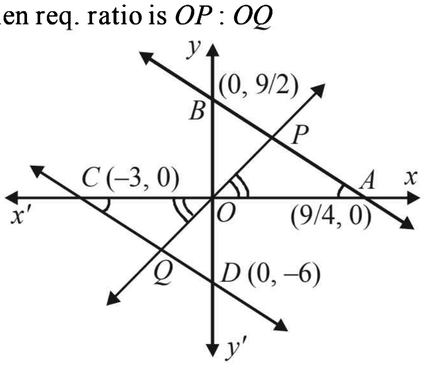


18. (b) The given lines are

$2x + y = 9/2$ .... (1)

and $2x + y = -6$ .... (2)

Signs of constants on R.H.S. show that two lines lie on opp. sides of origin. Let any line through origin meets these lines in P and Q respectively then req. ratio is $OP : OQ$



Now in ΔOPA and ΔOQC,

$\angle POA = \angle QOC$ (ver. opp. $\angle$' s)

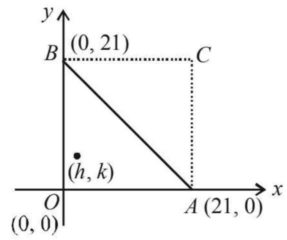
$\angle PAO = \angle OCQ$ (alt. int. $\angle$' s)

$\therefore$ $\Delta OPA \sim \Delta OQC$ (by AA similarly)

$$\therefore \quad \frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$$

$\therefore$ Req. ratio is 3 : 4.

19. (b) Total no. of points within the square $OABC$ $= 20 \times 20 = 400$

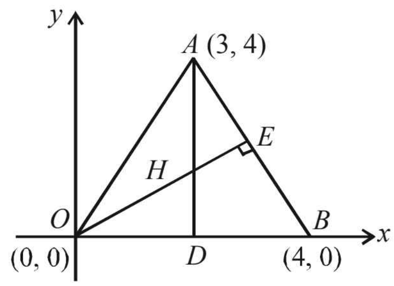


Points on line $AB = 20$ ((1, 1), (2, 2), ... (20, 20))

$\therefore$ Points within ΔOBC and ΔABC = 400 – 20 = 380

By symmetry points within ΔOAB $= \frac{380}{2} = 190$

20. (c) We know that orthocentre is the meeting point of altitudes of a Δ.



Equation of alt. AD

$\Rightarrow$ line parallel to y-axis through (3, 4)

$\Rightarrow$ $x = 3$ .... (1)

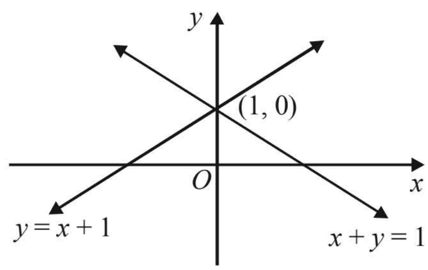
Similarly eq^n of $OE \perp AB$ is

$$y = -\frac{3-4}{4-0}x$$

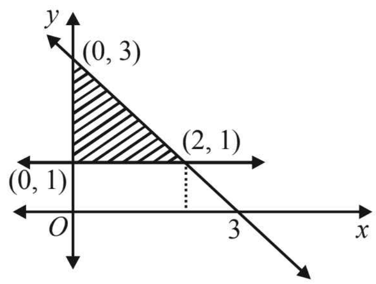
$\Rightarrow$ $y = x/4$ .... (2)

Solving (1) and (2), we get orthocentre as (3, 3/4).

21. (a) $x^2 - y^2 + 2y = 1 \Rightarrow \quad x = \pm(y - 1)$



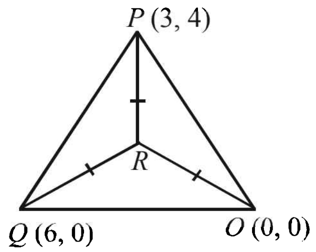
Bisectors of above lines are x = 0 and y = 1.



So area between x = 0, y = 1 and x + y = 3 is shaded region shown in figure.

Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units.

22. **(c)** $\because \quad Ar(\Delta OPR) = Ar(\Delta PQR) = Ar(\Delta OQR)$



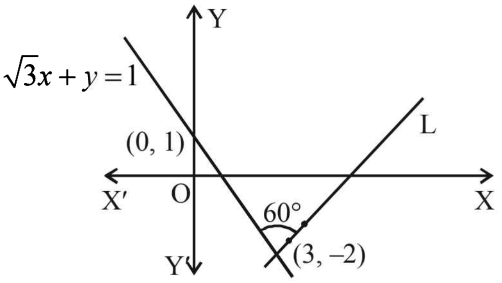
$\therefore$ By simply geometry

R should be the centroid of ΔPQO

$$\Rightarrow R\left(\frac{3+6+0}{3}, \frac{4+0+0}{3}\right) = \left(3, \frac{4}{3}\right)$$

23. **(b)** Let the slope of line L be m.

Then $\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$



$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3} \Rightarrow m = 0$ or $m = \sqrt{3}$

$\because$ L intersects x-axis, $\quad \therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}\,(x-3)$

or $\sqrt{3}\,x - y - (2 + 3\sqrt{3}) = 0$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(a, b, c)**

For concurrency of three lines

$px + qy + r = 0;\ qx + ry + p = 0;$

$rx + py + q = 0$

We must have,

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$\Rightarrow C_1 + C_2 + C_3, \begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = 0$$

$$\Rightarrow (p+q+r)\begin{vmatrix} 1 & q & r \\ 1 & r & p \\ 1 & p & q \end{vmatrix} = 0$$

$\Rightarrow C_1 - C_2,\ C_2 - C_3,$

$$\Rightarrow (p+q+r)\begin{vmatrix} 0 & q-r & r-p \\ 0 & r-p & p-q \\ 1 & p & q \end{vmatrix} = 0$$

$\Rightarrow (p+q+r)(pq - q^2 - rp + rq - r^2 + pr + pr - p^2) = 0$

$\Rightarrow (p+q+r)(p^2 + q^2 + r^2 - pq - pr - rq) = 0$

$\Rightarrow p^3 + q^3 + r^3 - 3pqr = 0$

It is clear that a, b, c are correct options.

2. **(e)** Let $A(0, 8/3)$, $B(1, 3)$ and $C(82, 30)$.

Now, slope of line $AB = \frac{3 - 8/3}{1-0} = \frac{1}{3}$

Slope of line $BC = \frac{30-3}{82-1} = \frac{27}{81} = \frac{1}{3}$

$\Rightarrow AB \parallel BC$ and B is common point.

$\Rightarrow A, B, C$ are collinear.

3. **(a, c)** Substituting the co-ordinates of the points (1, 3), (5, 0) and (– 1, 2) in $3x + 2y$, we obtain the value 8, 15 and 1 which are all +ve. Therefore, all the points lying inside the triangle formed by given points satisfy $3x + 2y \geq 0$. Hence (a) is correct answer.

Substituting the co-ordinates of the given points in $2x + y - 13$, we find the values – 8, – 3 and – 13 which are all –ve.

So, (b) is not correct.

Again substituting the given points in $2x - 3y - 12$ we get – 19, – 2, – 20 which are all –ve.

It follows that all points lying inside the triangle formed by given points satisfy $2x - 3y - 12 \leq 0$.

So, (c) is the correct answer.

Finally substituting the co-ordinates of the given points in $-2x + y$, we get 1, – 10 and 4 which are not all +ve.

So, (d) is not correct.

Hence, (a) and (c) are the correct answers.

4. **(b)** Consider $\vec{a} = 2p\hat{i} + \hat{j}$ with respect to original axes and $a = (p+1)i + j$ with respect to new axes.
Now, as length of vector will remain the same

$\therefore \quad |\vec{a}| = \sqrt{(2p)^2 + 1} = \sqrt{(p+1)^2 + 1^2}$

$\Rightarrow p^2 + 2p + 2 = 4p^2 + 1 \Rightarrow 3p^2 - 2p - 1 = 0$

$\Rightarrow p = 1$ or $-1/3$

$\therefore$ (b) is the correct answer.

5. **(c)** $PQRS$ will represent a parallelogram if and only if the mid-point of PR is same as that of the mid-point of QS. That is, if and only if

$$\frac{1+5}{2} = \frac{4+a}{2} \text{ and } \frac{2+7}{2} = \frac{6+b}{2}$$

$\Rightarrow a = 2$ and $b = 3$.

6. **(d)** Slope of $x + 3y = 4$ is $-1/3$ and slope of $6x - 2y = 7$ is 3. Therefore, these two lines are perpendicular which shows that both diagonals are perpendicular. Hence $PQRS$ must be a rhombus.

7. **(a, c, d)** Since the co-ordinates of in the centre depend on lengths of side of Δ. $\therefore$ it can have irrational coordinates

8. **(b, c)** We know that length of intercept made by a circle on a line is given by $= 2\sqrt{r^2 - p^2}$

where $p = \perp$ distance of line from the centre of the circle.

Here circle is $x^2 + y^2 - x + 3y = 0$ with centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$

and radius $= \frac{\sqrt{10}}{2}$

$L_1:\ y = mx$ (any line through origin)

$L_2:\ x + y - 1 = 0$ (given line)

ATQ circle makes equal intercepts on L_1 and L_2

$$\Rightarrow 2\sqrt{\frac{10}{4} - \frac{\left(\frac{m}{2} + \frac{3}{2}\right)^2}{m^2 + 1}} = 2\sqrt{\frac{10}{4} - \frac{\left(\frac{1}{2} - \frac{3}{2} - 1\right)^2}{2}}$$

$$\Rightarrow \frac{\left(\frac{m+3}{2}\right)^2}{m^2 + 1} = 2$$

$\Rightarrow m^2 + 6m + 9 = 8m^2 + 8 \Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow 7m^2 - 7m + m - 1 = 0 \Rightarrow (7m + 1)(m - 1) = 0$

$\Rightarrow m = 1, -1/7$

$\therefore$ The required line L_1 is $y = x$ or $y = -\frac{x}{7}$,

i.e., $x - y = 0$ or $x + 7y = 0$.

9. **(a)** The intersection point of two lines is $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Distance between (1, 1) and $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right) < 2\sqrt{2}$

$$\Rightarrow 2\left(1 + \frac{c}{a+b}\right)^2 < 8 \Rightarrow 1 + \frac{c}{a+b} < 2$$

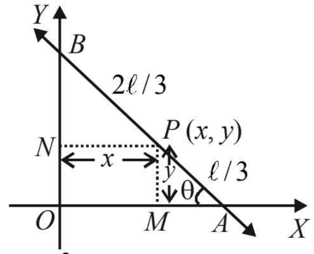
$\Rightarrow a + b - c > 0$

E. Subjective Problems

1. Let P (x, y) divides line segment AB in the ratio 1 : 2, so that AP = $\ell/3$ and BP = $2\ell/3$ where AB = ℓ.
Then PN = x and PM = y
Let $\angle$PAM = θ = $\angle$BPN

In ΔPMA, $\sin\theta = \frac{y}{\ell/3} = \frac{3y}{\ell}$

In ΔPNB, $\cos\theta = \frac{x}{2\ell/3} = \frac{3x}{2\ell}$



Now, $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{9y^2}{\ell^2} + \frac{9x^2}{4\ell^2} = 1 \Rightarrow 9x^2 + 36y^2 = 4\ell^2$$

2. As C lies on the line $y = x + 3$, let the co-ordinates of C be $(\lambda, \lambda + 3)$. Also A (2, 1), B (3, –2).
Then area of ΔABC is given by

$$\frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 3 & -2 & 1 \\ \lambda & \lambda+3 & 1 \end{vmatrix} = \pm 5$$

$\Rightarrow |2(-2 - \lambda - 3) - 1(3 - \lambda)(3\lambda + 9 + 2\lambda)| = 10$

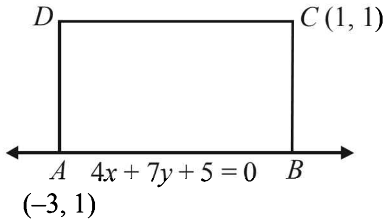
$\Rightarrow |-2\lambda - 10 - 3 + \lambda + 5\lambda + 9| = 10 \Rightarrow |4\lambda - 4| = 10$

$\Rightarrow 4\lambda - 4 = 10$ or $4\lambda - 4 = -10$

$\Rightarrow \lambda = 7/2$ or $\lambda = -3/2$

$\therefore$ Coordinates of C are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

3. Let side AB of rectangle $ABCD$ lies along $4x + 7y + 5 = 0$.
As $(-3, 1)$ lies on the line, let it be vertex A.
Now (1, 1) is either vertex C or D.



If (1, 1) is vertex D then slope of $AD = 0$

$\Rightarrow$ AD is not perpendicular to AB.

But it is a contradiction as $ABCD$ is a rectangle.

$\therefore$ (1, 1) are the co-ordinates of vertex C.

CD is a line parallel to AB and passing through C, therefore equation of CD is

$$y-1=-\frac{4}{7}(x-1)\Rightarrow 4x+7y-11=0$$

Also BC is a line perpendicular to AB and passing through C, therefore equation of BC is

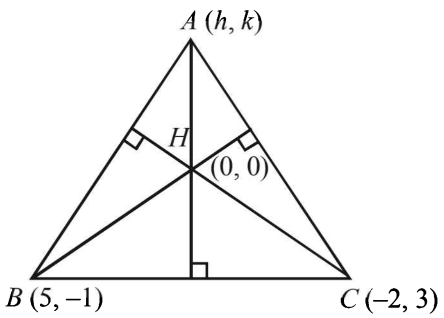
$$y-1=\frac{7}{4}(x-1)\Rightarrow 7x-4y-3=0$$

Similarly, AD is a line perpendicular to AB and passing through A (– 3, 1), therefore equation of line AD is

$$y-1=\frac{7}{4}(x+3)\Rightarrow 7x-4y+25=0$$

4. (a) $AH \perp BC \Rightarrow m_{AH}\times m_{BC}=-1$

$$\Rightarrow \frac{k}{h}\times\frac{3+1}{-2-5}=-1$$



$\Rightarrow 4k-7h=0$ .... (1)

Also, $BH \perp AC$

$$\Rightarrow \frac{-1}{5}\times\frac{3-k}{-2-h}=-1 \Rightarrow 3-k=-10-5h$$

$\Rightarrow 5h-k+13=0$ .... (2)

Solving (1) and (2), we get $h=-4$, $k=-7$

$\therefore$ Third vertex is (– 4, – 7).

(b) The given lines are $x-2y+4=0$ .... (1)

and $4x-3y+2=0$ .... (2)

Both the lines have constant terms of same sign.

$\therefore$ The equation of bisectors of the angles between the given lines are

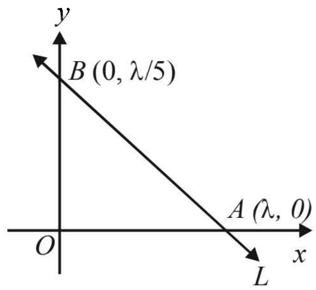
$$\frac{x-2y+4}{\sqrt{1+4}}=\pm\frac{4x-3y+2}{\sqrt{16+9}}$$

Here $a_1a_2+b_1b_2>0$ therefore, taking +ve sign on RHS, we get obtuse angle bisector as

$(4-\sqrt{5})x+(2\sqrt{5}-3)y-(4\sqrt{5}-2)=0$ ....(3)

5. The given line is $5x-y=1$

$\therefore$ The equation of line L which is perpendicular to the given line is $x+5y=\lambda$. This line meets co-ordinate axes at A $(\lambda, 0)$ and B $(0, \lambda/5)$.



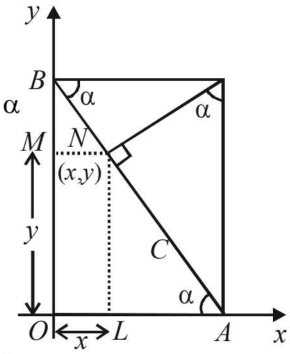
$\therefore$ Area of $\Delta OAB=\frac{1}{2}\times OA\times OB$

$\Rightarrow 5=\frac{1}{2}\times\lambda\times\frac{\lambda}{5} \Rightarrow \lambda^2=5^2\times 2\Rightarrow \lambda=\pm 5\sqrt{2}$

$\therefore$ The equation of line L is $x+5y-5\sqrt{2}=0$

or $x+5y+5\sqrt{2}=0$.

6. From figure,

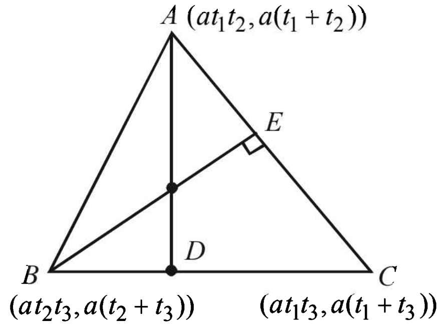


x = OA – AL
= c cos α – AN cos α
= c cos α – (AP sin α.) cos α
= c cos α – c sin α . sin α cos α
= c cos α (1 – $\sin^2$ α)
= c $\cos^3$ α

y = OB – MB
= c sin α – BN sin α
= c sin α – BP cos α sin α
= c sin α – c cos α . cos α sin α
= c sin α (1 – $\cos^2$ α) = c $\sin^3$ α

$\therefore$ Locus of (x, y) is $\left(\frac{x}{c}\right)^{\frac{2}{3}}+\left(\frac{y}{c}\right)^{\frac{2}{3}}=1$ or $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$

7.



Slope of $BC=\frac{a(t_1+t_3)-a(t_2+t_3)}{at_1t_3-at_2t_3}$

$$=\frac{a(t_1+t_3-t_2-t_3)}{a\,t_3(t_1-t_2)}=\frac{1}{t_3}$$

$\therefore$ Slope of $AD=-t_3$

$\therefore$ Eq. of AD,

$y-a(t_1+t_2)=-t_3(x-at_1t_2)$

or $x t_3+y=a\,t_1t_2t_3+a(t_1+t_2)$ .... (1)

Similarly, by symm. equation of BE is

$\Rightarrow \quad xt_1 + y = at_1t_2t_3 + a(t_2 + t_3)$ .... (2)

Solving (1) and (2), we get $x = -a$

$y = a(t_1 + t_2 + t_3) + at_1t_2t_3)$

$\therefore$ Orthocentre $H(-a, a(t_1 + t_2 + t_3) + at_1t_2t_3)$

8. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \frac{1}{2}[6(7) + 3(5) + 4(-2)] = \frac{49}{2}$

Area of $\Delta PBC = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \frac{1}{2}(7x + 7y - 14) - \frac{7}{2}|x + y - 2|$

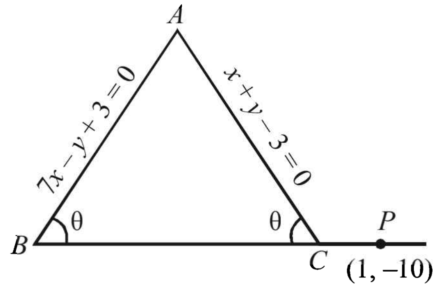
$ATQ, \frac{Ar(\Delta PBC)}{Ar(\Delta ABC)} = \frac{\frac{7}{2}|x + y - 2|}{\frac{49}{2}} = \left|\frac{x + y - 2}{7}\right|$

9. Let equations of equal sides AB and AC of isosceles ΔABC are

$7x - y + 3 = 0$ .... (1)

and $x + y - 3 = 0$ .... (2)

The third side BC of Δ passes through the point $(1, -10)$. Let its slope be m.



As $AB = AC$

$\therefore \quad \angle B = \angle C$

$\Rightarrow \quad \tan B = \tan C$ .... (3)

Now slope of $AB = 7$ and slope of $AC = -1$

Using $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$, we get

$\tan B = \left|\frac{7 - m}{1 + 7m}\right|$ and $\tan C = \left|\frac{-1 - m}{1 - m}\right|$

From eq. (3), we get

$\left|\frac{7 - m}{1 + 7m}\right| = \left|\frac{-1 - m}{1 - m}\right|$

$\Rightarrow \quad \frac{7 - m}{1 + 7m} = \pm\left(\frac{-1 - m}{1 - m}\right)$

Taking '+' sign, we get

$(7 - m)(1 - m) = -(1 + m)(1 + 7m)$

$\Rightarrow \quad 7 - 8m + m^2 + 7m^2 + 8m + 1 = 0$

$\Rightarrow \quad 8m^2 + 8 = 0 \Rightarrow m^2 + 1 = 0$

It has no real solution.

Taking '–' sign, we get

$(7 - m)(1 - m) = (1 + m)(1 + 7m)$

$\Rightarrow \quad 7 - 8m + m^2 - 7m^2 - 8m - 1 = 0$

$\Rightarrow \quad -6m^2 - 16m + 6 = 0 \Rightarrow 3m^2 + 8m - 3 = 0$

$\Rightarrow \quad (3m - 1)(m + 3) = 0 \Rightarrow m = 1/3, -3$

$\therefore$ The required line is

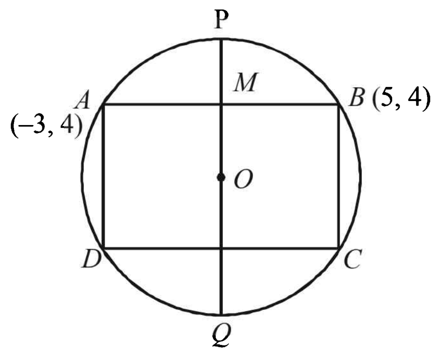
$y + 10 = \frac{1}{3}(x - 1)$ or $y + 10 = -3(x - 1)$

i.e. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$.

10. Let O be the centre of the circle. M is the mid point of AB. Then

$OM \perp AB$

Let OM when produced meets the circle at P and Q.



$\therefore$ PQ is a diameter perpendicular to AB and passing through M.

$M = \left(\frac{-3 + 5}{2}, \frac{4 + 4}{2}\right) = (1, 4)$

Slope of $AB = \frac{4 - 4}{5 + 3} = 0$

$\therefore$ PQ, being perpendicular to AB, is a line parallel to y-axis passing through $(1, 4)$.

$\therefore$ Its equation is

$x = 1$ .... (1)

Also eq. of one of the diameter given is

$4y = x + 7$ .... (2)

Solving (1) and (2), we get co-ordinates of centre O

$O(1, 2)$

Also let co-ordinates of D be (α, β)

Then O is mid point of BD, therefore

$\left(\frac{\alpha + 5}{2}, \frac{\beta + 4}{2}\right) = (1, 2) \Rightarrow \alpha = -3, \beta = 0$

$\therefore \quad D(-3, 0)$

Using the distance formula we get

$AD = \sqrt{(-3 + 3)^2 + (4 - 0)^2} = 4$

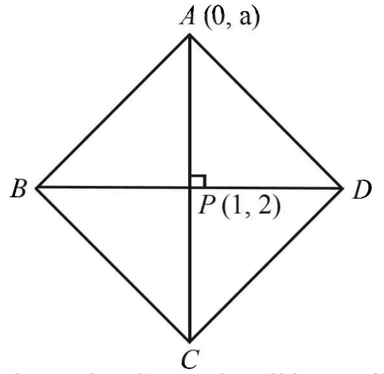
$AB = \sqrt{(5 + 3)^2 + (4 - 4)^2} = 8$

$\therefore$ Area of rectangle $ABCD = AB \times AD = 8 \times 4$

$= 32$ square units.

11. A being on y-axis, may be chosen as $(0, a)$.

The diagonals intersect at $P(1, 2)$.



Again we know that diagonals will be parallel to the angle bisectors of the two sides $y = x + 2$ and $y = 7x + 3$

i.e., $\dfrac{x - y + 2}{\sqrt{2}} = \pm \dfrac{7x - y + 3}{5\sqrt{2}}$

$\Rightarrow\ 5x - 5y + 10 = \pm(7x - y + 3)$

$\Rightarrow\ 2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$m_1 = -1/2 \qquad m_2 = 2$

Let diagonal d_1 be parallel to $2x + 4y - 7 = 0$ and diagonal d_2 be parallel to $12x - 6y + 13 = 0$. The vertex A could be on any of the two diagonals. Hence slope of AP is either $-1/2$ or 2.

$\Rightarrow\ \dfrac{2 - a}{1 - 0} = 2 \quad$ or $\quad \dfrac{-1}{2}$

$\Rightarrow\ a = 0 \quad$ or $\quad \dfrac{5}{2}$

$\therefore\ A$ is (0, 0) or (0, 5/2)

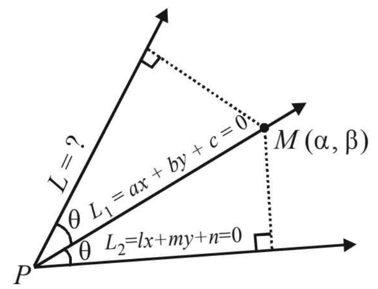
12. Let the equation of other line L, which passes through the point of intersection P of lines

$L_1 \equiv ax + by + c = 0 \qquad$ ....(1)

and $L_2 \equiv \ell x + my + n = 0 \qquad$ ....(2)

be $L_1 + \lambda L_2 = 0$

i.e. $(ax + by + c) + \lambda(\ell x + my + n) = 0$ ....(3)



From figure it is clear that L_1 is the bisector of the angle between the lines given by (2) and (3) [i.e. L_2 and L]

Let $M(\alpha, \beta)$ be any point on L_1 then

$a\alpha + b\beta + c = 0 \qquad$ ......(4)

Also from M, lengths of perpendiculars to lines L and L_2 given by equations (3) and (4), are equal

$$\frac{\ell\alpha + m\beta + n}{\sqrt{\ell^2 + m^2}} = \pm \frac{(a\alpha + b\beta + c) + \lambda(l\alpha + m\beta + n)}{\sqrt{(a + \lambda)^2 + (b + \lambda m)^2}}$$

$$\Rightarrow\ \frac{1}{\sqrt{\ell^2 + m^2}} = \pm \frac{\lambda}{\sqrt{(\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2)}}$$

[Using 4]

$\Rightarrow\ (\ell^2 + m^2)\lambda^2 + 2(a\ell + bm)\lambda + (a^2 + b^2) = \lambda^2(\ell^2 + m^2)$

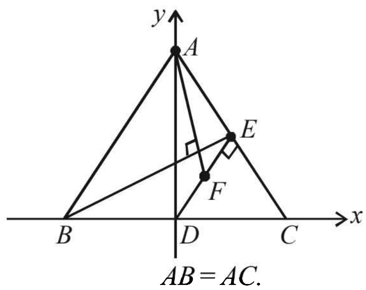
$\Rightarrow\ \lambda = -\dfrac{a^2 + b^2}{2(a\ell + bm)}$

Substituting this value of λ in eq. (3), we get L as

$$(ax + by + c) - \frac{(a^2 + b^2)}{2(a\ell + bm)}(\ell x + my + n) = 0$$

$\Rightarrow\ (a^2 + b^2)(\ell x + my + n) - 2(a\ell + bm)(ax + by + c) = 0$

13. Let BC be taken as x-axis with origin at D, the mid-point of BC, and DA will be y-axis.



$AB = AC.$

Let $BC = 2a$, then the co-ordinates of B and C are $(-a, 0)$ and $(a, 0)$.

Let $DA = h$, so that co-ordinates of A are $(0, h)$.

Then equation of AC is $\dfrac{x}{a} + \dfrac{y}{h} = 1 \qquad$ ......(1)

And equation of $DE \perp$ to AC and passing through origin is

$\dfrac{x}{h} - \dfrac{y}{a} = 0 \Rightarrow x = \dfrac{hy}{a} \qquad$ ....(2)

Solving (1) and (2) we get the co-ordinates of pt E as follows

$\dfrac{hy}{a^2} + \dfrac{y}{h} = 1 \ \Rightarrow\ h^2 y + a^2 y = a^2 h$

$\Rightarrow\ y = \dfrac{a^2 h}{a^2 + h^2} \ \Rightarrow\ x = \dfrac{ah^2}{a^2 h^2}$

$\therefore\ E\left(\dfrac{ah^2}{a^2 + h^2}, \dfrac{a^2 h}{a^2 + h^2}\right)$

Since F is mid pt. of DE, therefore, its co-ordinates are

$F\left(\dfrac{ah^2}{2(a^2 + h^2)}, \dfrac{a^2 h}{2(a^2 + h^2)}\right)$

$$\therefore\ \text{Slope of } AF = \frac{h - \dfrac{a^2 h}{2(a^2 + h^2)}}{0 - \dfrac{ah^2}{2(a^2 + h^2)}} = \frac{2h(a^2 + h^2) - a^2 h}{-ah^2}$$

$\Rightarrow\ m_1 = -\dfrac{a^2 + 2h^2}{ah} \qquad$ ....(i)

$$\text{And slope of } BE = \frac{\dfrac{a^2 h}{a^2 + h^2} - 0}{\dfrac{ah^2}{a^2 + h^2} + a} = \frac{a^2 h}{ah^2 + a^3 + ah^2}$$

$\Rightarrow\ m_2 = \dfrac{ah}{a^2 + 2h^2} \qquad$ ....(ii)

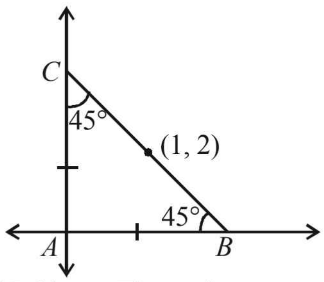
From (i) and (ii), we observe that

$m_1 m_2 = -1 \Rightarrow AF \perp BE.$ **Hence Proved.**

14. The given st. lines are $3x+4y=5$ and $4x-3y=15$. Clearly these st. lines are perpendicular to each other $(m_1\, m_2 = -1)$, and intersect at A. Now B and C are pts on these lines such that $AB = AC$ and BC passes through (1, 2)

From fig. it is clear that

$\angle B = \angle C = 45°$



Let slope of BC be m. Then using

$$\tan B = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \text{ we get } \tan 45° = \left|\frac{m + 3/4}{1 - \frac{3}{4}m}\right|$$

$\Rightarrow 4m + 3 = \pm(4 - 3m)$

$\Rightarrow 4m + 3 = 4 - 3m$ or $4m + 3 = -4 + 3m$

$\Rightarrow m = 1/7$ or $m = -7$

$\therefore$ Eq. of BC is, $y - 2 = \frac{1}{7}(x-1)$

or $y - 2 = -7\,(x-1)$

$\Rightarrow 7y - 14 = x - 1$ or $y - 2 = -7x + 7$

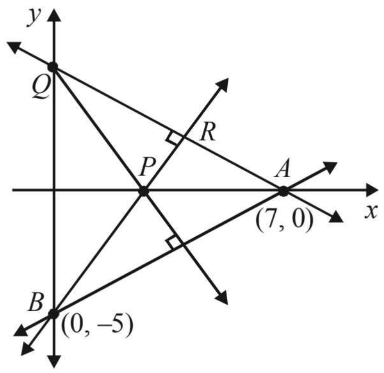
$\Rightarrow x - 7y + 13 = 0$ or $7x + y - 9 = 0$

15. Eq. of the line AB is

$$\frac{x}{7} - \frac{y}{5} = 1 \qquad [A\,(7,0), B(0,-5)]$$

$\Rightarrow 5x - 7y - 35 = 0$

Eq. of line $PQ \perp AB$ is $7x + 5y + \lambda = 0$ which meets axes of x and y at pts $P(-\lambda/7, 0)$ and $Q\,(0, -\lambda/5)$ resp.



Eq. of AQ is,

$$\frac{x}{y} + \frac{y}{-\lambda/5} = 1 \Rightarrow \lambda x - 35y - 7\lambda = 0 \qquad ....(2)$$

Eq. of BP is,

$$\frac{-7x}{\lambda} - \frac{y}{5} = 1 \Rightarrow 35x + \lambda y + 5\lambda = 0 \qquad ....(3)$$

Locus of R the pt. of intersection of (2) and (3) can be obtained by eliminating λ from these eq. ' s, as follows

$$35x + (5 + y)\left(\frac{35y}{x-7}\right) = 0$$

$\Rightarrow 35x\,(x-7) + 35y\,(5+y) = 0 \Rightarrow x^2 + y^2 - 7x + 5y = 0$

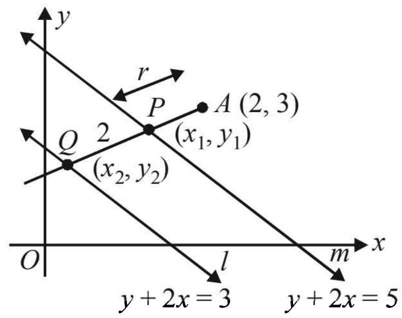
16. Let the equation of line through A which makes an intercept of 2 units between.

$2x + y = 3$ .... (1)

and $2x + y = 5$ .... (2)

be $\dfrac{x-2}{\cos\theta} = \dfrac{y-3}{\sin\theta} = r$

Let $AP = r$ then $AQ = r + 2$



Then for pt $P\,(x_1, y_1)$,

$$\frac{x_1 - 2}{\cos\theta} = \frac{y_1 - 3}{\sin\theta} = r \Rightarrow \frac{2(x_1-2) + (y_1 - 3)}{2\cos\theta + \sin\theta} = r$$

$$\left(\text{Using } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{\lambda a_1 + \mu b_1}{\lambda a_2 + \mu b_2}\right)$$

$$\Rightarrow \frac{(2x_1 + y_1) - 7}{2\cos\theta + \sin\theta} = r \Rightarrow \frac{5 - 7}{2\cos\theta + \sin\theta} = r$$

[Using $2x_1 + y_1 = 5$ as $P\,(x_1, y_1)$ lies on $2x + y = 5$]

$$\frac{-2}{2\cos\theta + \sin\theta} = r \qquad \ldots(i)$$

For pt $Q\,(x_2, y_2)$,

$$\frac{x_2 - 2}{\cos\theta} = \frac{y_2 - 3}{\sin\theta} = r + 2$$

$$\Rightarrow \frac{2(x_2 - 2) + (y_2 - 3)}{2\cos\theta + \sin\theta} = r + 2$$

$$\Rightarrow \frac{-4}{2\cos\theta + \sin\theta} = r + 2 \qquad \ldots(ii)$$

(ii) – (i)

$$\Rightarrow \frac{-2}{2\cos\theta + \sin\theta} = 2$$

$\Rightarrow 2\cos\theta + \sin\theta = -1$ ... (3)

$\Rightarrow 2\cos\theta = -(1 + \sin\theta)$

Squaring on both sides, we get

$\Rightarrow 4\cos^2\theta = 1 + 2\sin\theta + \sin^2\theta$

$\Rightarrow (5\sin\theta - 3)(\sin\theta + 1) = 0 \Rightarrow \sin\theta = 3/5, -1$

$\Rightarrow \cos\theta = -4/5, 0$ [Using eq. (3)]

GP_3480

$\therefore$ The required equation is either

$$\frac{x-2}{-4/5}=\frac{y-3}{3/5} \text{ or } \frac{x-2}{0}=\frac{y-3}{-1}$$

$\Rightarrow$ either $3x-6=-4y+12$ or $x-2=0$

$\Rightarrow$ either $3x+4y-18=0$ or $x-2=0$

17. The given curve is

$3x^2-y^2-2x+4y=0 \quad \ldots(1)$

Let $y = mx + c$ be the chord of curve (1) which subtends an $\angle$ of 90° at origin.

Then the combined eq. of lines joining points of intersection of curve (1) and chord $y = mx + c$ to the origin, can be obtained by making the eq. of curve homogeneous with the help of eq. of chord, as follows.

$$3x^2-y^2-2x\left(\frac{y-mx}{c}\right)+4y\left(\frac{y-mx}{c}\right)=0$$

$\Rightarrow (3c+2m)x^2-2(1+2m)xy+(4-c)y^2=0$

As the lines represented by this pair are perpendicular to each other, therefore we must have

coeff. of x^2 + coeff. of $y^2 = 0$

$\Rightarrow 3c+2m+4-c=0$

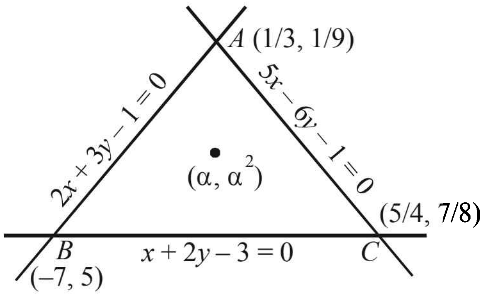
$\Rightarrow -2=m\,.\,1+c$

Which on comparision with eq. of chord, implies that $y = mx + c$ passes though $(1, -2)$.

Hence the family of chords must pass through $(1, -2)$.

18. The points of intersection of given lines are

$$A\left(\frac{1}{3},\frac{1}{9}\right), B(-7,5), C\left(\frac{5}{4},\frac{7}{8}\right)$$



If (α, α^2) lies inside the Δ formed by the given lines, then $\left(\frac{1}{3},\frac{1}{9}\right)$ and (α, α^2) lie on the same side of the line $x + 2y - 3 = 0$

$$\Rightarrow \frac{\alpha+2\alpha^2-3}{\frac{1}{3}+\frac{2}{9}-3}>0 \Rightarrow 2\alpha^2+\alpha-3<0 \ldots(1)$$

Similarly $\left(\frac{5}{4},\frac{7}{8}\right)$ and (α, α^2) lie on the same side of the line $2x + 3y - 1 = 0$.

$$\Rightarrow \frac{2\alpha+3\alpha^2-1}{\frac{10}{4}+\frac{21}{8}-1}>0 \Rightarrow 3\alpha^2+2\alpha-1>0 \ldots(2)$$

$(-7, 5)$ and (α, α^2) lie on the same side of the line $5x - 6y - 1 = 0$.

$$\Rightarrow \frac{5\alpha+6\alpha^2-1}{-35-30-1}>0 \Rightarrow 6\alpha^2-5\alpha+1>0 \ldots(3)$$

Now common solution of (1), (2) and (3) can be obtained as in the previous method,

$$\therefore \quad \alpha \in \left(-\frac{3}{2},-1\right)\cup\left(\frac{1}{2},1\right)$$

19. The given curve is

$y = x^3 \quad \ldots(1)$

Let the pt, P_1 be (t, t^3), $t \neq 0$

Then slope of tangent at $P_1 = \frac{dy}{dx} = (3x^2)_{x=t} = 3t^2$

$\therefore$ Equation of tangent at P_1 is

$y - t^3 = 3t^2(x-t) \Rightarrow y = 3t^2x - 2t^3$

$\Rightarrow 3t^2x - y - 2t^3 = 0 \quad \ldots(2)$

Now this tangent meets the curve again at P_2 which can be obtained by solving (1) and (2)

i.e., $3t^2x - x^2 - 2t^3 = 0$ or $x^3 - 3t^2x + 2t^3 = 0$

$(x-t)^2(x+2t) = 0 \Rightarrow x = -2t$ as $x = t$ is for P_1

$\therefore \quad y = -8t^3$

Hence pt P_2 is $(-2t, -8t^2) = (t_1, t_1^3)$ say.

Similarly, we can find that tangent at P_2 which meets the curve again at $P_3\,(2t_1, -8t_1^3)$ i.e., $(4t, 64t^3)$.

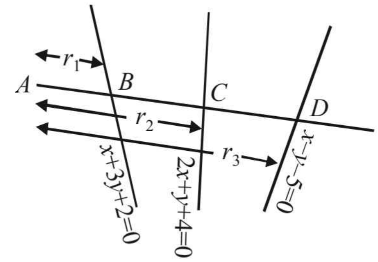
Similarly, $P_4 \equiv (-8t, -512t^3)$ and so on.

We observe that abscissae of pts. $P_1, P_2, P_3\ldots$ are $t, -2t, 4t, \ldots$ which form a GP with common ratio -2. Also ordinates of these pts. $t^3, -8t^3, 64t^3, \ldots$ also form a GP with common ratio -8.

$$\text{Now, } \frac{Ar(\Delta P_1P_2P_3)}{Ar(\Delta P_2P_3P_4)} = \frac{\begin{vmatrix} 1 & t & t^3 \\ 1 & -2t & -8t^3 \\ 1 & 4t & 64t^3 \end{vmatrix}}{\begin{vmatrix} 1 & -2t & -8t^3 \\ 1 & 4t & -64t^3 \\ 1 & -8t & -512t^3 \end{vmatrix}}$$

$$= \frac{t^4\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -8 \\ 1 & 4 & 64 \end{vmatrix}}{(-2)(-8)t^4\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & -8 \\ 1 & 4 & -64 \end{vmatrix}} = \frac{1}{16} \text{ sq. units.}$$

20. Let θ be the inclination of line through $A(-5, -4)$. Therefore equation of this line is



$$\frac{x+5}{\cos\theta}=\frac{y+4}{\sin\theta}=r_1, r_2, r_3$$

$\Rightarrow$ $B(r_1\cos\theta-5, r_1\sin\theta-4)$
$C(r_2\cos\theta-5, r_2\sin\theta-4)$
$D(r_3\cos\theta-5, r_3\sin\theta-4)$

But B lies on $x+3y+2=0$. therefore

$r_1\cos\theta-5+3r_1\sin\theta-12+2=0$

$$\Rightarrow r_1=\frac{15}{\cos\theta+3\sin\theta}=\text{AB}$$

$$\Rightarrow \frac{15}{AB}=\cos\theta+3\sin\theta \quad \ldots(1)$$

As C lies on $2x+y+4=0$, therefore

$2(r_2\cos\theta-5)+(r_2\sin\theta-4)+4=0$

$$\Rightarrow r_2=\frac{10}{2\cos\theta+\sin\theta}=AC$$

$$\Rightarrow \frac{10}{AC}=2\cos\theta+\sin\theta \quad \ldots(2)$$

Similarly D lines on $x-y-5=0$, therefore

$r_3\cos\theta-5-r_3\sin\theta+4-5=0$

$$\Rightarrow r_3=\frac{6}{\cos\theta-\sin\theta}=AD$$

$$\Rightarrow \frac{6}{AD}=\cos\theta-\sin\theta \quad \ldots(3)$$

Now, ATQ, $\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2$

$\Rightarrow$ $(\cos\theta+3\sin\theta)^2+(2\cos\theta+\sin\theta)^2$
$=(\cos\theta-\sin\theta)^2$ [Using (1), (2) and (3)]

$\Rightarrow$ $4\cos^2\theta+9\sin^2\theta+12\sin\theta\cos\theta=0$

$\Rightarrow$ $2\cos\theta+3\sin\theta=0$

$\Rightarrow$ $\tan\theta=-\frac{2}{3}$

$\therefore$ Equation of req. line is $y+4=-\frac{2}{3}(x+5)$

$\Rightarrow$ $2x+3y+22=0$

21. Let the co-ordinates of Q be (b, α) and that of S be $(-b, \beta)$. Let PR and SQ intersect each other at G.

$\because$ G is the mid pt of SQ.

($\because$ diagonals of a rectangle bisect each other)

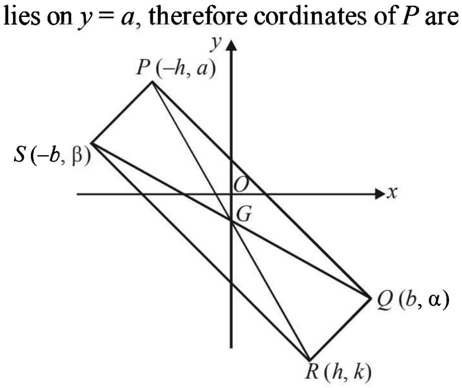
$\therefore$ x co-ordinates of G must be a.

Let the co-ordinates of R be (h, k).

$\therefore$ The x-coordinates of P is $-h$

($\because$ G is the mid point of PR)

As P lies on $y=a$, therefore cordinates of P are $(-h, a)$.



$\because$ PQ is parallel to $y=mx$,
Slope of $PQ=m$

$$\therefore \frac{\alpha-a}{b+h}=m\Rightarrow\alpha=a+m(b+h) \quad \ldots(1)$$

Also $RQ\perp PQ\Rightarrow$

Slope of $RQ=\frac{-1}{m}$

$$\therefore \frac{k-\alpha}{h-b}=\frac{-1}{m}\Rightarrow\alpha=k+\frac{1}{m}(h-b) \quad \ldots(2)$$

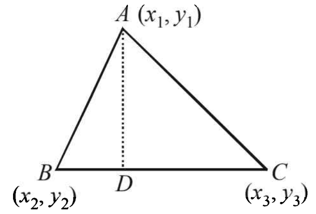
From (1) and (2) we get

$$a+m(b+h)=k+\frac{1}{m}(h-b)$$

$\Rightarrow$ $(m^2-1)h-mk+b(m^2+1)+am=0$

$\therefore$ Locus of vertex $R(h, k)$ is
$(m^2-1)x-my+b(m^2+1)+am=0.$

22. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ be the vertices of ΔABC



Then equation of alt. AD is

$$y-y_1=-\left[\frac{x_2-x_3}{y_2-y_3}\right](x-x_1)$$

or $(x-x_1)(x_2-x_3)+(y-y_1)(y_2-y_3)=0$ … (1)

Similarly equations of other two attitudes are

$(x-x_2)(x_3-x_1)+(y-y_2)(y_3-y_1)=0$ …(2)

and $(x-x_3)(x_1-x_2)+(y-y_3)(y_1-y_2)=0$ …(3)

Now, above three lines will be concurrent if

$$\begin{vmatrix} x_2-x_3 & y_2-y_3 & -x_1(x_2-x_3)-y_1(y_2-y_3) \\ x_3-x_1 & y_3-y_1 & -x_2(x_3-x_1)-y_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & -x_3(x_1-x_2)-y_3(y_1-y_2) \end{vmatrix}=0$$

On L.H.S.

Operating $R_1+R_2+R_3$, R_1 becomes row of zeros.

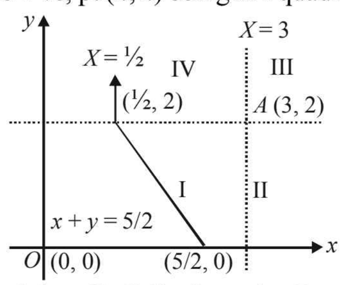
$\therefore$ Value of determinant $=0=$ R.H.S.

Hence the altitudes are concurrent.

23. Let $P=(h, k)$ be a general point in the first quadrant such that $d(P, A)=d(P, O)$

$\Rightarrow$ $|h-3|+|k-2|=|h|+|k|=h+k$ … (1)

[h and k are + ve, pt (h, k) being in I quadrant.]



If $h<3, k<2$ then (h, k) lies in region I.

It $h>3, k<2$, (h, k) lies in region II.

If $h>3, k>2$ (h, k) lies in region III.

If $h < 3, k > 2$ (h, k) lies in region IV.
In region I, eq. (1)

$\Rightarrow\ 3-h+2-k=h+k \Rightarrow h+k=\frac{5}{2}$

In region II, eq. (1) becomes

$\Rightarrow\ h-3+2-k=h+k \Rightarrow k=-\frac{1}{2}$ not possible.

In region III, eq. (1) becomes
$\Rightarrow\ h-3+k-2=h+k \Rightarrow -5=0$ not possible.
In region IV, eq. (1) becomes
$\Rightarrow\ 3-h+k-2=h+k \Rightarrow h=1/2$
$\Rightarrow$ Hence required set consists of line segment $x+y=5/2$ of finite length as shown in the first region and the ray $x=1/2$ in the fourth region.

24. Let the co-ordinates of the vertices of the ΔABC be $A\ (a_1, b_1)$, $B(a_2, b_2)$ and $C\ (a_3, b_3)$ and co-ordinates of the vertices of the ΔPQR be

$P\ (x_1, y_1)$, $B\ (x_2, y_2)$ and $R\ (x_3, y_3)$

Slope of $QR=\frac{y_2-y_3}{x_2-x_3}$

$\Rightarrow$ Slope of straight line perpendicular to

$QR=-\frac{x_2-x_3}{y_2-y_3}$

Equation of straight line passing through $A\ (a_1, b_1)$ and perpendicular to QR is

$$y-b_1=-\frac{x_2-x_3}{y_2-y_3}(x-a_1)$$

$\Rightarrow\ (x_2-x_3)x+(y_2-y_3)y-a_1(x_2-x_3)-b_1(y_2-y_3)=0 \quad \ldots(1)$

Similarly equation of straight line from B and perpendicular to RP is

$(x_3-x_1)x+(y_3-y_1)y-a_2(x_3-x_1)-b_2(y_3-y_1)=0 \quad \ldots(2)$

and eq^n of straight line from C and perpendicular to PQ is

$(x_1-x_2)x+(y_1-y_2)y-a_3(x_1-x_2)-b_3(y_1-y_2)=0 \quad \ldots(3)$

As straight lines (1), (2) and (3) are given to be concurrent, we should have

$$\begin{vmatrix} x_2-x_3 & y_2-y_3 & a_1(x_2-x_3)+b_1(y_2-y_3) \\ x_3-x_1 & y_3-y_1 & a_2(x_3-x_1)+b_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & a_1(x_1-x_2)+b_3(y_1-y_2) \end{vmatrix}=0 \quad \ldots(4)$$

Operating $R_1 \rightarrow R_1+R_2+R_3$, we get

$$\begin{vmatrix} 0 & 0 & S \\ x_3-x_1 & y_3-y_1 & a_2(x_3-x_1)+b_2(y_3-y_1) \\ x_1-x_2 & y_1-y_2 & a_3(x_1-x_2)+b_3(y_1-y_2) \end{vmatrix}=0$$

where

$[S=a_1(x_2-x_3)+b_1(y_2-y_3)+a_2(x_3-x_1)+b_2(y_3-y_1)+a_3(x_1-x_2)+b_3(y_1-y_2)]$

Expanding along R_1

$\Rightarrow\ [(x_3-x_1)(y_1-y_2)-(x_1-x_2)(y_3-y_1)]S=0$

$\Rightarrow\ \left[\frac{y_1-y_2}{x_1-x_2}-\frac{y_3-y_1}{x_3-x_1}\right]S=0$

$\Rightarrow\ [m_{PQ}-m_{PR}]S=0 \Rightarrow S=0$

$[m_{PQ}=m_{PR} \Rightarrow PQ \parallel PR$

which is not possible in $\Delta PQR]$

$\Rightarrow\ a_1(x_2-x_3)+b_1(y_2-y_3)+a_2(x_3-x_1)+b_2(y_3-y_1)+a_3(x_1-x_2)+b_3(y_1-y_2)=0 \ldots(5)$

$\Rightarrow\ x_1(a_3-a_2)+y_1(b_3-b_2)+x_2(a_1-a_3)+y_2(b_1-b_3)+x_3(a_2-a_1)+y_3(b_2-b_1)=0 \quad \ldots(6)$

(Rearranging the equation (5))

But above condition shows

$$\begin{vmatrix} a_3-a_2 & b_3-b_2 & x_1(a_3-a_2)+y_1(b_3-b_2) \\ a_1-a_3 & b_1-b_3 & x_2(a_1-a_3)+y_2(b_1-b_3) \\ a_2-a_1 & b_2-b_1 & x_3(a_2-a_1)+y_3(b_2-b_1) \end{vmatrix}=0 \quad \ldots(7)$$

[Using the fact that as (4) $\Leftrightarrow$ (5) in the same way (6) $\Leftrightarrow$ (7)]

Clearly equation (7) shows that lines through P and perpendicular to BC, through Q and perpendicular to AB are concurrent. **Hence Proved.**

25. $C_1 \rightarrow aC_1$

$$\Delta=\frac{1}{a}\begin{vmatrix} a^2x-aby-ac & bx+ay & cx+a \\ abx+a^2y & -ax+by-c & cy+b \\ acx+a^2 & cy+b & -ax-by+c \end{vmatrix}$$

Applying $C_1 \rightarrow C_1+bC_2+cC_3$

$$\Delta=\frac{1}{a}\begin{vmatrix} (a^2+b^2+c^2)x & ay+bx & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & cy+b \\ (a^2+b^2+c^2) & b+cy & -ax-by+c \end{vmatrix}$$

$$=\frac{1}{a}\begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix},$$

as $a^2+b^2+c^2=1$

$C_2 \rightarrow C_2-bC_1$ and $C_3 \rightarrow C_3-cC_1$

then $\Delta=\frac{1}{a}\begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$

$$=\frac{1}{ax}\begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$$

$R_1 \rightarrow R_1+yR_2+R_3$

$$\Delta=\frac{1}{ax}\begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$$

On expanding along R_1

$$\Delta = \frac{(x^2+y^2+1)}{ax} ax(ax+by+c)$$
$$= (x^2+y^2+1)(ax+by+c)$$

Given $\Delta = 0$

$\Rightarrow ax + by + c = 0$, which represents a straight line.

[$\because x^2+y^2+1 \neq 0$, being + ve].

26. The line $y = mx$ meets the given lines in

$P\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$ and $Q\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$

Hence equation of L_1 is

$$y - \frac{m}{m+1} = 2\left(x - \frac{1}{m+1}\right)$$

$$\Rightarrow y - 2x - 1 = -\frac{3}{m+1} \quad \ldots(1)$$

and that of L_2 is

$$y - \frac{3m}{m+1} = -3\left(x - \frac{3}{m+1}\right)$$

$$\Rightarrow y + 3x - 3 = \frac{6}{m+1} \quad \ldots(2)$$

From (1) and (2)

$$\frac{y-2x-1}{y+3x-3} = -\frac{1}{2}$$

$\Rightarrow x - 3y + 5 = 0$ which is a straight line.

27. Let the equation of the line be

$(y-2) = m(x-8)$ where $m < 0$

$\Rightarrow P \equiv \left(8 - \frac{2}{m}, 0\right)$ and $Q \equiv (0, 2-8m)$

Now, $OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$

$$= 10 + \frac{2}{-m} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times 8(-m)} \geq 18$$

28. A line passing through $P(h, k)$ and parallel to x-axis is

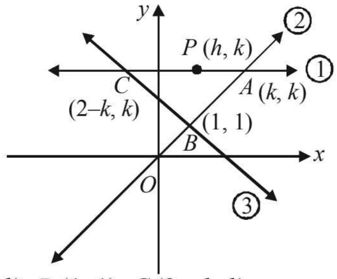
$y = k$. $\ldots(1)$

The other two lines given are

$y = x$ $\ldots(2)$

and $x + y = 2$ $\ldots(3)$

Let ABC be the Δ formed by the points of intersection of the lines (1), (2) and (3), as shown in the figure.



Then $A(k, k)$, $B(1, 1)$, $C(2-k, k)$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2-k & k & 1 \end{vmatrix} = 4h^2$$

Operating $C_1 - C_2$ we get

$$\frac{1}{2}\begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2-2k & k & 1 \end{vmatrix} = 4h^2$$

$\Rightarrow \frac{1}{2}|(2-2k)(k-1)| = 4h^2 \Rightarrow (k-1)^2 = 4h^2$

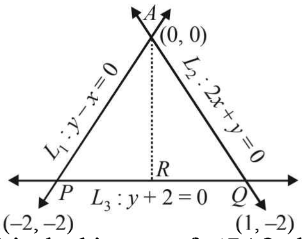
$\Rightarrow k - 1 = 2h$ or $k - 1 = -2h$

$\Rightarrow k = 2h + 1$ or $k = -2h + 1$

$\therefore$ Locus of (h, k) is, $y = 2x + 1$ or $y = -2x + 1$.

H. Assertion & Reason Type Questions

1. **(c)** Point of intersection of L_1 and L_2 is $A(0, 0)$. Also $P(-2, -2)$, $Q(1, -2)$



$\because$ AR is the bisector of $\angle PAQ$, therefore R divides PQ in the same ratio as $AP : AQ$.

Thus $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$

$\therefore$ Statement-1 is true.

Statement-2 is clearly false.

I. Integer Value Correct Type

1. **(6)** Let the point P be (x, y)

Then $d_1(P) = \left|\frac{x-y}{\sqrt{2}}\right|$ and $d_2(P) = \left|\frac{x+y}{\sqrt{2}}\right|$

For P lying in first quadrant $x > 0, y > 0$.

Also $2 \leq d_1(P) + d_2(P) \leq 4$

$$\Rightarrow 2 \leq \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \leq 4$$

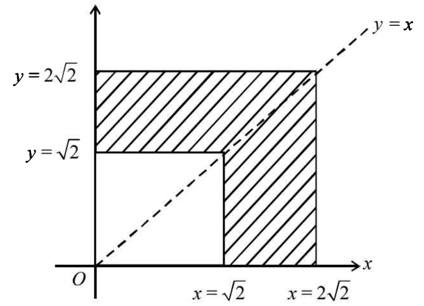
If $x > y$, then $2 \leq \frac{x-y+x+y}{\sqrt{2}} \leq 4$

or $\sqrt{2} \leq x \leq 2\sqrt{2}$

If $x < y$, then

$2 \leq \frac{y-x+x+y}{\sqrt{2}} \leq 4$ or $\sqrt{2} \leq y \leq 2\sqrt{2}$

The required region is the shaded region in the figure given below.



$\therefore$ Required area $= \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 8 - 2 = 6$ sq units.

Section-B JEE Main/ AIEEE

1. **(a)** $AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$;

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$;

In isosceles triangle side $AB = CA$

For right angled triangle, $BC^2 = AB^2 + AC^2$

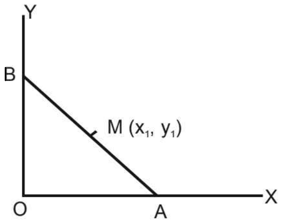
So, here $BC = \sqrt{52}$ or $BC^2 = 52$

or $(\sqrt{26})^2 + (\sqrt{26})^2 = 52$

So, the given triangle is right angled and also isosceles

2. **(d)** Equation of AB is

$x\cos\alpha + y\sin\alpha = p$;



$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$;

$\Rightarrow \frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$

So co-ordinates of A and B are

$\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$;

So coordinates of midpoint of AB are

$\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right) = (x_1, y_1)(let)$;

$x_1 = \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$;

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \frac{p^2}{4}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2}\right) = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

3. **(a)** Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0$.

For unique point of intersection $f^2 - bc = 0$

$\Rightarrow af^2 - abc = 0$.

Since $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 2fgh - bg^2 - ch^2 = 0$

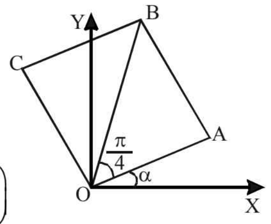
4. **(a)** $3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$.;

$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

5. **(a)** Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$

Equation of OB,

$y = \tan\left(\frac{\pi}{4} + \alpha\right)x$



$CA \perp^r$ to OB

$\therefore$ slope of $CA = -\cot\left(\frac{\pi}{4} + \alpha\right)$

Equation of CA

$y - a\sin\alpha = -\cot\left(\frac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$

$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) = (a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)\left(\frac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right)(a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)(1 + \tan\alpha) = (a\cos\alpha - x)(1 - \tan\alpha)$

$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha) = (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$

$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a$.

6. **(a)** Equation of bisectors of second pair of straight lines

is, $qx^2 + 2xy - qy^2 = 0$ ....(1)

It must be identical to the first pair

$x^2 - 2pxy - y^2 = 0$ ....(2)

from (1) and (2) $\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

7. **(c)** $x = \frac{a\cos t + b\sin t + 1}{3} \Rightarrow a\cos t + b\sin t = 3x - 1$

$y = \frac{a\sin t - b\cos t}{3} \Rightarrow a\sin t - b\cos t = 3y$

Squaring & adding, $(3x-1)^2 + (3y)^2 = a^2 + b^2$

8. **(b)** Taking co-ordinates as

$\left(\frac{x}{r}, \frac{y}{r}\right); (x, y) \,\&\, (xr, yr)$.

Then slope of line joining

$\left(\frac{x}{r}, \frac{y}{r}\right), (x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$

and slope of line joining (x, y) and (xr, yr)

$= \frac{y(r-1)}{x(r-1)} = \frac{y}{x}$ $\therefore m_1 = m_2$

$\Rightarrow$ Points lie on the straight line.

9. **(b)** $(x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$

$(a_1 - a_2)x + (b_1 - b_2)y$

$+ \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$

$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

10. **(d)** Let the vertex C be (h, k), then the centroid of ΔABC is $\left(\frac{2-2+h}{3}, \frac{-3+1+k}{3}\right)$

or $\left(\frac{h}{3}, \frac{-2+k}{3}\right)$. It lies on $2x + 3y = 1$

$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$

= Locus of C is $2x + 3y = 9$

11. **(a)** Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(1)

then $a + b = -1$ ....(2)

(1) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(3)

Eliminating b from (2) and (3), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

12. **(c)** Let the lines be $y = m_1 x$ and $y = m_2 x$ then

$m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$

Given $m_1 + m_2 = 4\, m_1 m_2$

$\Rightarrow \frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

13. **(a)** $3x + 4y = 0$ is one of the lines of the pair

$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$,

we get $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$

$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

14. **(a)** The line passing through the intersection of lines $ax + 2by = 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

As this line is parallel to x-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$\Rightarrow \quad ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow \quad ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

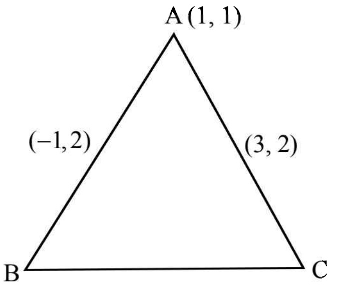
$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3/2 units below x-axis.

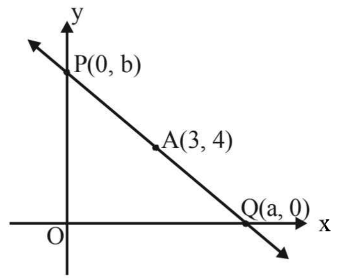
15. **(c)** Vertex of tringle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)



$\Rightarrow$ vertex B and C come out to be (– 3, 3) and (5, 3)

$\therefore$ Centroid is $\frac{1-3+5}{3}, \frac{1+3+5}{3} \Rightarrow \left(1, \frac{7}{3}\right)$

16. **(c)**

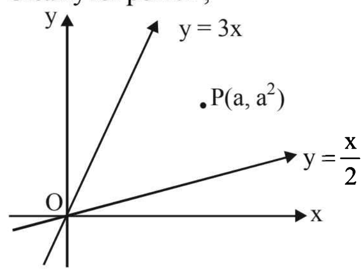


$\because A$ is the mid point of PQ, therefore

$\frac{a+0}{2} = 3, \ \frac{0+b}{2} = 4 \Rightarrow a = 6, b = 8$

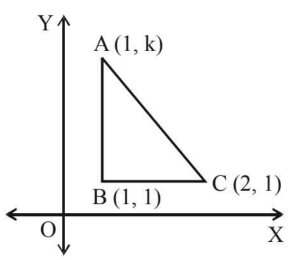
$\therefore$ Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$ or $4x + 3y = 24$

17. **(c)** Clearly for point P,



$a^2 - 3a < 0$ **and** $a^2 - \frac{a}{2} > 0 \Rightarrow \frac{1}{2} < a < 3$

18. **(a)** **Given :** The vertices of a right angled triangle A(l, k), B(1, 1) and C(2, 1) and Area of $\Delta ABC = 1$ square unit

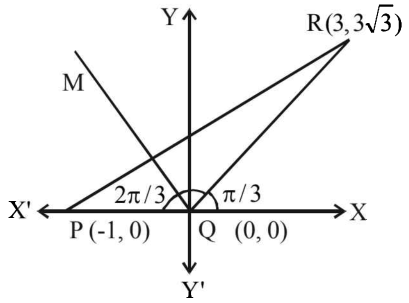


We know that, area of right angled triangle

$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(1)|(k-1)|$

$\Rightarrow \pm(k-1) = 2 \Rightarrow k = -1, 3$

19. (c) **Given :** The coordinates of points P, Q, R are (–1, 0), (0, 0), $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \quad \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

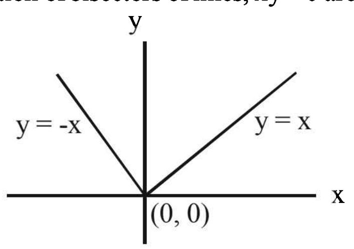
Let QM bisects the $\angle PQR$,

$\therefore$ Slope of the line QM $= \tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y-0) = -\sqrt{3}(x-0)$

$\Rightarrow \quad y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

20. (a) Equation of bisectors of lines, $xy = 0$ are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation

$my^2 + (1-m^2)xy - mx^2 = 0$

$\therefore \quad mx^2 + (1-m^2)x^2 - mx^2 = 0$

$\Rightarrow \quad 1 - m^2 = 0 \Rightarrow m = \pm 1$

21. (d) Slope of $PQ = \frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ = (k –1)

Also mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow \quad 2y - 7 = 2(k-1)x - (k^2 - 1)$

$\Rightarrow \quad 2(k-1)x - 2y + (8 - k^2) = 0$

$\therefore \quad$ y-intercept $= -\frac{8-k^2}{-2} = -4$

$\Rightarrow \quad 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

22. (d) Let (a^2, a) be the point of shortest distance on $x = y^2$

Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \frac{a^2 - a + 1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left[(a - \frac{1}{2})^2 + \frac{3}{4}\right]$$

It is min when $a = \frac{1}{2}$ and $D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

23. (a) If the lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2 x + (p^2+1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore \quad m_1 = m_2 \Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2+1)(p+1) = 0$

$\Rightarrow p = -1$

$\therefore \quad p$ can have exactly one value.

24. (a) Given that

$P(1, 0), Q(-1, 0)$ and $\frac{AP}{AQ} = \frac{BP}{BQ} = \frac{CP}{CQ} = \frac{1}{3}$

$\Rightarrow \quad 3AP = AQ$

Let $A = (x, y)$ then $3AP = AQ \Rightarrow 9AP^2 = AQ^2$

$\Rightarrow \quad 9(x-1)^2 + 9y^2 = (x+1)^2 + y^2$

$\Rightarrow \quad 9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow \quad 8x^2 - 20x + 8y^2 + 8 = 0$

$\Rightarrow \quad x^2 + y^2 - \frac{5}{2}x + 1 = 0 \quad$ ....(1)

$\therefore \quad$ A lies on the circle given by eq (1). As *B* and *C* also follow the same condition, they must lie on the same circle.

$\therefore \quad$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) $= \left(\frac{5}{4}, 0\right)$

25. (c) Slope of line L $= -\frac{b}{5}$

Slope of line K $= -\frac{3}{c}$

Line L is parallel to line *k*.

$\Rightarrow \frac{b}{5} = \frac{3}{c} \Rightarrow bc = 15$

(13, 32) is a point on L.

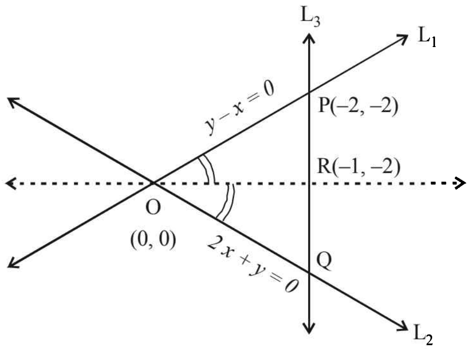
$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5}$

$\Rightarrow b = -20 \Rightarrow c = -\frac{3}{4}$

Equation of K : $y - 4x = 3 \Rightarrow 4x - y + 3 = 0$

Distance between L and K $= \frac{|52 - 32 + 3|}{\sqrt{17}} = \frac{23}{\sqrt{17}}$

26. (b)



$L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$

On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.

On solving the equation of line L_1 and L_3,
we get $P = (-2, -2)$.
Similarly, we get $Q = (-1, -2)$

We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

27. (c) Let the joining points be A (1,1) and B (2,4).
Let point C divides line AB in the ratio 3 : 2.
So, by section formula we have

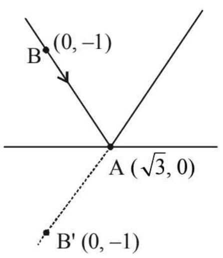
$$C = \left(\frac{3\times 2 + 2\times 1}{3+2}, \frac{3\times 4 + 2\times 1}{3+2}\right) = \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\therefore$ C satisfies the equation $2x + y = k$.

$$\Rightarrow \frac{2+8}{5} + \frac{14}{5} = k \Rightarrow k = 6$$

28. (b) Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of

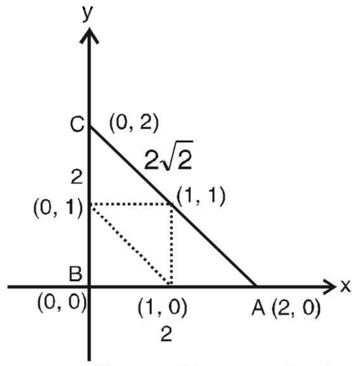


Reflected Ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$

$$\Rightarrow \sqrt{3}y = x - \sqrt{3}$$

29. (b) From the figure, we have

$a = 2, b = 2\sqrt{2}, c = 2$
$x_1 = 0, x_2 = 0, x_3 = 2$



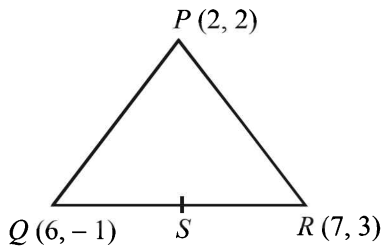
Now, x-co-ordinate of incentre is given as

$$\frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$\Rightarrow x\text{-coordinate of incentre} = \frac{2\times 0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}}$$

$$= \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$$

30. (d) Let P, Q, R, be the vertices of ΔPQR



Since PS is the median, S is mid-point of QR

So, $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of $PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

Since, required line is parallel to PS therefore slope of required line = slope of PS Now, eqn of line passing through $(1, -1)$ and having slope $-\frac{2}{9}$ is

$$y - (-1) = -\frac{2}{9}(x-1)$$

$$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$$

31. (a) Given lines are

$4ax + 2ay + c = 0$
$5bx + 2by + d = 0$

The point of intersection will be

$$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$$

$$\Rightarrow x = \frac{2(ad-bc)}{-2ab} = \frac{bc-ad}{ab}$$

$$\Rightarrow y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$$

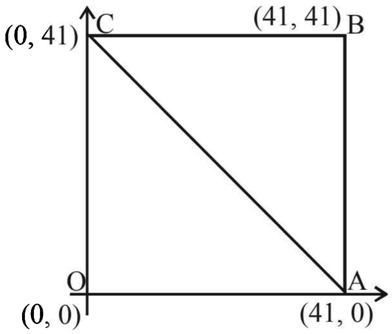
$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.
Also distance from axes is same
So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)

$$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab}$$

$\Rightarrow \; 3bc - 2ad = 0$

32. (b) Total number of integral points inside the square OABC $= 40 \times 40 = 1600$

No. of integral points on AC



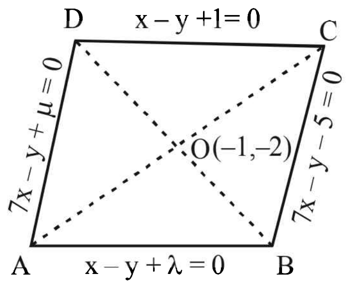
= No. of integral points on OB

= 40 [namely (1, 1), (2, 2) ... (40, 40)]

$\therefore$ No. of integral points inside the ΔOAC

$$= \frac{1600 - 40}{2} = 780$$

33. (a)



Let other two sides of rhombus are
$x - y + \lambda = 0$
and $7x - y + \mu = 0$
then O is equidistant from AB and DC and from AD and BC

$\therefore \; |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda = -3$

and $|-7+2-5| = |-7+2+\mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$
On solving the eq^{n}s of sides pairwise, we get

the vertices as $\left(\frac{1}{3}, \frac{-8}{3}\right), (1, 2), \left(\frac{-7}{3}, \frac{-4}{3}\right), (-3, -6)$