

Chapter – 06

Squares and Square Roots

Exercises 6.3

Question 1. What could be the possible ‘one’s’ digits of the square root of each of the following numbers?

(i) 9801 (ii) 99856

(iii) 998001 (iv) 657666025

Answer: We need to find the number's unit digit whose square are given to us.

For example square root of 400 is 20 and thus the unit digit of 20 is 0. We need to know some properties. When unit digit of a number is 1, The square of the number will also contain 1 as the unit digit. When unit digit of the number is 2, the square of the number will contain 4 as the unit digit. When unit digit of the number is 3, the square of the number will contain 9 as the unit digit. When unit digit of the number is 4, the square of the number will contain 6 as the unit digit. When unit digit of the number is 5, the square of the number will also contain 5 as the unit digit. When unit digit of the number is 6, The square of the number will contain 6 as the unit digit. When unit digit of the number is 7, the square of the number will contain 9 as the unit digit. When unit digit of the number is 8, the square of the number will contain 4 as the unit digit. When unit digit of the number is 9, the square of the number will contain 1 as the unit digit. When unit digit of the number is 0, the square of the number will contain 0 as the unit digit. (i) If the number ends with 1, then the one's digit of the

square root of that number may be 1 or 9. Hence, one's digit of the square root of 9801 is either 1 or 9

(ii) If the number ends with 6, then the one's digit of the square root of that number may be 4 or 6. Hence, one's digit of the square root of 99856 is either 4 or 6

(iii) If the number ends with 1, then the one's digit of the square root of that number may be 1 or 9. Hence, one's digit of the square root of 998001 is either 1 or 9

(iv) If the number ends with 5, then the one's digit of the square root of that number will be 5. Hence, the one's digit of the square root of 657666025 is 5

Question 2. Without doing any calculation, find the numbers which are surely not perfect squares.

(i) 153 (ii) 257

(iii) 408 (iv) 441

Answer:

The perfect squares of a number may end with any of the following digits at unit's place:

0, 1, 4, 5, 6, or 9

And, a perfect square will end with even number of zeroes

(i) Since the unit place digit of number 153 is 3, hence it is not a perfect square

(ii) Since the unit place digit of number 257 is 7, hence it is not a perfect square

(iii) Since the number 408 has its unit place digit as 8, it is not a perfect square

(iv) Since the number 441 has its unit place digit as 1, it is a perfect square

Question 3. Find the square roots of 100 and 169 by the method of repeated subtraction

Answer: The square root of 100 can be obtained by the method of repeated subtraction as:

$$(i) 100 - 1 = 99 \quad (ii) 99 - 3 = 96 \quad (iii) 96 - 5 = 91$$

$$(iv) 91 - 7 = 84 \quad (v) 84 - 9 = 75 \quad (vi) 75 - 11 = 64$$

$$(vii) 64 - 13 = 51 \quad (viii) 51 - 15 = 36 \quad (ix) 36 - 17 = 19$$

$$(x) 19 - 19 = 0$$

In this we have subtracted successive odd numbers starting from 1 to 100, and obtained 0 at 10th step

Therefore,

$$\sqrt{100} = 10$$

The square root of 169 can be obtained by the method of repeated subtraction as:

$$(i) 169 - 1 = 168 \quad (ii) 168 - 3 = 165 \quad (iii) 165 - 5 = 160$$

$$(iv) 160 - 7 = 153 \quad (v) 153 - 9 = 144 \quad (vi) 144 - 11 = 133$$

$$(vii) 133 - 13 = 120 \quad (viii) 120 - 15 = 105 \quad (ix) 105 - 17 = 88$$

$$(x) 88 - 19 = 69 \quad (xi) 69 - 21 = 48 \quad (xii) 48 - 23 = 25$$

$$(xiii) 25 - 25 = 0$$

We have subtracted successive odd numbers starting from 1 to 169, and obtained 0 at 13th step

Hence,

$$\sqrt{169} = 13$$

Question 4. Find the square roots of the following numbers by the Prime Factorization Method

(i) 729 (ii) 400

(iii) 1764 (iv) 4096

(v) 7744 (vi) 9604

(vii) 5929 (viii) 9216

(ix) 529 (x) 8100

Answer:

(i) The prime factorization of 729 is as follows:

$$729 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

Now, Square root of 3×3 is 3, Taking square root both side, we get

$$\sqrt{729} = 3 * 3 * 3$$

$$= 27$$

(ii) The prime factorization of 400 is as follows:

$$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$

Now, Square root of 2×2 is 2 and 5×5 is 5, Taking square root both side, we get

$$\sqrt{400} = 2 * 2 * 5$$

$$= 20$$

(iii) The prime factorization of 1764 is as follows:

$$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Now, Square root of 2×2 is 2, 3×3 and 7×7 is 7, Taking square root both side, we get

$$\sqrt{1764} = 2 * 3 * 7$$

$$= 42$$

(iv) The prime factorization of 4096 is as follows:

$$4096 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$$

Now, Square root of 2×2 is 2, Taking square root both side, we get

$$\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 64$$

(v) The prime factorization of 7744 is as follows:

$$7744 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{11 \times 11}$$

Now, Square root of 2×2 is 2 and 11×11 is 11, Taking square root both side, we get

$$\sqrt{7744} = 2 \times 2 \times 2 \times 11$$

$$= 8$$

(vi) The prime factorization of 9604 is as follows:

$$9604 = \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}$$

Now, Square root of 2×2 is 2 and 7×7 is 7, Taking square root both side, we get

$$\sqrt{9604} = 2 \times 7 \times 7$$

$$= 98$$

(vii) The prime factorization of 5929 is as follows:

$$5929 = 11 \times 11 \times 7 \times 7$$

Now, Square root of 11×11 is 11 and 7×7 is 7, Taking square root both side, we get

$$\sqrt{5929} = 11 \times 7$$

$$= 77$$

(viii) 9216 can be factorized as follows:

$$9216 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$$

$$\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 396$$

(ix) The prime factorization of 529 is as follows:

$$529 = \underline{23 \times 23}$$

$$\sqrt{529} = 23$$

(x) The prime factorization of 8100 is as follows:

$$8100 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}$$

$$\sqrt{8100} = 2 \times 5 \times 3 \times 3$$

$$= 90$$

Question 5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252 (ii) 180

(iii) 1008 (iv) 2028

(v) 1458 (vi) 768

Answer:

(i) The prime factorization of 252 is follows:

$$252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$$

Here,

Prime factor 7 does not have its pair.

If 7 gets a pair, then the number will become a perfect square.

Therefore, 252 has to be multiplied with 7 to obtain a perfect square

$$252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Hence, $252 \times 7 = 1764$ is a perfect square

$$\sqrt{1764} = 2 * 3 * 7$$

$$= 42$$

(ii) The prime factorization of 180 is as follows:

$$180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$$

Here, prime factor 5 does not have its pair.

If 5 gets a pair, then the number will become a perfect square.

Therefore, 180 has to be multiplied with 5 to obtain a perfect square.

$$180 \times 5 = 900 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}$$

Therefore,

$180 \times 5 = 900$ is a perfect square

$$\sqrt{900} = 2 * 3 * 5$$

$$= 30$$

(iii) The prime factorization of 1008 is as follows:

$$1008 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 7$$

Here, prime factor 7 does not have its pair.

If 7 gets a pair, then the number will become a perfect square.

Therefore, 1008 can be multiplied with 7 to obtain a perfect square.

$$1008 \times 7 = 7056 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times 7$$

Therefore,

$1008 \times 7 = 7056$ is a perfect square

$$\sqrt{7056} = 2 * 2 * 3 * 7$$

$$= 84$$

(iv) The prime factorization of 2028 is as follows:

$$2028 = \underline{2 * 2} * 3 * \underline{13 * 13}$$

Here, prime factor 3 does not have its pair. If 3 gets a pair, then the number will become a perfect square. Therefore, 2028 can be multiplied with 3 to obtain a perfect square.

$$2028 \times 3 = 6084 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{13 \times 13}$$

Therefore,

$2028 \times 3 = 6084$ is a perfect square

$$\sqrt{6084} = 2 * 3 * 13$$

$$= 78$$

(v) The prime factorization of 1458 is as follows:

$$1458 = 2 * \underline{3 * 3} * \underline{3 * 3} * \underline{3 * 3}$$

Here, prime factor 2 does not have its pair.

If 2 gets a pair, then the number will become a perfect square.

Therefore, 1458 can be multiplied with 2 to obtain a perfect square.

$$1458 \times 2 = 2916 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

Therefore,

$1458 \times 2 = 2916$ is a perfect square

$$\sqrt{2916} = 2 * 3 * 3 * 3$$

$$= 54$$

(vi) The prime factorization of 768 is as:

$$768 = \underline{2 * 2} * \underline{2 * 2} * \underline{2 * 2} * \underline{2 * 2} * 3$$

Here, prime factor 3 does not have its pair.

If 3 gets a pair, then the number will become a perfect square.

Therefore, 768 can be multiplied with 3 to obtain a perfect square.

$$768 \times 3 = 2304 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$$

Therefore,

$768 \times 3 = 2304$ is a perfect square

$$\sqrt{2304} = 2 * 2 * 2 * 2 * 3$$

$$= 48$$

Question 6.

For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

(i) 252 (ii) 2925

(iii) 396 (iv) 2645

(v) 2800 (vi) 1620

Answer:

(i) 252 can be factorized as follows:

$$252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$$

Here, prime factor 7 does not have its pair

If we divide this number by 7, then the number will become a perfect square.

Therefore, 252 has to be divided by 7 to obtain a perfect square

$$\frac{252}{7} = 36 \text{ is a perfect square}$$

$$36 = \underline{2 \times 2} \times \underline{3 \times 3}$$

$$\sqrt{36} = 2 * 3$$

$$= 6$$

(ii) The prime factorization of 2925 is as follows:

$$2925 = \underline{3 \times 3} \times \underline{5 \times 5} \times 13$$

Here, prime factor 13 does not have its pair

If we divide this number by 13, then the number will become a perfect square.

Therefore, 2925 has to be divided by 13 to obtain a perfect square

$$\frac{2925}{13} = 225 \text{ is a perfect square}$$

$$225 = \underline{3 \times 3} \times \underline{5 \times 5}$$

$$\sqrt{225} = 3 * 5$$

$$= 15$$

(iii) The prime factorization of 396 is as follows:

$$396 = 2 \times 2 \times 3 \times 3 \times 11$$

Here, prime factor 11 does not have its pair

If we divide this number by 11, then the number will become a perfect square.

Therefore, 396 has to be divided by 11 to obtain a perfect square

is a perfect square

$$\frac{396}{11} = 36 \text{ is a perfect square}$$

$$36 = \underline{2 \times 2} \times \underline{3 \times 3}$$

$$\sqrt{36} = 2 * 3$$

$$= 6$$

(iv) The prime factorization of 2645 is as follows:

$$2645 = 5 \times \underline{23 \times 23}$$

Here, prime factor 5 does not have its pair

If we divide this number by 5, then the number will become a perfect square. Therefore, 2645 has to be divided by 5 to obtain a perfect square

$$\frac{2645}{5} = 529 \text{ is a perfect square}$$

$$529 = \underline{23 \times 23}$$

$$\sqrt{529} = 23$$

$$= 23$$

(v) 2800 can be factorized as follows:

$$2800 = \underline{2 \times 2} \times 7 \times \underline{10 \times 10}$$

Here, prime factor 7 does not have its pair

If we divide this number by 7, then the number will become a perfect square. Therefore, 2800 has to be divided by 7 to obtain a perfect square

is a perfect square

$$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$

$$\sqrt{400} = 2 \times 2 \times 5$$

$$= 20$$

(vi) The prime factorization of 1620 is as follows:

$$1620 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times 5$$

Here, prime factor 5 does not have its pair

If we divide this number by 5, then the number will become a perfect square.

Therefore, 1620 has to be divided by 5 to obtain a perfect square

is a perfect square

$$\frac{1620}{5} = 324 \text{ is a perfect square}$$

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

$$\sqrt{324} = 2 * 3 * 3$$

$$= 18$$

Question 7. The students of Class VIII of a school donated Rs 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Answer: Let the number of students be x .

Each student donated as many rupees as the number of students of the class.

The donation amount $w = \text{no. of students} \times \text{amount of donation}$

$$2401 = x(x) \quad 2401 = x^2$$

Hence,

$$\text{Number of students in the class} = \sqrt{2401}$$

$$2401 = \underline{7 \times 7} \times \underline{7 \times 7}$$

$$\sqrt{2401} = 7 * 7$$

$$= 49$$

Hence, the number of students in the class is 49.

Question 8.

2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row

Answer: Given:

In the garden, each row contains as many plants as the number of rows.

Hence,

Number of rows = Number of plants in each row

Total number of plants = Number of rows \times Number of plants in each row

Number of rows \times Number of plants in each row = 2025

$$(\text{Number of rows})^2 = 2025$$

$$\text{Number of rows} = \sqrt{2025}$$

$$2025 = \underline{5 * 5} * \underline{3 * 3} * \underline{3 * 3}$$

$$\sqrt{2025} = 5 * 3 * 3$$

$$= 45$$

Hence, the number of rows and the number of plants in each row is 45

Question 9.

Find the smallest square number that is divisible by each of the numbers 4, 9 and 10

Answer:

We know that the number that will be perfectly divisible by each one of 4, 9, and 10 is their LCM.

The LCM of these numbers is:

$$\text{LCM of 4, 9, 10} = \underline{2 \times 2} \times \underline{3 \times 3} \times 5 = 180$$

Here, prime factor 5 does not have its pair

Therefore, 180 is not a perfect square.

If we multiply 180 with 5, then the number will become a perfect square.

Hence,

The required square number is $180 \times 5 = 900$

Question 10.

Find the smallest square number that is divisible by each of the numbers 8, 15 and 20

Answer:

We know that the number that is perfectly divisible by each of the numbers 8, 15, and 20 is their LCM.

Therefore,

LCM of 8, 15, and 20 is:

$$\underline{2 \times 2 \times 2 \times 3 \times 5} = 120$$

Here, prime factors 2, 3, and 5 do not have their respective pairs.

Therefore, 120 is not a perfect square

Therefore, 120 should be multiplied by $2 \times 3 \times 5$, i.e. 30, to obtain a perfect square

Hence,

$$\begin{aligned} \text{The required square number} &= 120 \times 2 \times 3 \times 5 \\ &= 3600 \end{aligned}$$