ELECTROMAGNETIC INDUCTION

Contents

JEE (Advance) Syllabus

Electro Magnetic induction : Faraday's law, Lenz's law; Self and mutual inductance; RC, LR and LC circuits with d.c. and a.c. sources.

JEE (Main) Syllabus

Electromagnetic induction; Faraday’s law, induced emf and current; Lenz’s Law, Eddy currents. Self and mutual inductance.

Note: ✍ Marked Questions can be used for Revision.

ELECTROMAGNETIC INDUCTION

1. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

(i) When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf. If the circuit is closed then the current will be called induced current.

$$\text{magnetic flux} = \int \vec{B}.d\vec{s}$$

(ii) The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

For a coil that consists of *N* loops, the total induced emf would be *N* times as large:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.
SI unit of magnetic flux = Weber.

Thus, we see that an emf may be induced in the following ways:

(a) By varying the magnitude of $\vec{B}$ with time (illustrated in Figure)

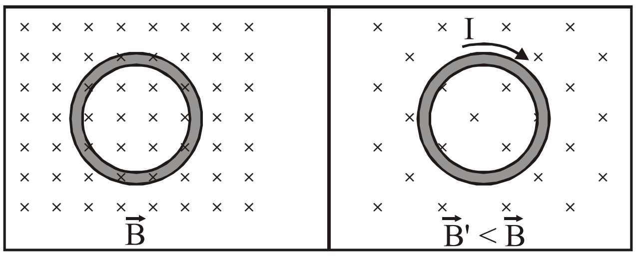


Figure : Inducing emf by varying the magnetic field strength

(b) By varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)

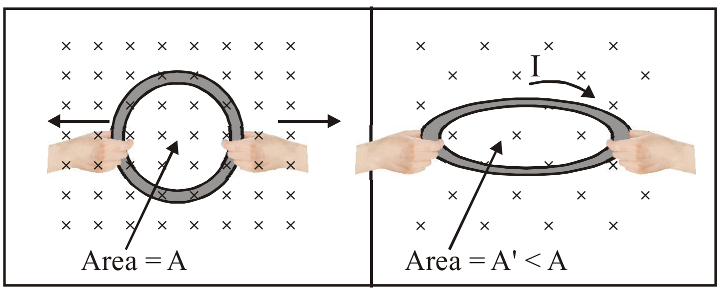


Figure : Inducing emf by changing the area of the loop

(c) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)

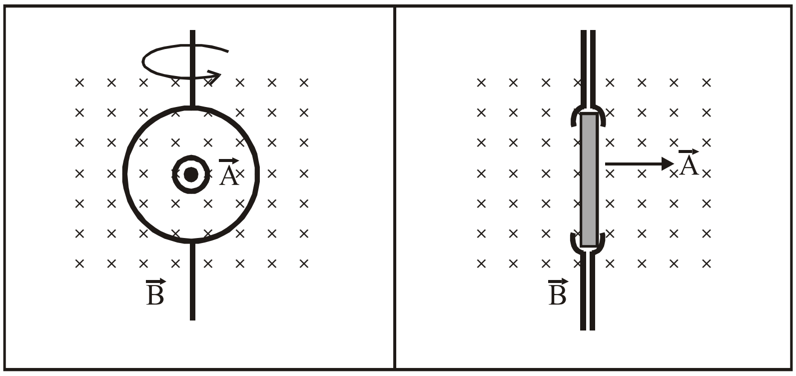
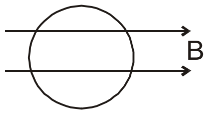


Figure : Inducing emf by varying the angle between **B** and **A**

SOLVED EXAMPLE

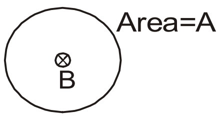
Example 1. A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil .



Solution : $\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ emf = 0

Example 2. Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.

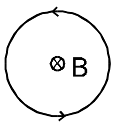


Solution : $\phi = BA$ (always)

= const.

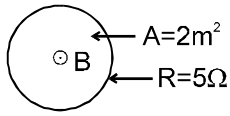
$\therefore$ emf = 0

Example 3. Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



Solution : Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

Example 4. Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction

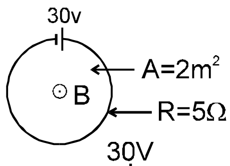


Solution : $\phi = B.A$

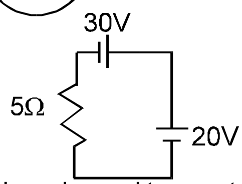
$$\text{emf} = A \cdot \frac{dB}{dt} = 2 \times 10 = 20 \text{ v}$$

$\therefore$ i = 20/ 5 = 4 amp. From Lenz's law direction of current will be anticlockwise.

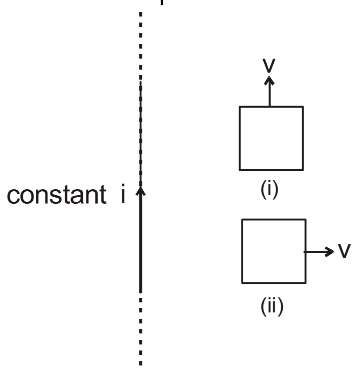
Example 5. Figure shows a coil placed in a magnetic field decreasing at a rate of 10T/s. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.



Solution : Induce emf = 20V
equivalent
i = 2A clockwise



Example 6. Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



Solution : In loop (i) no emf will be induced because there is no flux change.
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

Example 7. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m² acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the 1μV

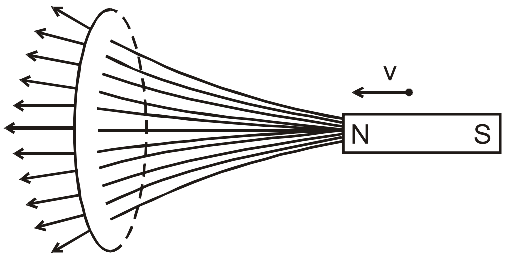
Solution : Induced emf $e = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r \frac{dr}{dt}$ radius of coil

$$r = \frac{e}{2B\pi\left(\frac{dr}{dt}\right)} = \frac{1 \times 10^{-6}}{2 \times 10^{-3} \times \pi \times 10^{-2}} = \frac{5}{\pi} \text{ cm}$$

2 LENZ'S LAW (CONSERVATION OF ENERGY PRINCIPLE)

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

Figure shows a magnet approaching a ring with its north pole towards the ring.



We know that magnetic field lines come out of the north pole and magnetic field intensity decreases as we move away from magnet. So the magnetic flux (here towards left) will increase with the approach of magnet. This is the cause of flux change. To oppose it, induced magnetic field will be towards right. For this the current must be anticlockwise as seen by the magnet.

If we consider the approach of North pole to be the cause of flux change, the Lenz's law suggests that the side of the coil towards the magnet will behave as North pole and will repel the magnet. We know that a current carrying coil will behave like North pole if current flows anticlockwise. Thus as seen by the magnet, the current will be anticlockwise.

If we consider the approach of magnet as the cause of the flux change, Lenz's law suggest that a force opposite to the motion of magnet will act on the magnet, whatever be the mechanism.

Lenz's law tells that if the coil is set free, it will move away from magnet, because in doing so it will oppose the 'approach' of magnet.

If the magnet is given some initial velocity towards the coil and is released, it will slow down .It can be explained as the following .

The current induced in the coil will produce heat. From the energy conservation, if heat is produced there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus the magnet must slow down. So we can justify that the **Lenz's law is conservation of energy principle**.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{A}$.

2. Assuming that $\vec{B}$ is uniform, take the dot product of $\vec{B}$ and $\vec{A}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} : \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure.

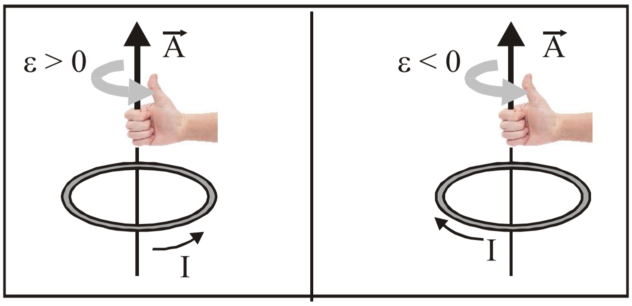


Figure : Determination of the direction of induced current by the right-hand rule

In Figure we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current *I*.

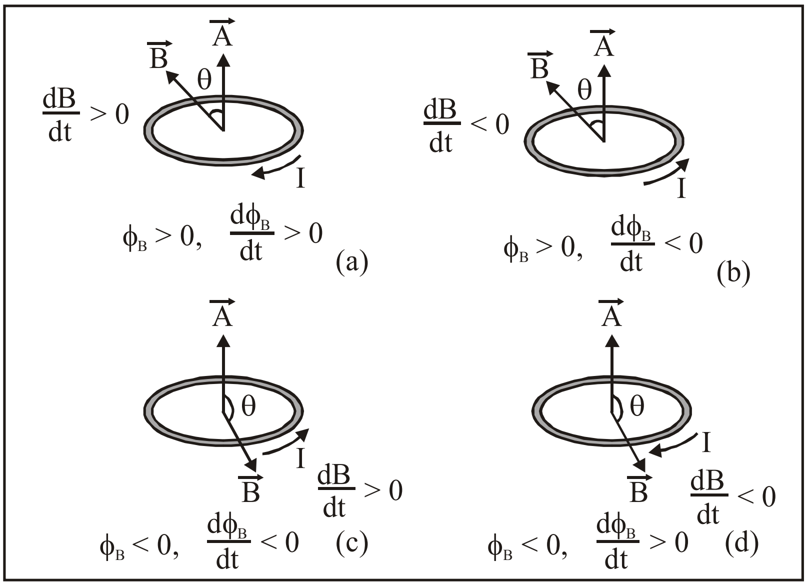
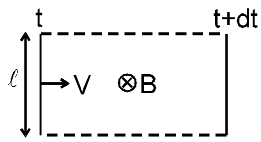


Figure : Direction of the induced current using Lenz's law

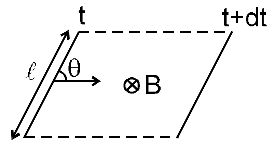
3. MOTIONAL EMF

We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length ℓ moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to ℓv and hence it will cut B ℓ v lines per unit time.



Hence emf induced between the ends of the rod = $Bv\ell$

Also emf = $\frac{d\phi}{dt}$. Here ϕ denotes flux passing through the area,swept by the rod.The rod sweeps an area equal to ℓ vdt in time interval dt.Flux through this area = Bℓ vdt. Thus $\frac{d\phi}{dt} = \frac{B\ell vdt}{dt} = Bv\ell$



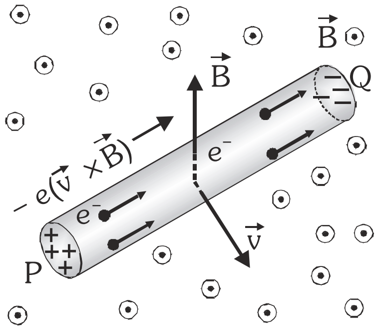
If the rod is moving as shown in the following figure,

it will sweep area per unit time = v ℓ sinθ

and hence it will cut B v ℓ sinθ lines per unit time.

Thus emf = $Bv\ell\sin\theta$.

EXPLANATION OF EMF INDUCED IN ROD ON THE BASIS OF MAGNETIC FORCE:



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = -e(\vec{v}\times\vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative chagre accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = -e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

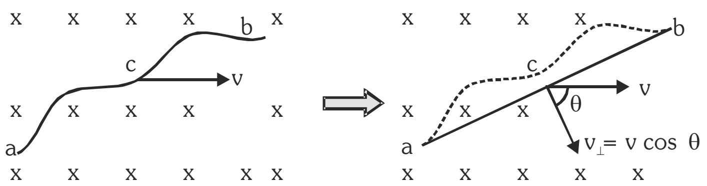
so $\vec{F}_m = -\vec{F}_e \Rightarrow -e(\vec{v}\times\vec{B}) = e\vec{E} \Rightarrow \vec{E} = -(\vec{v}\times\vec{B})$

The potential difference between the ends P and Q is V = $\vec{E}.\vec{\ell}$ = $(\vec{v}\times\vec{B}).\vec{\ell}$ .It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E}$ = B ℓ v (for $\vec{B}\perp\vec{v}\perp\vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution '$d\varepsilon$' to the emf is the magnitude dℓ multiplied by the component of $\vec{v}\times\vec{B}$ parallel to $\vec{d\ell}$, that is $d\varepsilon = (\vec{v}\times\vec{B}).\vec{d\ell}$

For any two points a and b the motional emf in the direction from b to a is,

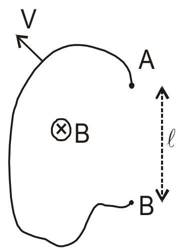


$$\varepsilon = \int_b^a (\vec{v}\times\vec{B}).\vec{d\ell}$$

Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $\varepsilon_{acb} = \varepsilon_{ab}$ = (length of ab) ($v_\perp$) (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a$ = (ab) (v cosθ) (B)

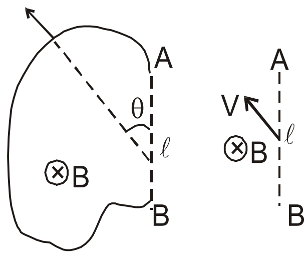
SOLVED EXAMPLE

Example 8. Figure shows an irregular shaped wire AB moving with velocity v, as shown.



Find the emf induced in the wire.

Solution : The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin \theta$



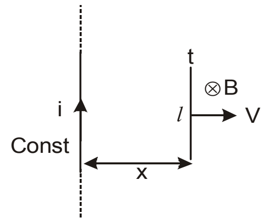
Example 9. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

Solution : $E = B\,l\,V = \dfrac{\mu_0 i l v}{2\pi x}$

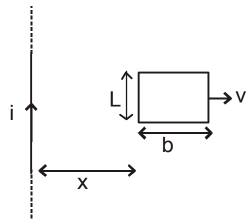
OR

Emf is equal to the rate with which magnetic field lines are cut . In dt time the area swept by the rod is l v dt. The magnetic field lines cut in dt time $= B\,l\,v dt = \dfrac{\mu_0 i l v dt}{2\pi x}$.

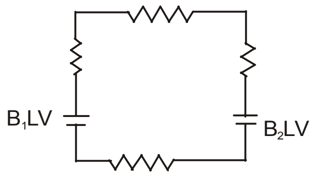
$\therefore$ The rate with which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$



Example 10. A rectangular loop ,as shown in the figure ,moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



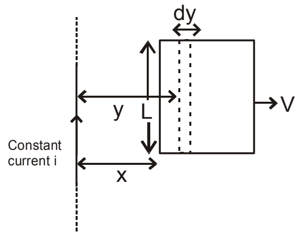
Solution : $E = B_1 LV - B_2Lv = \frac{\mu_0 i}{2\pi x}Lv - \frac{\mu_0 i}{2\pi(x+b)}Lv = \frac{\mu_0 iLbv}{2\pi x(x+b)}$



Aliter:

Consider a small segment of width dy at a distance y from the wire.

Let flux through the segment be $d\phi$.



$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} L\,dy$

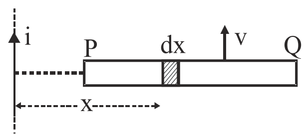
$\therefore \quad \phi = \frac{\mu_0 iL}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 iL}{2\pi}\left(\ell n(x+b) - \ell nx\right)$

Now $\frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right] = \frac{\mu_0 iL}{2\pi}\left[\frac{(-b)}{x(x+b)}\right] v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$

$\therefore \quad$ induced emf $= \frac{\mu_0 ibLv}{2\pi x(x+b)}$

Example 11. A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i, the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take v = 5.0 m/s, i = 100 amp, r = 1.0 cm and L = 19 cm.

Solution : The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B\,v\,dx = \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$$

$\therefore$ emf $\mathcal{E}$ is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$

Now x = r at P, and x = r + L at Q. hence

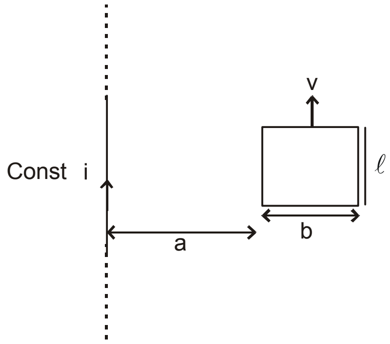
$$\mathcal{E} = \frac{\mu_0 i v}{2\pi}\int_r^{r+L}\frac{dx}{x} = \frac{\mu_0 i v}{2\pi}\left[\log_e x\right]_r^{r+L}$$

$$= \frac{\mu_0 i v}{2\pi}[\log_e(r+L) - \log_e r] = \frac{\mu_0 i v}{2\pi}\log\frac{r+L}{r}$$

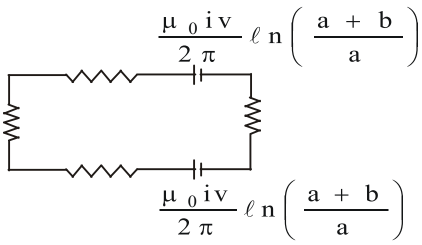
Putting the given values :

$$\mathcal{E} = (2 \times 10^{-7})(100)(5.0)\log_e \frac{1.0+19}{1.0} = 10^{-4}\log_e 20 \text{ Wb/s} = 3 \times 10^{-4} \text{ volt}$$

Example 12. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop , if its nearest end is at a distance 'a' from the wire. Draw equivalent electrical diagram.

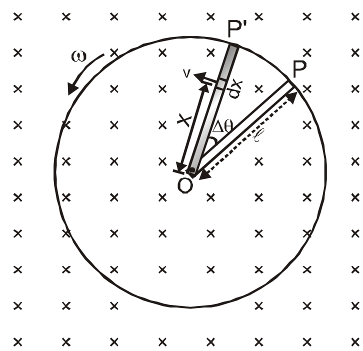


Solution : emf = 0 ;



4. INDUCED EMF DUE TO ROTATION

ROTATION OF THE ROD



Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element

$$d\varepsilon = B\, v.\, dx$$

$\because$ This small element dx is at distance x from O (axis of rotation)

$\therefore$ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eq^n (i) $d\varepsilon = B\,\omega\, x\, dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega\ x\ dx = \omega\ B\left(\frac{x^2}{2}\right)_0^{\ell}$

or $\quad \varepsilon = \frac{1}{2}B\omega\ell^2$

SOLVED EXAMPLE

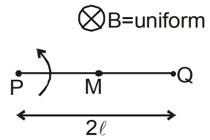
Example 13. A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's mangetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

Solution : $\omega = 2\pi n = 2\pi\times\frac{120}{60} = 4\pi$, B = 0.4 G = 4 × 10^{-5} T, length of each spoke = 0.5 m

induced emf $e = \frac{1}{2}B\,\omega\,\ell^2 = \frac{1}{2}\times 4\times 10^{-5}\times 4\pi\times(0.5)^2 = 6.28\times 10^{-5}$ volt

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Example 14. A rod PQ of length 2l is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .

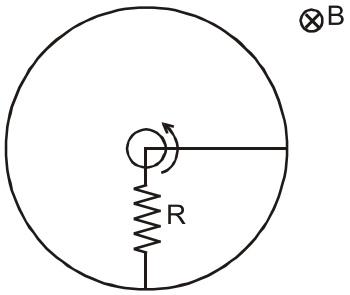


Solution : $E_{MQ} + E_{PM} = E_{PQ}$

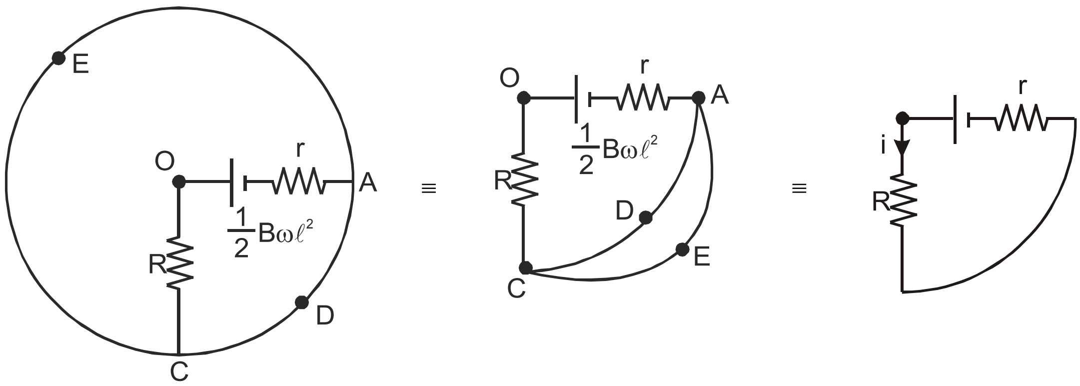
$E_{PQ} = \frac{Bw(2\ell)^2}{2} = 100$

$E_{MQ} + \frac{B\omega(\ell)^2}{2} = \frac{B\omega(2\ell)^2}{2} \quad \Rightarrow \quad E_{MQ} = \frac{3}{2}B\omega\ell^2 = \frac{3}{2}\times 50\text{ V} = 75\text{ V}$

Example 15. A rod of length ℓ and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

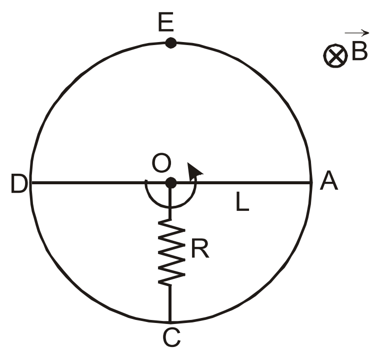


Solution :

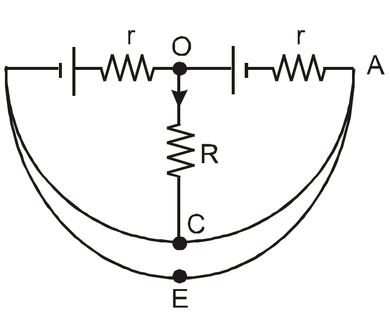


current $i = \dfrac{\frac{1}{2}B\omega\ell^2}{R+r}$

Example 16. Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring



Solution :



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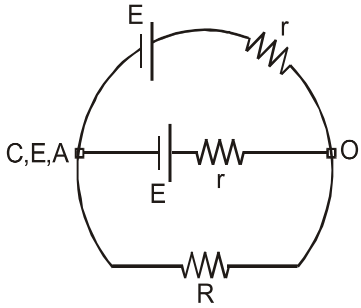

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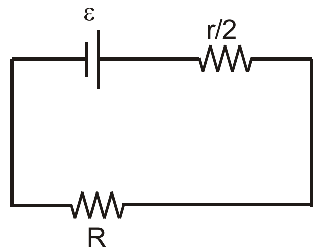
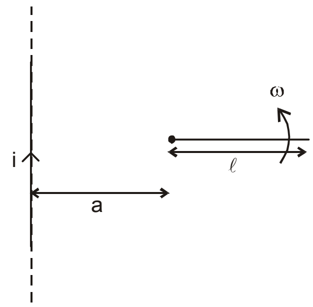
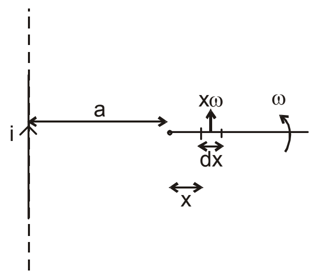


$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Example 17. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.



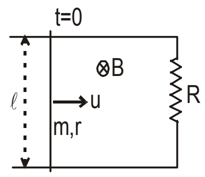
Solution : Consider a small segment of rod of length dx , at a distance x from one end of the rod. Emf induced in the segment



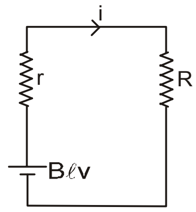
$$dE = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$\therefore\ E = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a.\ell n\left(\frac{\ell + a}{a}\right)\right].$$

Example 17. A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u .Find its velocity as a function of time.

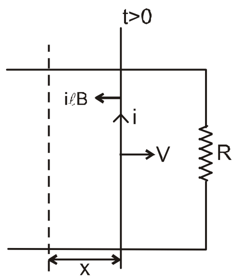


Solution : Let at an instant the velocity of the rod be v .The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \frac{B\ell v}{R+r}$

At time t



Magnetic force acting on the rod is F = $i\,\ell\,B$, opposite to the motion of the rod.

$$i\ell B = -m\frac{dV}{dt} \qquad ...(1)$$

$$i = \frac{B\ell v}{R+r} \qquad ...(2)$$

Now solving these two equation

$$\frac{B^2\ell^2 V}{R+r} = -m\,.\,\frac{dV}{dt}$$

$$-\frac{B^2\ell^2}{(R+r)m}\,.\,dt = \frac{dV}{V}$$

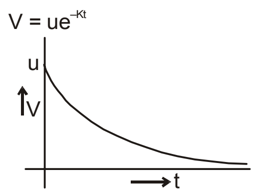
let $\frac{B^2\ell^2}{(R+r)m} = k$

$$-K\,.\,dt = \frac{dV}{V}$$

$$\int_u^v \frac{dV}{V} = \int_0^t -K.dt$$

$$\ln\left(\frac{v}{u}\right) = -Kt$$

$V = ue^{-Kt}$



Example 18. In the above question find the force required to move the rod with constant velocity v, and also find the power delivered by the external agent .

Solution : The force needed to keep the velocity constant

$$F_{ext} = i\ell B = \frac{B^2\ell^2 v}{R+r}$$

Power due to external force $= \frac{B^2\ell^2 v^2}{R+r} = \frac{\varepsilon^2}{R+r} = i^2(R+r)$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

Example 19. In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Solution :

$$m\frac{dv}{dt} = F - i\,\ell\,B \qquad ....(1)$$

$$i = \frac{B\ell v}{R+r} \qquad ....(2)$$

$$m\frac{dv}{dt} = F - \frac{B^2\ell^2 v}{R+r}$$

$$\text{let } K = \frac{B^2\ell^2}{R+r}$$

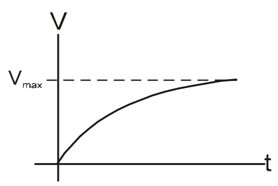
$$\int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$$

$$-\frac{1}{K}\left[\ell n(F-KV)\right]_0^V = \frac{t}{m}$$

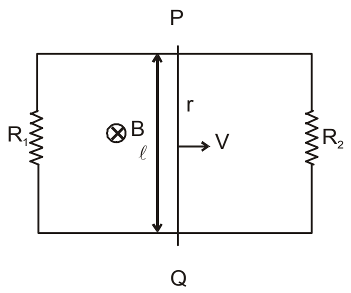
$$\ell n\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m}$$

$$F - KV = F\,e^{-kt/m}$$

$$V = \frac{F}{K}\,(1 - e^{-kt/m})$$

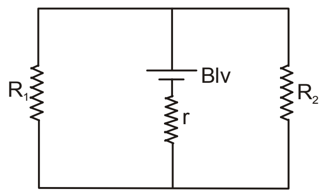


Example 20. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2) .Find the current in the rod at the instant its velocity is v.



Solution : $$i = \frac{B\ell V}{r + \frac{R_1R_2}{R_1 + R_2}}$$

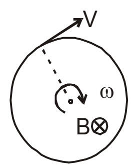
this circuit is equivalent to the following diagram.



EMF INDUCED DUE TO ROTATION OF A COIL

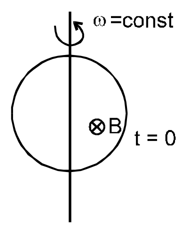
SOLVED EXAMPLE

Example 21. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring



Solution : Flux passing through the ring ϕ = B.A is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

Example 22. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



Solution : At any time t , $\phi = BA\cos\theta \quad = BA\cos\omega t$

Now induced emf in the loop

$$e = \frac{-d\phi}{dt} = BA\,\omega\sin\omega t$$

If there are N turns

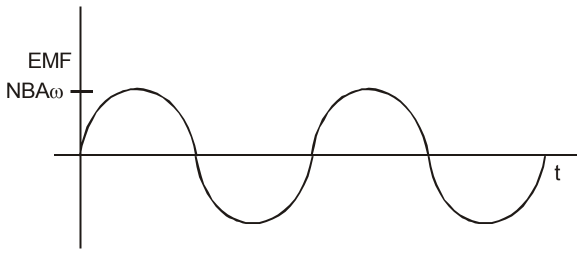
emf = $BA\omega N\sin\omega t$

BA ωN is the amplitude of the emf

$e = e_m\sin\omega t$

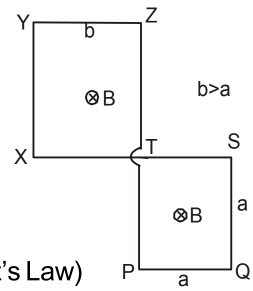
$$i = \frac{e}{R} = \frac{e_m}{R}\ \sin\omega t = i_m\sin\omega t$$

$$i_m = \frac{e_m}{R}$$



The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Example 23. Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B=\beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.

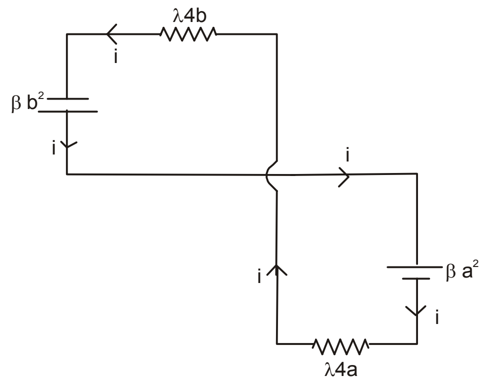


Solution : Induced emf in part PQST = βa^2 (in anticlockwise direction, from Lenz's Law)

Similarly Induced emf in part TXYZ = βb^2 (in anticlockwise direction, from Lenz's Law)

Total resistance of the part PQST =$\lambda 4a$.

Total resistance of the part TXYZ = $\lambda 4b$. The equivalent circuit is as shown in the following diagram.



writing KVL along the current flow

$\beta b^2 - \beta a^2 - \lambda\,4ai - \lambda\,4bi = 0$

$$i = \frac{\beta}{4\lambda}\ (b-a)$$

EMF INDUCED IN A ROTATING DISC :

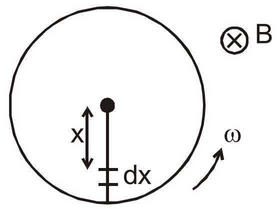
Consider a disc of radius r rotating in a magnetic field B.

Consider an element dx at a distance x form the centre. This element is moving with speed $v = \omega x$.

$\therefore$ Induced emf across dx

$$= B(dx)\,v = B(dx)\omega x = B\omega x dx$$

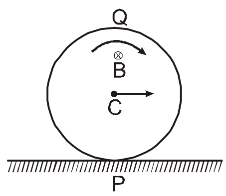
$\therefore$ emf between the centre and the edge of disc.



$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$

SOLVED EXAMPLE

Example 24. A conducting disc of radius R is rolling without sliding on a horizontal surface with a constant velocity ' v ' . A uniform magnetic field of strength B is applied normal to the plane of the disc. Find the EMF induced between (at this moment)



(a) P & Q (b) P & C. (c) Q & C

(C is centre, P&Q are opposite points on vertical diameter of the disc)

Solution : (a) $\varepsilon_{PQ} = \frac{1}{2}B\omega(2R)^2$

$= \frac{1}{2}B\left(\frac{v}{R}\right)(2R)^2 = 2BvR$

(b) $\varepsilon_{PC} = \frac{1}{2}B\omega R^2 = \frac{1}{2}B\left(\frac{v}{R}\right)R^2 = \frac{BvR}{2}$.

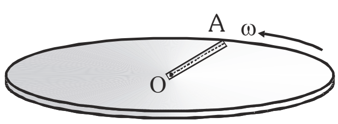
(c) $\varepsilon_{QC} = 2BvR - \frac{BvR}{2} = \frac{3}{2}vBR$

Example 25. A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference between its centre and rim in volts.

Solution : B = 100 gauss = 100 × 10^{-4} Wb/m² = 10^{-2},

r = 10 cm = 0.10 m, frequency of rotaion = 10 rot/sec

The emf induced between centre and rim $\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}B\omega r^2$ ($\because$ r = ℓ)

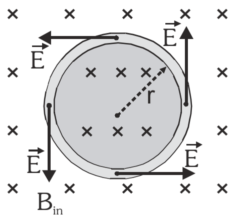


$\omega = 2\pi f = 2 \times 3.14 \times 10 = 62.8\ s^{-1}$

$\therefore\ \mathcal{E} = \frac{1}{2} \times 10^{-2} \times 62.8 \times (0.1)^2$ = 3.14 × 10^{-3} V = 3.14 mV.

5. INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E}.\vec{d\ell} = e = -\frac{d\phi}{dt}$ ...(i)

This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

(iv) This field is different from the conservative electrostatic field produced by stationary charges.

(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field. (vi) This field can vary with time.

- For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

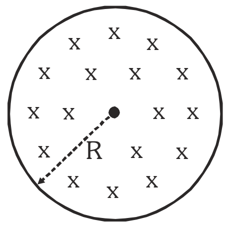
  ℓ = the length of closed loop in which electric field is to be calculated

  A = the area in which magnetic field is changing.

  Direction of induced electric field is the same as the direction of included current.

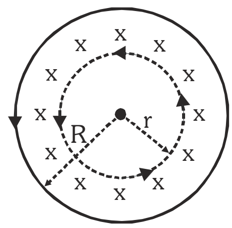
SOLVED EXAMPLE

Example 26. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha \frac{\text{tesla}}{\text{second}}$. Find the magnitude of electric field as a function of r, the distance from the geomatric centre of the region.



Solution : **For r ≤ R :**

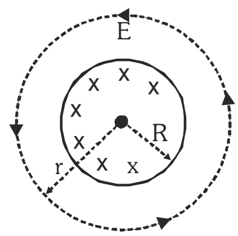
$$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$$



$\therefore$ E (2πr) = (πr²) α ⇒ $E = \frac{r\alpha}{2}$ ⇒ $E \propto r$

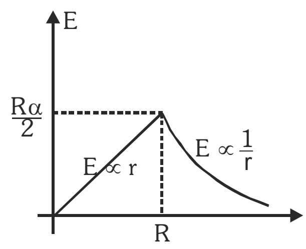
E-r graph is straight line passing through origin.

At r = R, $E = \frac{R\alpha}{2}$



For r ≥ R :

$$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$$

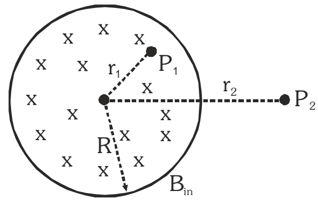


$\therefore$ E (2πr) = (πR²) α ⇒ $E = \frac{\alpha R^2}{2r}$ ⇒ $E \propto \frac{1}{r}$

Example 27. For the situation described in figure the magnetic field changes with time according to,
$B = (2.00\,t^3 - 4.00\,t^2 + 0.8)$ T and $r_2 = 2R = 5.0$ cm

(a) Calculate the force on an electron located at P_2 at t = 2.00 s

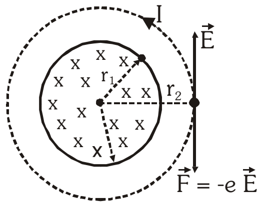
(b) What are the magnetude and direction of the electric field at P_1 when t = 3.00 s and r_1 = 0.02m.



Solution : $E\ell = A\left|\frac{dB}{dt}\right| \Rightarrow E = \frac{\pi R^2}{2\pi r_2}\frac{d}{dt}(2t^3 - 4t^2 + 0.8) = \frac{R^2}{2r_2}(6t^2 - 8t)$

(a) Force on electron at P_2 is F = eE

$\therefore$ at t = 2 s $F = \frac{1.6\times10^{-19}\times(2.5\times10^{-2})^2}{2\times5\times10^{-2}}\times[6(2)^2 - 8(2)]$



$= \frac{1.6}{4}\times2.5\times10^{-21}\times(24-16) = 8\times10^{-21}\,N$ at t = 2s,

$\frac{dB}{dt}$ is positive so it is increasing.

$\therefore$ direction of induced current and E are as shown in figure and hence force of electron having charge –e is right perpendicular to r_2 downwards

(b) For r_1 = 0.02 m and at t = 3s, $E = \frac{\pi r_1^2}{2\pi r_1}(6t^2 - 8t) = \frac{0.02}{2}\times[6(3)^2 - 8(3)]$

= 0.3 V/m at t = 3sec,

$\frac{dB}{dt}$ is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

Example 28. A uniform field of induction B is changing in magnitude at a constant rate dB/dt. You are given a mass *m* of copper which is to be drawn into a wire of radius *r* & formed into a circular loop of radius R. Show that the induced current in the loop does not depend on the size of the wire or of the loop. Assuming B perpendicular to the loop prove that the induced current $i = \frac{m}{4\pi\rho\delta}\frac{dB}{dt}$, where ρ is the resistivity and δ the density of copper.

Solution : $m = \pi r^2 L\delta$ ....(i)

Here $L = 2\pi R$

$$\varepsilon = \pi R^2 . \frac{dB}{dt} = \pi\left(\frac{L}{2\pi}\right)^2\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

$$i = \frac{\varepsilon}{\text{Resistance}} = \frac{\varepsilon 2\pi R}{\rho\pi r^2}$$

After solving,

$$i = \frac{m}{4\pi\rho\delta}.\frac{dB}{dt}.$$

6. SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change. Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = L i$ where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as $\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$

S.I Unit of inductance is wb/amp or Henry(H)

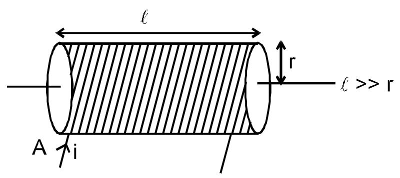
L - self inductance is +ve quantity .

L depends on : (1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current.

L is a scalar quantity.

SELF INDUCTANCE OF SOLENOID



Let the volume of the solenoid be V , the number of turns per unit length be n.

Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B=\mu_0 ni$. The magnetic flux through one turn of solenoid $\phi = \mu_0 n\, i\, A$.

The total magnetic flux through the solenoid = $N\phi = N\mu_0 ni A = \mu_0 n^2 i A \ell$

$\therefore \quad L = \mu_0 n^2 \ell A = \mu_0 n^2 V$

$\phi = \mu_0 n\, i\, \pi r^2 (n\ell)$

$$L = \frac{\phi}{i} = \mu_0 n^2 \pi r^2 \ell.$$

Inductance per unit volume = $\mu_0 n^2 A$.

Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant. Current can not change suddenly in the inductor.

SOLVED EXAMPLE

Example 29. Self-Inductance of a Toroid :

Calculate the self-inductance of a toroid which consists of *N* turns and has a rectangular cross section, with inner radius *a*, outer radius *b* and height *h*, as shown in Figure (a).

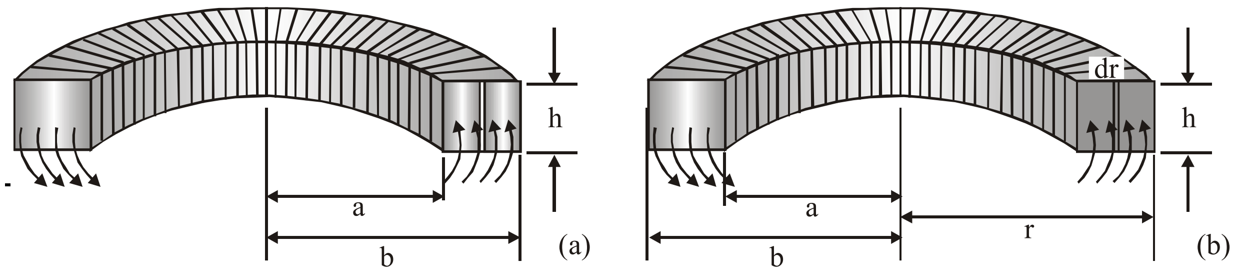


Figure : A toroid with *N* turns

Solution : According to Ampere's law discussed in section, the magnetic field is given by

$$\oint \vec{B}.d\vec{s} = \oint B ds = B\oint ds = B(2\pi r) = \mu_0 NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

The magnetic flux through one turn of the torid may be obtained by integrating over the rectangular cross section, with dA = hdr as the differential area element (figure-b)

$$\Phi_B = \int \vec{B}.d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right) hdr = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where a >>(b – a). In this limit, the logarithmic term in the equation above may be expanded as

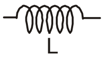
$$\ln\left(\frac{b}{a}\right) = \ln\left(1+\frac{b-a}{a}\right) \approx \frac{b-a}{a}$$

and the self-inductance becomes

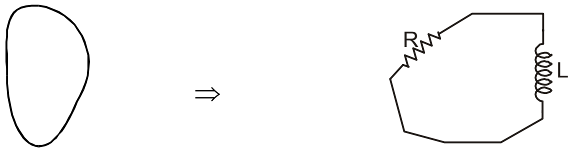
$$L \approx \frac{\mu_0 N^2 h}{2\pi}\cdot\frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{\ell}$$

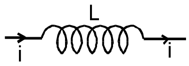
where A = h(b – a) is the cross-sectional area, and $\ell = 2\pi a$. We see that the self inductance of the torid in this limit has the same form as that of a solenoid.

7 INDUCTOR :

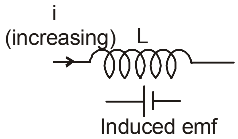
It is represent by 

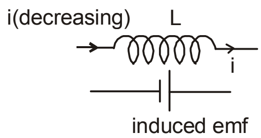
electrical equivalence of loop



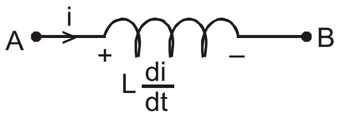


If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current.If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.



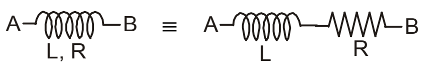


Over all result



$$V_A - L\frac{di}{dt} = V_B$$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :



SOLVED EXAMPLE

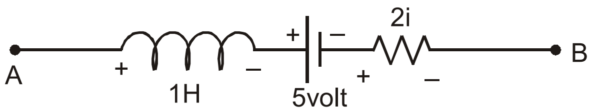
Example 30. A B is a part of circuit. Find the potential difference $v_A - v_B$ if

(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution :



$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B .$$

(i) Put i = 2, $\frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$

$\therefore V_A - V_B = 9$ volt

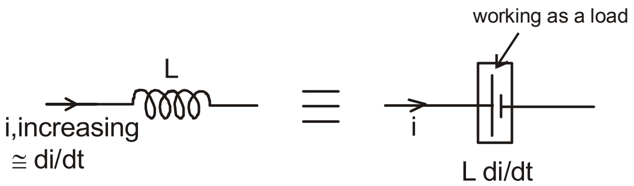
(ii) Put i = 2 , $\frac{di}{dt} = 1$; $V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10$ volt

(iii) Put i = 2 , $\frac{di}{dt} = -1$; $V_A + 1 - 5 - 2 \times 2 = V_B$ or $V_A = 8$ volt.

ENERGY STORED IN AN INDUCTOR:

The energy of a capacitor is stored in the electric field between its plates. Similary, an inductor has the capability of storing energy in its magnetic field.An increasing current in an inductor causes an emf between its terminals.

If current in an inductor at an instant is i and is increasing at the rate di/dt,the induced emf will oppose the current . Its behaviour is shown in the figure.



Power P = The work done per unit time $= \frac{dW}{dt} = -ei = -\left[L\frac{di}{dt}\right]i = -L\,i\frac{di}{dt}$

here i = instanteneous current and L = inductance of the coil

From dW = – dU (energy stored) so $\frac{dW}{dt} = -\frac{dU}{dt}$ $\therefore \frac{dU}{dt} = Li\frac{di}{dt} \Rightarrow dU = Li\,di$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L\int_0^I i\,di = \frac{1}{2}L(i^2)_0^I \quad \therefore U = \frac{1}{2}LI^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2}LI^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I, $\frac{di}{dt} = 0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc in the switch.

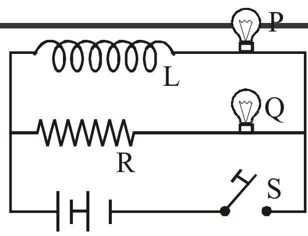
Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy U = $\int \frac{B^2}{2\mu_0\mu_r}dV$

SOLVED EXAMPLE

Example 31. Figure shows an inductor L a resistor R connected in paralled to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.



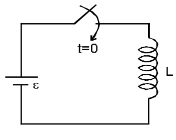
(a) Which of two bulbs lights up earlier when S is closed?

(b) Will the bulbs be equally bright after some time?

Solution : (i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.

(ii) Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

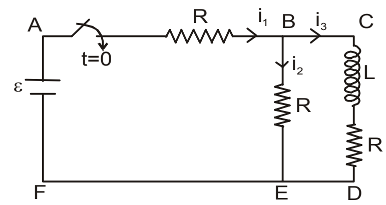
Example 32. A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open .It is closed at t=0.Find the current as a function of time.



Solution : $\varepsilon = L\frac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i L di$

$\varepsilon t = Li \Rightarrow i = \frac{\varepsilon t}{}$

Example 33. In the following circuit, the switch is closed at t = 0.



Find the currents i_1, i_2, i_3 and $\frac{di_3}{dt}$ at t=0 and at t = ∞.

Initially all currents are zero.

Solution : At **t = 0**

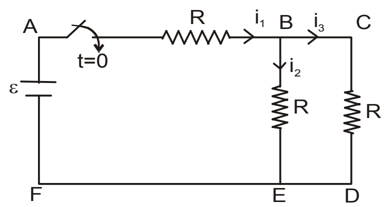
i_3 is zero ,since current cannot suddenly change due to the inductor.

∴ $i_1 = i_2$ (from KCL)

applying KVL in the part ABEF we get $i_1 = i_2 = \frac{\varepsilon}{2R}$.

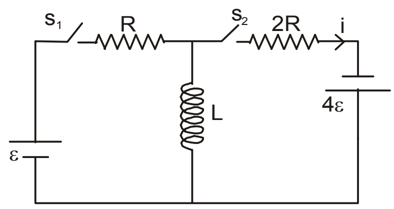
At **t=∞**

i_3 will become constant and hence potential difference across the inductor will be zero.It is just like a simple wire and the circuit can be solved assuming it to be like shown in the following diagram.



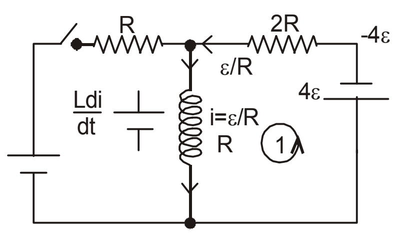
$i_2 = i_3 = \frac{\varepsilon}{3R}$, $i_1 = \frac{2\varepsilon}{3R}$.

Example 34. In the circuit shown in the figure,S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.



Solution : Before S_2 is closed and S_1 is opened current in the left part of the circuit $= \frac{\varepsilon}{R}$.

Now when S_2 closed S_1 opened, current through the inductor can not change suddenly , current $\frac{\varepsilon}{R}$ will continue to move in the inductor.



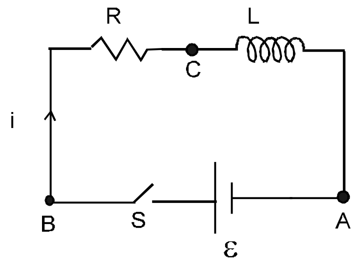
Applying KVL in loop 1.

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0$$

$$\frac{di}{dt} = -\frac{6\varepsilon}{L}$$

GROWTH OF CURRENT IN SERIES R-L CIRCUIT :

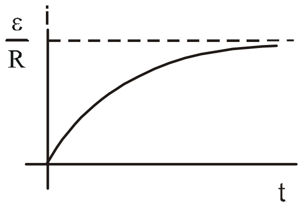
Figure shows a circuit consisting of a cell, an inductor L and a resistor R ,connected in series. Let the switch S be closed at t=0.Suppose at an instant current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit , we have $\varepsilon - L\frac{di}{dt} - iR = 0$

On solving we get, $i = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.



Note : 1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.

2. After one time constant , current in the circuit =63% of the final current (verify yourself)

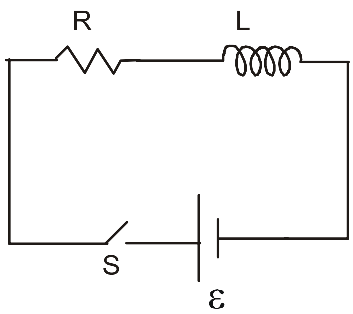
3. More time constant in the circuit implies slower rate of change of current.

4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$L_1 i_1 = L_2 i_2$

SOLVED EXAMPLE

Example 35. At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser ($\frac{L}{\eta}$) then its initial value, find out instant current just after the operation.

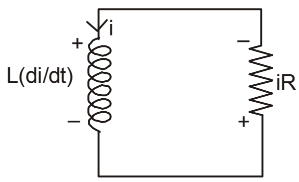


Solution : Using above result (note 4)

$L_1 i_1 = L_2 i_2 \quad \Rightarrow \quad i_2 = \frac{\eta\varepsilon}{R}$

DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR:

Let the initial current in the circuit be I_0.At any time t,let the current be i and let its rate of change at this instant be $\frac{di}{dt}$.



$$L.\frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} = -\frac{iR}{L}$$

$$\int_{I_0}^{i}\frac{di}{i} = -\int_0^t \frac{R}{L}.dt$$

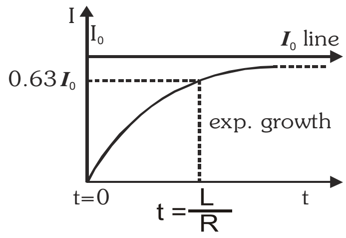
$$\ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L} \text{ or } i=I_0 e^{\frac{-Rt}{L}}$$

Current after one time constant :$i=I_0 e^{-1}$ = 37% of initial current.

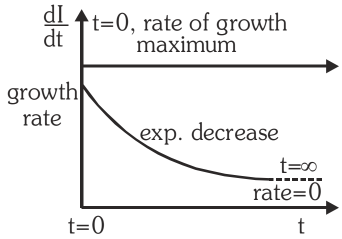
Graph for R–L circuit :–

Current Growth :–

(a)

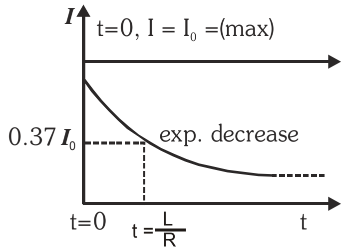


(b)

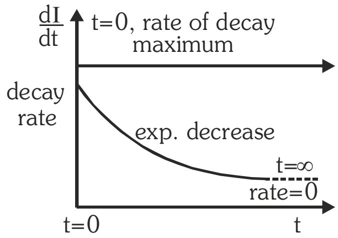


Current decay :–

(a)

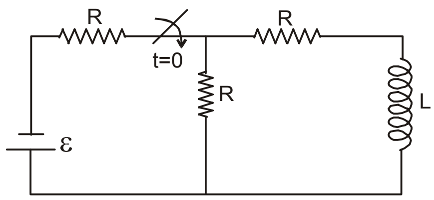


(b)

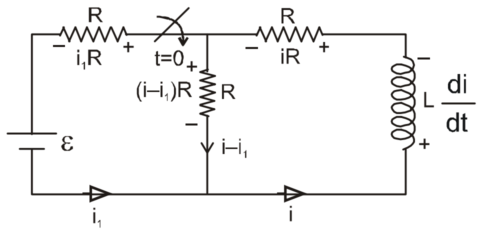


SOLVED EXAMPLE

Example 36. In the following circuit the switch is closed at t = 0.Intially there is no current in inductor. Find out current in the inductor coil as a function of time.



Solution : At any time t



$-\varepsilon + i_1R - (i - i_1) R = 0$

$-\varepsilon + 2i_1R - iR = 0$

$$i_1 = \frac{iR + \varepsilon}{2R}$$

Now, $-\varepsilon + i_1R + iR + L.\dfrac{di}{dt} = 0$

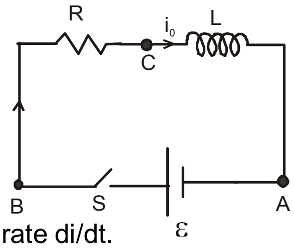
$$-\varepsilon + \left(\frac{iR + \varepsilon}{2}\right) + iR + L\,.\,\frac{di}{dt} = 0 \qquad \Rightarrow \qquad -\frac{\varepsilon}{2} + \frac{3iR}{2} = -L\,.\,\frac{di}{dt}$$

$$\left(\frac{-\varepsilon + 3iR}{2}\right)dt = -L\,.\,di \qquad \Rightarrow \qquad -\frac{-dt}{2L} = \frac{di}{-\varepsilon + 3iR}$$

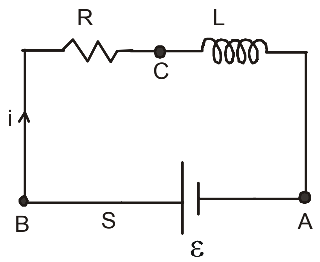
$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon+3iR} \quad \Rightarrow \quad -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right) = \frac{3Rt}{2L} \quad \Rightarrow \quad i = +\frac{\varepsilon}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$$

Example 37. Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R ,connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Solution : Let an instant t current in the circuit is i which is increasing at the rate di/dt.



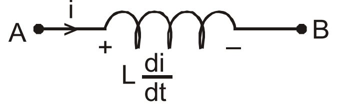
Writing KVL along the circuit , we have $\varepsilon - L\frac{di}{dt} - iR = 0$

$$\Rightarrow \quad L\frac{di}{dt} = \varepsilon - iR \quad \Rightarrow \quad \int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow \quad \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L}$$

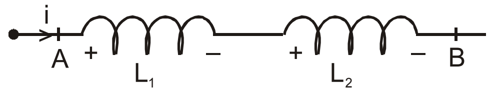
$$\Rightarrow \varepsilon - iR = (\varepsilon - i_0R)\, e^{-Rt/L} \quad \Rightarrow \quad i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

Equivalent self inductance :



$$L = \frac{V_A - V_B}{di/dt} \qquad ..(1)$$

Series combination

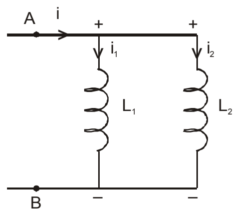


$$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B \qquad ....(2)$$

from (1) and (2)

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination :



From figure $V_A - V_B = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$ ..... (3)

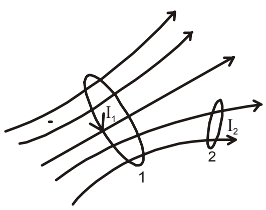
also $i = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

or $\frac{V_A - V_B}{L} = \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2}$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \qquad \text{(neglecting mutual inductance)}$$

8. MUTUAL INDUCTANCE



Consider two arbitrary conducting loops 1 and 2. Suppose that I_1 is the instantaneous current flowing around loop 1. This current generates a magnetic field $\mathbf{B_1}$ which links the second circuit, giving rise to a magnetic flux ϕ_2 through that circuit. If the current I_1doubles, then the magnetic field $\mathbf{B_1}$ doubles in strength at all points in space, so the magnetic flux ϕ_2 through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux ϕ_2 through the second circuit is *directly proportional* to the current I_1 flowing around the first circuit. Hence, we can write $\phi_2 = M_{21}I_1$ where the constant of proportionality M_{21} is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux ϕ_2 through the first circuit due to the instantaneous current I_2 flowing around the second circuit is directly proportional to that current, so we can write $\phi_1 = M_{12} I_2$ where M_{12} is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that **$M_{21} = M_{12}$ (Reciprocity Theorem)**. Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The S.I. unit of mutual inductance is called Henry (H). One Henry is equivalent to a volt-second per ampere:

Suppose that the current flowing around circuit 1 changes by an amount ΔI_1 in a small time interval Δt. The flux linking circuit 2 changes by an amount $\Delta\phi_2 = M\Delta I_1$ in the same time interval. According to Faraday's law, an emf $\varepsilon_2 = -\dfrac{\Delta\phi_2}{\Delta t}$ is generated around the second circuit due to the changing magnetic flux linking that circuit.

Since, $\Delta\phi_2 = M\Delta I_1$, this emf can also be written $\varepsilon_2 = -M\dfrac{\Delta I_1}{\Delta t}$

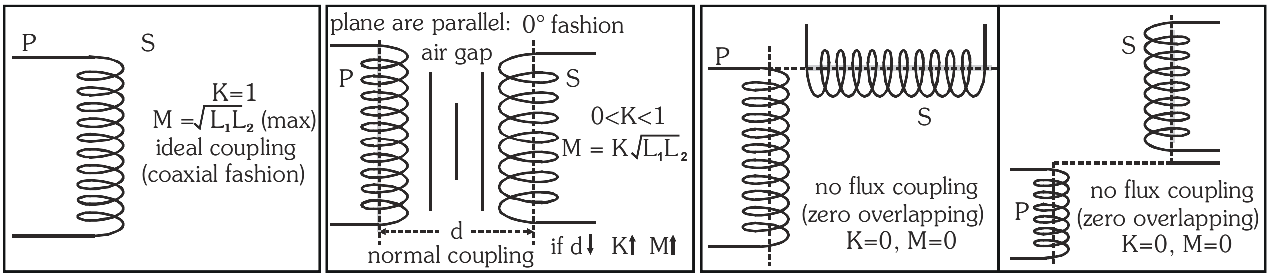
Thus, the emf generated around the second circuit due to the current flowing around the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current I_2 flowing around the second circuit changes by an amount ΔI_1 in a time interval Δt then the emf generated around the first circuit is $\varepsilon_1 = -M\dfrac{\Delta I_2}{\Delta t}$ Note that there is no direct physical connection(coupling) between the two circuits: the coupling is due entirely to the magnetic field generated by the currents flowing around the circuits.

Note : (1) $M \le \sqrt{L_1 L_2}$

(2) For two coils in series if mutual inductance is considered then

$L_{eq} = L_1 + L_2 \pm 2M$

- **Different fashion of coupling**



'K' also defined as $K = \dfrac{\phi_s}{\phi_p} = \dfrac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

SOLVED EXAMPLE

Example 38. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let l be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids,neglecting the end effects.

Solution : If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\ell}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is B_1A. Thus, the flux linking all N_2 turns of the second wire is

$$\phi_2 = N_2B_1A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = MI_1 .$$

$$\therefore \quad M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

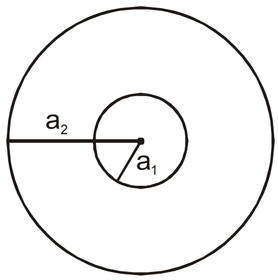
As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

Example 39. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Solution :
$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3}\,\text{H}$$

$$\mathcal{E} = -M\frac{dI}{dt} = -3 \times 10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3}\,\text{V} = 48 \text{ mV}$$

Example 40. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution : Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2} \pi a_1^2$

or $\quad Mi = \frac{\mu_0 i}{2a_2} \pi a_1^2 \qquad$ or $\quad M = \frac{\mu_0 \pi a_1^2}{2a_2}$

Example 41. Solve the above question, if the planes of coil are perpendicular.

Solution : Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0.

Example 42. Solve the above problem if the planes of coils make θ angle with each other.

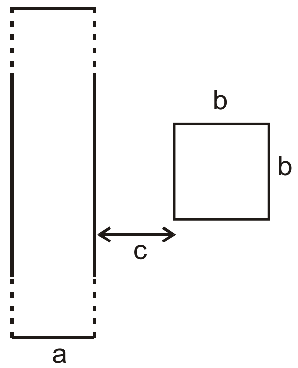
Solution : If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

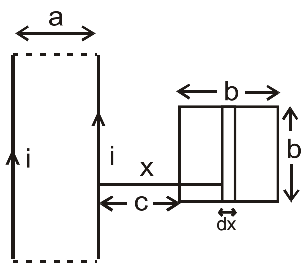
Thus flux = $\vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2}.\pi a_1^2.\cos\theta$

or $$M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$$

Example 43. Find the mutual inductance between two rectangular loops, shown in the figure



Solution :

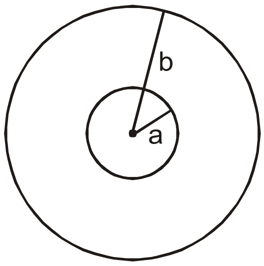


Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx .$$

$$\Rightarrow \quad \phi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx = \frac{\mu_0 ib}{2\pi}\left[\ln\frac{c+b}{c} - \ln\frac{a+b+c}{a+c}\right].$$

Example 44. Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils

(b) Find the emf induced in the larger coil

(c) If the resistance of the larger loop is R find the current in it as a function of time

Solution :

(a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 i}{2b}$.

flux through the smaller coil = $\frac{\mu_0 i}{2b}\pi a^2$

$\therefore\ M = \frac{\mu_0}{2b}\pi a^2$

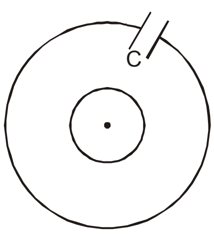
(b) $|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$

$= \frac{\mu_0}{2b}\pi a^2\ (2) \qquad = \frac{\mu_0 \pi a^2}{b}$

(c) current in the larger coil

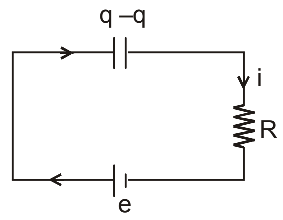
$= \frac{\mu_0 \pi a^2}{bR}$.

Example 45. If the current in the inner loop changes according to i = 2t² then, find the current in the capacitor as a function of time.



Solution : $M = \frac{\mu_0}{2b}\pi a^2$

$$\left|\text{emf induced in larger coil}\right| = M\left[\left(\frac{di}{dt}\right)\text{in smaller coil}\right]$$



$$e = \frac{\mu_0}{2b}\pi a^2\ (4t) \quad = \frac{2\mu_0\pi a^2 t}{b}$$

Applying KVL :-

$$+e - \frac{q}{c} - iR = 0$$

$$\frac{2\mu_0\pi a^2 t}{b} - \frac{q}{c} - iR = 0$$

differentiate wrt time :-

$$\frac{2\mu_0\pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$$

on solving it

$$i = \frac{2\mu_0\pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$$

9. LC Oscillations :

Consider an *LC* circuit in which a capacitor is connected to an inductor, as shown in Figure.

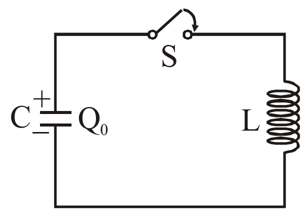


Figure *LC* Circuit

Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the *LC* circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{} + \frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $I = -dQ/dt$ (and $dI/dt = -d^2Q/dt^2$). Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff's loop rule clockwise.

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

followed by our definition of current.

$$\frac{d^2Q}{dt^2} = -\left(\frac{1}{LC}\right)Q$$

The general solution to equation is $Q(t) = Q_0 \cos(\omega_0 t + \phi)n$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

where $I_0 = \omega_0 Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t) = Q_0 \cos \omega_0 t$$

and $I(t) = I_0 \sin w_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

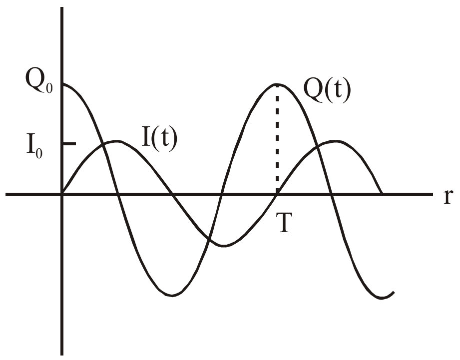


Figure : Charge and current in the *LC* circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t$$

and $$U_B = \frac{1}{2}LI^2 = \frac{LI_0^2}{2}\sin^2 \omega t = \frac{L(-\omega_0 Q_0)^2}{2}\sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2 \omega_0 t$$

$$U_B + U_E = \frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

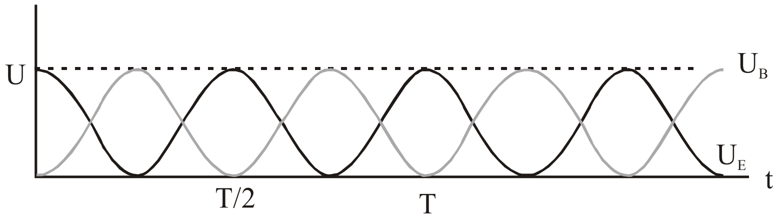


Figure : Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.

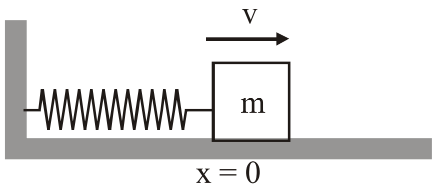


Figure : Mass-spring oscillations

If the mass is moving with a speed *v* and the spring having a spring constant *k* is displaced from its equilibrium by *x*, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

Using v = dx/dt and dv/dt = d^2x/dt^2, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2} m x_0^2 \omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2} k x_0^2 \cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2} k x_0^2 [\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi)] = \frac{1}{2} k x_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

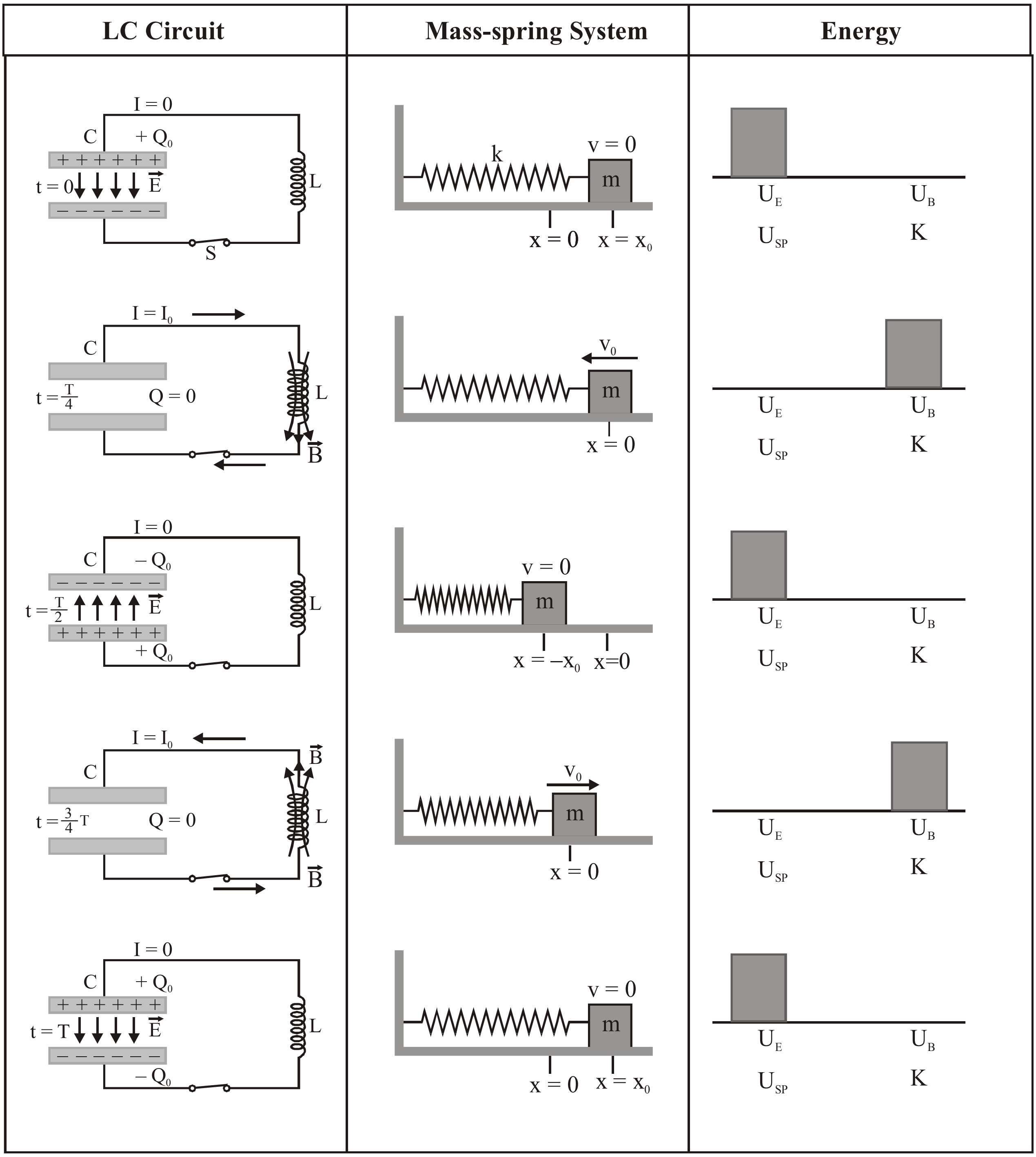


Figure : Energy oscillations in the *LC* Circuit and the mass-spring system

SOLVED EXAMPLE

Example 46. Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.

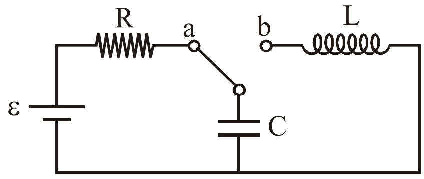


Figure : *LC* circuit

Find the following quantities :

(a) the frequency of oscillation of the *LC* circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy the circuit possesses at any time *t*.

Solution :

(a) The (angular) frequency of oscillation of the *LC* circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is

$Q = C\varepsilon$

(c) The energy stored in the capacitor before the switch is thrown is :

$$U_E = \frac{1}{2}C\varepsilon^2$$

On the other hand, the magnetic energy stored in the inductor is :

$$U_B = \frac{1}{2}LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor :

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = \varepsilon\sqrt{\frac{C}{L}}$$

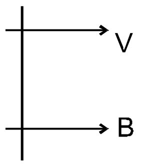
(d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2}C\varepsilon^2$$

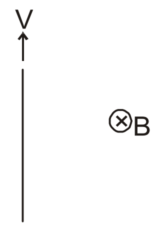
MISCELLANEOUS SOLVED EXAMPLE

Problem 1. Find the emf induced in the rod in the following cases. The figures are self explanatory.

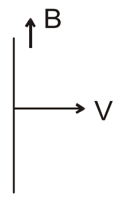
(a)



(b)



(c)



Solution : (a) here $\vec{v} \parallel \vec{B}$ so $\vec{v} \times \vec{B} = 0$

emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

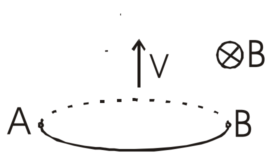
(b) here $\vec{v} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(c) here $\vec{B} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

Problem 2. A circular coil of radius R is moving in a magnetic field **B** with a velocity **v** as shown in the figure.

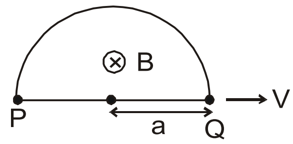


Find the emf across the diametrically opposite points A and B.

Solution : emf = $BVl_{effective}$

$= 2\,R\,v\,B$

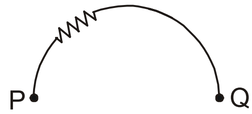
Problem 3. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown.Also draw the electrical equivalent circuit of each branch.



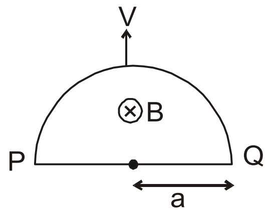
Solution : here $\vec{v} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

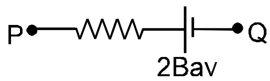
Induced emf = 0

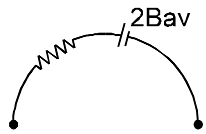


Problem 4. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalence of each branch.

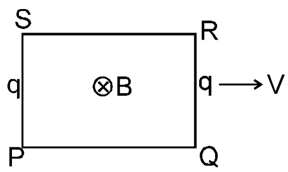


Solution : Induced emf = 2Bav

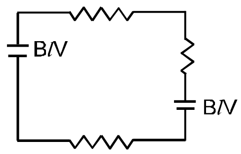




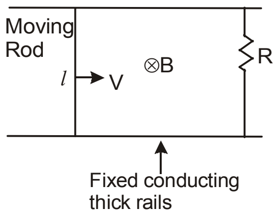
Problem 5. Figure shows a rectangular loop moving in a uniform magnetic field .Show the electrical equivalence of each branch.



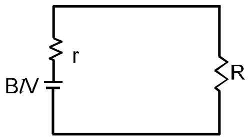
Solution :



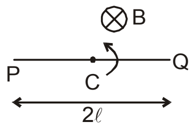
Problem 6. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R.A uniform magnetic field B is present normal to the plane of rod and rails .Show the electrical equivalence of each branch.



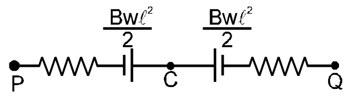
Solution :



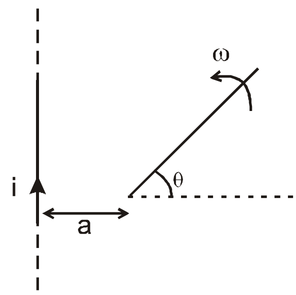
Problem 7. A rod PQ of length 2l is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced emf between P Q and PC. Draw the circuit diagram of parts PC and CQ.



Solution : $emf_{PQ} = 0$; $emf_{PC} = \frac{B\omega\ell^2}{2}$



Problem 8. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.

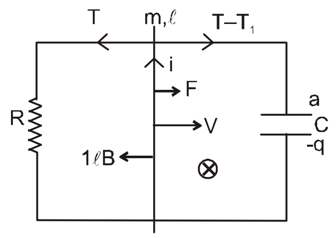


Solution : $E = \int \frac{\mu_0 i}{2\pi(a + r\cos\theta)} \times (r\omega) . (dr)$

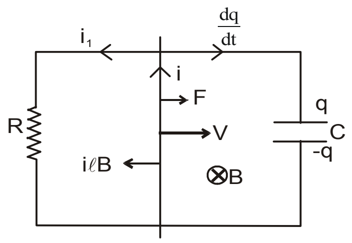
$$E = \frac{\mu_0 \omega i}{2\pi} \int_0^{\ell} \frac{r}{a + r\cos\theta} dr$$

$$E = \frac{\mu_0 i\omega}{2\pi\cos\theta} \left[\ell - \frac{a}{\cos\theta} \ell n\left(\frac{a + \ell\cos\theta}{a}\right)\right]$$

Problem 9. Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.



Solution : At any time t, let the velocity of the rod be v .



Applying Newtons law: F – ilB = ma ... (1)

Also $B\,l\,v = i_1R = \dfrac{q}{c}$

Applying Kcl,

$$i = i_1 + \frac{dq}{dt} = \frac{B\ell V}{R} + \frac{d}{dt}(BlvC)$$

or $$i = \frac{B\ell V}{R} + B\ell C\,a$$

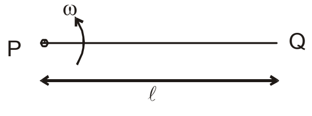
Putting the value of i in eq (1), $F - \dfrac{B^2\ell^2V}{R} = (m + B^2\ell^2C)a = (m + B^2\ell^2C)\dfrac{dv}{dt}$

$$(m + B^2\ell^2C)\frac{dv}{F - \frac{B^2l^2v}{R}} = dt$$

Integrating both sides, and solving we get

$$v = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{tB^2\ell^2}{R(m+CB^2\ell^2)}}\right)$$

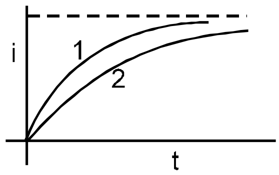
Problem 10. A rod PQ of length l is rotating about end P, with an angular velocity ω . Due to centrifugal forces the free electrons in the rod move towards the end Q and an emf is created. Find the induced emf.



Solution : The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state $m_e\omega^2x = e\,E$.

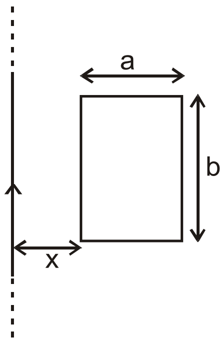
$$V_P - V_Q = \int_{x=0}^{x=\ell} \overline{E}.d\overline{x} = \int_0^{\ell} \frac{m_e\omega^2x}{e}dx = \frac{m_e\omega^2\ell^2}{2e}$$

Problem 11. Which of the two curves shown has less time constant.

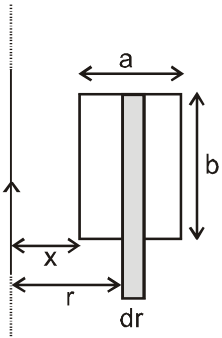


Solution curve1

Problem 12. Find the mutual inductance of a straight long wire and a rectangular loop, as shown in the figure



Solution :



$$d\phi = \frac{\mu_0 i}{2\pi r} \times bdr$$

$$\phi = \int_{x}^{x+a} \frac{\mu_0 i}{2\pi r} \times bdr$$

$M = \phi/i$

$$M = \frac{\mu_0 b}{2\pi} \ln\left(1 + \frac{a}{x}\right)$$

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

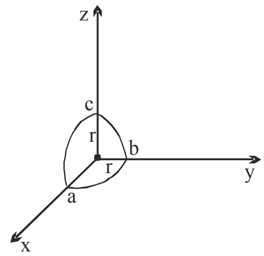
SECTION (A) : FLUX AND FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

A 1. The flux of magnetic field through a closed conducting loop of resistance 0.4 Ω changes with time according to the equation $\Phi = 0.20t^2 + 0.40t + 0.60$ where t is time in seconds. Find (i) the induced emf at t = 2s. (ii) the average induced emf in t = 0 to t = 5 s. (iii) charge passed through the loop in t = 0 to t = 5s (iv) average current in time interval t = 0 to t = 5 s (v) heat produced in t = 0 to t = 5s.

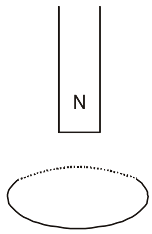
A 2. A wire is bent into 3 circular segments of radius r = 10 cm as shown in figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane & ca lying in the zx plane.

(i) if a magnetic field B points in the positive x direction, what is the magnitude of the emf developed in the wire, when B increases at the rate of 3 mT/s ?

(ii) what is the direction of the current in the segment bc.

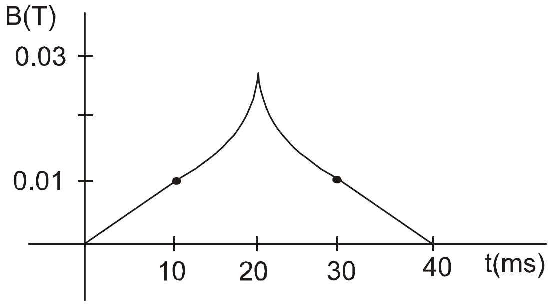


A 3. The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure)As a result the flux through the coil changes from 0.4 Weber to 0.9 Weber in an interval of half of a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet?

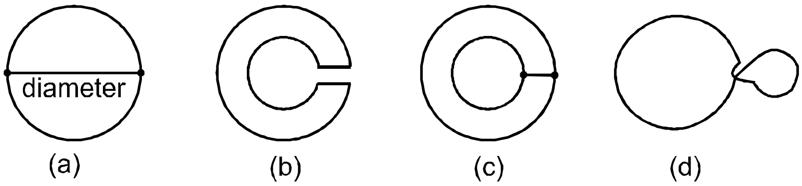


A 4. (a) The magnetic field in a region varies as shown in figure. Calculate the average induced emf in a conducting loop of area 10^{-3} m^2 placed perpendicular to the field in each of the 10 ms intervals shown.

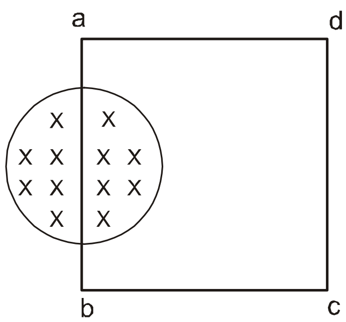
(b) In which interval(s) is the emf not constant? Neglect the behavior near the ends of 10 ms intervals.



A 5. Figure illustrates plane figures made of thin conductors which are located in a uniform magnetic field directed away from a reader beyond the plane of the drawing. The magnetic induction starts diminishing. Find how the currents induced in these loops are directed.



A 6. A uniform magnetic field B exists in a cylindrical region or radius 1 cm as shown in figure. A uniform wire of length 16 cm and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameters of the cylindrical region. If the magnetic field increases at a constant rate of 1 T/s find the current induced in the frame.

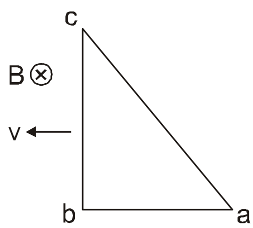


SECTION (B) : LENZ'S LAW

B 1. Two straight long parallel conductors are moved towards each other. A constant current i is flowing through one of them. What is the direction of the current induced in other conductor? What is the direction of induced current when the conductors are drawn apart.

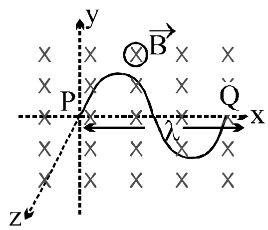
SECTION (C) : INDUCED EMF IN A MOVING ROD IN UNIFORM MAGNETIC FIELD

C 1. A right angled triangle abc, made of a metallic wire, moves at a uniform speed v in its plane as shown in the figure. A uniform magnetic field B exists in the perpendicular direction of plane of triangle. Find the emf induced (a) in the loop abc, (b) in the segment bc, (c) in the segment ac and (d) in the segment ab.



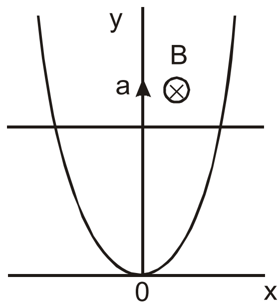
C 2. The two rails, separated by 1m, of a railway track are connected to a voltmeter. What will be the reading of the voltmeter when a train travels on the rails with speed 5 m/s. The earth's magnetic field at the place is 4×10^{-4} T, and the angle of dip is 30º .

C 3. A wire forming one cycle of sine curve is moved in x-y plane with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j}$. There exist a magnetic field $\vec{B} = -B_0\hat{k}$. Find the motional emf develop across the ends PQ of wire.



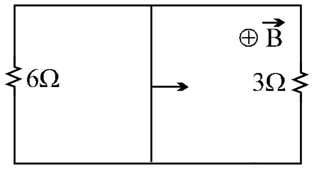
C 4. A circular conducting-ring of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring. (a) Between which pair of points is the emf maximum? (b) Between which pair of points is the emf minimum? What is the value of this minimum emf?

C 5. A wire bent as a parabola $y = kx^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the plane xy. At the moment t = 0 a connector starts sliding translation wise from the parabola apex with a constant acceleration a (figure). Find the emf of electromagnetic induction in the loop thus formed as a function of y.

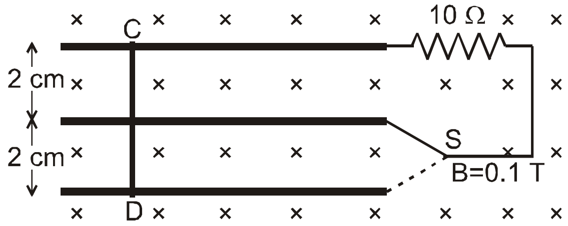


SECTION (D) : CIRCUIT PROBLEMS AND MECHANICS

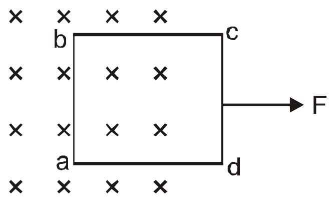
D 1. A rectangular loop with a sliding connector of length l = 1.0 m is situated in a uniform magnetic field B = 2T perpendicular to the plane of loop. Resistance of connector is r = 2Ω. Two resistances of 6Ω and 3Ω are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity v = 2m/s.



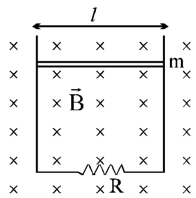
D 2. Consider the situation shown in figure. The wire CD has a negligible resistance and is made to slide on the three rails with a constant speed of 50 cm/s. Find the current in the 10 Ω resistor when the switch S is thrown to (a) the middle rail (b) bottom rail. (Neglect resistance of rails)



D 3. A square frame of wire abcd of side 1 m has a total resistance of 4 Ω. It is pulled out of a magnetic field B = 1 T by applying a force of 1 N (figure). It is found that the frame moves with constant speed. Find (a) this constant speed, (b) the emf induced in the loop, (c) the potential difference between the points a and b and (d) the potential difference between the points c and d.

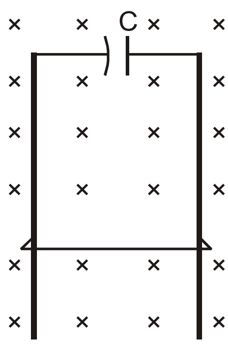


D 4. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length *l* and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.



D 5. Consider the situation of the previous problem. (a) Calculate the force needed to keep the sliding wire moving with a constant velocity v. (b) If the force needed just after t = 0 is F_0, find the time at which the force needed will be $F_0/2$.

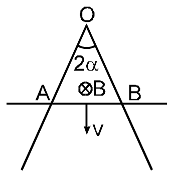
D 6. A wire of mass m and length ℓ can slide freely on a pair of fixed, smooth, vertical rails (figure). A magnetic field B exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by an initially uncharged capacitor of capacitance C. Find the velocity of the wire at any time (t) after released. Neglecting any electric resistance. (initial velocity of wire is zero)



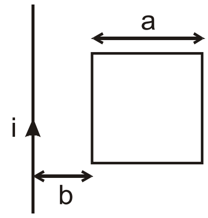
SECTION (E) : EMF INDUCED IN A ROD OR LOOP IN NON UNIFORM MAGNETIC FIELD

E 1. The magnetic field in a region is given by $\vec{B} = \frac{B_0}{L} x\hat{k}$, where L is a fixed length. A conducting rod of length L lies along the X-axis between the origin and the point (L, 0, 0). If the rod moves with a velocity $\vec{v} = v_0 \hat{j}$, find the emf induced between the ends of the rod.

E 2. A straight wire with a resistance of r per unit length is bent to form an angle 2α. A rod of the same wire perpendicular to the angle bisector (of 2α) forms a closed triangular loop. This loop is placed in a uniform magnetic field of induction B. Calculate the current in the wires when the rod moves at a constant speed V.



E 3. Figure shows a fixed square frame of wire having a total resistance r placed coplanarly with a long, straight wire. The wire carries a current i given by $i = i_0 \cos(2\pi t/T)$. Find (a) the flux of the magnetic field through the square frame, (b) the emf induced in the frame and (c) the heat developed in the frame in the time interval 0 to 10 T.

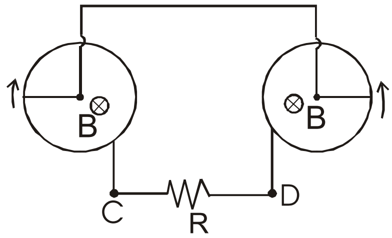


SECTION (F) : INDUCED EMF IN A ROD, RING, DISC ROTATING IN A UNIFORM MAGNETIC FIELD

F 1. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown In the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet.

F 2. A metal rod of length 15×10^{-2} m rotates about an axis passing through one end with a uniform angular velocity of 60 rad s^{-1}. A uniform magnetic field of 0.1 Tesla exists in the direction of the axis of rotation. Calculate the EMF induced between the ends of the rod.

F 3. In the figure there are two identical conducting rods each of length 'a' rotating with angular speed ω in the directions shown. One end of each rod touches a conducting ring. Magnetic field B exists perpendicular to the plane of the rings. The rods, the conducting rings and the lead wires are resistanceless. Find the magnitude and direction of current in the resistance R.



F 4. A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 50 revolutions per minute. If the length of each spoke is 30.0 cm and the horizontal component of the earth's magnetic field is 4×10^{-5} T, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

F 5. A thin wire of negligible mass & a small spherical bob constitute a simple pendulum of effective length ℓ. If this pendulum is made to swing through a semi-vertical angle θ, under gravity in a plane normal to a uniform magnetic field of induction B, find the maximum potential difference between the ends of the wire.

F 6. A closed coil having 50 turns is rotated in a uniform magnetic field $B = 2 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 100 cm ^{2} and its resistance is 4 Ω. Find (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field, (b) the average emf in a full turn , (c) the net charge flown in part (a) and (d) the emf induced as a function of time if it is zero at t=0 and is increasing in positive direction.(e) the maximum emf induced. (f) the average of the squares of emf induced over a long period

SECTION (G) : FIXED LOOP IN A TIME VARYING MAGNETIC FIELD & INDUCED ELECTRIC FIELD

G 1. A circular loop of radius 1m is placed in a varying magnetic field given as B = 6t Tesla, where t is time in sec.

(a) Find the emf induced in the coil if the plane of the coil is perpendicular to the magnetic field.

(b) Find the electric field in the tangential direction, induced due to the changing magnetic field.

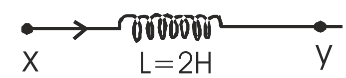
(c) Find the current in the loop if its resistance is1Ω/m.

G 2. The current in an ideal, long solenoid is varied at a uniform rate of 0.01 A/s. The solenoid has 2000 turns/m and its radius is 6.0 cm. (a) Consider a circle of radius 1.0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2.0 seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance 8.0 cm from its axis.

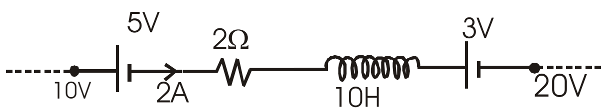
G 3. There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as B = kt. If an electron is released from rest in this field at a distance of 'r' from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)

SECTION (H) : SELF INDUCTION, SELF INDUCTANCE SELF INDUCED EMF & MAGNETIC ENERGY DENSITY

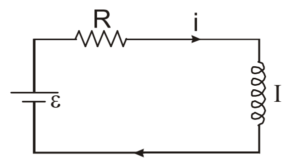
H 1. The figure shows an inductor of 2 H through which a current increasing at the rate of 5A/sec, is flowing. Find the potential difference V_X-V_Y.



H 2. Figure shows a part of a circuit. Find the rate of change of the current, as shown.



H 3. Suppose the EMF of the battery, the circuit shown varies with time t so the current is given by $i(t) = 3 + 5t$, where i is in amperes & t is in seconds. Take R = 4 Ω, L = 6 H & find an expression for the battery EMF as a function of time.



H 4. What is the magnetic energy density (in terms of standard constant & r) at the centre of a circulating electron in the hydrogen atom in first orbit. (Radius of the orbit is r)

H 5. Find the energy stored in the magnetic field inside a volume of 1.00 mm^3 at a distance of 10.0 cm from a long wire carrying a current of 4 A.

SECTION (I) : CIRCUIT CONTAINING INDUCTANCE, RESISTANCE & BATTERY, GROWTH AND DECAY OF CURRENT IN A CIRCUIT CONTAINING INDUCTOR

I 1. A solenoid of resistance 50 Ω and inductance 80 Henry is connected to a 200 V battery. How long will the current take to reach 50 % of its final equilibrium value? Calculate the maximum energy stored.

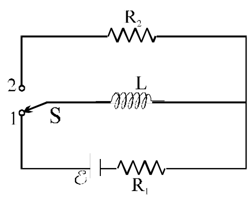
I 2. A coil having resistance 20 Ω and inductance 2 H is connected to a battery of emf 4.0 V. Find (a) the current at 0.20 s after the connection is made and (b) the magnetic field energy in the coil at this instant.

I 3. A coil of resistance 4 Ω is connected across a 0.4 V battery. The current in the coil is 63 mA , 1sec after the battery is connected. Find the inductance of the coil. [$e^{-1} \simeq 0.37$]

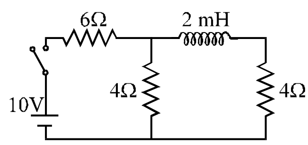
I 4. A coil of negligible resistance and inductance 5H, is connected in series with a 100 Ω resistor and a battery of emf 2.0 V. Find the potential difference across the resistor 20 ms after the circuit is switched on.

($e^{-0.4} = 0.67$)

I 5. In the circuit shown, initially the switch is in position 1 for a long time. Then the switch is shifted to position 2 for a long time. Find the total heat produced in R_2.



I 6. In the given circuit, find the ratio of i_1 to i_2 where i_1 is the initial (at t = 0) current and i_2 is steady state (at t = ∞) current through the battery.



I 7. An inductor-coil of inductance 20 mH having resistance 10 Ω is joined to an ideal battery of emf 5.0 V. Find the rate of change of the magnitude of induced emf at (a) t = 0, (b) t = 10 ms.

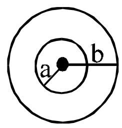
I 8. A superconducting loop of radius R has self inductance L. A uniform & constant magnetic field B is applied perpendicular to the plane of the loop. Initially current in this loop is zero. The loop is rotated about its diameter by 180º. Find the current in the loop after rotation.

SECTION (J) : MUTUAL INDUCTION & MUTUAL INDUCTANCE

J 1. The mutual inductance between two coils is 0.5 H. It the current in one coil is changed at the rate of 5 A/s, what will be the emf induced in the other coil?

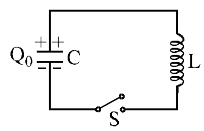
J 2. The average emf induced in the secondary coil is 0.1 V when the current in the primary coil changes from 1 to 2 A in 0.1 s . What is the mutual inductance of the coils.

J 3. Two concentric and coplanar circular coils have radii a and b(>>a)as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then find the total charge circulating the inner coil.



SECTION (K) : LC OSCILLATIONS

K 1. A capacitor C with a charge Q_0 is connected across an inductor through a switch S. If at t = 0, the switch is closed, then find the instantaneous charge q on the upper plate of capacitor.



K 2. An inductor of inductance 2.0mH,is connected across a charged capacitor of capacitance 5.0μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor, and I the current in the circuit .It is found that the maximum value of Q is 200μC.

(a) when Q = 100μC, what is the value of $|dI/dt|$?

(b) when Q = 200 μC, what is the value of I ?

(c) Find the maximum value of I.

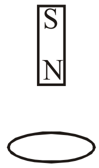
(d) when I is equal to one half its maximum value, what is the value of $|Q|$

PART - II : OBJECTIVE QUESTIONS

*** Marked Questions are having more than one correct option.**

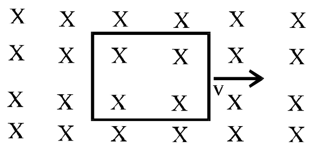
SECTION (A) : FLUX AND FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

A 1. A short circuited coil is kept on the ground and a magnet is dropped on it as shown. The coil shows (when viewed from top)



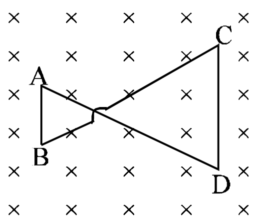
(A) anticlockwise current that increases in magnitude
(B) anticlockwise current that remains constant
(C) clockwise current that remains constant
(D) clockwise current that increases in magnitude

A 2. A square frame of wire of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop in fig. The current induced in the loop is -



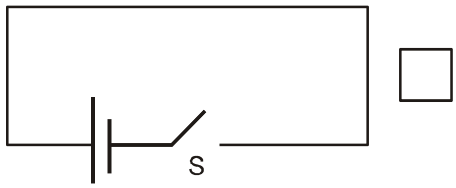
(A) $2Bl\upsilon/R$ anticlockwise (B) $Bl\upsilon/R$ anticlockwise
(C) $Bl\upsilon/R$ clockwise (D) zero

A 3. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



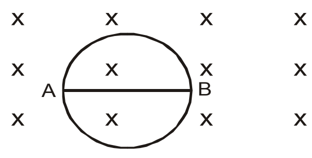
(A) B to A and D to C (B) A to B and C to D
(C) A to B and D to C (D) B to A and C to D

A 4. Consider the conducting square loop shown in fig. If the switch is closed and after some time it is opened again, the closed loop will show



(A) a clockwise current-pulse
(B) an anticlockwise current-pulse
(C) an anticlockwise current-pulse and then a clockwise current-pulse
(D) a clockwise current-pulse and then an anticlockwise current-pulse

A 5. The radius of the circular conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ. Then current in wire AB is (AB is one of the diameters)



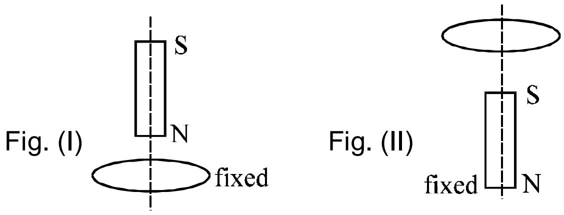
(A) $\frac{R\alpha}{2\rho}$ from A to B (B) $\frac{R\alpha}{2\rho}$ from B to A (C) $\frac{2R\alpha}{\rho}$ from A to B (D) Zero

A 6. A small, circular loop of wire is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is

(A) anticlockwise (B) clockwise (C) zero
(D) clockwise or anticlockwise depending on whether the resistance in increased or decreased.

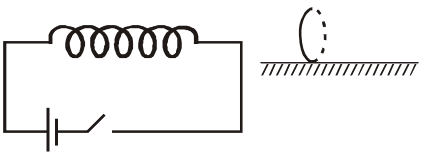
SECTION (B) : LENZ'S LAW

B 1. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in fig - I. In fig.II the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then :-



(A) $a_1 > g$, $a_2 > g$ (B) $a_1 > g$, $a_2 < g$ (C) $a_1 < g$, $a_2 < g$ (D) $a_1 < g$, $a_2 > g$

B 2. A horizontal solenoid is connected to a battery and a switch (figure). A conducting ring is placed on a frictionless surface, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will



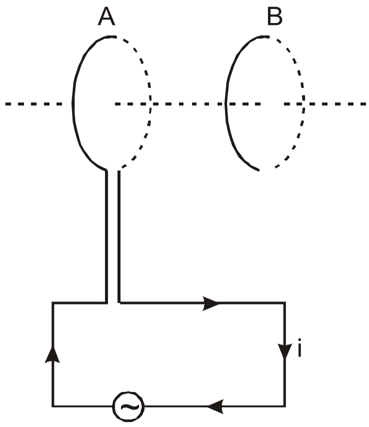
(A) move towards the solenoid

(B) remain stationary

(C) move away from the solenoid

(D) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

B 3. Two circular coils A and B are facing each other as shown in figure. The current i through A can be altered



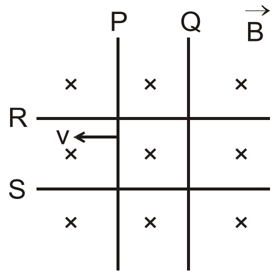
(A) there will be repulsion between A and B if i is increased

(B) there will be attraction between A and B if i is increased

(C) there will be neither attraction nor repulsion when i is changed

(D) attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.

B 4. Two identical conductors P and Q are placed on two frictionless fixed conducting rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



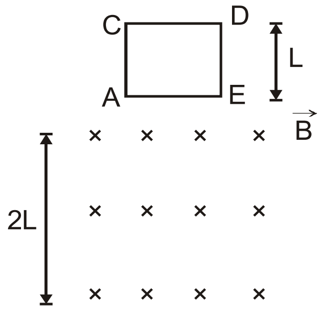
(A) will be attracted towards P

(B) will be repelled away from P

(C) will remain stationary

(D) may be repelled or attracted towards P

B 5. A square coil ACDE with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length 2L. The acceleration of the coil is



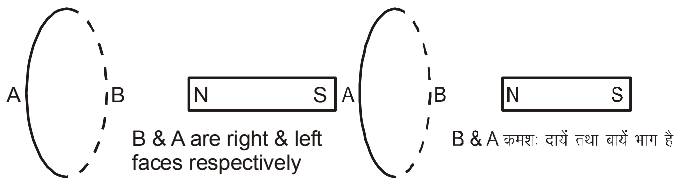
(A) less than g for all the time till the loop crosses the magnetic field completely

(B) less than g when it enters the field and greater than g when it comes out of the field

(C) g all the time

(D) less than g when it enters and comes out of the field but equal to g when it is within the field

B 6. In the figure shown, the magnet is pushed towards the fixed ring along the axis of the ring and it passes through the ring.



(A) when magnet goes towards the ring the face B becomes south pole and the face A becomes north pole

(B) when magnet goes away from the ring the face B becomes north pole and the face A becomes south pole

(C) when magnet goes away from the ring the face A becomes north pole and the face B becomes south pole

(D) the face A will always be a north pole.

B 7. A neutral metallic ring is placed in a circular symmetrical uniform magnetic field with its plane perpendicular to the field. If the magnitude of field starts increasing with time, then:

(A) the ring starts translating

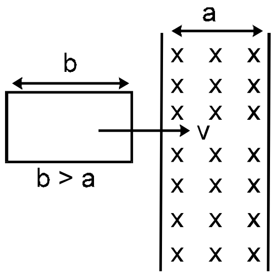
(B) the ring starts rotating about its axis

(C) the ring slightly contracts

(D) the ring starts rotating about a diameter

SECTION (C) : INDUCED EMF IN A MOVING ROD IN UNIFORM MAGNETIC FIELD

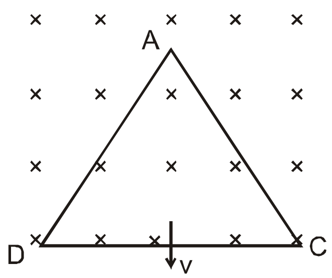
C 1. In the given arrangement, the loop is moved with constant velocity v in a uniform magnetic field B in a restricted region of width a. The time for which the emf is induced in the circuit is:



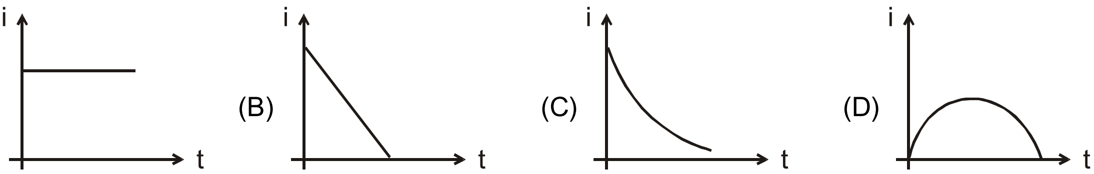
(A) $\frac{2b}{v}$ (B) $\frac{2a}{v}$ (C) $\frac{(a+b)}{v}$ (D) $\frac{2(a-b)}{v}$

C 2. A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m is placed along y–axis is moved along x–axis with constant speed 1 m/sec. Then induced e.m.f. in the rod will be:
(A) zero (B) 25 v (C) 20 v (D) 15 v

C 3. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper. At time t = 0, side DC of the loop is at edge of the magnetic field. The induced current (i) versus time (t) graph will be as



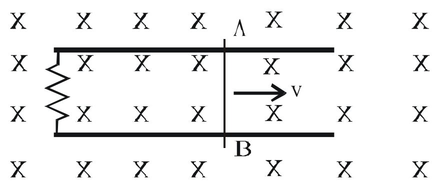
(A) (B) (C) (D)



C 4. A wire of length ℓ is moved with a constant velocity $\vec{v}$ in a magnetic field. A potential difference appears across the two ends

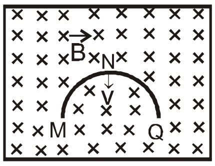
(A) if $\vec{v} \parallel \vec{\ell}$ (B) if $\vec{v} \parallel \vec{B}$ (C) if $\vec{\ell} \parallel \vec{B}$ (D) none of these

C 5. The resistanceless wire AB (in figure) is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a resistanceless semicircular wire, the magnitude of the induced current will



(A) decrease
(B) remain the same
(C) increase
(D) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

C 6. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v then the potential difference developed across the ring is: **[JEE - 1996]**

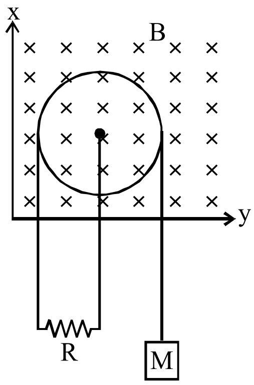


(A) zero (B) $\frac{Bv\pi R^2}{2}$ and M is at higher potential

(C) π RBV and Q is at higher potential (D) 2 RBV and Q is at higher potential.

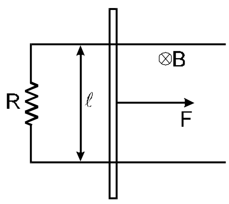
SECTION (D) : CIRCUIT PROBLEMS WITH DYNAMICS

D 1. The block of mass (M) is connected by thread which is wound on a pulley, free to rotate about fixed horizontal axis as shown. A uniform magnetic field B exists in a horizontal plane. The disc is connected with the resistance R as shown. Calculate the terminal velocity of the block if it was released from rest. Treat pulley as uniform metallic disc of radius L.



(A) $\frac{4mgR}{B^2L^2}$ (B) $\frac{3mgR}{4B^2L^2}$ (C) $\frac{2mgR}{B^2L^2}$ (D) $\frac{3mgR}{2B^2L^2}$

D 2. A constant force F is being applied on a rod of length 'ℓ' kept at rest on two parallel conducting rails connected at ends by resistance R in uniform magnetic field B as shown.



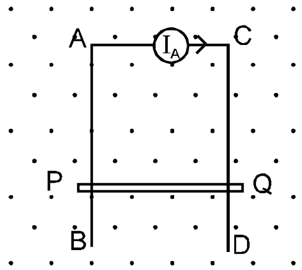
(A) the power delivered by force will be constant with time

(B) the power delivered by force will be increasing first and then it will decrease

(C) the rate of power delivered by the external force will be increasing continuously

(D) the rate of power delivered by external force will be decreasing continuously before becoming zero.

D 3. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then,



(A) The rod PQ may move downward with constant acceleration

(B) The rod PQ may move upward with constant acceleration

(C) The rod will move downward with decreasing acceleration and finally acquire a constant velocity

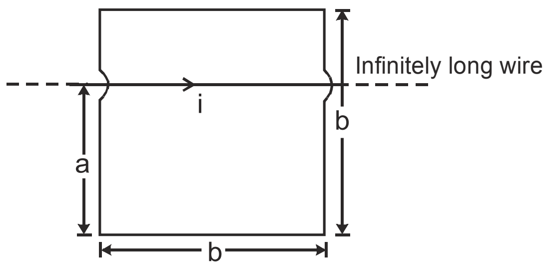
(D) either A or B.

D 4. A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field) ,with a velocity v through a distance x . The work done is :

(A) $\frac{B\ell^2vx}{R}$ (B) $\frac{2B^2\ell^2vx}{R}$ (C) $\frac{4B^2\ell^2vx}{R}$ (D) 0

SECTION (E) : EMF INDUCED IN A ROD OR LOOP IN NONUNIFORM MAGNETIC FIELD

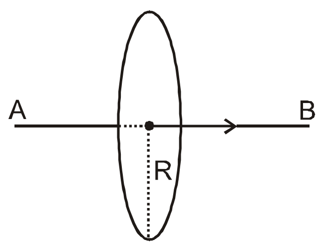
E 1. For the situation shown in the figure, flux through the square loop is :



(A) $\left(\frac{\mu_0 ia}{2\pi}\right)\ell n\left(\frac{a}{2a-b}\right)$ (B) $\left(\frac{\mu_0 ib}{2\pi}\right)\ell n\left(\frac{a}{2b-a}\right)$

(C) $\left(\frac{\mu_0 ib}{2\pi}\right)\ell n\left(\frac{a}{b-a}\right)$ (D) $\left(\frac{\mu_0 ia}{2\pi}\right)\ell n\left(\frac{2a}{b-a}\right)$

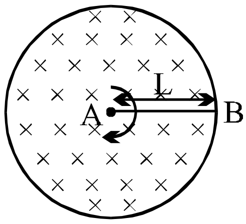
E 2. A long conductor AB lies along the axis of a circular loop of radius R. If the current in the conductor AB varies at the rate of I ampere/second, then the induced emf in the loop is



(A) $\frac{\mu_0 IR}{2}$ (B) $\frac{\mu_0 IR}{4}$ (C) $\frac{\mu_0 \pi IR}{2}$ (D) zero

SECTION (F) : INDUCED EMF IN A ROD, RING, DISC ROTATING IN A UNIFORM MAGNETIC FIELD

F 1. A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angles to a uniform magnetic field of induction B. The e.m.f developed between the mid point C of the rod and end B is

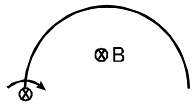


(A) $\frac{B\omega L^2}{4}$ (B) $\frac{B\omega L^2}{2}$ (C) $\frac{3B\omega L^2}{4}$ (D) $\frac{3B\omega L^2}{8}$

F 2. A conducting rod of length ℓ rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

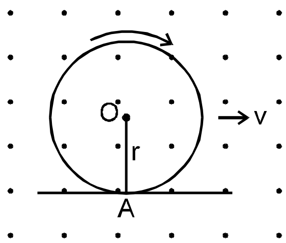
(A) $2B\omega\ell^2$ (B) $\frac{1}{2}\omega B\ell^2$ (C) $B\omega\ell^2$ (D) zero

F 3. A semicircular wire of radius R is rotated with constant angular velocity ω about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced e.m.f. between the ends is:



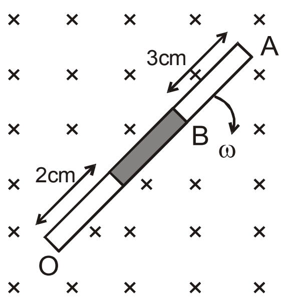
(A) $B\omega R^2/2$ (B) $2B\omega R^2$ (C) is variable (D) none of these

F 4. A conducting ring of radius r with a conducting spoke is in pure rolling on a horizontal surface in a region having a uniform magnetic field B as shown, v being the velocity of the centre of the ring. Then the potential difference $V_0 - V_A$ is



(A) $\frac{Bvr}{2}$ (B) $\frac{3Bvr}{2}$ (C) $\frac{-Bvr}{2}$ (D) $\frac{3Bvr}{2}$

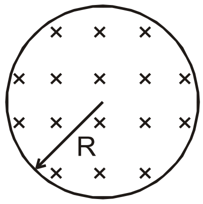
F 5. A rod of length 10 cm made up of conducting and non-conducting material (shaded part is non-conducting). The rod is rotated with constant angular velocity 10 rad/sec about point O, in constant and uniform magnetic field of 2 Tesla as shown in the figure. The induced emf between the point A and B of r od will be

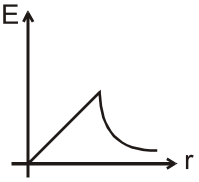


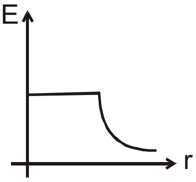
(A) 0.029 v (B) 0.1 v (C) 0.051 v (D) 0.064 v

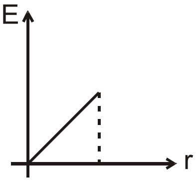
SECTION (G) : FIXED LOOP IN A TIME VARYING MAGNETIC FIELD & INDUCED ELECTRIC FIELD

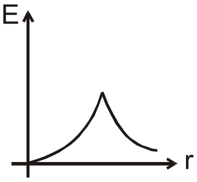
G 1. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is



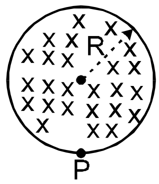
(A) 

(B) 

(C) 

(D) 

G 2. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (Tesla/second). An electron of charge q, placed at the point P on the periphery of the field experiences an acceleration :



(A) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward left

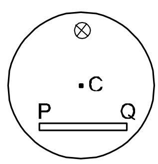
(B) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward right

(C) $\frac{eR}{m}\frac{dB}{dt}$ toward left

(D) zero

G 3. In a cylindrical region uniform magnetic field which is perpendicular to the plane of the figure is increasing with time and a conducting rod PQ is

placed in the region. If C is the centre of the circle then



(A) P will be at higher potential than Q.

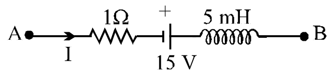
(B) Q will be at higher potential than P.

(C) Both P and Q will be equipotential.

(D) no emf will be developed across rod as it is not crossing / cutting any line of force.

SECTION (H) : SELF INDUCTION, SELF INDUCTANCE SELF INDUCED EMF & MAGNETIC ENERGY DENSITY

H 1. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of $10^3\ As^{-1}$ then $V_B - V_A$ equals



(A) 20 V (B) 15 V (C) 10 V (D) 5 V

H 2. Two different coils have self-inductance L_1 = 8 mH, L_2 = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then **[JEE - 1994]**

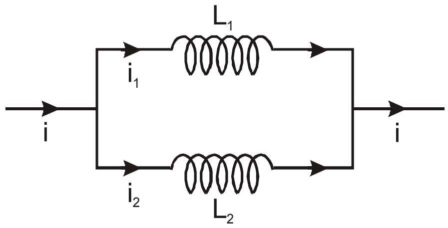
(A) $\frac{i_1}{i_2} = \frac{1}{4}$ (B) $\frac{i_1}{i_2} = 4$ (C) $\frac{W_2}{W_1} = 4$ (D) $\frac{V_2}{V_1} = \frac{1}{4}$

H 3. A wire of fixed length is wound on a solenoid of length 'ℓ' and radius 'r'. Its self inductance is found to be L. Now if same wire is wound on a solenoid of length $\frac{\ell}{2}$ and radius $\frac{r}{2}$, then the self inductance will be:

(A) 2 L (B) L (C) 4 L (D) 8 L

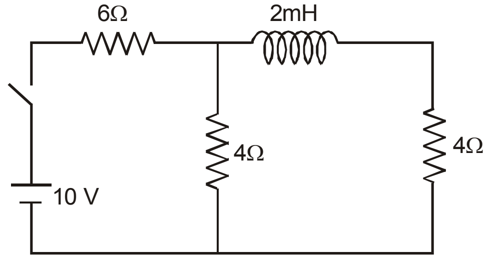
SECTION (I) : CIRCUIT CONTAINING INDUCTANCE, RESISTANCE & BATTERY, GROWTH AND DECAY OF CURRENT IN A CIRCUIT CONTAINING INDUCTOR

I 1. Two inductors L_1 and L_2 are connected in parallel and a time varying current i flows as shown. The ratio of currents i_1/i_2 at any time t is



(A) L_1/L_2 (B) L_2/L_1 (C) $\frac{L_1^2}{(L_1+L_2)^2}$ (D) $\frac{L_2^2}{(L_1+L_2)^2}$

I 2. In the given circuit find the ratio of i_1 to i_2. Where i_1 is the initial (at t = 0) current, and i_2 is steady state (at t = ∞) current through the battery :



(A) 1.0 (B) 0.8 (C) 1.2 (D) 1.5

I 3. In a series L–R growth circuit, if maximum current and maximum induced emf in an inductor of inductance 3mH are 2A and 6V respectively, then the time constant of the circuit is :

(A) 1 ms. (B) 1/3 ms. (C) 1/6 ms (D) 1/2 ms

SECTION (J) : MUTUAL INDUCTION & MUTUAL INDUCTANCE

J 1. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s the e.m.f. in coil 1 in 25.0 mV, when coil 2 has no current and coil 1 has a current of 3.6 A, flux linkage in coil 2 is

(A) 16 mWb (B) 10 mWb (C) 4.00 mWb (D) 6.00 mWb

J 2. A rectangular loop of sides 'a' and 'b' is placed in xy plane. A very long wire is also placed in xy plane such that side of length 'a' of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is 'd'. The mutual inductance of this system is proportional to:

(A) a (B) b (C) 1/d (D) current in wire

J 3. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon-

(A) the rates at which currents are changing in the two coils

(B) relative position and orientation of the two coils

(C) the materials of the wires of the coils

(D) the currents in the two coils

J 4. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is

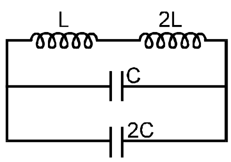
(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0

SECTION (K) : LC OSCILLATIONS

K 1. The inductor in a L–C oscillation has a maximum potential difference of 16 V and maximum energy of 640 μJ. Find the value of capacitor in μF in L–C circuit.

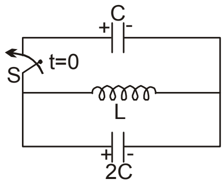
(A) 5 (B) 4 (C) 3 (D) 2

K 2. The frequency of oscillation of current in the inductor is:



(A) $\frac{1}{3\sqrt{LC}}$ (B) $\frac{1}{6\pi\sqrt{LC}}$ (C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{1}{2\pi\sqrt{LC}}$

K 3. In the given LC circuit if initially capacitor C has charge Q on it and 2C has charge 2Q .The polarities are as shown in the figure. Then after closing switch S at t = 0



(A) energy will get equally distributed in both the capacitor just after closing the switch.

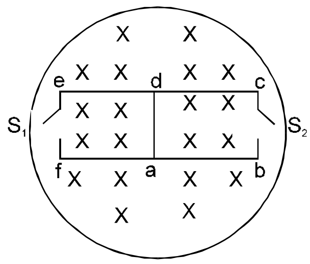
(B) initial rate of growth of current in inductor will be $2Q/3CL$

(C) maximum energy in the inductor will be $3Q^2/2C$

(D) none of these

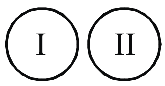
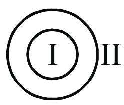
PART - III : MATCH THE COLUMN

1. The magnetic field in the cylindrical region shown in figure increases at a constant rate of 10.0 mT/s Each side of the square loop abcd and defa has a length of 2.00 cm and resistance of 2.00 Ω . Correctly match the current in the wire 'ad' in four different situations as listed in column-I with the values given in column-II.



Column-I	Column-II
(A) the switch S_1 is closed but S_2 is open	(p) 5×10^{-7} A, d to a
(B) S_1 is open but S_2 is closed	(q) 5×10^{-7} A, a to d
(C) both S_1 and S_2 are open	(r) 2.5×10^{-8} A, d to a
(D) both S_1 and S_2 are closed.	(s) 2.5×10^{-8} A, a to d
	(t) No current flows

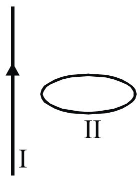
2. In all the situations current in loop-I is i_1 and current in loop-II is i_2. Consider the infinite wire as the side of infinite large loop. Column-II describes the variation in current i_1 in different arrangements and column-I describes the various effects.

Column I	Column-II	
(A) Current is clockwise in loop-II.	(P) Current i_1 is clockwise and decreasing at constant rate.	
(B) Current is anticlockwise in loop-II.	(Q) Current i_1 is clockwise and decreasing at constant rate.	

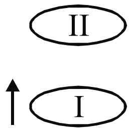
(C) Flux of current i_1 through loop-II is less than flux of current i_1 through loop-I

(D) Loop-II tends to reduce its area due to magnetic force applied by magnetic field of i_1

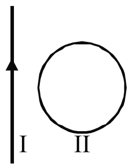
(R) Current in infinite wire is i_1 and decreasing at constant rate. The wire is perpendicular to plane of loop.



(S) Loop-I having constant current in clockwise direction moving upward with retardation. Both loops are co-axial.



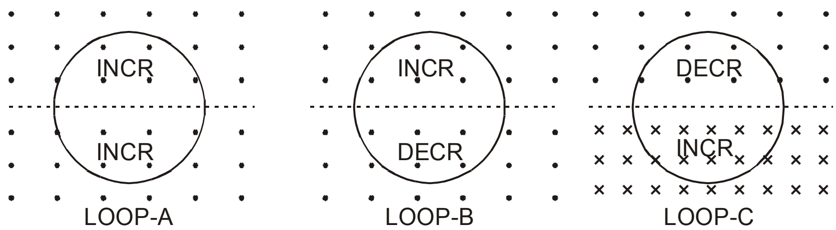
(T) Current in infinite wire is i_1 upward and decreasing at constant rate. The wire is parallel to the plane of loop.



Exercise # 2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Three identical conducting circular loops are placed in uniform magnetic fields. Inside each loop, there are two magnetic field regions, separated by dashed line that coincides with a diameter, as shown. Magnetic fields may either be increasing (marked as INCR) or decreasing (marked as DECR) in magnitude at the same rates. If I_A, I_B and I_C are the magnitudes of the induced currents in the loops A, B and C respectively then choose the **CORRECT** relation :-



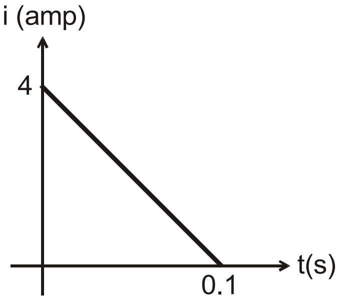
(A) $I_A > I_B = I_C$ (B) $I_A = I_C > I_B$ (C) $I_A = I_B = I_C$ (D) $I_C > I_A > I_B$

2. A bar magnet is released at one end from rest coaxially along the axis of a very long fixed vertical copper tube. After some time the magnet

(A) will move with an acceleration g (B) will move with almost constant speed

(C) will stop in the tube (D) will oscillate

3. Some magnetic flux is changed in a coil of resistance 10 ohm. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Webers is (Neglect self inductance of the coil)



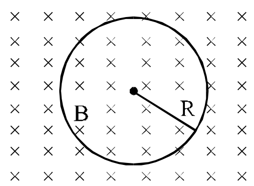
(A) 2 (B) 4

(C) 6 (D) 8

4. A metal rod of resistance 20 Ω is fixed along a diameter of conducting ring of radius 0.1 m and lies in x-y plane. There is a magnetic field $\vec{B} = (50T)\hat{k}$. The ring rotates with an angular velocity ω = 20 rad/s about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. The current through external resistance is

(A) $\frac{1}{4}$A (B) $\frac{1}{2}$A (C) $\frac{1}{3}$A (D) zero

5. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The e.m.f induced in the loop is



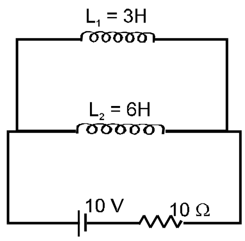
(A) $2\pi(R_0 + t)B$ clockwise (B) $\pi(R_0 + t)B$ clockwise

(C) $2\pi(R_0 + t)B$ anticlockwise (D) zero

6. An inductor coil stores energy U when a current i is passed through it and dissipates heat energy at the rate of P. The time constant of the circuit when this coil is connected across a battery of zero internal resistance is :

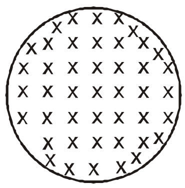
(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

7. Two inductor coils of self inductance 3H and 6H respectively are connected with a resistance 10Ω and a battery 10 V as shown in figure. The ratio of total energy stored at steady state in the inductors to that of heat developed in resistance in 10 seconds at the steady state is(neglect mutual inductance between L_1 and L_2):



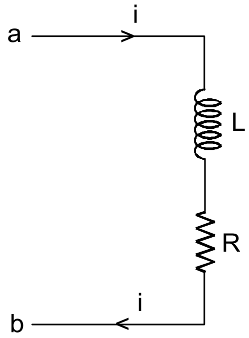
(A) $\frac{1}{10}$ (B) $\frac{1}{100}$ (C) $\frac{1}{1000}$ (D) 1

8. A non conducting ring of radius R and mass m having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying uniform magnetic field $B = 4t^2$ is switched on at time t=0. The coefficient of friction between the ring and the table, if the ring starts rotating at t =2 sec, is :



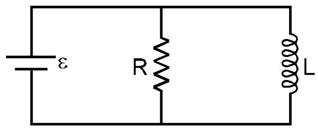
(A) $\frac{4qmR}{g}$ (B) $\frac{2qmR}{g}$ (C) $\frac{8qR}{mg}$ (D) $\frac{qR}{2mg}$

✍**9.** When the current in the portion of the circuit shown in the figure is 2A and increasing at the rate of 1A/s, the measured potential difference $V_a - V_b$ = 8V. However when the current is 2A and decreasing at the rate of 1A/s, the measured potential difference $V_a - V_b$ = 4V. The values of R and L are :



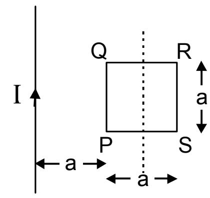
(A) 3 ohm and 2 Henry respectively (B) 2 ohm and 3 Henry respectively
(C) 10 ohm and 6 Henry respectively (D) 6 ohm and 1 Henry respectively

✍**10.** The battery shown in the figure is ideal. The values are ε = 10 V, R = 5Ω, L = 2H . Initially the current in the inductor is zero. The current through the battery at t = 2s is



(A) 12 A (B) 7 A (C) 3 A (D) none of these

11. In the figure shown a square loop PQRS of side 'a' and resistance 'r' is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid points of the side QR and PS. The total amount of charge which passes through any point of the loop during rotation is :



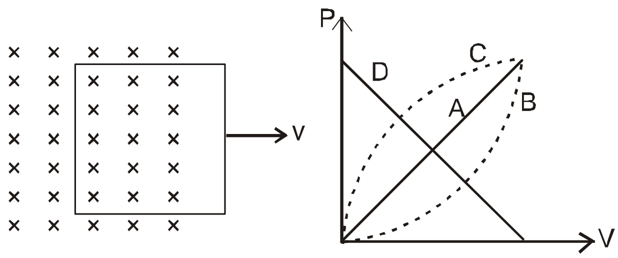
(A) $\frac{\mu_0 I a}{2\pi r} \ell n2$

(B) $\frac{\mu_0 I a}{\pi r} \ell n2$

(C) $\frac{\mu_0 I a^2}{2\pi r}$

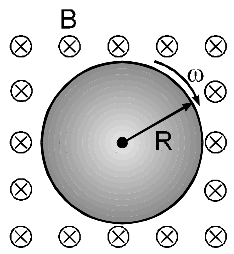
(D) cannot be found because time of rotation not give.

12. Fig. shows a conducting loop being pulled out of a magnetic field with a constant speed v. Which of the four plots shown in fig. may represent the power delivered by the pulling agent as a function of the constant speed v.



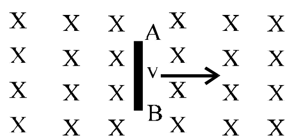
(A) A (B) B (C) C (D) D

13. A conducting disc of radius R is placed in a uniform and constant magnetic field B parallel to the axis of the disc. With what angular speed should the disc be rotated about its axis such that no electric field develops in the disc.(the electronic charge and mass are e and m)



(A) $\frac{eB}{2m}$ (B) $\frac{eB}{m}$ (C) $\frac{2\pi m}{eB}$ (D) $\frac{\pi m}{eB}$

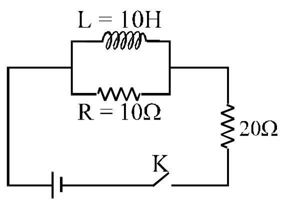
14. A conducting rod AB moves with a uniform velocity v in a constant magnetic field as shown in fig.



(A) The rod becomes hot because of Joule heating (B) The end A becomes positively charged

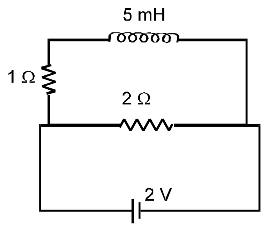
(C) The end B become positively charged (D) The rod becomes electrically charged

15. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2V battery as shown. The key K is shorted at time t = 0. Find the initial (t = 0) and final (t → ∞) currents through battery.



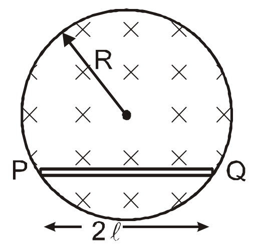
(A) $\frac{1}{15}A, \frac{1}{10}A$ (B) $\frac{1}{10}A, \frac{1}{15}A$ (C) $\frac{2}{15}A, \frac{1}{10}A$ (D) $\frac{1}{15}A, \frac{2}{25}A$

16. When induced emf in inductor coil is 50% of its maximum value then stored energy in inductor coil in the given circuit at that instant will be : -



(A) 2.5 mJ (B) 5mJ (C) 15 mJ (D) 20 mJ

✍17. A uniform magnetic field, $B = B_0 t$ (where B_0 is a positive constant), fills a cylindrical volume of radius R, then the potential difference in the conducting rod PQ due to electrostatic field is :



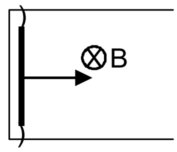
(A) $B_0\ell\sqrt{R^2+\ell^2}$ (B) $B_0\ell\sqrt{R^2-\frac{\ell^2}{4}}$ (C) $B_0\ell\sqrt{R^2-\ell^2}$ (D) $B_0R\sqrt{R^2-\ell^2}$]

18. When the current in a certain inductor coil is 5.0 A and is increasing at the rate of 10.0 A/s, the magnitude of potential difference across the coil is 140 V. When the current is 5.0 A and decreasing at the rate of 10.0 A/s, the potential difference is 60 V. The self inductance of the coil is :

(A) 2H (B) 4H (C) 10H (D) 12H

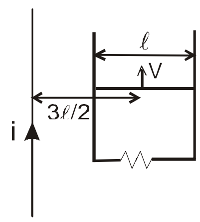
PART - II : INTEGER TYPE (SINGLE AND DOUBLE VALUE)

1. A Π-shaped conductor is located in a uniform magnetic field perpendicular to the plane of the conductor and varying with time at the rate $\frac{dB}{dt}$ = 0.10 T/s. A conducting connector starts moving with a constant acceleration w = 10 cm/s² along the parallel bars of the conductor. The length of the connector is equal to ℓ = 20 cm. Find the emf induced (in mV) in the loop at t = 2.0 s after the beginning of the motion, if at the moment t = 0 the loop area and the magnetic induction (B) are equal to zero. The self inductance of the loop is to be neglected.

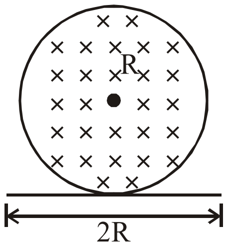


2. A conducting rod slides on a pair of thick fixed, metallic rails laid parallel to an infinitely long fixed wire carrying a constant current i. The center of the rod is at a distance 3ℓ/2 from the wire. The ends of the rails are connected by resistor of resistance R. If a force is needed to keep the rod sliding at a constant speed v, as shown in figure. Then the power delivered by the external agent exerting the force on the rod is $\frac{1}{R}\left(\frac{\mu_0 iv}{2\pi}\ln\lambda\right)^2$.

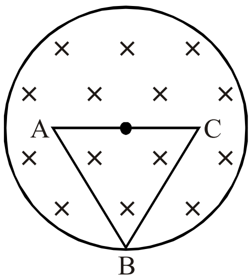
Then value of λ is:



3. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length 2R placed as shown in figure. Then magnitude of induced emf across the rod is $\frac{\pi R^2\alpha}{n}$. Find value of n.

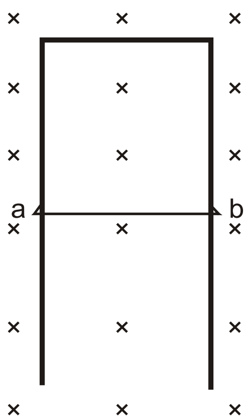


4. An equilateral triangle ABC of side a is placed in the magnetic field with side AC and its centre coinciding with the centre of the magnetic field. The magnetic field varies with time as B = ct. The emf induced (in V) across side AB is (given a = 4m, c = $\frac{1}{\sqrt{3}} Ts^{-1}$)

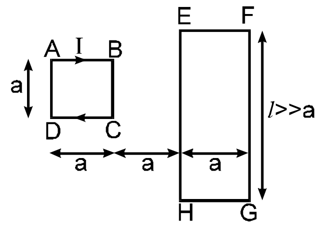


5. A copper wire ab of length ℓ, resistance r and mass m starts sliding at t = 0 down a smooth, vertical, thick pair of connected condcuting rails as shown in figure. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the rails. If the velocity of the wire as a function of time is $\frac{mgr}{B^2l^2}\left(1-e^{\frac{-gtB^2l^2}{xmgr}}\right)$.

Then value of x is:

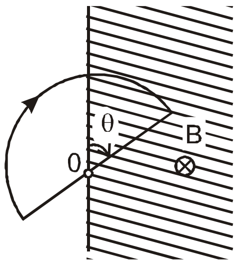


6. In the figure shown two loops ABCD & EFGH are in the same plane . The smaller loop carries time varying current $I = b\,t$, where b is a positive constant and t is time . The resistance of the smaller loop is r and that of the larger loop is R. If induced current in the larger loop is $\frac{\mu_0\, a\, b}{2\pi\, R}\,\ell n\,\frac{4}{c}$. Then value of c is (Neglect the self inductance of large loop)

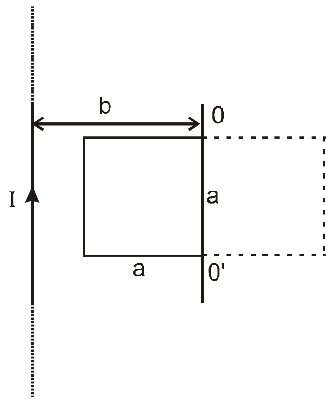


7. A very small circular loop of radius a is initially coplanar & concentric with a much larger circular loop of radius b (> > a). A constant current I is passed in the large loop which is kept fixed in space & the small loop is rotated with constant angular velocity ω about a diameter. The resistance of the small loop is R & its inductance is negligible. If torque exerted on smaller loop to rotate is $\frac{\omega}{R}\left(\frac{\pi a^2 \mu_0 I \sin \omega t}{\lambda b}\right)^2$, then value of λ is:

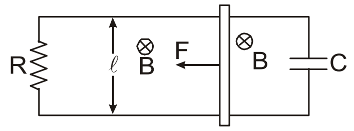
8. A wire loop enclosing a semi-circle of radius a is located on the boundary of a uniform magnetic field of induction B (Figure). At the moment t = 0 the loop is set into rotation with a constant angular acceleration β about an axis O coinciding with a line of vector B on the boundary. If the emf induced in the loop as a function of time t is $\varepsilon_i = \frac{1}{x}\,(-1)^n\,Ba^2\beta t$. The arrow in the figure shows the emf direction taken to be positive. Then value of x is: (at t = 0 loop was completely outside)



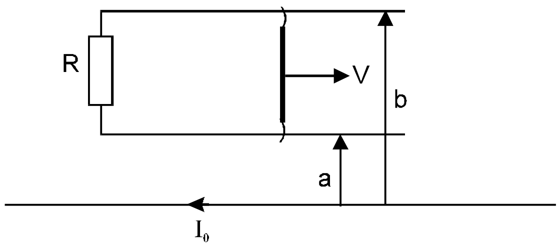
9. A square wire frame (initially current is zero) with side a and a straight conductor carrying a constant current I are located in the same plane (figure). The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current-carrying conductor by a distance b. If the total electric charge having flown through the frame if i = 0 at t = 0 in the loop is $\frac{\beta \mu_0 a I}{8\pi R}\,\ell n\,\frac{b+a}{b-a}$. Then value of β is



10. Two parallel long smooth conducting rails separated by a distance ℓ are connected by a movable conducting connector of mass 'm'. Terminals of the rails are connected by the resistor R & the capacitor C as shown. A uniform magnetic field B perpendicular to the plane of the rails is switched on. The connector is dragged by a constant force F. The speed of the connector as function of time is $\frac{FR}{B^2\ell^2}\left(1-e^{\left[\frac{-B^2\ell^2 t}{R(m+B^2\ell^2 C)}\right]}\right)^x$. Then the value of x is. If the force F is applied at t = 0.



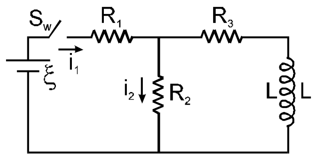
11. A long straight wire carries a current I_0. at distance a and b from it there are two other wires, parallel to the former one, which are interconnected by a resistance R (figure). A connector slides without friction along the wires with a constant velocity v. Assuming the resistances of the wires, the connector, the sliding contacts, and the self-inductance of the frame to be negligible, (use b = 2a, I_0 = 1 A, v = 1 m/s, R = 3 × $10^{-2}\,\Omega$, ln2 = 0.6)



(a) Find the magnitude of the current (in μA) induced in the connector;

(b) If the force required to maintain the connector's velocity constant is a × 10^{-14} N, then value of a is:

12. In figure, ξ = 60 V, $R_1 = R_2 = R_3 = 10\ \Omega$ and L = 2 H. Find i_1 & i_2 (in μA).



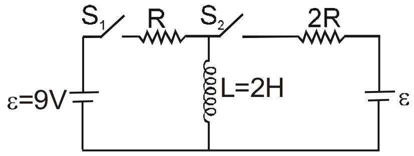
(a) immediately after switch S_w is closed

(b) a long time after

(c) immediately after S_w is opened again

(d) a long time later.

13. In the circuit shown S_1 & S_2 are switches. S_2 remains closed for a long time and S_1 open. Now S_1 is also closed. Just after S_1 is closed,

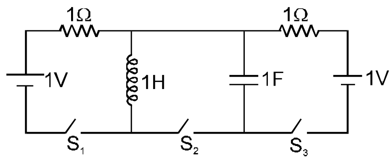


find :

(a) the potential difference (V) across R

(b) $\frac{di}{dt}$ (with sign) in L.

14. In the circuit shown switches S_1 and S_3 have been closed for 1 sec and S_2 remained open. Just after 1 second is over switch S_2 is closed and S_1, S_3 are opened. After that instant



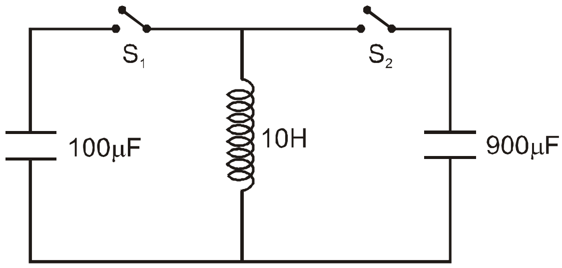
(a) If the maximum current in the circuit containing inductor and capacitor only is x.

(b) If the maximum charge on the capacitor is y.

(c) If the charge on the upper plate of the capacitor as function of time taking the instant of switching on of S_2 and switching off all the switches to be t = 0 is $\sqrt{2}\,(1-e^{-1})\cos\left(t+\frac{\pi}{z}\right)$.

Then value of $\frac{xz}{y}$ is:

15. Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged in the figure shown. Then the switch S_2 is closed for a time t_1, after which it is opened and at the same instant switch S_1 is closed for a time t_2 and then opened. It is now found that the 100 μF capacitor is charged to 300 V. If t_1 and t_2 are minimum possible values of the time intervals. Then t_1 / t_2 is ($\pi^2 = 10$)



PART - III : ONE OR MORE THAN ONE OPTION CORRECT

1. A conducting ring is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the ring if

(A) it is rotated about its axis
(B) it is translated
(C) it is rotated about a diameter
(D) it is deformed

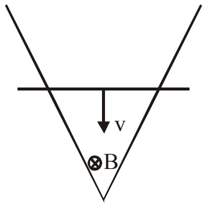
2. A super conducting loop having an inductance 'L' is kept in a magnetic field which is varying with respect to time. If ϕ is the total flux, ε = total induced emf, then:

(A) ϕ = constant
(B) I = 0
(C) $\varepsilon = 0$
(D) $\varepsilon \neq 0$

3. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field in a direction perpendicular to that fixed diameter.

(A) The emf will be maximum at the moment when flux is zero.
(B) The emf will be '0' at the moment when flux is maximum.
(C) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(D) The phase difference between the flux and the emf is $\pi/2$

4. A wire is bent in the form of a V shape and placed in a horizontal plane. There exists a uniform magnetic field B perpendicular to the plane of the wire. A uniform conducting rod starts sliding over the V shaped wire with a constant speed v as shown in the figure. If the wire has uniform resistance then choose the **CORRECT** option(s).



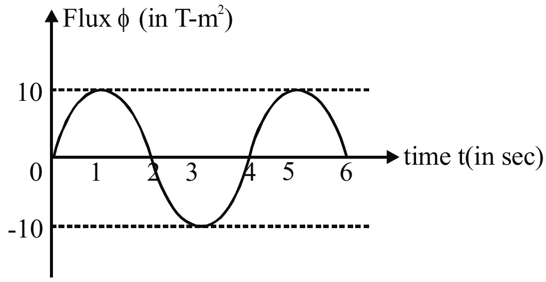
(A) the emf in the circuit will remain constant

(B) the emf in the circuit will change with time

(C) the current in the rod will remain constant

(D) the current in the rod will change with time

5. A closed conducting loop, having resistance R, is being rotated about an axis perpendicular to the magnetic field. Magnetic flux through the closed conducting loop is continuously changing according to the graph shown in the adjacent figure. Then, which of the following statement(s) is/are correct?



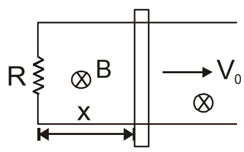
(A) The electric current through the loop is minimum (zero) at t = 1 s, 3s and 5s.

(B) The electric current through the loop is minimum (zero) at t = 0 s, 2s and 6s.

(C) Total charge flown through any cross-section of a closed conducting loop between 0 and 6 s is zero

(D) Total work done in rotating the loop in the magnetic field is zero

6. A conducting rod of length ℓ is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct :



(A) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.

(B) If applied external force is doubled than a part of external power increases the velocity of rod.

(C) Lenz's Law is not satisfied if the rod is accelerated by external force

(D) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

7.* An LR series circuit with a battery is connected at t =0. Which of the following quantities are zero just after the connection ?

(A) current in the circuit (B) magnetic field energy in the inductor

(C) power delivered by the battery (D) emf induced in the inductor

8. An LR series circuit has L = 1 H and R = 1 Ω. It is connected across an emf of 2 V. The maximum rate at which energy is stored in the magnetic field is :

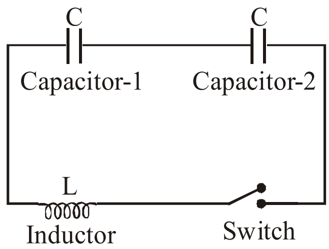
(A) The maximum rate at which energy is stored in the magnetic field is 1W

(B) The maximum rate at which energy is stored in the magnetic field is 2W

(C) The current at that instant is 1 A

(D) The current at that instant is 2 A

9. Consider the circuit shown with respective specifications of elements marked in the figure. Capacitor-1 is charged such that charge on it is Q_0 and it's left plate is positively charged. While capacitor-2 is uncharged. The switch is closed at t = 0.



(A) Frequency of oscillation of charge on left plate of capacitor-1 is $\frac{1}{\pi\sqrt{2LC}}$

(B) Frequency of oscillation of charge on left plate of capacitor-1 is $\frac{1}{\pi}\sqrt{\frac{2}{LC}}$

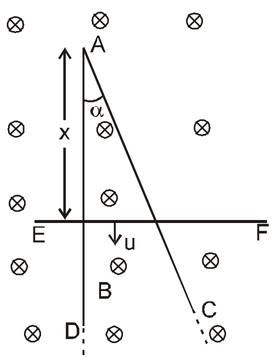
(C) Maximum current through the inductor is $\frac{Q_0}{\sqrt{2LC}}$

(D) Maximum current through the inductor is $\frac{Q_0}{\sqrt{LC}}$

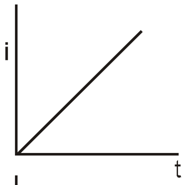
PART - IV : COMPREHENSION

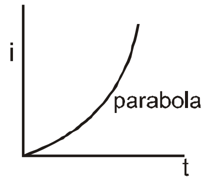
Comprehension # 1

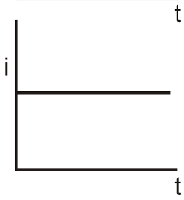
A long straight wire having uniform resistance per unit length λ (ohms/metre) is bent into V- shape as shown (angle DAC = α). A uniform and constant magnetic field B (tesla) exists in space which is perpendicular to the plane of V-shaped wire. Another straight wire EF of same uniform resistance per unit length λ (ohms/ metre) is pulled with constant velocity u (m/s) such that the wire EF is always perpendicular to side AD of V-shape(the wire EF is always in conducting contact with the V-shaped wire at two points). At t=0 (t is time in seconds) the value of x= 0 (x= distance of wire EF from end A, in metres)

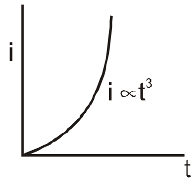


1. The variation of current 'i' induced in the triangular loop with time 't' is

(A) 

(B) 

(C) 

(D) 

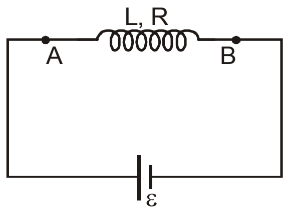
2. The magnetic force acting on the wire EF due to uniform magnetic field B at any time t (in N) is

(A) $\dfrac{B^2tu^2\tan^2\alpha}{\lambda(1+\tan\alpha+\sec\alpha)}$ (B) $\dfrac{B^2u^2\tan^2\alpha}{\lambda(1+\tan\alpha+\sec\alpha)}$

(C) $\dfrac{B^2tu^2}{\lambda(2+\tan\alpha)}$ (D) none of these

Comprehension # 2

An inductor having self inductance *L* with its coil resistance *R* is connected across a battery of emf ε. When the circuit is in steady state at *t* = 0 an iron rod is inserted into the inductor due to which its inductance becomes n *L* (*n* > 1).



3. After insertion of rod which of the following quantities will change with time ?

(1) Potential difference across terminals *A* and *B*.

(2) Inductance.

(3) Rate of heat produced in coil

(A) only (1) (B) (1) & (3) (C) Only (3) (D) (1), (2) & (3)

4. After insertion of the rod, current in the circuit :

(A) Increases with time

(B) Decreases with time

(C) Remains constant with time

(D) First decreases with time then becomes constant

5. When again circuit is in steady state, the current in it is :

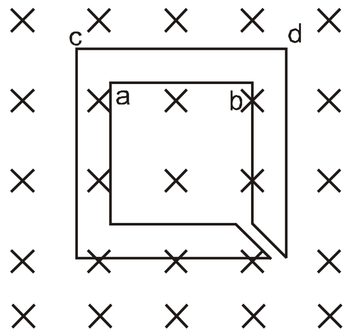
(A) I < ε/R (B) I > ε/R (C) I = ε/R (D) None of these

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions are having more than one correct option.**

1. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then, **[JEE 2009; 3/160, –1]**



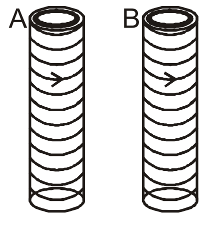
(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction **ba** and I_2 is in the direction **cd**

(D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

2. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are) **[JEE 2009 4/160, –1]**



(A) $\rho_A > \rho_B$ and $m_A = m_B$

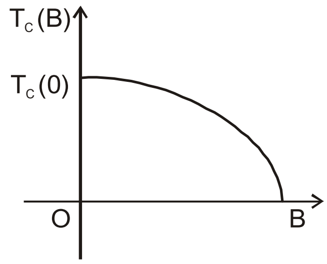
(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

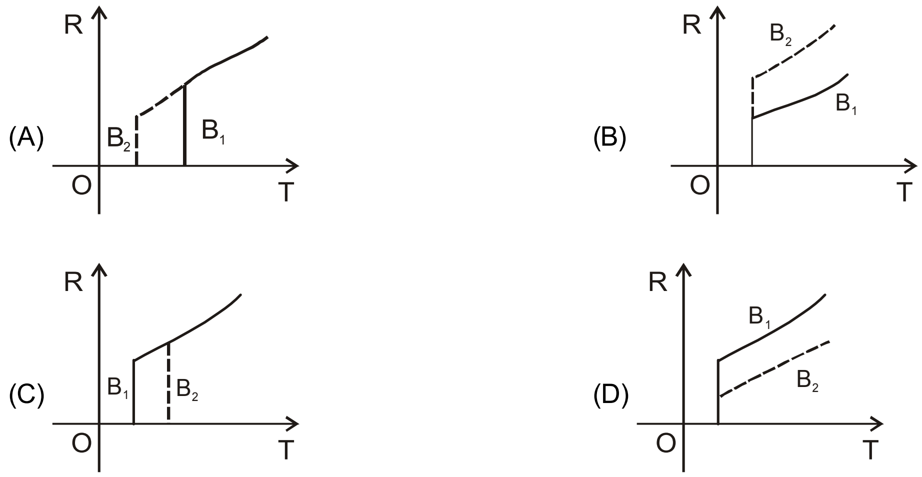
(D) $\rho_A < \rho_B$ and $m_A < m_B$

Paragraph for Question No. 3 to 4

Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure. **[JEE - 2010' 3 × 3/163, –1]**



3. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

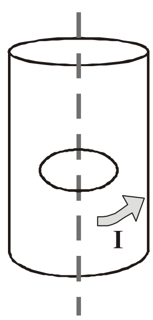


4. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when :

(A) B = 5 Tesla, $T_C(B) = 80$ K (B) B = 5 Tesla, 75 K < $T_C(B)$ < 100 K

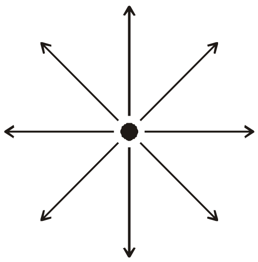
(C) B = 10 Tesla, 75 K < $T_C(B)$ < 100 K (D) B = 10 Tesla, $T_C(B) = 70$ K

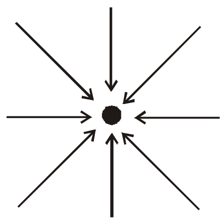
5. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N \mu_0 I_0 \sin(300t)$, then 'N' is **[JEE - 2011' 4/160]**

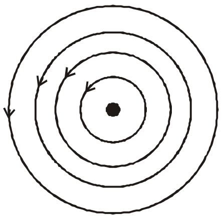


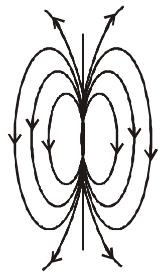
6. Which of the field patterns given below is valid for electric field as well as for magnetic field?

[JEE - 2011' 3/160, –1]

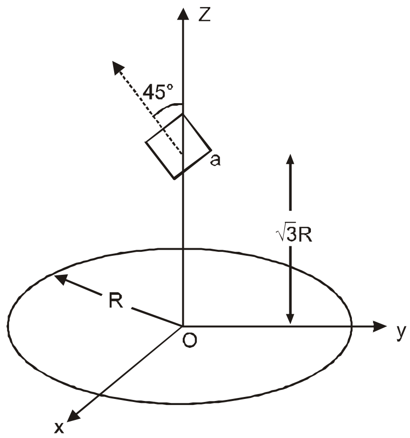
(A) 

(B) 

(C) 

(D) 

7. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a (a << R) having two turns is placed with its center at z = $\sqrt{3}$ R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is **[IIT-JEE-2012, Paper-1; 4/70]**



8. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are) : **[IIT-JEE-2012, Paper-2; 4/66]**

(A) the emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is finite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) The emf induced in the loop is finite if the current decreases at a steady rate.

Paragraph for Questions 9 and 10

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionally constant γ. **[JEE ADVANCED_2013, 3×2/60]**

9. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is :

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

10. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is :

(A) $-\gamma BQR^2$ (B) $-\gamma\frac{BQR^2}{2}$ (C) $\gamma\frac{BQR^2}{2}$ (D) γBQR^2

Paragraph for Questions 11 and 12

A thermal power plant produces electric power of 600 kW and 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values. **[JEE Advance-2013]**

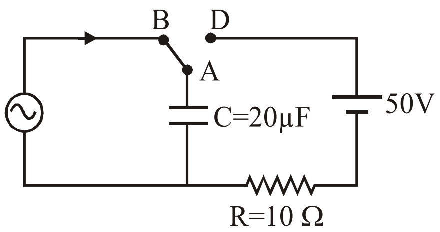
11. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

12. If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30 (C) 40 (D) 50

13. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1A$ and $\omega = 500$ rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 µF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement (s). **[JEE Advance-2014]**



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

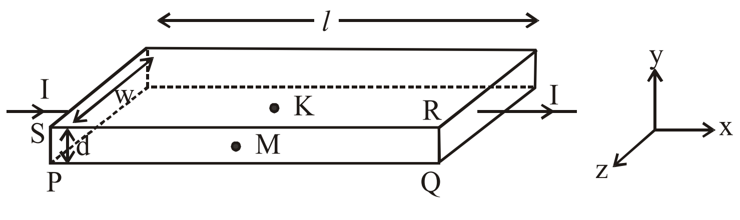
(C) Immediately after A is connected to D, the current in R is 10A

(D) $Q = 2 \times 10^{-3}$ C

Paragraph for Question No. 14 and 15

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width the thickness of the strip are *l*, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons. **[JEE Advance-2015]**



14. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

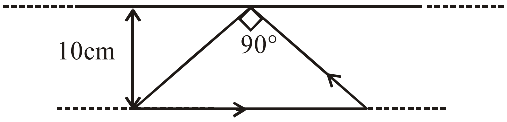
(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

15. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$ (B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$ (D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

16. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of $10\ \mathrm{A\,s^{-1}}$. Which of the following statement(s) is(are) true? **[JEE Advance-2016]**



(A) The induced current in the wire is in opposite direction to the current along the hypotenuse.

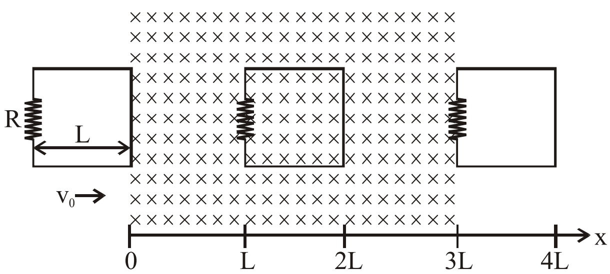
(B) There is a repulsive force between the wire and the loop

(C) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

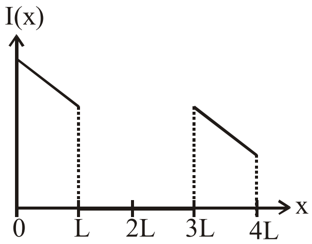
(D) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

17. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is. **[JEE Advance-2016]**

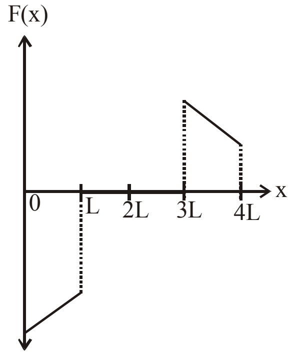
18. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. Which of the following schematic plot(s) is(are) correct ? (Ignore gravity) **[JEE Advance-2016]**



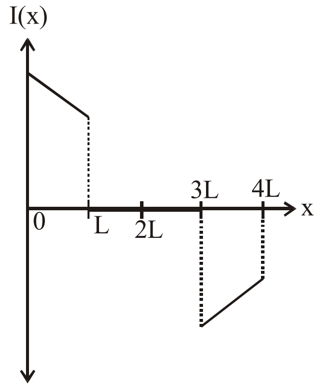
(A)



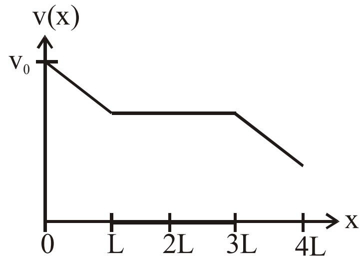
(B)



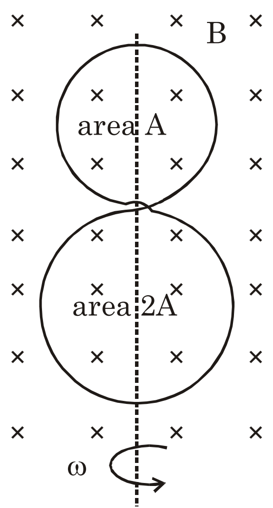
(C)



(D)



19. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other*. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **[JEE Advance-2017]**



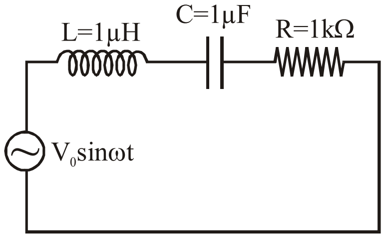
(A) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(B) The net emf induced due to both the loops is proportional to cos ωt

(C) The emf induced in the loop is proportional to the sum of the areas of the two loops

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

20. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct ? **[JEE Advanced-2017]**



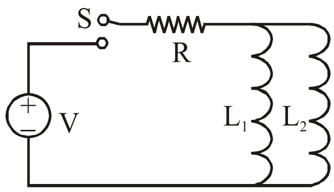
(A) The frequency at which the current will be in phase with the voltage is independent of R.

(B) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

(C) At $\omega >> 10^6$ rad.s^{-1}, the circuit behaves like a capacitor.

(D) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}.

21. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct? **[JEE Advanced-2017]**



(A) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

(B) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(C) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(D) At t = 0, the current through the resistance R is $\frac{V}{R}$

22. The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_X = V_0 \sin \omega t$

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and

$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:- **[JEE Advanced-2017]**

(A) $V_{XY}^{rms} = V_0$

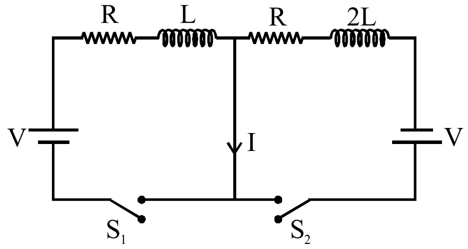
(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) Independent of the choice of the two terminals

(D) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

23. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statement(s) is (are) true?

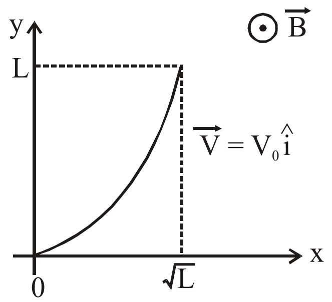
[JEE Advanced-2017]



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

24. A conducting wire of parabolic shape, initially y = x^2, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are:

[JEE ADVANCED_2019,+4,–1]



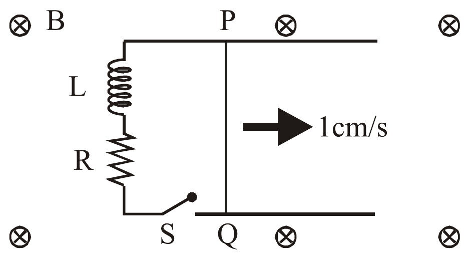
(A) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, y = x initially, of length $\sqrt{2}L$

(B) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(C) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for β = 0

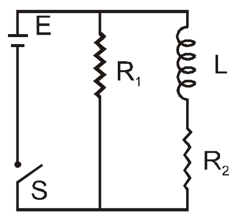
(D) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for β = 2

25. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is x × 10^{-3}A, where the value of x is_______.
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed.
Given : e^{-1} = 0.37 , where e is base of the natural logarithm] **[JEE ADVANCED_2019,+3]**



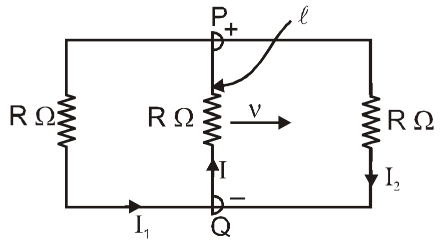
PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. An inductor of inductance L = 400 mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is : **[AIEEE 2009; 4/120, –1]**



(1) $\frac{12}{t}e^{-3t}$ V (2) $6\left(1-e^{-t/0.2}\right)$ V (3) $12\ e^{-5t}$ V (4) $6\ e^{-5t}$ V

2. A rectangular loop has a sliding connector PQ of length ℓ and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are : **[AIEEE 2010, 8/144, –2]**



(1) $I_1 = -I_2 = \frac{B\ell v}{R}$, $I = \frac{2B\ell v}{R}$ (2) $I_1 = I_2 = \frac{B\ell v}{3R}$, $I = \frac{2B\ell v}{3R}$

(3) $I_1 = I_2 = I = \frac{B\ell v}{R}$ (4) $I_1 = I_2 = \frac{B\ell v}{6R}$, $I = \frac{B\ell v}{3R}$

3. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :

[AIEEE - 2011, 1 May, 4/120, –1]

(1) $\pi\sqrt{LC}$ (2) $\frac{\pi}{4}\sqrt{LC}$ (3) $2\pi\sqrt{LC}$ (4) $\sqrt{LC}$

4. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is : **[AIEEE - 2011, 1 May, 4/120, –1]**

(1) 1 mV (2) 0.75 mV (3) 0.50 mV (4) 0.15 mV

5. A horizontal straight wire 20 m long extending from to east to west falling with a speed of 5.0 m\s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb \ m^2. The instantaneous Value of the e.m. f. induced in the wire will be : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) 3 mV (2) 4.5 mV (3) 1.5 mV (4) 6.0mV

6. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to : **[AIEEE 2012 ; 4/120, –1]**

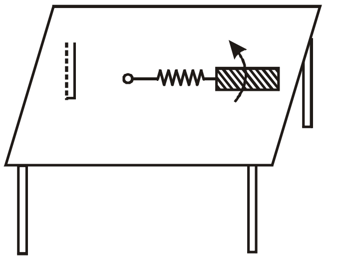
(1) developement of air current when the plate is placed.

(2) induction of electrical charge on the plate

(3) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(4) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

7. A metallic rod of length 'l' is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is: **[JEE-Mains 2013, 4/120]**

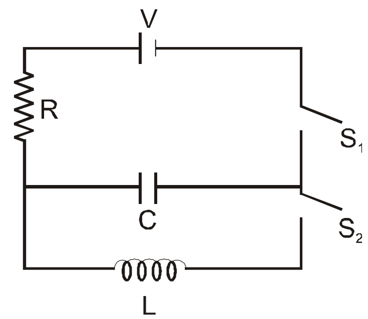


(1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$

8. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :

(1) 9.1×10^{-11} weber (2) 6×10^{-11} weber (3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber

9. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct ? **[JEE-Mains 2013, 4/120]**



(1) Work done by the battery is half of the energy dissipated in the resistor

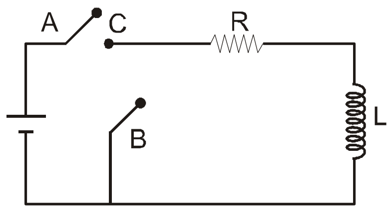
(2) At $t = \tau$, $q = CV/2$

(3) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(4) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

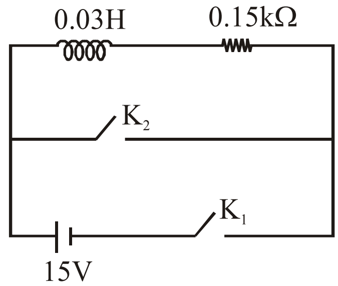
10. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to :

[JEE-Main 2014, 4/120, –1]



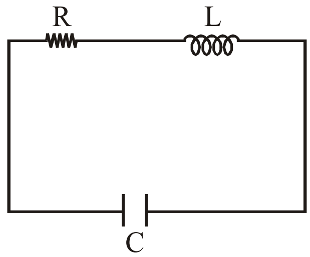
(1) $\frac{e}{1-e}$ (2) 1 (3) –1 (4) $\frac{1-e}{e}$

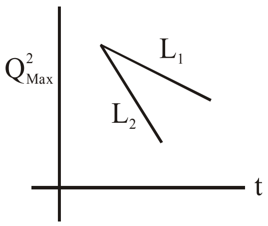
11. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1ms, the current in the circuit will be ($e^5 \cong 150$): **[JEE Main 2015; 4/120, –1]**

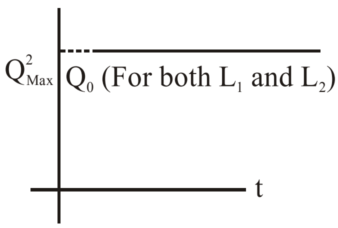


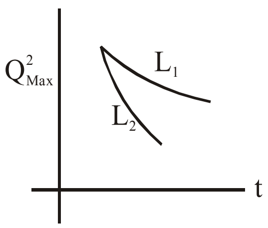
(1) 6.7 mA (2) 0.67 mA (3) 100 mA (4) 67 mA

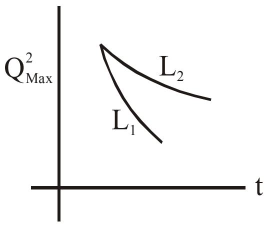
12. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below. If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale) **[JEE Main 2015; 4/120, –1]**



(1)


(2)


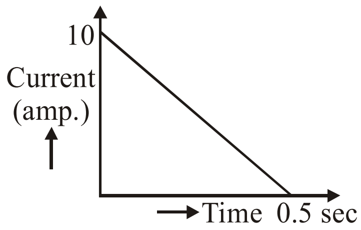
(3)


(4)


13. An arc lamp requires a direct current of 10A at 80V to function. If it is connected to a 220V (rms), 50Hz AC supply, the series inductor needed for it to work is close to :- **[JEE Main 2016; 4/120, –1]**

(1) 0.065 H (2) 80 H (3) 0.08 H (4) 0.044 H

14. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :- **[JEE Main 2017; 4/120, –1]**



(1) 250 Wb (2) 275 Wb (3) 200 Wb (4) 225 Wb

15. For an RLC circuit driven with voltage of amplitude v_m and frequency $w_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :- **[JEE Main 2018; 4/120, –1]**

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$ (3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

16. In an a. c. circuit, the instantaneous e.m.f. and current are given by

e = 100 sin 30 t

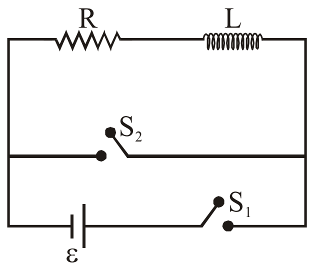
$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively.

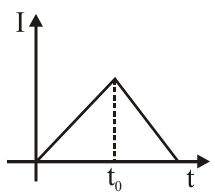
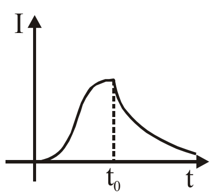
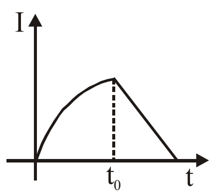
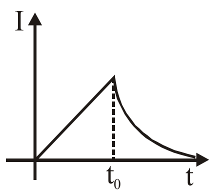
e = 100 sin 30 t **[JEE Main-2018; 4/120, –1]**

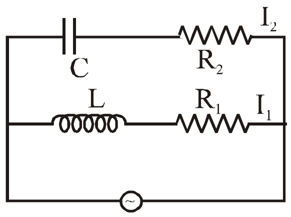
(1) $\frac{1000}{\sqrt{2}}$, 10 (2) $\frac{50}{\sqrt{2}}$, 0 (3) 50, 0 (4) 50, 10

17. In the circuit shown, **[JEE Main-2019, Jan; 4/120, –1]**



the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behavious of the current I as a function of time 't' is given by:

(1) 
(2) 
(3) 
(4) 

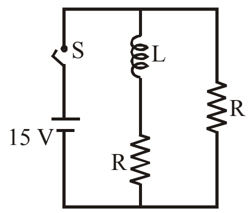
18. 

In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by **[JEE Main 2019 Jan; 4/120, –1]**

V= $200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is :

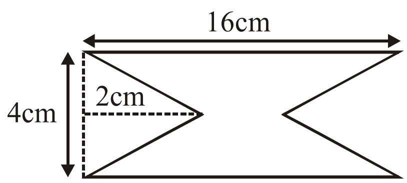
(1) 30° (2) 0° (3) 90° (4) 60°

19. In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed? **[JEE-Main 2019 April; 4/120, –1]**



(1) 6A (2) 7.5A (3) 5.5A (4) 3A

20. At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is : **[JEE-Main 2020 Jan; 4/100, –1]**



(1) 36 μV (2) 48 μV (3) 56 μV (4) 28 μV

21. A long solenoid of radius R carries a time (t)-dependent current $I(t) = I_0 t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \leq t \leq 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as : **[JEE-Main 2020, Jan, 4/100, –1]**

(1) At t = 0.5 direction of I_R reverses and V_R is zero

(2) Direction of I_R remains unchanged and V_R is zero at t = 0.25

(3) Direction of I_R remains unchanged and V_R is maximum at t = 0.5

(4) At t = 0.25 direction of I_R reverses and V_R is maximum

Answers

Exercise # 1

PART - I

SECTION (A) :

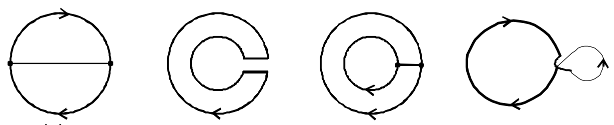
A 1. (i) 1.2 Volt (ii) 1.4 volt (iii) 17.5 C
(iv) 3.5 A (v) 86/3 joule.

A 2. (i) 2.4×10^{-5} V (ii) from c to b

A 3. 1.0 V ,anticlockwise.

A 4. (a) –1 mV, –2 mV, 2 mV, 1mV
(b)10 ms to 20 ms and 20 ms to 30 ms.

A 5. (a) In the round conductor the current flows clockwise, there is no current in the connector;
(b) in the outside conductor clockwise;
(c) in both round conductors, clockwise; no current in the connector,
(d) in the left-hand side of the figure eight, clockwise.



(a) (b) (c) (d)

A 6. $\frac{\pi}{8} \times 10^{-4}$ A

SECTION (B) :

B 1. Opposite direction, Same direction.

SECTION (C) :

C 1. (a) zero
(b) vB (bc), positive at b
(c) vB(bc), positive at a
(d) zero

C 2. 1 mV **C 3.** $\lambda V_y B_0$

C 4. (a) at the ends of the diameter perpendicular to the velocity, 2rvB (b) at the ends of the diameter parallel to the velocity, zero.

C 5. B y $\sqrt{8a/k}$

SECTION (D) :

D 1. 2 N **D 2.** (a) 0.1 mA (b) 0.2 mA

D 3. (a) 4 m/s (b) 4 V (c) 3 V (d) 1 V.

D 4. $\frac{mgR}{B^2\ell^2}$ **D 5.** (a) $\frac{B^2\ell^2 v}{2r(\ell + vt)}$; (b) ℓ/v.

D 6. $\frac{mgt}{m + CB^2\ell^2}$

SECTION (E) :

E 1. $\frac{B_0 v_0 L}{2}$

E 2. $(BV \sin\alpha) / r(1 + \sin\alpha)$

E 3. (a) $\phi = \frac{\mu_0 i a}{2\pi} \ell n\left(\frac{a+b}{b}\right)$;

(b) $\varepsilon = \frac{\mu_0 i_0 a}{T} \ell n\left(\frac{a+b}{b}\right) \sin\left(\frac{2\pi t}{T}\right)$

(c) heat $= \left(\frac{5\mu_0^2 i_0^2 a^2}{Tr}\right) \left[\ell n\left(\frac{a+b}{b}\right)\right]^2$

SECTION (F) :

F 1. 0.75 T **F 2.** 67.5 mV

F 3. $\frac{B\omega a^2}{R}$ from C to D **F 4.** $3\pi \times 10^{-6}$ V

F 5. $B\ell\sqrt{g\ell} \sin\frac{\theta}{2}$

F 6. (a) 2.0×10^{-3} V (b) zero
(c) 50 μC (d) $\pi \times 10^{-3} \sin(10\pi t)$
(e) π mV (f) $\frac{\pi^2}{2} \times 10^{-6}$ V

SECTION (G) :

G 1. (a) 6π Volt **(b)** 3 N/C **(c)** 3 A

G 2 (a) $16\pi^2 \times 10^{-10} = 1.6 \times 10^{-8}$ Weber

(b) $4\pi \times 10^{-8}$ V/m

(c) $18\pi \times 10^{-8} = 5.6 \times 10^{-7}$ V/m

G 3. $\frac{erk}{2m}$ directed along tangent to the circle of radius r, whose centre lies on the axis of cylinder.

SECTION (H) :

H 1. 10V **H 2.** 2.2 A/s, decreasing

H 3. 42 + 20 t volt **H 4.** $\frac{\mu_0 e^4}{128\pi^3 \varepsilon_0 m R^5}$

H 5. 2.55×10^{-14} J

SECTION (I) :

I 1. (L/R) $ln\,2$ = 1.109 s, 640 J

I 2. (a) $\frac{1}{5}(1-e^{-2}) \simeq 0.17$ A

(b) $\frac{1}{25}(1-e^{-2})^2 \simeq 0.03$ J

I 3. 4.0 H **I 4.** $2\,[1 - e^{-0.4}] = 0.66$ V

I 5. $\frac{LE^2}{2R_1^2}$ **I 6.** 0.8

I 7. (a) -2.5×10^3 V/s (b) $-2.5\times 10^3 \times e^{-5}$ V/s

I 8. $\frac{2\,B\,\pi R^2}{L}$

SECTION (J) :

J 1. 2.5 V **J 2.** 0.01 H

J 3. $\frac{\mu_0 i a^2 \pi}{2Rb}$

SECTION (K) :

K 1. $q = Q_0 \sin\left(\sqrt{\frac{1}{LC}}\,t + \frac{\pi}{2}\right)$

K 2. (a) 10^4 A/s (b) 0 (c) 2A (d) $100\sqrt{3}\ \mu C$

PART - II

A 1. (A) **A 2.** (D) **A 3.** (A) **A 4.** (D)

A 5. (D) **A 6.** (C)

SECTION (B) :

B 1. (C) **B 2.** (C) **B 3.** (A) **B 4.** (A)

B 5. (D) **B 6.** (C) **B 7.** (C)

SECTION (C) :

C 1. (B) **C 2.** (B) **C 3.** (B) **C 4.** (D)

C 5. (B) **C 6.** (D)

SECTION (D) :

D 1. (A) **D 2.** (D) **D 3.** (D) **D 4.** (D)

SECTION (E) :

E 1. (C) **E 2.** (D)

SECTION (F) :

F 1. (D) **F 2.** (D) **F 3.** (B) **F 4.** (C)

F 5. (C)

SECTION (G) :

G 1. (A) **G 2.** (A) **G 3.** (B)

SECTION (H) :

H 1. (B) **H 2.** (A,C,D) **H 3.** (A)

SECTION (I) :

I 1. (B) **I 2.** (B) **I 3.** (A)

SECTION (J) :

J 1. (D) **J 2.** (A) **J 3.** (B) **J 4.** (D)

SECTION (K) :

K 1. (A) **K 2.** (B) **K 3.** (C)

PART - III

1. (A) $\rightarrow$ q ; (B) $\rightarrow$ p; (C) $\rightarrow$ t; (D) $\rightarrow$ t

2. (A) $\rightarrow$ QT; (B) $\rightarrow$ PS; (C) $\rightarrow$ PQRST; (D) $\rightarrow$ QS

Exercise # 2

PART - I

1.	(B)	**2.**	(B)	**3.**	(A)
4.	(C)	**5.**	(C)	**6.**	(C)
7.	(B)	**8.**	(C)	**9.**	(A)
10.	(A)	**11.**	(B)	**12.**	(B)
13.	(B)	**14.**	(B)	**15.**	(A)
16.	(A)	**17.**	(C)	**18.**	(B)

PART - II

1.	12	**2.**	2	**3.**	4
4.	2	**5.**	1	**6.**	3
7.	2	**8.**	2	**9.**	4
10.	1	**11.**	(a) 4		(b) 48

12. (a) $i_1 = i_2 = 3$; (b) $i_1 = 4$; $i_2 = 2$

(c) $i_1 = 0$, $i_2 = -2$; (d) $i_1 = i_2 = 0$

13. (a) 3 ; (b) 3 **14.** 4 **15.** 3

PART - III

1.	(C, D)	**2.**	(A, C)
3.	(A, B, C, D)	**4.**	(B, C)
5.	(A, C)	**6.**	(A, B, D)
7.	(A, B, C)	**8.**	(A, C)
9.	(A, C)		

PART - IV

1.	(C)	**2.**	(A)	**3.**	(B)
4.	(A)	**5.**	(C)		

Exercise # 3

PART - I

1.	(D)	**2.**	(B,D)	**3.**	(A)
4.	(B)	**5.**	6	**6.**	(C)
7.	7	**8.**	(A, C)	**9.**	(B)
10.	(B)	**11.**	(A)	**12.**	(B)
13.	(C, D)	**14.**	(A, D)	**15.**	(A, C)
16.	(B, D)	**17.**	8	**18.**	(C, D)
19.	(A, D)	**20.**	(A,B)	**21.**	(A,B,C)
22.	(C,D)	**23.**	(B,D)	**24.**	(A,B,D)
25.	(0.62 to 0.64)				

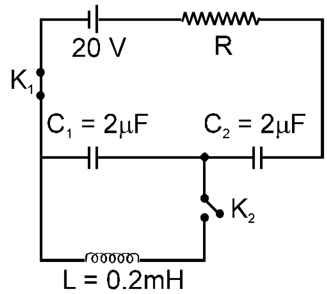
PART - II

1.	(3)	**2.**	(2)	**3.**	(2)
4.	(4)	**5.**	(1)	**6.**	(4)
7.	(4)	**8.**	(1)	**9.**	(3)
10.	(3)	**11.**	(2)	**12.**	(3)
13.	(1)	**14.**	(1)	**15.**	(4)
16.	(1)	**17.**	(2)	**18.**	(3)
19.	(1)	**20.**	(3)	**21.**	(1)

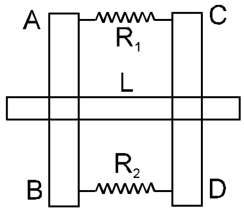
RANKER PROBLEMS

SUBJECTIVE QUESTIONS

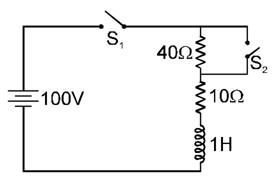
1. A circuit containing capacitors C_1 and C_2 as shown in the figure are in steady state with key K_1 closed. At the instant t = 0, if K_1 is opened and K_2 is closed then the maximum current in the circuit will be :



2. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistance R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of bar L and value R_1 and R_2. (g = 9.8 m/s^2) **[JEE - 1994]**

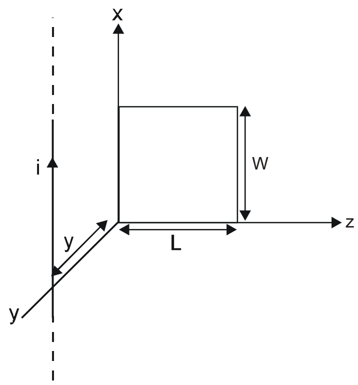


3. In the circuit diagram shown in the figure the switches S_1 and S_2 are closed at time t = 0. After time t = (0.1) ℓn 2 sec, switch S_2 is opened. Find the current in the circuit at time , t = (0.2) ln 2 sec.

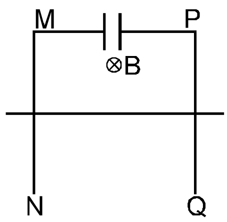


4. A closed circuit consists of a source of constant emf E and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R. It is in steady state. At the moment t = 0 the choke coil inductance was decreased abruptly η times. Find the current in the circuit as a function of time t.

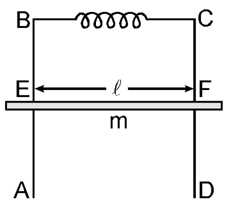
5. In the figure, a long thin wire carrying a varying current $i = i_0 \sin \omega t$ lies at a distance y above one edge of a rectangular wire loop of length L and width W lying in the x-z plane. What emf is induced in the loop.



6. In the figure shown a conducting rod of length ℓ, resistance R & mass m can move vertically downward due to gravity. Other parts are kept fixed. B = constant = B_0. MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is C. The rod is released from rest. Find the maximum current in the circuit.



7. A conducting frame ABCD is kept fixed in a vertical plane. A conducting rod EF of mass m can slide smoothly on it remaining horizontal always. The resistance of the loop is negligible and inductance is constant having value L. The rod is left from rest and allowed to fall under gravity and inductor has no initial current. A uniform magnetic field of magnitude B is present throughout the loop pointing inwards. Determine.

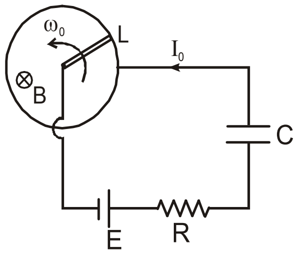


(a) position of the rod as a function of time assuming initial position of the rod to be x = 0 and vertically downward as the positive X-axis.

(b) maximum current in the circuit

(c) maximum velocity of the rod.

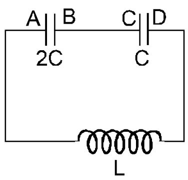
8. L is s smooth conducting loop of radius ℓ = 1.0 m & fixed in a horizontal plane. A conducting rod of mass m = 1.0 kg and length slightly greater than ℓ hinged at the centre of the loop can rotate in the horizontal plane such that the free end slides on the rim of the loop. There is a uniform magnetic field of strength B = 1.0 T directed vertically downward. The rod is rotated with angular velocity ω_0 = 1.0 rad/s and left. The fixed end of the rod and the rim of the loop are connected through a battery of e.m.f. E, a resistor of resistance R = 1.0 Ω, and initially uncharged capacitor of capacitance C = 1.0 F in series. Find :



(i) the time dependence of e.m.f. E such that the current I_0 = 1.0 A in the circuit is constant.

(ii) energy supplied by the battery by the time rod stops .

9. Two capacitors of capacitances 2C and C are connected in series with an inductor of inductance L. Initially capacitors have charge such that $V_B - V_A = 4V_0$ and $V_c - V_D = V_0$. Initial current in the circuit is zero. Find:



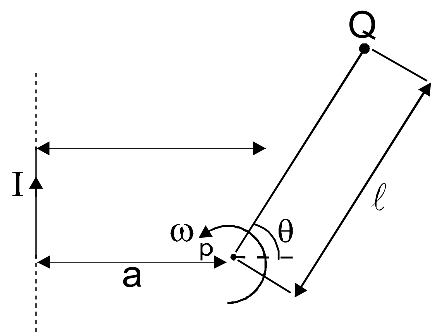
(a) Maximum current that will flow in the circuit.

(b) Potential difference across each capacitor at that instant.

(c) equation of current flowing towards left in the inductor.

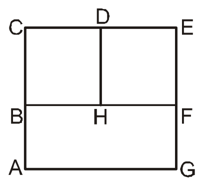
10. A horizontal rod AB of length l is rotated with constant angular velocity ω about a vertical axis passing through end A. A non-uniform charge of linear charge density $\lambda = \lambda_0 x$ is distributed on the rod, where x is the distance from the end A and λ_0 is a constant. Find the ratio of the energy density of magnetic field and electric field on the vertical line passing through A, at a point whose distance is l from the end A.

11. In the figure shown a long conductor carries constant current I. A rod PQ of length ℓ is in the plane of the rod. The rod is rotated about point P with constant angular velocity ω as shown in the figure. Find the e.m.f. induced in the rod in the position shown. Indicate which point is at high potential.



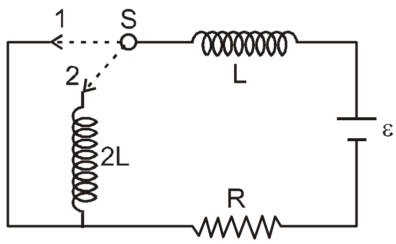
12. An infinitesimally small bar magnet of dipole moment M is moving with the speed v in the X-direction. A small closed circular conducting loop of radius 'a' and negligible self-inductance lies is the Y-Z plane with its centre at x = 0, and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a. **[JEE - 1997]**

13. In the figure shown ABCDEFGH is a square conducting frame of side 2m and resistance 1 Ω/m. A uniform magnetic field B is applied perpendicular to the plane and pointing inwards. It increases with time at a constant rate of 10 T/s. Find the rate at which heat is produced in the circuit, AB = BC = CD = BH.



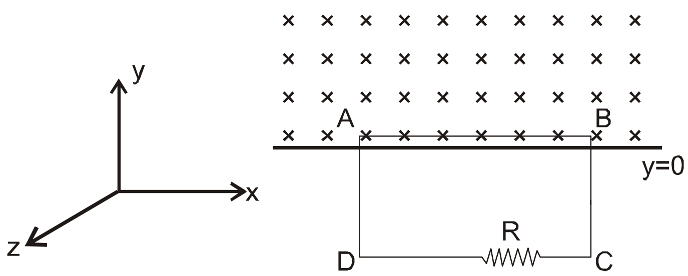
14. A square loop of side a = 12 cm with its sides parallel to x, and y-axis is moved with velocity, V = 8 cm/s in the positive x direction in a magnetic field along the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient $\partial B/\partial x = -10^{-3}$ T/cm along the x-direction, and it is changing in time at the rate $\partial B/\partial t$ = 7T/sec in the loop if its resistance is R = 4.5 Ω. Find the current.

15. In the circuit shown, the switch S is shifted to position 2 from position 1 at t = 0, having been in position 1 for a long time. Find the current in the circuit as a function of time.

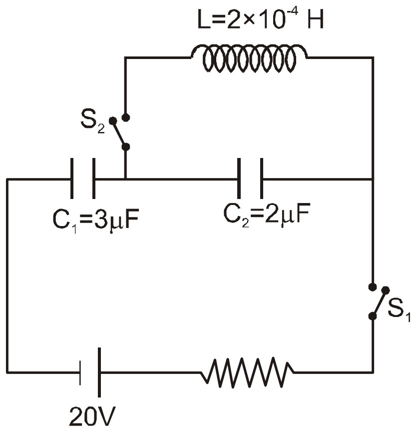


16. A square loop ABCD of side ℓ is moving in xy plane with velocity $\vec{v} = \beta t\hat{j}$. There exists a non-uniform magnetic field $\vec{B} = -B_0(1 + \alpha y^2)\hat{k}$ (y > 0), where B_0 and α are positive constants.

Initially, the upper wire of the loop is at y = 0. Find the induced voltage across the resistance R as a function of time. Neglect the magnetic force due to induced current.



17. The circuit shown in figure is in the steady state with switch S_1 closed. At t = 0, S_1 is opened and switch S_2 is closed.



(i) Derive an expression for the charge on the capacitor C_2 as a function of time.

(ii) Determine the first instant t, when energy in inductor becomes one third of that in capacitor.

Answers

1. 1A

2. $V = 1\ ms^{-1}$, $R_1 = \frac{36}{76} = 0.47\ \Omega$, $R_2 = \frac{36}{120} = 0.30\ \Omega$

3. $\frac{67}{32}$ A

4. $I = \frac{E}{R}[1 + (\eta - 1)\, e^{-t\eta R/L}]$.

5. $\frac{\mu_0 i_0 W \omega \cos \omega t}{4\pi} \ell n\left(\frac{L^2}{Y^2}+1\right)$

6. $i_{max} = \frac{mgB\ell c}{m + B^2\ell^2 c}$

7. (a) $x = \frac{g}{\omega^2}[1 - \cos\omega t]$, (b) $I_{max} = \frac{2mg}{B\ell}$, (c) $V_{max} = \frac{g}{\omega}$

8. **(i)** $\frac{1}{2} + \frac{7t}{4}$ (ii) $\frac{13}{18}$ J

9. (a) $I_{max} = \left(\sqrt{\frac{6C}{L}}\right) v_0$ (b) $3v_0$, $3v_o$ (c) $i = I_{max} \sin \omega t$; $\omega = \left(\sqrt{\frac{3}{2LC}}\right)$]

10. $\frac{B^2}{E^2}.c^2$

11. $\frac{\mu_0 i \omega}{2\pi \cos\theta}\left[\ell - \frac{a}{\cos\theta}\ell n\left(\frac{a + \ell\cos\theta}{a}\right)\right]$

12. $\frac{9\mu_0{}^2 M^2 a^4 v}{4Rx^8}$

13. 200 watt

14. 22.4 mA

15. $I = \frac{\varepsilon}{R}\left(1 - \frac{2}{3} \times e^{\frac{-2Rt}{3L}}\right)$

16. $\varepsilon = -B_0 l\beta\left(t + \frac{\alpha \times \beta^2 t^5}{4}\right)$

17. (i) $Q = Q_0 \cos \omega t$ where $Q_0 = 24\ \mu C$, $\omega = 5\times10^4$ rad/s

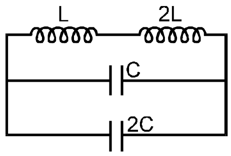
(ii) $t_1 = \frac{\pi}{6\omega} = 1.05 \times 10^{-5} = 10.5\ \mu S$

SELF ASSESSMENT PAPER

JEE (ADVANCED) PAPER

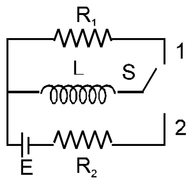
SECTION-1 : ONE OPTION CORRECT (Maximum Marks - 12)

1. The frequency of oscillation of current in the inductor is :



(A) $\frac{1}{3\sqrt{LC}}$ (B) $\frac{1}{6\pi\sqrt{LC}}$ (C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{1}{2\pi\sqrt{LC}}$

2. In the circuit shown switch S is connected to position 2 for a long time and then joined to position 1. The total heat produced in resistance R_1 is :

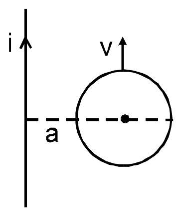


(A) $\frac{LE^2}{2R_2^2}$ (B) $\frac{LE^2}{2R_1^2}$ (C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2(R_1+R_2)^2}{2R_1^2R_2^2}$

3. A rectangular loop of sides of length ℓ and b is placed in x–y plane. A uniform but time varying magnetic field of strength $\vec{B} = 20\,t\,\hat{i} + 10\,t^2\,\hat{j} + 50\,\hat{k}$ where t is time elapsed. The magnitude of induced e.m.f. at time t is:

(A) 20 + 20 t (B) 20 (C) 20 t (D) zero

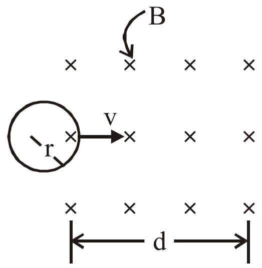
4. A circular loop of radius r is moved with a velocity v as shown in the diagram. The work needed to maintain its velocity constant is :



(A) $\frac{\mu_0 i v r}{2\pi a}$ (B) $\frac{\mu_0 i v r}{2\pi(a+r)}$ (C) $\frac{\mu_0 i v r}{2\pi}\ell n\left(\frac{2r+a}{a}\right)$ (D) zero

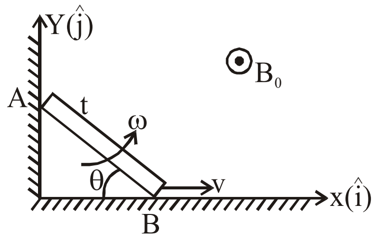
SECTION-2 : ONE OR MORE THAN ONE CORRECT (Maximum Marks - 32)

5. A conducting loop is pulled with a constant velocity towards a region of uniform magnetic field of induction B as shown in the figure. Then the current involved in the loop is (d > r)



(A) clockwise while entering
(B) anti-clockwise while entering
(C) zero when completely inside
(D) clockwise while leaving

6. A thin conducting rod of length ℓ is moved such that its end B moves along the X-axis while end A moves along the Y-axis. A uniform magnetic field $B = B_0\hat{k}$ exists in the region. At some instant, velocity of end B is v and the rod makes an angle of $\theta = 60°$ with the X-axis as shown in the figure. Then, at this instant



(A) angular speed of rod AB is $\omega = \dfrac{2v}{\sqrt{3}\ell}$
(B) angular speed of rod AB is $\omega = \dfrac{\sqrt{3}v}{2\ell}$

(C) e.m.f. induced in rod AB is $B\ell v\sqrt{3}$
(D) e.m.f. induced in rod AB is $B\ell v/2\sqrt{3}$

7. An infinite solenoid has radius R and n turns per unit length. The current grows linearly with time, according to $I_t = Ct$, in the solenoid. Here C is some constant. Let the induced electric field at distance r from axis of solenoid is E. Choose correct alternative(s).

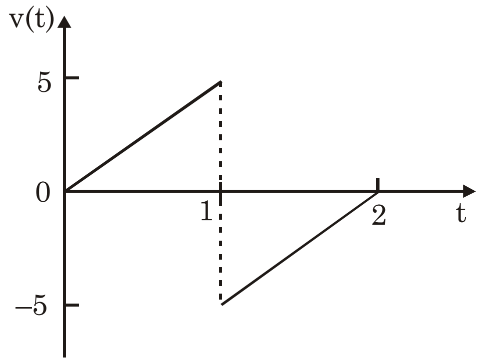
(A) $E \propto r$ for $r < R$

(B) $E \propto 1/r$ for $r > R$

(C) If an infinite line charge having uniform linear charge density λ is placed along the axis, then electrostatic field produced by line charge and induced electric field are perpendicular to each other

(D) The induced field and electrostatic field produced by line charge placed along axis of solenoid can be added vectorically to get net electric field at a point.

8. If the voltage waveform in figure is applied to a 10-mH inductor, find the inductor current Assume $i(0) = 0$.



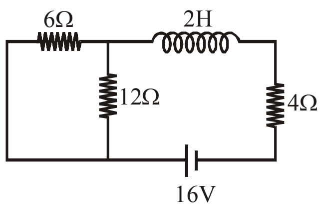
(A) The current through the inductor at t = 1 seconds is 5×10^{-2} A

(B) The current through the inductor at t = 2 seconds is 0 A

(C) The current through the inductor at t = 1 seconds is 250 A

(D) The current through the inductor at t = 2 seconds is 5×10^{-3} A

9. For the circuit shown. Which of the following statements is correct :-



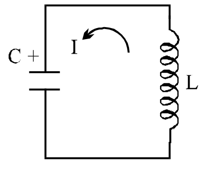
(A) Its time constant is 0.25 second

(B) In steady state, current through inductance will be equal to zero

(C) In steady state, current through inductor is 2A

(D) Current grows to 1.5A in t = $\ell n\sqrt{2}$

10. At time t = 0, the LC circuit shown in the figure has equal amount of energy stored in the capacitor C = 20 μF and in the inductor, each equal to 200 μJ. The current amplitude in the circuit is $1/\sqrt{10}$ A.



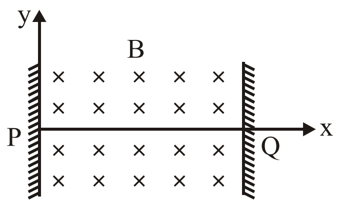
(A) the natural frequency of oscillation is 398 Hz

(B) the natural frequency of oscillation is 199 Hz

(C) the equation for current in terms of t is $\frac{1}{\sqrt{10}}\cos\left(2500\ \ t+\frac{\pi}{4}\right)$

(D) the equation for current in terms of t is $\frac{1}{\sqrt{10}}\cos\left(2500\ \ t+\frac{3\pi}{4}\right)$

11. A standing wave y = 2 A sin kx cos ωt is setup in the conducting wire PQ fixed at both ends by two vertical walls (see the figure). The region between the walls contains a constant magnetic field B. The wire is found to vibrate in the 3rd harmonic (where PQ = L) :-



(A) The maximum emf induced is $\frac{4AB\omega}{k}$

(B) The time when the emf becomes zero for the first time is $\frac{\pi}{2\omega}$

(C) The total emf induced is always zero between x = 0 and x = 2L/3

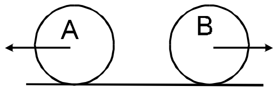
(D) At t = 0, the emf in the entire wire is zero.

12. Two different coils have self inductance 8mH and 2mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2, V_2 and W_2 respectively. Then:

(A) $\frac{I_1}{I_2}=\frac{1}{4}$ (B) $\frac{I_1}{I_2}=4$ (C) $\frac{W_2}{W_1}=4$ (D) $\frac{V_2}{V_1}=\frac{1}{4}$

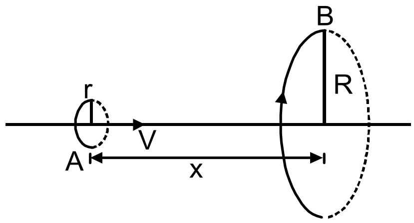
SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. Two identical conducting rings A & B of radius r are in pure rolling over a horizontal conducting plane with same speed (of center of mass)υ but in opposite direction. A constant magnetic field B is present pointing inside the plane of paper. Then the potential difference between the highest points of the two rings, is xBvr. Then value of x is:

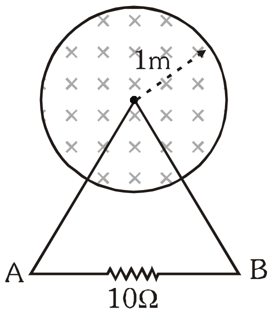


14. A solenoid of length 1 m, area of cross-section 4.0 cm^2 and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area 6 cm ^{2} and length 2 m. Then the mutual inductance between the solenoids is $\lambda\pi \times 10^{-4}$ H. Then the value of λ is

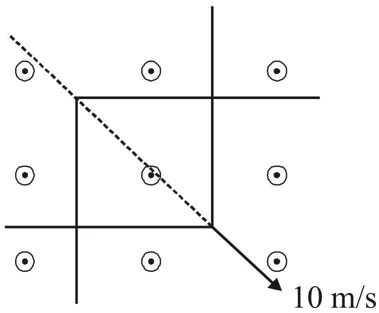
15. Loop A of radius (r << R) moves towards loop B with a constant velocity V in such a way that their planes are always parallel. Then the distance between the two loops when the induced emf in loop A is maximum is R/x. Then the value of x is.



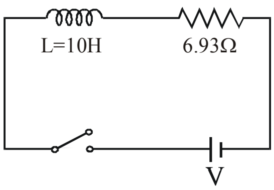
16. A uniform magnetic field of intensity $B = \frac{6}{\pi}$ sin 2t directed into the plane of the paper exists in the cylindrical region of radius 1 m. A loop of resistance 10Ω folded in the form of equilateral triangle of side length 2m is placed as shown in figure. Find the maximum potential drop in wire AB.



17. The L-shaped conductor as shown in figure moves a 10 m/s across a stationary L-shaped conductor in a 0.10 T magnetic field. The two vertices overlap so that the enclosed area is zero at t = 0. The conductor has resistance of 0.010 ohms per meter. What is current (in Amp.) at t = 0.10 sec. (Round off to nearest integer.)



18. A resistor and an inductor in series are connected to a battery through a switch. After the switch has been closed, at what time (in sec) will the Joule heat dissipated in the resistor change at the fastest rate?



Answers

1.	(B)	2.	(A)	3.	(D)	4.	(D)	5.	(B,C,D)
6.	(A, D)	7.	(A, B, C, D)	8.	(B, C)	9.	(A, C, D)	10.	(A, C)
11.	(A, C, D)	12.	(A, C, D)	13.	04.00	14.	06.40	15.	02.00
16.	02.00	17.	35.00	18.	01.00				