15

Electric Charges and Fields

TOPIC 1

Electric Charges and Coulomb's Law

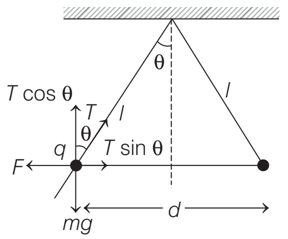
01 Two identical tennis balls each having mass m and charge q are suspended from a fixed point by threads of length l. What is the equilibrium separation when each thread makes a small angle θ with the vertical ? **[2021, 27 July Shift-I]**

(a) $d=\left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$ (b) $d=\left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(c) $d=\left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$ (d) $d=\left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

Ans. *(b)*

The given situation is shown below



$\because$ According to Coulomb's law,

$$F=\frac{kq_1q_2}{r^2}$$

Here, $q_1=q_2=q$

$\therefore$ Force due to charge,

$$F=\frac{k\times q\times q}{d^2} \qquad \{\because\ r=d \text{ here}\}$$

$$\Rightarrow \quad F=\frac{kq^2}{d^2} \qquad \text{...(i)}$$

Using above diagram, we can write

$T\cos\theta=mg$ and $T\sin\theta=F$

$$\because \quad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{F}{mg}=\frac{\frac{kq^2}{d^2}}{mg}$$

{using Eq (i)}

$$\Rightarrow \quad \tan\theta=\frac{kq^2}{d^2mg} \qquad \text{...(ii)}$$

$\because$ We know that, $\tan\theta\approx\sin\theta\approx\frac{d}{2l}$ ...(iii)

$\therefore$ From Eqs. (ii) and (iii), we get

$$\frac{d}{2l}=\frac{kq^2}{d^2mg}$$

$$\Rightarrow \quad d^3mg=kq^2 2l \ \Rightarrow d^3=\frac{kq^2 2l}{mg}$$

$$\because \quad k=\frac{1}{4\pi\varepsilon_0}$$

$$\Rightarrow \quad d^3=\frac{q^2 2l}{4\pi\varepsilon_0 mg}$$

$$\Rightarrow \quad d^3=\frac{q^2 l}{2\pi\varepsilon_0 mg}$$

$$\Rightarrow \quad d=\left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

02 A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance 2 m when each is carrying same charge q. If the free charged particle is displaced from its equilibrium position through distance x (x << 1 m), the particle executes SHM. Its angular frequency of oscillation will be ...... $\times 10^5$ rad/s, if $q^2=10C^2$. **[2021, 25 July Shift-I]**

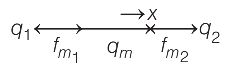
Ans. *(6000)*

Given,

Mass, $m=1\text{ mg}=1\times10^{-6}\text{ kg}$,

Separation between two charges, $d=2\text{m}$ and $q^2=10C^2$

When free charge is displaced from its position, then from given diagram,



$$F_{\text{net}}=F_{m_1}-f_{m_2}=-m\omega^2x$$

where, F_{net} = net electrostatic force,

F_{m_1}, F_{m_2} = force on m due to q_1 and q_2 respectively,

ω and x = angular frequency and equilibrium distance, respectively.

By using Coulomb's law,

$$F=\frac{kq_1q_2}{r^2}$$

where, k = Coulomb's constant

$=9\times10^9\text{ N-m}^2\text{C}^{-2}$

r = distance between two charges

$=\frac{d}{2}=1\text{ m}$

$$\therefore \quad F_{\text{net}}=\frac{kq_mq_1}{(r+x)^2}-\frac{kq_mq_2}{(r-x)^2}=-m\omega^2x$$

$$\Rightarrow \frac{kq^2}{m}\left[\frac{(r-x)^2-(r+x)^2}{(r^2-x^2)^2}\right]=-\omega^2x$$

$(\because q_1=q_2=q_m=q)$

$$\Rightarrow \frac{9\times10^9\times10}{10^{-6}}\left[\frac{-4rx}{r^4}\right]=-\omega^2x$$

$(\because\ x<<1\text{m},\ \therefore r^2-x^2=r^2)$

$$\Rightarrow \ \omega=\sqrt{36\times10^{16}}=6\times10^8\text{ rads}^{-1}$$

From question $\omega=x\times10^5$ rad/s

$$\therefore \qquad x=6000$$

03 An electric dipole is placed on X-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on

Z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin, respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

[2021, 22 July Shift-II]

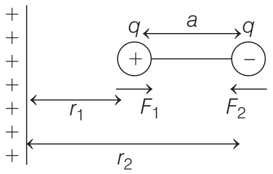
(a) 815.1 nC (b) 8.8 μC

(c) 0.485 nC (d) 4.44 μC

Ans. *(d)*

Given, linear charge density,

$$\lambda = 3\times10^{-6}\ \text{C/m}$$



$r_1 = 10\ \text{mm} = 10\times10^{-3}\text{m}$,
$r_2 = 12\text{mm} = 12\times10^{-3}\text{m}$

and the separation between charges

$$= r_2 - r_1 = 12 - 10 = 2\ \text{mm} = 2\times10^{-3}\ \text{m}$$

Electric field due to line charge, $E = \dfrac{2k\lambda}{x}$

Now, force on dipole due to line charge be F_1 and F_2 from +ve and −ve charge respectively, then the net force.

$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 - \mathbf{F}_2 = q\mathbf{E}_1 - q\mathbf{E}_2$$

$$\Rightarrow \quad 4 = q\left(\frac{2k\lambda}{r_1} - \frac{2k\lambda}{r_2}\right)$$

$$\Rightarrow \quad 4 = 2k\lambda q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow 4 = 2\times9\times10^{9}\times3\times10^{-6}\times q \times10^{3}\left(\frac{1}{10} - \frac{1}{12}\right)$$

$$\Rightarrow \quad 4 = 54\times10^{5}\left(\frac{2}{12}\right)q$$

$$\Rightarrow \quad 4 = 9\times10^{5}q$$

$$\Rightarrow \quad q = \frac{4}{9}\times10^{-5}$$

$$= 0.44\times10^{-5}$$

$$= 4.44\times10^{-6}$$

$$= 4.44\mu\text{C}$$

04 A certain charge Q is divided into two parts q and $(Q-q)$. How should the charges Q and q be divided, so that q and $(Q-q)$ placed at a certain distance apart experience maximum electrostatic repulsion ?

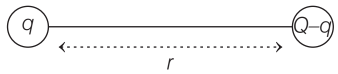
[2021, 20 July Shift-I]

(a) $Q = \dfrac{q}{2}$ (b) $Q = 2q$

(c) $Q = 4q$ (d) $Q = 3q$

Ans. *(d)*

(b) Let's say the charge q and $(Q-q)$ are at r distance from each other. This can be shown as



According to Coulomb's law, force between both the parts can be given as

$$F = \frac{kq(Q-q)}{r^2}$$

$$F = \frac{k}{r^2}(qQ - q^2)$$

As we know that $\dfrac{dF}{dq} = 0$, for maximum force.

$$\Rightarrow \quad \frac{dF}{dq} = \frac{d}{dq}\left[\frac{k}{r^2}(qQ - q^2)\right] = 0$$

$$\Rightarrow \quad \frac{k}{r^2}(Q - 2q) = 0 \Rightarrow Q = 2q$$

05 An infinite number of point charges, each carrying 1 μC charge, are placed along the Y-axis at $y = 1\text{m}, 2\text{m}, 4\text{m}, 8\text{m}$.

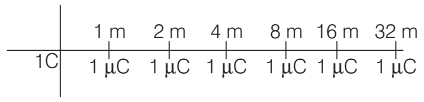
The total force on a 1 C point charge, placed at the origin, is $x\times10^{3}$ N. The value of x to the nearest integer, is ....

(Take, $\dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \text{N-m}^2/\text{C}^2$)

[2021, 18 March Shift-II]

Ans. *(12)*

The 1 C charge present in the origin and other
1μC charge are placed at 1 m, 2 m, 4 m, 8 m ...



Using the Coulomb's law,

$$F = \frac{kq_1q_2}{r^2}$$

$$F = 9\times10^{9}\times(1)\times10^{-6}\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \ldots\right]$$

$$= 9\times10^{3}\times\left[\frac{1}{1-\frac{1}{4}}\right] = 12\times10^{3}\ \text{N}$$

The value of x to the nearest integer is 12.

06 Find out the surface charge density at the intersection of point $X = 3$ m plane and X-axis, in the region of uniform line charge of 8 nC/m lying along the Z-axis in free space.

[2021, 16 March Shift-II]

(a) 0.424 nC m^{-2} (b) 47.88 nC m^{-2}

(c) 0.07 nC m^{-2} (d) 4.0 nC m^{-2}

Ans. *(a)*

Given,

Linear charge density, $\lambda = 8\ \text{nC/m} = 8\times10^{-9}\text{C/m}$

The relation between surface charge density and linear charge density can be given as

$$\frac{2k\lambda}{r} = \frac{\sigma}{\varepsilon_0} \quad \ldots\text{(i)}$$

where,
k = Coulomb's constant,
λ = linear charge density,
σ = surface charge density,
ε_0 = absolute electrical permittivity of free space
and r = distance.

Substituting the values in Eq. (i), we get

$$\sigma = \frac{2k\lambda\varepsilon_0}{r}$$

$$= \frac{2\times9\times10^{9}\times8\times10^{-9}\times8.85\times10^{-12}}{3}$$

$$= 0.424\times10^{-9}\ \text{Cm}^{-2} = 0.424\ \text{nCm}^{-2}$$

07 Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\dfrac{a}{21}\times10^{-8}$ C. The value of a will be ....

[Given, $g = 10\ \text{ms}^{-2}$]

[2021, 25 Feb Shift-II]

Ans. *(630)*

Given, mass of each spheres, $m = 10$ mg $= 10\times10^{-3}$ g

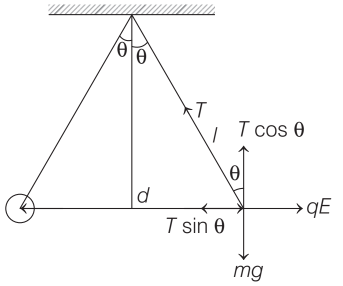
Length of thread $(l) = 0.5$ m

Separation between charges, $d = 0.2$ m

Charge of each sphere, $q = \dfrac{a}{21}\times10^{-8}$ C

Acceleration due to gravity $(g) = 10\ \text{ms}^{-2}$

The situation can be shown as below,



Taking component of tension (T)

$$T\cos\theta = mg \quad \text{...(i)}$$

$$T\sin\theta = qE = \frac{kq^2}{d^2} \quad \text{...(ii)}$$

$$\sin\theta = \frac{d/2}{l} = \frac{0.1}{0.5} = \frac{1}{5}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-1/25} = \frac{\sqrt{24}}{5}$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{kq^2}{d^2 mg}$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \quad \frac{1/5}{\sqrt{24}/5} = \frac{kq^2}{d^2 mg}$$

$$\Rightarrow \quad \frac{1}{\sqrt{24}} = \frac{9\times10^9 \times q^2}{(0.2)^2\times 10\times10^{-3}\times 10}$$

$$\Rightarrow \quad q = \sqrt{\frac{(0.2)^2\times10^{-1}}{\sqrt{24}\times 9\times10^9}}$$

$$\Rightarrow \quad q = 3\times10^{-7}\ \text{C}$$

$$\Rightarrow \frac{a}{21}\times10^{-8} = 30\times10^{-8}$$

$$a = 630$$

08 Two identical conducting spheres with negligible volume have 2.1 nC and – 0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is .... × 10^{-9} N.

[Given, $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$ SI unit]

[2021, 25 Feb Shift-II]

Ans. *(7.56)*

Given, $q_1 = 2.1\ \text{nC} = 2.1\times10^{-9}$ C,
$q_2 = -0.1\ \text{nC} = -0.1\times10^{-9}$ C

Separation $(d) = 0.5$ m

By Coulomb's law,

$$\text{Force}\,(F) = \frac{kq_1q_2}{d^2}$$

where, $k = 9\times10^9\ \text{N-m}^2\text{C}^{-2}$

= Coulomb's constant

$$\therefore \quad F = \frac{9\times10^9\times2.1\times10^{-9}\times(-0.1\times10^{-9})}{(0.5)^2}$$

$$= -7.56\times10^{-9}\ \text{N}$$

09 Two electrons each are fixed at a distance $2d$. A third charge proton placed at the mid-point is displaced slightly by a distance $x(x << d)$ perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency? (m = mass of charged particle)

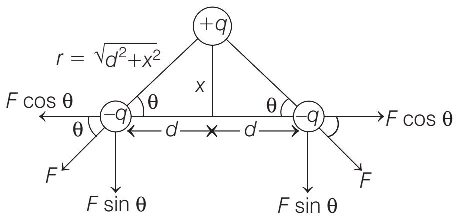
[2021, 24 Feb Shift-II]

(a) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{1/2}$ (b) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{1/2}$

(c) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{1/2}$ (d) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{1/2}$

Ans. *(c)*

The arrangement of charges is shown below



As we know that,

Coulomb's force between two charges. i.e. q_1 and q_2,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{(d^2+x^2)} \quad \text{... (i)}$$

Here, $q_1 = q_2 = q$

Force in SHM, $F = m\omega^2 x$ ...(ii)

Since, in order to have SHM $+q$ should move downwards and force responsible for this will be only

$$F' = F\sin\theta + F\sin\theta = 2F\sin\theta \quad \text{...(iii)}$$

Using Eqs. (ii) and (iii), we get

$$2F\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\sin\theta = m\omega^2 x$$

$$\Rightarrow \quad \frac{2}{4\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)}\cdot\frac{x}{(d^2+x^2)^{1/2}} = m\omega^2 x$$

$$\Rightarrow \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{(d^2+x^2)^{3/2}m}\right)^{1/2}$$

As, $x << d$

$$\therefore \quad \omega = \left(\frac{1}{2\pi\varepsilon_0}\frac{q^2}{md^3}\right)^{1/2}$$

10 A charge Q is distributed over three concentric spherical shells of radii a, b, c $(a < b < c)$ such that their surface charge densities are equal to one another.

The total potential at a point at distance r from their common centre, where $r < a$ would be

[2019, 10 Jan Shift-I]

(a) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

(b) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

(c) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

(d) $\frac{Q}{12\pi\varepsilon_0}\cdot\frac{ab+bc+ca}{abc}$

Ans. *(b)*

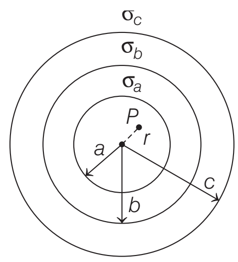
Given charge distribution is shown in the figure below

Given surface charge densities of each shell are same.

$$\therefore \quad \sigma_a = \sigma_b = \sigma_c \quad \text{...(i)}$$

As, surface charge density of shell of radius 'r' and having charge 'Q' is given as

$$\sigma = \frac{Q}{4\pi r^2}$$



So relation (i) can be rewritten as

$$\frac{Q_a}{4\pi a^2} = \frac{Q_b}{4\pi b^2} = \frac{Q_c}{4\pi c^2}$$

$$\Rightarrow \quad Q_a : Q_b : Q_c = a^2 : b^2 : c^2$$

where Q_a, Q_b and Q_c are charges on shell of radius a, b and c, respectively.

Also, $\quad Q_a + Q_b + Q_c = Q$

Hence, $\quad Q_a = \frac{a^2}{a^2+b^2+c^2}\cdot Q$

$$Q_b = \frac{b^2}{a^2+b^2+c^2}\cdot Q$$

$$\Rightarrow \quad Q_c = \frac{c^2}{a^2+b^2+c^2}\cdot Q$$

As we know for charged spherical shell with charge Q of radius 'R', the potential at a point

'P' at distance r such that $r < R$ is,

$$V_P = \frac{kQ}{R}$$

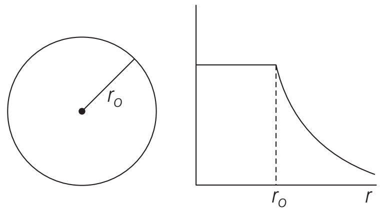
$\therefore$ potential at point P at a distance 'r' = Potential due to Q_a + Potential due to Q_b + Potential due to Q_c

$$= \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

Substituting values of Q_a, Q_b and Q_c, we get

$$V = \frac{Q\,(a+b+c)}{4\pi\varepsilon_0\,(a^2+b^2+c^2)}.$$

11 The given graph shows variation (with distance r from centre) of **[2019, 11 Jan Shift-I]**



(a) electric field of a uniformly charged spherical shell
(b) potential of a uniformly charged spherical shell
(c) electric field of a uniformly charged sphere
(d) potential of a uniformly charged sphere

Ans. ***(b)***

For a uniformly charged spherical shell, electric potential inside it is given by

$$V_{\text{inside}} = V_{\text{surface}} = kq/r_0 = \text{constant},$$

(where r_0 = radius of the shell).

and electric potential outside the shell at a distance r is

$$V_{\text{outside}} = \frac{kq}{r} \Rightarrow V \propto 1/r$$

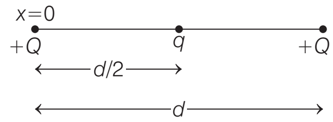
$\therefore$ The given graph represents the variation of r and potential of a uniformly charged spherical shell.

12 Three charges $+Q, q, +Q$ are placed respectively at distance 0, $d/2$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, then value of q is **[2019, 9 Jan Shift-I]**

(a) $\frac{+Q}{2}$ (b) $\frac{+Q}{4}$
(c) $\frac{-Q}{2}$ (d) $\frac{-Q}{4}$

Ans. ***(d)***

The given condition is shown in the figure given below,



Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as,

$$F = \frac{1 \cdot q_1 q_2}{4\pi\varepsilon_0 \cdot r^2}$$

$\therefore$Net force on charge 'Q' placed at origin i.e. at $x = 0$ in accordance with the principle of superposition can be given as

$$F_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2}$$

Since, it has been given that, $F_{\text{net}} = 0$.

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2} = 0$$

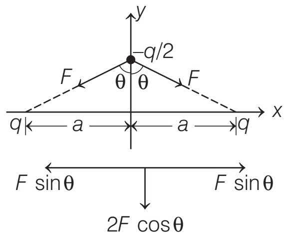
$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2}$$

or $q = -\frac{Q}{4}$

13 Two charges each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given, a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to **[JEE Main 2013]**

(a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

Ans. ***(b)***



$$F_{\text{net}} = 2F\cos\theta$$

$$\therefore \quad F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2+a^2})^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

[negative sign indictate the net force is towards the mean position]

$$F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}$$

$$\Rightarrow \frac{kq^2 y}{a^3} \propto -y$$

14 Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d, (d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result, charges approach each other with a velocity v. Then, as a function of distance x between them, is **[AIEEE 2011]**

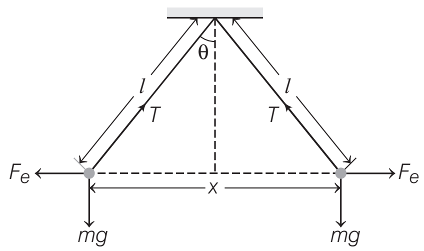
(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

Ans. ***(d)***

At any instant,

$$T\cos\theta = mg \quad \text{...(i)}$$

$$T\sin\theta = F_e = \frac{kq^2}{x^2} \quad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{kq^2}{x^2} = mg\tan\theta$$

$$\Rightarrow \quad q^2 = \frac{mg}{k}\cdot\frac{x}{2l}\cdot x^2 \quad \left[\because \tan\theta \approx \frac{x}{2l}\right]$$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3 \quad \text{...(iii)}$$

$$\Rightarrow \quad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v$$

$$\left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

$$\Rightarrow vx^{1/2} = \text{constant} \Rightarrow v \propto x^{-1/2}$$

15 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8\ \text{g cm}^{-3}$, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, then dielectric constant of the liquid is **[AIEEE 2010]**

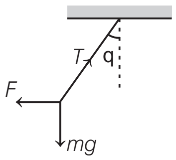
(a) 4 (b) 3 (c) 2 (d) 1

Ans. *(c)*

From FBD of sphere, using Lami's theorem,

$$\frac{F}{mg} = \tan\theta \quad ...(i)$$

When suspended in liquid, as θ remains same.



$$\therefore \quad \frac{F'}{mg\left(1-\frac{\rho}{d}\right)} = \tan\theta \quad ...(ii)$$

Using Eqs. (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg\left(1-\frac{\rho}{d}\right)}, \text{ where } F' = \frac{F}{K}$$

$$\therefore \quad \frac{F}{mg} = \frac{F}{mgK\left(1-\frac{\rho}{d}\right)}$$

$$\text{or} \quad K = \frac{1}{1-\frac{\rho}{d}} = \frac{1}{\left(1-\frac{0.8}{1.6}\right)} = 2$$

16 The question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement II The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement I is true, Statement II is false
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is false, Statement II is true

Ans. *(b)*

Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

17 A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals **[AIEEE 2009]**

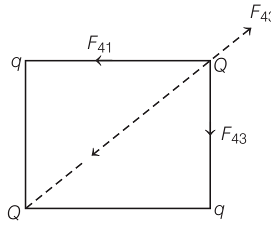
(a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$

Ans. *(a)*

Three forces F_{41}, F_{42} and F_{43} acting on Q as shown.

Resultant of $F_{41} + F_{43} = \sqrt{2}F_{\text{each}}$

$$= \sqrt{2}\,\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$$



Resultant on Q becomes zero only when q charges are of negative nature.

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(\sqrt{2}\,d)^2}$$

$$\Rightarrow \quad \sqrt{2}\,\frac{dQ}{d^2} = \frac{Q\times Q}{2d^2} \Rightarrow \sqrt{2}\times q = \frac{Q\times Q}{2}$$

$$\therefore \quad q = \frac{Q}{2\sqrt{2}} \quad \text{or} \quad \frac{Q}{q} = -2\sqrt{2}$$

18 Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[AIEEE 2004]**

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$ (c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

Ans. *(d)*

Let the spherical conductors B and C have same charge as q. The electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

Here, r being the distance between them.

When third uncharged conductor A is brought in contact with B, then charge on each conductor

$$q_A = q_B = \frac{q_A + q_B}{2} = \frac{0+q}{2} = \frac{q}{2}$$

When this conductor A is now brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2} = \frac{(q/2)+q}{2} = \frac{3q}{4}$$

Hence, electric force acting between B and C is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(3q/4)}{r^2}$$

$$= \frac{3}{8}\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] = \frac{3F}{8}$$

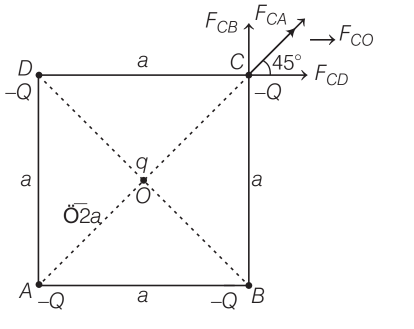
19 Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is **[AIEEE 2004]**

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

Ans. *(b)*

The system is in equilibrium means the force experienced by each charge is zero. It is clear that charge placed at centre would be in equilibrium for any value of q, so we are considering the equilibrium of charge placed at any corner.



$$F_{CD} + F_{CA}\cos 45° + F_{CO}\cos 45° = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}\,a)^2}\times\frac{1}{\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{(\sqrt{2}\,a/2)^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\text{or} \quad \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}} = 0$$

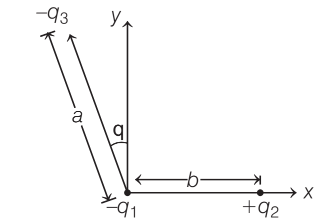
or $Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0$

or $2\sqrt{2}\,Q + Q - 4q = 0$

or $4q = (2\sqrt{2} + 1)\,Q$

or $q = (2\sqrt{2} + 1)\,\frac{Q}{4}$

20 Three charges $-q_1, +q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to **[AIEEE 2003]**

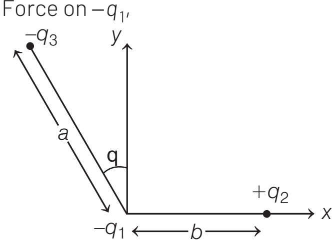


(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

Ans. *(b)*

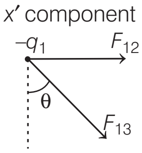
Force on $-q_1$,



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{b^2}\,\mathrm{i} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{a^2}$$

$$[\sin\theta\,\mathrm{i} - \cos\theta\,\mathrm{j}]$$

From above, x' component of force is



$$F_x = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

$$F_x \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

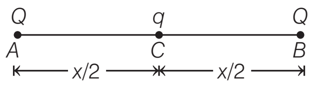
21 If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is **[AIEEE 2002]**

(a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$

(c) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$

Ans. *(d)*

Let charge q be placed at mid-point of line AB as shown below.



Also, $AB = x$ [say]

$\therefore$ $AC = \frac{x}{2},\ BC = \frac{x}{2}$

For the system to be in equilibrium,

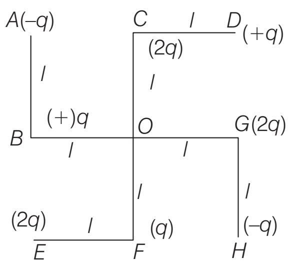
$$F_{Qq} + F_{QQ} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(x/2)^2} + \frac{1}{4\pi\varepsilon_0}\frac{QQ}{x^2} = 0$$

or $$q = -\frac{Q}{4}$$

TOPIC 2
Electric and Field Lines

22 What will be the magnitude of electric field at point O as shown in figure? Each side of the figure is l and perpendicular to each other. **[2021, 27 July Shift-II]**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

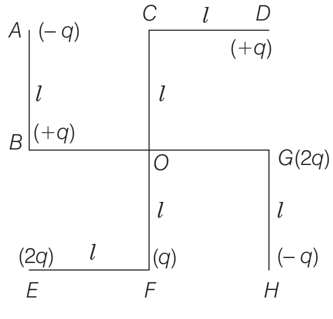
(b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2l^2)}(2\sqrt{2} - 1)$

(c) $\frac{q}{4\pi\varepsilon_0(2l)^2}$

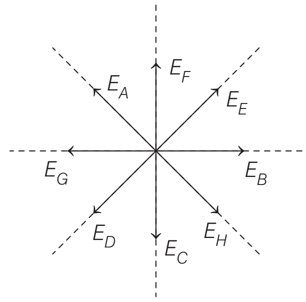
(d) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}(\sqrt{2})$

Ans. *(d)*

Consider the given figure,



At centre O, different electric fields, will be acting. These can be shown as follows



The electric field in different directions can be given as

$$E_A = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2};\ E_B = \frac{kq}{(l)^2} = \frac{kq}{l^2};$$

$$E_C = \frac{2kq}{l^2};\ E_D = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2}$$

$$E_E = \frac{2kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2};$$

$$E_F = \frac{kq}{l^2};\ E_G = \frac{2kq}{l^2};$$

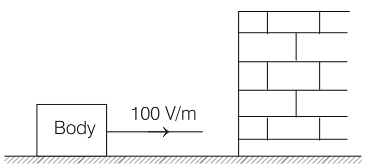
$$E_H = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2}$$

$$\therefore \quad E_{\text{net}} = \sqrt{(E_C - E_F)^2 + (E_G - E_B)^2}$$

$$= \sqrt{\left(\frac{kq}{l^2}\right)^2 + \left(\frac{kq}{l^2}\right)^2}$$

$$= \frac{kq\sqrt{2}}{l^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q(\sqrt{2})}{2l^2}$$

23 A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be ...... s. **[2021, 20 July Shift-I]**



Ans. *(1)*

Given, $q = 8\mu\text{C/g} = 8\times10^{-6}\,\text{C/g}$

$= 8\times10^{-3}\,\text{C/kg}$

$s = 10\,\text{cm} = 0.1\,\text{m}$

$\Rightarrow$ $E = 100\ \text{V/m}$

We know that, acceleration,

$$a = \frac{\text{force}(F)}{\text{mass}(m)}$$

$\Rightarrow \quad a = \dfrac{qE}{m} \qquad [\because F = qE]$

$$= \frac{8 \times 10^{-6} \times 100}{10^{-3}} = 0.8 \text{ ms}^{-2}$$

As per question, when electric field is switched on, the body strikes to the wall and then returns back.

For one oscillation,

$$s = ut + \frac{1}{2}at^2$$

$\Rightarrow \quad 0.1 = \dfrac{1}{2} \times 0.8t^2 \qquad [\because u = 0]$

$\Rightarrow \quad 0.2 = 0.8t^2$

$\Rightarrow \quad \dfrac{2}{8} = t^2$

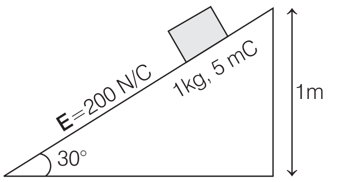
$\Rightarrow \quad t^2 = \dfrac{1}{4} \Rightarrow t = \dfrac{1}{2}$

$\therefore$ Time period $= 2 \times \dfrac{1}{2} = 1$ s

Therefore, if the collision of the body is perfectly elastic, the time period of motion will be 1s.

24 An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field 200 N/C as shown in the figure. A body of mass 1kg and charge 5 mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom. [Take, $g = 9.8$ m/s^2, $\sin 30° = \dfrac{1}{2}, \cos 30° = \dfrac{\sqrt{3}}{2}$]

[2021, 26 Feb Shift-II]



(a) 0.92 s (b) 0.46 s (c) 2.3 s (d) 1.3 s

Ans. *(d)*

Given, mass of block, $m = 1$ kg

Acceleration due to gravity, $g = 9.8$ ms^{-2}

Inclination, $\theta = 30°$

Electric field, $E = 200$ N/C

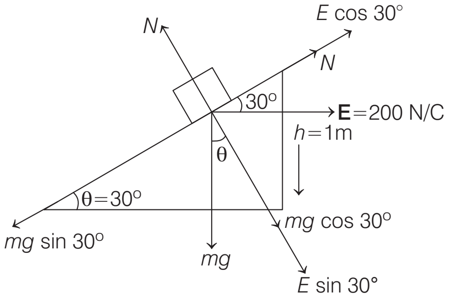
Coefficient of friction, $\mu = 0.2$

Charge, $q = 5 \text{ mC} = 5 \times 10^{-3}$ C

Let friction force, $f = \mu N$

where, N be the normal reaction.

Since, net force is zero along the perpendicular direction of incline.



Therefore, force along Y-axis will be zero.

$\Rightarrow \quad N = mg\cos 30° + qE \sin 30°$

$\Rightarrow \quad N = 1 \times 9.8 \times \sqrt{3}/2 + 5 \times 10^{-3} \times 200 \times 1/2$

$= 8.49 + 0.5$

$= 8.99 \text{ N} \simeq 9 \text{ N}$

$\therefore \quad f = \mu N = \dfrac{2}{10} \times 9$

$= \dfrac{18}{10} = 1.8$ N

Now, total force along the plane of incline, $mg \sin 30° - f - qE\cos 30° = ma$

$\Rightarrow 1 \times 9.8 \times \dfrac{1}{2} - 1.8 - 5 \times 10^{-3} \times \dfrac{200\sqrt{3}}{2} = a$

$\Rightarrow \quad 5 - 1.8 - 1.732/2 = a$

$\Rightarrow \quad a = 2.34$ ms^{-2}

Since, initial velocity of body, $u = 0$ ms^{-1}

and distance along incline, $s = h/\sin 30° = 1/\sin 30° = 2$

By using second equation of motion,

$$s = ut + 1/2at^2$$

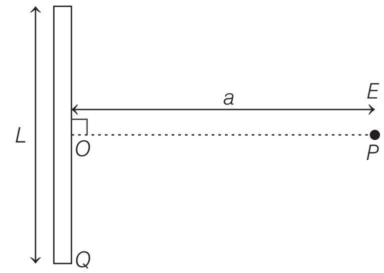
$\Rightarrow \quad 2 = 0 + 1/2 \times 2.34 \times t^2$

$\Rightarrow \quad t^2 = \dfrac{4}{2.34}$

$\Rightarrow \quad t = \dfrac{2}{\sqrt{2.34}} = 0.65 \times 2 = 1.3$ s

25 Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \dfrac{\sqrt{3}}{2}L$.

[2021, 26 Feb Shift I]



(a) $\dfrac{\sqrt{3}\,Q}{4\pi\varepsilon_0 L^2}$ (b) $\dfrac{Q}{3\pi\varepsilon_0 L^2}$

(c) $\dfrac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ (d) $\dfrac{Q}{4\pi\varepsilon_0 L^2}$

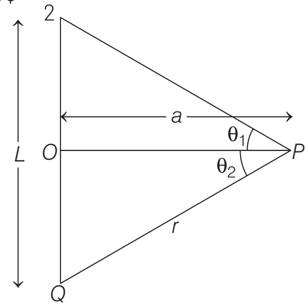
Ans. *(c)*

Given, length of conductor $= L$

Charge on conductor $= Q$

According to figure, $OP = a = \dfrac{\sqrt{3}}{2}L$,

$OQ = \dfrac{L}{2}$



Let $PQ = r = \sqrt{OP^2 + OQ^2}$

$\Rightarrow \quad PQ = \sqrt{\left(\dfrac{\sqrt{3}}{2}L\right)^2 + \left(\dfrac{L}{2}\right)^2}$

$= \sqrt{\dfrac{3}{4}L^2 + \dfrac{L}{4}} = L$

and E be the electric field at point P.

Since, E (due to finite wire)

$$= \frac{k\lambda}{a}(\sin\phi_1 + \sin\phi_2) \qquad \text{...(i)}$$

where, $k =$ Coulomb's constant $= \dfrac{1}{4\pi\varepsilon_0}$

$\lambda =$ linear charge density $= \dfrac{Q}{L}$

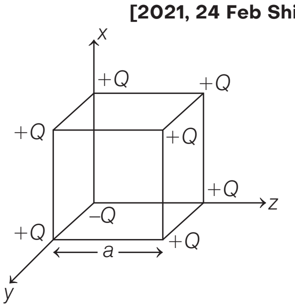
and $\sin\phi_1 = \sin\phi_2 = \dfrac{L/2}{L} = \dfrac{1}{2}$

Substituting the above value in Eq. (i), we get

$$E = \frac{k\lambda}{a} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{L \cdot \frac{\sqrt{3}L}{2}}$$

$$= \frac{1}{2\sqrt{3}\pi\varepsilon_0} \frac{Q}{L^2}$$

26 A cube of side a has point charges $+Q$ located at each of its vertices except at the origin, where the charge is $-Q$. The electric field at the centre of cube is **[2021, 24 Feb Shift-I]**



(a) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

(b) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

(c) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

(d) $\frac{2Q}{3\sqrt{3}\,\pi\varepsilon_0 a^2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

Ans. *(c)*

We can replace $-Q$ charge at origin by $+Q$ and $-2Q$. Now, due to $+Q$ charge at every corner of cube, electric field at centre of cube is zero. So, net electric field at centre is only due to $-2Q$ charge at origin. Vector form of electric field strength,

$$\mathbf{E}=\frac{Kq\mathbf{r}}{r^3}$$

Here, position vector, $r=\frac{a}{2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$

$$\Rightarrow |\mathbf{r}|=\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2}=\frac{\sqrt{3}a}{2}$$

$$\Rightarrow \mathbf{E}=\frac{1}{4\pi\varepsilon_0}\times\frac{(-2Q)\,a/2}{\left(\frac{a}{2}\sqrt{3}\right)^3}\cdot(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})$$

$$\mathbf{E}=\frac{-2Q\,(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})}{3\sqrt{3}\,\pi a^2\varepsilon_0}$$

27 An oil drop of radius 2 mm with a density 3 g cm^{-3} is held stationary under a constant electric field 3.55×10^5 Vm^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess? (Take, $g=9.81 m/s^2$) **[2021, 18 March Shift-I]**

(a) 48.8×10^{11} (b) 1.73×10^{10}
(c) 17.3×10^{10} (d) 1.73×10^{12}

Ans. *(b)*

Given, the radius of oil drop, $r=2$ mm $=0.002$ m

The density of oil drop, $\rho=3\,g/cm^3$ $=3\times10^3\,kg/m^3$

The constant electric field, $E=3.55\times10^5\,Vm^{-1}$

Under stationary condition of oil drop,

$$mg=qE$$

$$\Rightarrow \rho Vg=qE \qquad [\because m=\rho V]$$

$$\Rightarrow \rho\left(\frac{4}{3}\pi r^3\right)g=neE$$

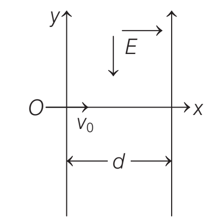
$$\left[\because V=\frac{4}{3}\pi r^3 \text{ and } q=ne\right]$$

$$n=\frac{\rho\left(\frac{4}{3}\pi r^3\right)g}{eE}$$

$$\Rightarrow n=\frac{3\times10^3\times\left(\frac{4}{3}\pi(0.002)^3\right)9.81}{1.6\times10^{-19}\times(3.55\times10^5)}$$

$$\Rightarrow n=1.73\times10^{10}$$

28 A charged particle (mass m and charge q) moves along X-axis with velocity v_0. When it passes through the origin it enters a region having uniform electric field $\mathbf{E}=-E\,\hat{\mathbf{j}}$ which extends upto $x=d$. Equation of path of electron in the region $(x>d)$ is **[2020, 2 Sep Shift-I]**

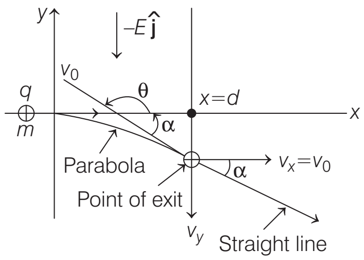


(a) $y=\frac{qEd}{mv_0^2}x$ (b) $y=\frac{qEd}{mv_0^2}(x-d)$

(c) $y=\frac{qEd}{mv_0^2}\left(\frac{d}{2}-x\right)$ (d) $y=\frac{qEd^2}{mv_0^2}x$

Ans. *(c)*

The path followed by charged particle is shown in figure below,



When particle is in the region of electric field $(0<x\le d)$; it has two velocity components

Along X-axis, $v_x=v_0\hat{i}$ ...(i)

Along Y-axis, $v_y=a_y\cdot t\,\hat{j}$

where, t = time in which particle crosses region of field.

$$\Rightarrow v_y=\frac{F}{m}\cdot t\,\hat{j} \Rightarrow v_y=\frac{-qEt}{m}\hat{j} \qquad \text{...(ii)}$$

Now, if particle crosses region of field in time t, then

$$d=v_xt \Rightarrow t=\frac{d}{v_0} \qquad \text{...(iii)}$$

So, from Eqs. (ii) and (iii), we get

$$v_y=\frac{-qEd}{mv_0}\cdot\hat{j}$$

Hence, angle α is given by

$$\tan\alpha=\frac{|\mathbf{v}_y|}{|\mathbf{v}_x|}=\frac{qEd}{mv_0^2}$$

So, slope of path (straight line) of particle when it comes out of region of field is

$$m'=\tan\theta=\tan(180°-\alpha)=-\tan\alpha$$

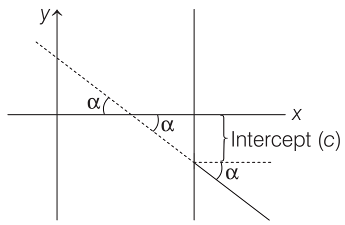
$$=\frac{-qEd}{mv_0^2}$$

Now, y-coordinate at point of exit is

$$y=\frac{1}{2}a_yt^2 \Rightarrow y=\frac{-1}{2}\left(\frac{qE}{m}\right)\cdot t^2=-\frac{1}{2}\frac{qEd^2}{mv_0^2}$$

So, the intercept length c is $-y$ as shown in figure.

$$c=\left(\frac{1}{2}\frac{qEd^2}{mv_0^2}\right)$$



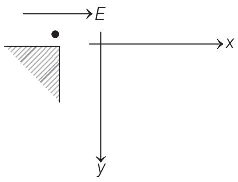
∴ Equation of path will be

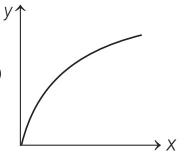
$$y=m'x+c$$

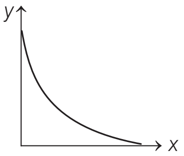
$$\Rightarrow y=\frac{-qEdx}{mv_0^2}+\frac{1}{2}\frac{qEd^2}{mv_0^2}=\frac{qEd}{mv_0^2}\left(\frac{d}{2}-x\right)$$

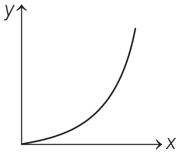
Hence, correct option is (c).

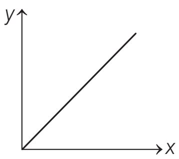
29 A small point mass carrying some positive charge on it, is released from the edge of a table. There is an uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale) **[2020, 2 Sep Shift-II]**



(a)


(b)


(c)


(d)


Ans. *(d)*

As gravitational force is considerably smaller than electric force. So, there is only one prominent acceleration in the direction of electric field.

Hence, path of charged particle of small mass is nearly a straight line.

Hence, the graph in option (d) is correct.

30 A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x-direction, where a and E_0 are constants. Initially, the particle was at rest at $x = 0$. Other than the initial position, the kinetic energy of the particle becomes zero when the distance of the particle from the origin is **[2020, 4 Sep Shift-II]**

(a) $\sqrt{\frac{1}{a}}$ (b) a (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{2}{a}}$

Ans. *(c)*

Here, initial kinetic energy, $K_i = 0$ [at $x = 0$]

Final kinetic energy, $K_f = 0$ [at $x = x_0$ (say)]

So, change in kinetic energy,

$$\Delta K = K_f - K_i = 0$$

From work-energy theorem,

Work done = Change in kinetic energy

i.e., $W = \Delta K \therefore W = 0$

$\Rightarrow \int F dx = 0 \Rightarrow \int qE\, dx = 0 \quad (\because F = qE)$

$\Rightarrow \int q[E_0(1 - ax^2)]dx = 0$

[given that, $E = E_0(1 - ax^2)$]

$$\Rightarrow \quad qE_0 \int_0^{x_0} (1 - ax^2) dx = 0$$

$$\Rightarrow \quad qE_0 \left[x - \frac{ax^3}{3} \right]_0^{x_0} = 0$$

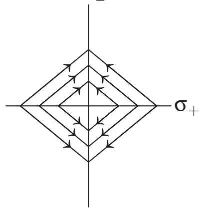
$$\Rightarrow \left[\left\{ x_0 - \frac{a x_0^3}{3} \right\} - \{0 - 0\} \right] = 0$$

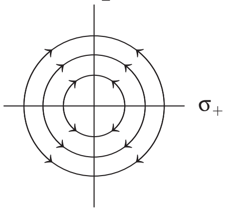
$$\Rightarrow x_0 - \frac{ax_0^3}{3} = 0 \Rightarrow x_0 = \frac{ax_0^3}{3}$$

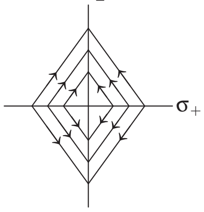
$$\Rightarrow 1 = \frac{ax_0^2}{3} \Rightarrow x_0^2 = \frac{3}{a} \Rightarrow x_0 = \sqrt{\frac{3}{a}}$$

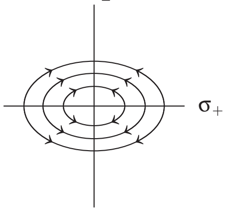
Hence, option (c) is correct.

31 Two charged thin infinite plane sheets of uniform surface charge densities σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system? **[2020, 4 Sep Shift-I]**

(a)


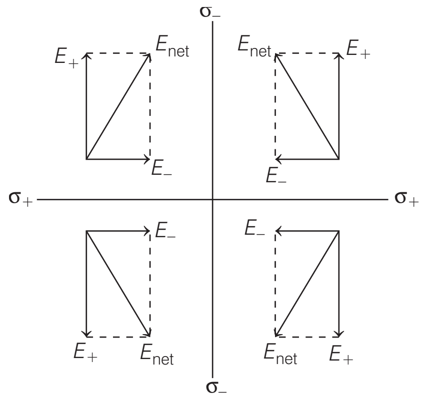
(b)


(c)


(d)


Ans. *(c)*

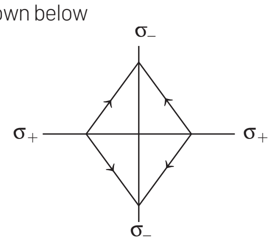
Electric field in each quadrant will look like this



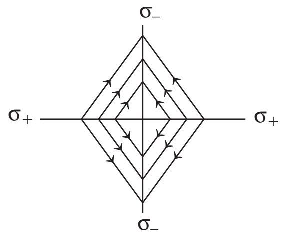
$\because |\sigma_+| > |\sigma_-|$

$\therefore |E_+| > |E_-|$

So, the final electric field will become as shown below



So, the nearest matching option is



Hence, correct option is (c).

32 Consider the force F on a charge q due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statement is true for F, if q is placed at distance r from the centre of the shell ? **[2020, 6 Sep Shift-II]**

(a) $F = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{R^2}$ (for $r < R$)

(b) $\frac{1}{4\pi\varepsilon_0} \frac{Qq}{R^2} > F > 0$ (for $r < R$)

(c) $F = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r^2}$ (for $r > R$)

(d) $F = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{R^2}$ (for all r)

Ans. *(c)*

To calculate force on a point charge q, we need to find electric field due to uniformly charged spherical shell at various points.

If $r < R$, i.e. inside the shell, then $E = 0$

$$\Rightarrow \quad F = qE = 0$$

If $r > R$, i.e., outside the shell, then

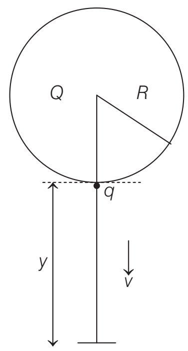
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$\Rightarrow \quad F = qE \Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

If $r = R$, i.e., at surface of shell, then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \Rightarrow F = \frac{Qq}{4\pi\varepsilon_0 R^2}$$

33 A solid sphere of radius R carries a charge $Q + q$ distributed uniformly over its volume. A very small point-like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then (Assume the remaining portion to be spherical.) **[2020, 5 Sep Shift-I]**



(a) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(b) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(c) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(d) $v^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

Ans. *(b)*

Using law of conservation of total energy,

$$\frac{1}{2}mv^2 = mgy + (\Delta\text{PE})$$

$$\Rightarrow \frac{1}{2}mv^2 = mgy + kQq\left[\frac{1}{R} - \frac{1}{(R+y)}\right]$$

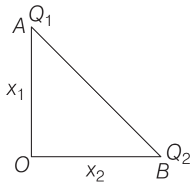
$$\Rightarrow \quad v^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

$$\Rightarrow \quad v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$$

$$\left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$$

Hence, correct option is (b).

34 Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1 / Q_2 is proportional to **[2020, 6 Sep Shift-I]**

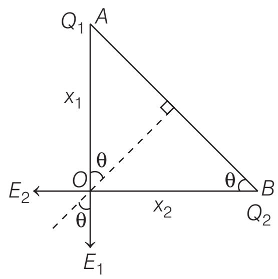


(a) $\frac{x_1^3}{x_2^3}$ (b) $\frac{x_2}{x_1}$

(c) $\frac{x_1}{x_2}$ (d) $\frac{x_2^2}{x_1^2}$

Ans. *(c)*

Let electric field produced by charges Q_1 and Q_2 at point O be E_1 and E_2, respectively. The direction of fields are shown in the figure below and a perpendicular is also drawn on side AB, that passes through point O.



If the resultant electric field at point O is perpendicular to hypotenuse this means resultant of E_1 and E_2 must be along it.

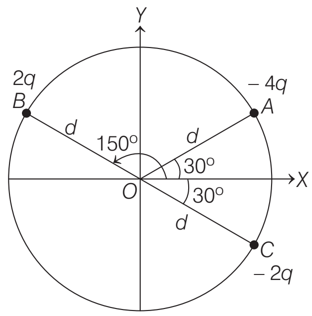
$$\therefore \quad \tan\theta = \frac{E_2}{E_1} = \frac{\frac{KQ_2}{x_2^2}}{\frac{KQ_1}{x_1^2}} = \frac{Q_2}{Q_1}\frac{x_1^2}{x_2^2} \quad \text{...(i)}$$

$$\text{In } \Delta OAB, \quad \tan\theta = \frac{x_1}{x_2} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{Q_2}{Q_1}\cdot\frac{x_1^2}{x_2^2} = \frac{x_1}{x_2} \quad \text{or} \quad \frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

35 Three charged particles A, B and C with charges $-4q, 2q$ and $-2q$ are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is **[2020, 8 Jan Shift-I]**

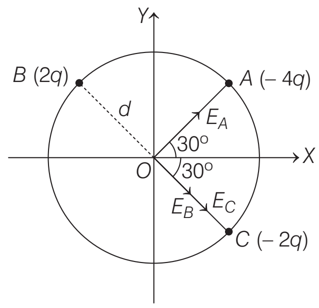


(a) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (b) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(c) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (d) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

Ans. *(d)*

Fields of charges at A, B and C are as shown below.



Magnitude of resultant field component directed along positive X-axis,

$$E_x = E_A\cos 30° + E_B\cos 30° + E_C\cos 30°$$

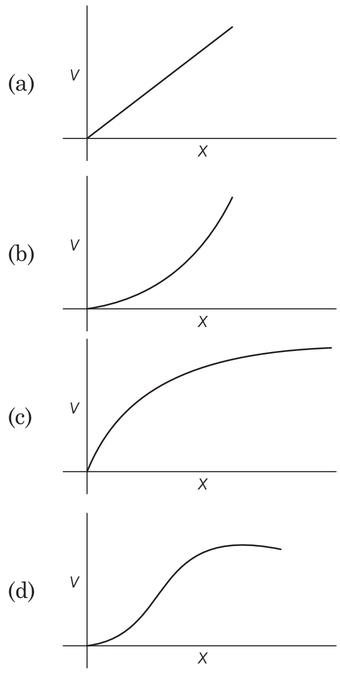
$$= (E_A + E_B + E_C)\cos 30°$$

$$= \left(\frac{k(4q)}{d^2} + \frac{k(2q)}{d^2} + \frac{k(2q)}{d^2}\right) \times \frac{\sqrt{3}}{2}$$

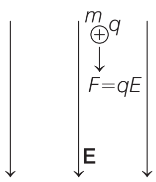
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{8q}{d^2} \times \frac{\sqrt{3}}{2} \Rightarrow E_x = \frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$$

36 A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the

distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)
[2020, 8 Jan Shift-II]



Ans. *(c)*



Acceleration of the particle,

$$a = \frac{F}{m} = \frac{q}{m} \cdot E$$

Velocity v and distance x can be related using

$$v^2 - u^2 = 2ax$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{q}{m}E\right)x} \qquad (\because u = 0)$$

or $\quad v^2 = 2\left(\frac{q}{m}E\right)x$

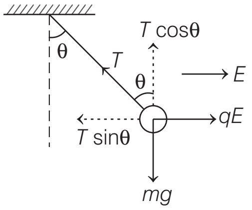
This equation resembles a parabola $y^2 = 4ax$. So, the graph between v and x will be as shown in option (c).

37 The bob of a simple pendulum has mass 2g and a charge of $5.0\mu C$. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is (take $g = 10\ m/s^2$)
[2019, 8 April Shift-I]

(a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$
(c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

Ans. *(d)*

Forces on the bob are as shown



For equilibrium,

$$T\cos\theta = mg \qquad \text{...(i)}$$

and $\quad T\sin\theta = qE \qquad$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{qE}{mg}$$

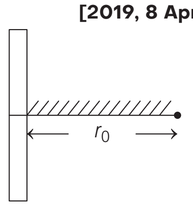
Here, $q = 5\mu C = 5\times10^{-6}$ C,
$E = 2000$ V/m,
$m = 2g = 2\times10^{-3}$ kg, $g = 10\ ms^{-2}$

$$\therefore \quad \tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2} = 0.5$$

So, the angle made by the string of the pendulum with the vertical is

$$\theta = \tan^{-1}(0.5)$$

38 A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to
[2019, 8 April Shift-II]



(a) $v \propto \left(\frac{r}{r_0}\right)$ (b) $v \propto e^{+\frac{r}{r_0}}$
(c) $v \propto \ln\left(\frac{r}{r_0}\right)$ (d) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

Ans. *(d)*

For a positive line charge or charged wire with uniform density λ, electric field at distance x is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x} \qquad \text{...(i)}$$

So, force on charge q which is at a distance r_0 due to this line charge is

$$F = qE = \frac{2kq\lambda}{x} \qquad \text{...(ii) [using Eq. (i)]}$$

Now, work done when charge is pushed by field by a small displacement dx is

$$dW = F\cdot dx = \frac{2kq\lambda}{x}\cdot dx \qquad \text{[using Eq. (ii)]}$$

$\therefore$ Total work done by field of wire in taking charge q from distance r_0 to distance r will be

$$W = \int_{r_0}^{r} dW = \int_{r_0}^{r} \frac{2kq\lambda}{x}\cdot dx$$

$$= 2kq\lambda[\log x]_{r_0}^{r}$$

$$= 2kq\lambda(\log r - \log r_0)$$

$$= 2kq\lambda \log\left|\frac{r}{r_0}\right| \qquad \text{...(iii)}$$

As we know, from work-kinetic energy theorem,

$$K_{\text{final}} - K_{\text{initial}} = W$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - 0 = 2kq \times \log\left|\frac{r}{r_0}\right|$$

[using Eq. (iii)]

$$\Rightarrow \quad v = \left(\frac{4kq\lambda}{m}\log\left|\frac{r}{r_0}\right|\right)^{\frac{1}{2}}$$

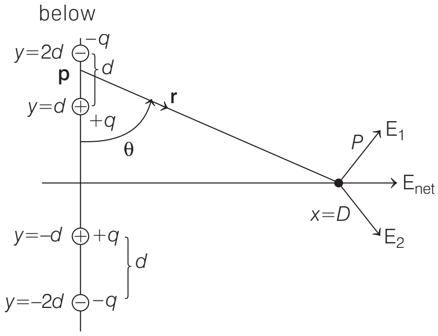
$$\therefore \quad v \propto \left(\log\left|\frac{r}{r_0}\right|\right)^{\frac{1}{2}}$$

39 Four point charges $-q, +q, +q$ and $-q$ are placed on Y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the X-axis at $x = D$, with $D >> d$, will behave as
[2019, 9 April Shift-II]

(a) $E \propto \frac{1}{D}$ (b) $E \propto \frac{1}{D^3}$
(c) $E \propto \frac{1}{D^2}$ (d) $E \propto \frac{1}{D^4}$

Ans. *(d)*

Given charge distribution is as shown below



So, we can view above point charges as combination of pair of dipoles or a quadrupole.

By symmetry, the field components parallel to quadrupole cancels and the resultant perpendicular field is

$$E = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{D^2} - \frac{D}{(D^2+d^2)^{3/2}}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0 D^2}\left(1 - \left(1+\frac{d^2}{D^2}\right)^{-3/2}\right)$$

As, $\left(1+\frac{d^2}{D^2}\right)^{-\frac{3}{2}} \approx \left(1 - \frac{3}{2}\frac{d^2}{D^2}\right)$

(using binomial expansion)

We have,

$$E = \frac{3qd^2}{4\pi\varepsilon_0 D^4}$$

$\Rightarrow \quad E \propto \frac{1}{D^4}$

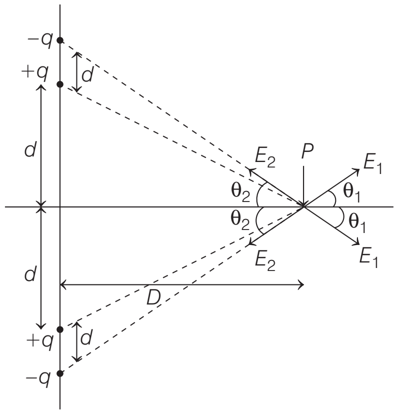
Note Dependence of field for a point charge is

$$E \propto \frac{1}{r^2}$$

For a dipole, it is, $\quad E \propto \frac{1}{r^3}$

For a quadrupole, it is, $E \propto \frac{1}{r^4}$ ..... etc.

Alternate Solution The given distribution of charges can be shown as the figure below



Electric field at point P,

$E = E_1\cos\theta_1$

$\quad + E_1\cos\theta_1 - E_2\cos\theta_2 - E_2\cos\theta_2$

$= 2E_1\cos\theta_1 - 2E_2\cos\theta_2$

$$= \frac{2kq}{(d^2+D^2)}\cos\theta_1 - \frac{2kq}{(2d)^2+D^2}\cos\theta_2$$

As, $\cos\theta_1 = \frac{D}{(d^2+D^2)^{1/2}}$

Similarly, $\cos\theta_2 = \frac{D}{[(2d)^2+D^2]^{1/2}}$

$= 2kqD[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}]$

$$= \frac{2kqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

As $D >> d$, then by applying binomial approximation, we get

$$= \frac{2kq}{D^2}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3}{2}\left(\frac{4d^2}{D^2}\right)\right)\right]$$

$$= \frac{2kq}{D^2}\left[\frac{9d^2}{2D^2}\right] = \frac{9kqd^2}{D^4} \Rightarrow E \propto \frac{1}{D^4}$$

40 Two point charges $q_1\ (\sqrt{10}\,\mu C)$ and $q_2\ (-25\,\mu C)$ are placed on the x-axis at $x = 1$ m and $x = 4$ m, respectively. The electric field (in V/m) at a point $y = 3$ m on Y-axis is

$\left(\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N-m}^2\text{C}^{-2}\right)$ **[2019, 9 Jan Shift-II]**

(a) $(63\,\hat{i} - 27\,\hat{j})\times10^2$

(b) $(81\,\hat{i} - 81\,\hat{j})\times10^2$

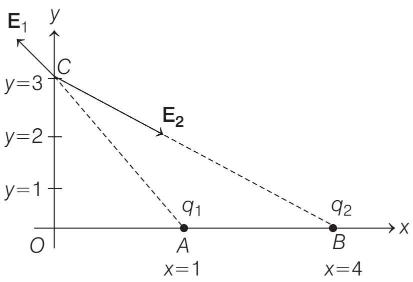
(c) $(-81\,\hat{i} + 81\,\hat{j})\times10^2$

(d) $(-63\,\hat{i} + 27\,\hat{j})\times10^2$

Ans. *(a)*

Here, $q_1 = \sqrt{10}\,\mu C = \sqrt{10}\times10^{-6}$ C

$\qquad q_2 = -25\,\mu C = -25\times10^{-6}$ C



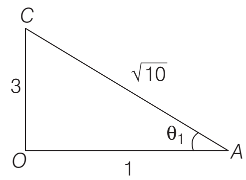
Let $\mathbf{E}_1$ and $\mathbf{E}_2$ are the values of electric field due to q_1 and q_2 respectively.

Here, $E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{AC^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$= 9\times10^9\times\sqrt{10}\times10^{-7} = 9\sqrt{10}\times10^2$

$\therefore\ \mathbf{E}_1 = 9\sqrt{10}\times10^2[\cos\theta_1(-\hat{i}) + \sin\theta_1\hat{j}]$

From ΔOAC,



$\sin\theta_1 = \frac{3}{\sqrt{10}}$

and $\cos\theta_1 = \frac{1}{\sqrt{10}}$

$\therefore \quad E_1 = 9\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}(-\hat{i}) + \frac{3}{\sqrt{10}}\hat{j}\right]$

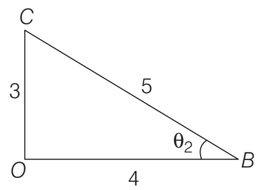
$= 9\times10^2[-\hat{i} + 3\hat{j}]$

$= (-9\,\hat{i} + 27\hat{j})\times10^2$ V/m

and $E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-25\times10^{-6}}{(4^2+3^2)}$

$= 9\times10^3$ V/m

From ΔOBC,



$\sin\theta_2 = \frac{3}{5}$

$\cos\theta_2 = \frac{4}{5}$

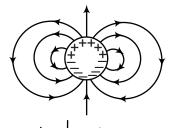
$\therefore\quad \mathbf{E}_2 = 9\times10^3[\cos\theta_2\hat{\mathbf{i}} - \sin\theta_2\hat{\mathbf{j}}]$

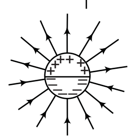
$\mathbf{E}_2 = 9\times10^3\left[\frac{4}{5}\hat{\mathbf{i}} - \frac{3}{5}\hat{\mathbf{j}}\right]$

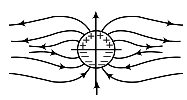
$= (72\,\hat{\mathbf{i}} - 54\,\hat{\mathbf{j}})\times10^2$

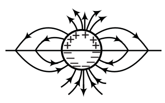
$\therefore\quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}})\times10^2$ V/m

41 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **[JEE Main 2015]**

(a) 

(b) 

(c) 

(d) 

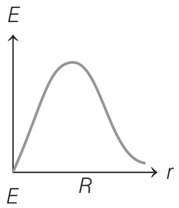
Ans. *(a)*

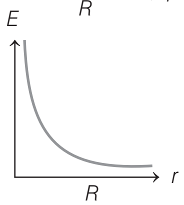
Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible.

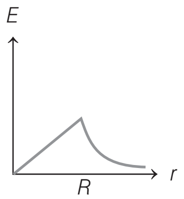
Electric field lines cannot form corners as shown in (d).

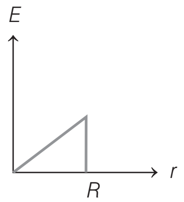
Thus, correct option is (a).

42 In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be **[AIEEE 2012]**

(a)


(b)


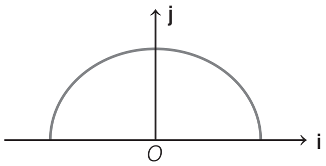
(c)


(d)


Ans. *(c)*

Electric field inside the uniformly charged sphere varies linearly, $E = \frac{kQ}{R^3} \cdot r$, $(r \le R)$, while outside the sphere, it varies as inverse square of distance, $E = \frac{kQ}{r^2}; (r \ge R)$ which is correctly represented in option (c).

43 A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\mathbf{E}$ at the centre O is **[AIEEE 2010]**

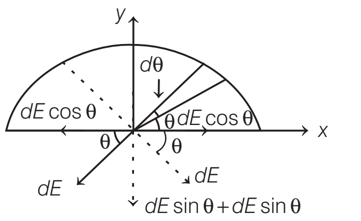


(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\mathbf{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\mathbf{j}$

(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\mathbf{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\mathbf{j}$

Ans. *(c)*

Linear charge density, $\lambda = \left(\frac{q}{\pi r}\right)$



$$E = \int dE \sin\theta(-\mathbf{j}) = \int_o^\pi \int \frac{K \cdot dq}{r^2} \sin\theta(-\mathbf{j})$$

$$E = \frac{K}{r^2}\int_0^\pi \frac{qr}{\pi r} d\theta \sin\theta(-\mathbf{j})$$

$$= \frac{K}{r^2}\frac{q}{\pi}\int_0^\pi \sin\theta \, d\theta(-\mathbf{j})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}[-\cos\theta]_0^\pi(-\mathbf{j})$$

$$= \frac{q}{2\pi^2\varepsilon_0 r^2}(-\mathbf{j})$$

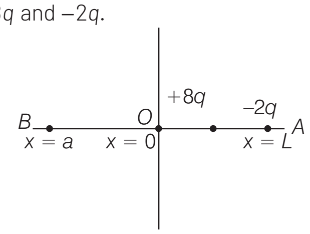
44 Two point charges $+8\,q$ and $-2\,q$ are located at $x = 0$ and $x = L$, respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero, is **[AIEEE 2005]**

(a) $2L$ (b) $\frac{L}{4}$

(c) $8L$ (d) $4L$

Ans. *(a)*

Suppose that a point B, where net electric field is zero due to charges $8q$ and $-2q$.



$$\mathbf{E}_{BO} = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{8q}{a^2}\mathbf{i}$$

$$\mathbf{E}_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{+2q}{(a+L)^2}\mathbf{i}$$

According to condition,

$$\mathbf{E}_{BO} + \mathbf{E}_{BA} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{8q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(a+L)^2}$$

or $\frac{2}{a} = \frac{1}{a+L}$ or $2a + 2L = a$ or $2L = -a$

Thus, at distance $2L$ from origin, net electric field will be zero.

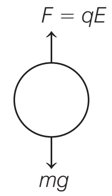
45 A charged oil drop is suspended in uniform field of 3×10^4 Vm^{-1} so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge $= 9.9 \times 10^{-15}$ kg and $g = 10$ ms^{-2}) **[AIEEE 2004]**

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C

(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

Ans. *(a)*

In steady state,



Electric force on drop = Weight of drop

i.e., $qE = mg$

$$\Rightarrow \quad q = \frac{mg}{E} = \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4}$$

$$= 3.3 \times 10^{-18} \text{ C}$$

TOPIC 3
Electric Dipole

46 Two identical electric point dipoles have dipole moments $\mathbf{p}_1 = p\hat{\mathbf{i}}$ and $\mathbf{p}_2 = -p\hat{\mathbf{i}}$ are held on the X-axis at distance a from each other. When released, they move along the X-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is m, their speed when they are infinitely far apart is **[2020, 6 Sep Shift-II]**

(a) $\frac{p}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (b) $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$

(c) $\frac{p}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$ (d) $\frac{p}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

Ans. *(b)*

When two electric dipoles of opposite dipole moments are placed on a line, they experience force of attraction along the same line as shown below.

Considering both dipoles as a system, we find that net external force on system is zero, i.e., $\mathbf{F}_{ext} = 0$

So, total mechanical energy = constant

$$(\text{ME})_i = (\text{ME})_f$$

or $(\text{KE})_i + (\text{PE})_i = (\text{KE})_f + (\text{PE})_f$

As, initially they are released from rest, so initial KE is zero and finally they are infinite apart, so final PE is zero.

$$0 + \left(-\frac{2kp_1p_2}{r^3}\cos 180°\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

Here, $m_1 = m_2 = m,\ p_1 = p_2 = p,\ r = a$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{2kp^2}{a^3} \quad \text{...(i)}$$

Using conservation of momentum,

$$\mathbf{p}_i = \mathbf{p}_f \Rightarrow 0 = m\mathbf{v}_1 + m\mathbf{v}_2$$

or $\mathbf{v}_1 = -\mathbf{v}_2 \Rightarrow v_1 = v_2 = v$ ...(ii)

Putting this value in Eq. (i), we get

$$mv^2 = \frac{2kp^2}{a^3} \text{ or } v = \frac{p}{a}\sqrt{\frac{2k}{ma}}$$

or $v = \frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ $\left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$

Hence, correct option is (b).

47 An electric dipole of moment $\mathbf{p} = (-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times 10^{-29}$ C-m is at the origin (0, 0, 0). The electric field due to this dipole at $\mathbf{r} = +\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ (note that $\mathbf{r} \cdot \mathbf{p} = 0$) is parallel to **[2020, 9 Jan Shift-I]**

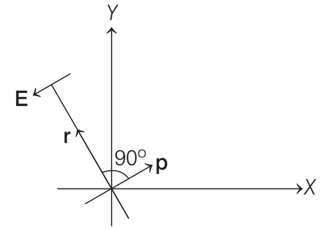
(a) $(+\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
(b) $(-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
(c) $(+\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
(d) $(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Ans. *(c)*

Given, $\mathbf{r} \cdot \mathbf{p} = 0$

So, $\mathbf{r} \perp \mathbf{p}$, i.e. we have following situation,

So, we have to find direction of electric field at equatorial line. As $\mathbf{E}$ is directed opposite to $\mathbf{p}$ at all equatorial points, direction of $\mathbf{E}$ is along $-\mathbf{p}$;.



So, $\mathbf{E} = \lambda(-\mathbf{p})$

$= \lambda[-(-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})]$

$= \lambda(\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

48 An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is **[2019, 8 April Shift-II]**

(a) $\sqrt{\frac{2qE}{md}}$ (b) $2\sqrt{\frac{qE}{md}}$ (c) $\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

Ans. *(a)*

Key Idea When an electric dipole is placed in an electric field **E** at some angle θ, then two forces equal in magnitude but opposite in direction acts on the +ve and –ve charges, respectively. These forces forms a couple which exert a torque, which is given as

$$\tau = \mathbf{p} \times \mathbf{E}$$

where, **p** is dipole moment.

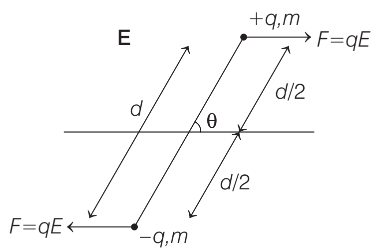
$\therefore$ Torque on the dipole can also be given as

$$\tau = I\alpha = -pE\sin\theta$$

where, I is the moment of inertia and α is angular acceleration. For small angles, $\sin\theta \approx \theta$

$$\therefore \quad \alpha = -\left(\frac{pE}{I}\right)\theta \quad \text{...(i)}$$

Moment of inertia of the given system is



$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$$\Rightarrow \quad \alpha = -\left(\frac{2pE}{md^2}\right)\cdot\theta \quad \text{...(ii)}$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii), with the general equation of angular SHM, i.e.

$$\alpha = -\omega^2\theta$$

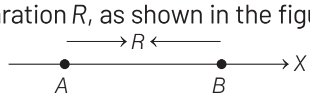
where, ω is the angular frequency, we get

$$\omega^2 = \frac{2pE}{md^2} \text{ or } \omega = \sqrt{\frac{2pE}{md^2}}$$

As $p = qd$

$$\therefore \quad \omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

49 Two electric dipoles, A, B with respective dipole moments $\mathbf{d}_A = -4qa\,\hat{\mathbf{i}}$ and $\mathbf{d}_B = -2qa\,\hat{\mathbf{i}}$ are placed on the X-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is **[2019, 10 Jan Shift-I]**

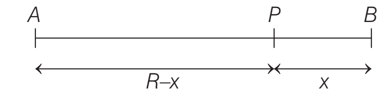
(a) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (c) $\frac{R}{\sqrt{2}+1}$ (d) $\frac{R}{\sqrt{2}-1}$

Ans. *(a)*

Key Idea As, dipole moments points in same direction

So, potential of both dipoles can be same at some point between A and B.

Let potentials are same at P, distant x from B as shown below



Then, $\frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$

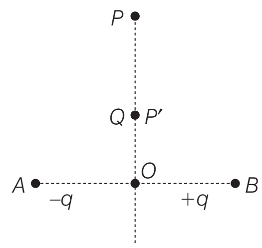
$$2x^2 = (R-x)^2$$

$$\Rightarrow \quad \sqrt{2}x = R - x \Rightarrow x = \frac{R}{\sqrt{2}+1}$$

Distance from A is

$$\Rightarrow \quad R - x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

50 Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F.



If Q is now moved along the equatorial line to P' such that

$OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2a\right)$ **[2019, 10 Jan Shift-II]**

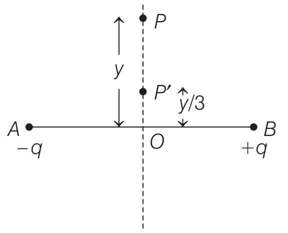
(a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

Ans. *(d)*

Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \quad \text{...(i)}$$

where, P is the dipole moment of the charges.



In first case $r = y$

$$\Rightarrow \quad E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

Here, $y^2 >> a^2$

$$\Rightarrow \quad y^2 + a^2 \approx y^2 \quad \text{or} \quad E_1 = \frac{2kP}{y^3} \quad \text{...(ii)}$$

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kPQ}{y^3} \quad \text{...(iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $\frac{y}{3} >> a$

$$\Rightarrow \quad \left(\frac{y}{3}\right)^2 + a^2 \approx \left(\frac{y}{3}\right)^2$$

$$\Rightarrow \quad E_2 = \frac{2kP}{(y/3)^3}$$

$$\Rightarrow \quad E_2 = 27 \times \frac{2kP}{y^3}$$

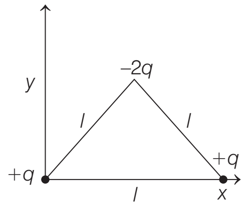
Force on charge in this position,

$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27F$$

51 Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. **[2019, 12 Jan Shift-I]**

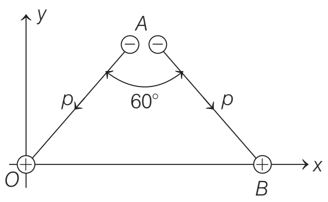


(a) $\sqrt{3}\, ql \frac{\hat{j} - \hat{i}}{\sqrt{2}}$ (b) $2ql\,\hat{j}$

(c) $-\sqrt{3}\, ql\,\hat{j}$ (d) $(ql)\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

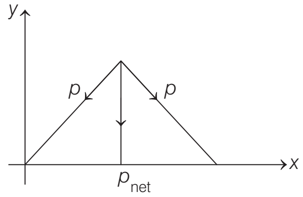
Ans. *(c)*

Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below



Now, magnitude of resultant of these dipole moments is

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p \cdot p \cos 60^\circ}$$
$$= \sqrt{3}p = \sqrt{3}ql$$

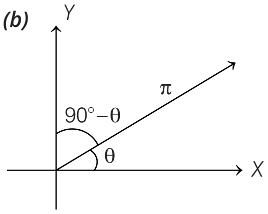


As, resultant is directed along negative y-direction $p_{\text{net}} = -\sqrt{3}p\hat{j} = -\sqrt{3}ql\,\hat{j}$

52 An electric dipole has a fixed dipole moment **p**, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E_1} = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **[JEE Main 2017]**

(a) 45° (b) 60° (c) 90° (d) 30°

Ans. *(b)*



Torque applied on a dipole $\tau = pE \sin\theta$

where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{i}$

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1 \sin\theta$$

In electric field $E_2 = \sqrt{3}\,E\hat{j}$,

it means field is directed along positive Y-axis, so angle between dipole and field will be $90° - \theta$.

Torque in this direction

$$\tau_2 = pE \sin(90° - \theta) = p\sqrt{3}\,E_1 \cos\theta$$

According to question

$$\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$$

$$\therefore \quad pE_1 \sin\theta = p\sqrt{3}\,E_1 \cos\theta$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$$

$$\therefore \quad \theta = 60°$$

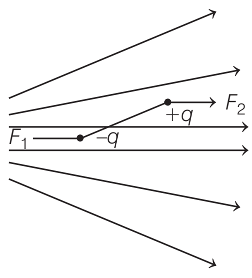
53 An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[AIEEE 2006]**

(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

Ans. *(c)*

In a non-uniform electric field, the dipole may experience both non-zero torque as well as translational force.

For example, as shown in figure



$F_1 \neq F_2$ as E is non-uniform.

Torque would also be non-zero.

TOPIC 4

Electric Flux and Gauss Laws

54 The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is ...... nC, if electric flux density of its field is found as $D = e^{-x}\sin y\hat{\mathbf{i}} - e^{-x}\cos y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}$ C/m^2. **[2021, 22 July Shift-II]**

Ans. *(4)*

Given, incremental volume,

$$dV = 2\times10^{-9}\,\mathrm{m}^3$$

Electric flux denisty,

$$D = e^{-x}\sin y\hat{\mathbf{i}} - e^{-x}\cos y\hat{\mathbf{j}} + 2z\hat{\mathbf{k}}\,\mathrm{C/m^2}$$

As we know that,

$$\rho(x,y,z) = \nabla\cdot\mathbf{D} = \frac{d}{dx}(e^{-x}\sin y) - \frac{d}{dy}(e^{-x}\cos y) + \frac{d}{dz}2z$$

$$= -e^{-x}\sin y + e^{-x}\sin y + 2$$

$$\rho(0,0,0) = 2 = \frac{Q}{dV}$$

$$\Rightarrow \quad Q = 2dV = 2\times2\times10^{-9} = 4\times10^{-9} = 4\,\mathrm{nC}$$

55 The electric field in a region is given by $\mathbf{E} = \frac{2}{5}E_0\hat{\mathbf{i}} + \frac{3}{5}E_0\hat{\mathbf{j}}$ with $E_0 = 4.0\times10^3$ N/C. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the *yz*-plane is .... N-m^2 C^{-1}. **[2021, 17 March Shift-II]**

Ans. *(640)*

Given,

The electric field in the region,

$$\mathbf{E} = \frac{2}{5}E_0\hat{\mathbf{i}} + \frac{3}{5}E_0\hat{\mathbf{j}}$$

Here, $E_0 = 4\times10^3\,\frac{\mathrm{N}}{\mathrm{C}}$

Area of the rectangular surface,

$$A = 0.4\mathrm{m}^2$$

The direction of electric field vector and area vector is same, so the angle between the electric field vector and area vector is 0.

As we know the expression of electric flux,

$$\phi = E\cdot A\cos\theta \qquad \text{...(i)}$$

Here, E is the electric field vector, and A is the surface area of the surfaces.

Consider the surface parallel to the Y-Z plane, so the area vector, $A = 0.4\hat{\mathbf{i}}\,\mathrm{m}^2$

Substituting the values in Eq. (i), we get

$$\phi = E\cdot A\cos0^\circ \Rightarrow \phi = \frac{2}{5}E_0(0.4)$$

$$\Rightarrow \quad \phi = \frac{2}{5}(4\times10^3)(0.4) = 640\,\mathrm{Nm^2C^{-1}}$$

Hence, the electric flux of the surface parallel to the Y-Z plane is $640\,\mathrm{Nm^2C^{-1}}$.

56 Given below are two statements:

Statement I An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius $r\,(<R)$ is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below. **[2021, 26 Feb Shift-II]**

(a) Both Statement I and Statement II are true.
(b) Statement I is true but Statement II is false.
(c) Both Statement I and Statement II are false.
(d) Statement I is false but Statement II is true.

Ans. *(b)*

Net charge on electyric dipole

$$= +q - q = 0$$

Hence, according to Gauss's law,

$$\text{Electric flux, } \phi = \frac{q_{\mathrm{net}}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

Electric field due to electric dipole is non-zero and varies at point to point.

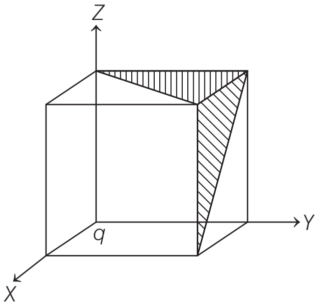
Hence, statement I is true.

Electric field due to charged solid sphere at a distance r from centre.

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3} \qquad \begin{bmatrix}\text{when } r<R\\ R\to\text{radius}\end{bmatrix}$$

which is non-zero.
Hence, statement II is false.
Hence, option (b) is the correct.

57 A charge q is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\mathbf{E}$ through the shaded area is **[25 Feb 2021 Shift-II]**



(a) $\frac{q}{48\varepsilon_0}$ (b) $\frac{q}{4\varepsilon_0}$
(c) $\frac{q}{8\varepsilon_0}$ (d) $\frac{q}{24\varepsilon_0}$

Ans. *(d)*

Given, charge q is at one of the corner of the cube.

$\therefore$ Contribution of q in cube will be $q_{\mathrm{enclosed}} = q/8$

As, only 3 faces of cube is allowing the flux lines to pass through it.

$$\therefore \text{Flux } (\phi) = \frac{q_{\mathrm{enclosed}}}{\varepsilon_0} = \frac{1}{3}\frac{q/8}{\varepsilon_0} = \frac{q}{24\varepsilon_0}$$

58 The electric field in a region is given $\mathbf{E} = \left(\frac{3}{5}E_0\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}\right)\frac{\mathrm{N}}{\mathrm{C}}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to YZ-plane) to that of the surface of area 0.3 m^2 (parallel to XZ- plane) is $a:b$, where a = ... [Here $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are unit vectors along X, Y and Z-axes, respectively] **[2021, 25 Feb Shift-I]**

Ans. *(1)*

Given, $\mathbf{E} = \frac{3E_0}{5}\hat{\mathbf{i}} + \frac{4}{5}E_0\hat{\mathbf{j}}$,

$$\mathbf{A}_1 = 0.2\,\mathrm{m}^2\,\hat{\mathbf{i}} \quad\text{and}\quad \mathbf{A}_2 = 0.3\,\mathrm{m}^2\,\hat{\mathbf{j}}$$

Let ϕ_1 and ϕ_2 be the flux linked with area A_1 and A_2, respectively.

As we know that,

$$\phi = \oint\mathbf{E}\cdot dS = \mathbf{E}\cdot\mathbf{A}$$

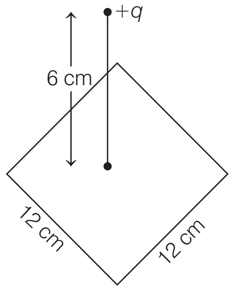
$$\Rightarrow \quad \phi_1 = (3/5E_0\hat{\mathbf{i}} + 4/5E_0\hat{\mathbf{j}})\cdot0.2\,\hat{\mathbf{i}} = 3/5\,E_0\times0.2$$

and similarly, $\phi_2 = 4/5\,E_0\times0.3$

Now, $\frac{\phi_1}{\phi_2} = \frac{3/5E_0 \times 0.2}{4/5\,E_0 \times 0.3} = \frac{0.6}{1.2} = \frac{1}{2}$

$\therefore \quad a = 1$

59 A point charge of $+12\mu C$ is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be .... $\times 10^3$ N-m^2/C. **[2021, 24 Feb Shift-II]**



Ans. *(226)*

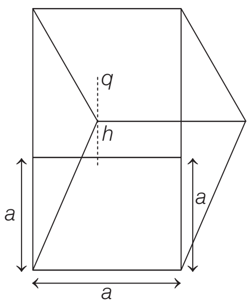
Given, charge, $q = 12\mu C = 12 \times 10^{-6}$ C

Height of charge from surface, $h = 6$ cm

$= 6 \times 10^{-2}$ m

and side of square, $a = 12$ cm $= 12 \times 10^{-2}$ m

From figure, it is clear that the given square is one of the face of a cube of side 12 cm and + $12\mu C$ charge is placed at its centre. Then, by Gauss's theorem,



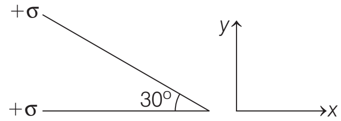
Flux through any face, $\phi = \frac{q}{6\varepsilon_0}$

$\Rightarrow \quad \phi = \frac{12 \times 10^{-6}}{6 \times 8.854 \times 10^{-12}}$

$= 0.226 \times 10^6$ N-m^2/C

$= 226 \times 10^3$ N-m^2/C

60 Two infinite planes each with uniform surface charged density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by **[2020, 7 Jan Shift-I]**



(a) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{y} - \frac{\hat{x}}{2}\right]$

(b) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$

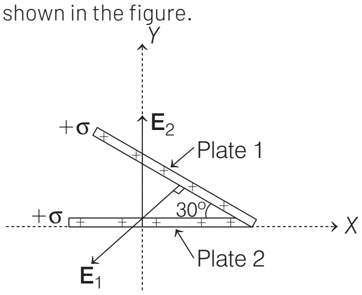
(c) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{y} + \frac{\hat{\xi}}{2}\right]$

(d) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y} + \frac{\hat{x}}{2}\right]$

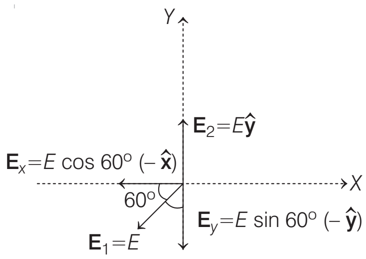
Ans. *(b)*

Electric field of an infinite plate is perpendicular to the plane of plate and its magnitude is $E = \frac{\sigma}{2\varepsilon_0}$.

We are given with two positively charged plates with a set of coordinate axes as shown in the figure.



From geometry of figure, net electric field in region between plates is resultant of fields of both plates,



Now, field of plate 1 can be resolved along X and Y-axes as shown in above figure.

Now, $\mathbf{E}_{net} = \mathbf{E}_1 + \mathbf{E}_2$

$= E\cos 60°(-\hat{x}) + E\sin 60°(-\hat{y}) + E\hat{y}$

Here, $E = \frac{\sigma}{2\varepsilon_0}$ and $\hat{x}$ and $\hat{y}$ are unit vectors along X and Y-axes.

$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{\hat{x}}{2} - \frac{\sqrt{3}}{2}\hat{y} + \hat{y}\right)$

$= \frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$

61 In finding the electric field using Gauss law the formula $|\mathbf{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula, ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation? **[2020, 8 Jan Shift-I]**

(a) Only when the Gaussian surface is an equipotential surface and $|\mathbf{E}|$ is constant on the surface.

(b) Only when the Gaussian surface is an equipotential surface.

(c) For any choice of Gaussian surface.

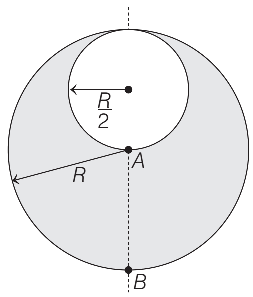
(d) Only when $|\mathbf{E}|$ = constant on the surface.

Ans. *(a)*

Equation $|\mathbf{E}| = \frac{q_{en}}{\varepsilon_0 |A|}$ Gives $\int \mathbf{E} \cdot d\mathbf{A} = \frac{q_{en}}{\varepsilon_0}$

Now, in finding the electric field by above equation, the integral is easy to evaluate, if $|\mathbf{E}|$ = constant. Also, if $|\mathbf{E}|$ = constant for the surface, then surface is equipotential.

62 Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown in the figure the ratio $\frac{|\mathbf{E}_A|}{|\mathbf{E}_B|}$ of magnitude of electric field $\mathbf{E}_A$ and $\mathbf{E}_B$ respectively, at points A and B due to the remaining portion is **[2020, 9 Jan Shift-I]**



(a) $\frac{21}{34}$ (b) $\frac{18}{54}$ (c) $\frac{17}{54}$ (d) $\frac{18}{34}$

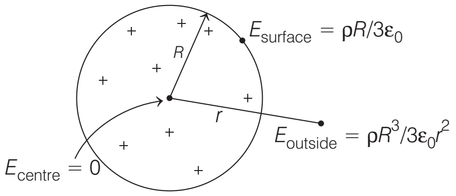
Ans. *(d)*

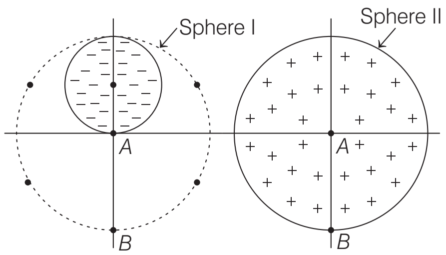
Electric field at points A and B can be viewed as a superposition of

(i) electric field due to complete solid sphere of radius R and charge density ρ.

(ii) electric field due to sphere of radius $\frac{R}{2}$ and charge density $-\rho$.

Now, we use standard result of electric field due to a solid sphere which are





Now, electric field due to sphere I at points A and B are

$$E_{IA} = \frac{\rho\left(\frac{R}{2}\right)}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0}$$

$$E_{IB} = \frac{\rho\left(\frac{R}{2}\right)^3}{3\varepsilon_0\left(\frac{3R}{2}\right)^2} = \frac{\rho R}{54\varepsilon_0}$$

and similarly due to sphere II,

$$E_{IIA} = 0$$

$$E_{IIB} = \frac{\rho R}{3\varepsilon_0}$$

So, net field magnitude at point A,

$$E_A = E_{IA} + E_{II\,A} = \frac{\rho R}{6\varepsilon_0} + 0 = \frac{\rho R}{6\varepsilon_0}$$

and net field magnitude to point B,

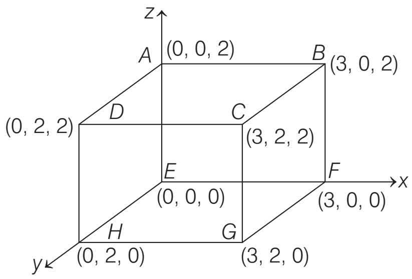
$$E_B = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17}{54}\frac{\rho R}{\varepsilon_0}$$

Hence, ratio

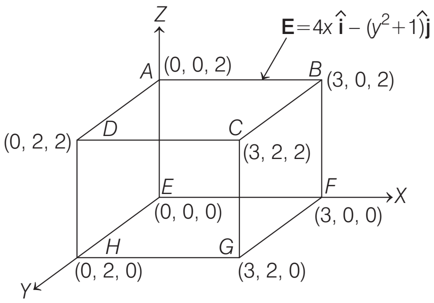
$$\frac{E_A}{E_B} = \frac{\rho R}{6\varepsilon_0} \times \frac{54\varepsilon_0}{17\rho R} = \frac{18}{34}$$

63 An electric field $\mathbf{E} = 4x\hat{\mathbf{i}} - (y^2 + 1)\hat{\mathbf{j}}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces $ABCD$ and $BCGF$ are marked as ϕ_{I} and ϕ_{II}, respectively. The difference between $(\phi_{\mathrm{I}} - \phi_{\mathrm{II}})$ is (in N-m^2/C) ....

[2020, 9 Jan Shift-II]



Ans. *(–48)*



Area vector of face $ABCD$,

$$\mathbf{A}_1 = 2 \times 3\hat{\mathbf{k}} = 6\hat{\mathbf{k}}$$

Area vector of face $BCGF$,

$$\mathbf{A}_2 = 2 \times 2\hat{\mathbf{i}} = 4\hat{\mathbf{i}}$$

So, flux through face $ABCD$,

$$\phi_1 = \mathbf{E} \cdot \mathbf{A}_1 = (4x\hat{\mathbf{i}} - (y^2 + 1)\hat{\mathbf{j}}) \cdot 6\hat{\mathbf{k}} = 0$$

Flux through face $BCGF$,

$$\phi_2 = \mathbf{E} \cdot \mathbf{A}_2 = (4x\hat{\mathbf{i}} - (y^2 + 1)\hat{\mathbf{j}}) \cdot 4\hat{\mathbf{i}} = 16x$$

At face $BCGF$, $x = 3$

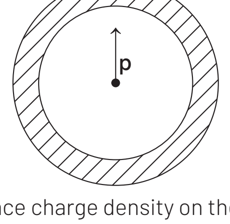
So, $\phi_2 = 16 \times 3 = 48$ units

$\therefore \quad \phi_1 - \phi_2 = 0 - 48$

$= -48$ N-m^2 C^{-1}

64 Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carries charge Q. At its centre is a dipole $\mathbf{p}$ as shown.

In this case, **[2019, 12 April Shift-I]**



(a) surface charge density on the inner surface is uniform and equal to $\frac{\left(\frac{Q}{2}\right)}{4\pi a^2}$

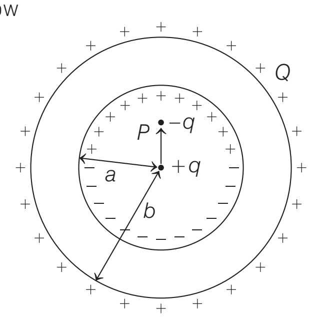
(b) electric field outside the shell is the same as that of a point charge at the centre of the shell

(c) surface charge density on the outer surface depends on $|\mathbf{p}|$

(d) surface charge density on the inner surface of the shell is zero everywhere

Ans. *(b)*

Electric charge distribution at inner and outer surface of spherical shell due to the electric dipole can be shown as below



Here, we need to consider two different factors

(i) charge on the spherical shell is $+Q$ which will be distributed on its outer surface as shown in figure.

(ii) Electric dipole will create non-uniform electric field inside the shell which will distribute the charges on inner surface as shown in figure. But its net contribution to the outer side of the shell will be zero as net charge of a dipole is zero.

$\therefore$ Net charge on outer surface of shell will be $+Q$.

Hence, using (ii), option (a) is incorrect as field inside shell is not uniform. Option (b) is correct, as net charge on outer surface is $+Q$ even in the presence of dipole.

Option (c) is incorrect, as surface charge density at outer surface is uniform $\left(= \frac{Q}{A} = \frac{Q}{4\pi b^2}\right)$.

Option (d) is incorrect, as surface charge density at inner surface is non-zero.

So, option (b) is correct.

Alternate Solution Using Gauss' law at outer surface, let charge on dipole is q ,

$$\phi = \frac{\Sigma q}{\varepsilon_0} = E \cdot A \text{ or } E = \frac{1}{A\varepsilon_0}\Sigma q$$

$$= \frac{(+Q + q - q)}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \text{constant}$$

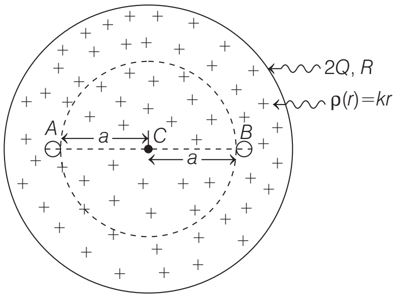
65 Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a, from the centre. If A and B do not experience any force, then **[2019, 12 April Shift-II]**

(a) $a = 8^{-1/4}R$ (b) $a = \frac{3R}{2^{1/4}}$

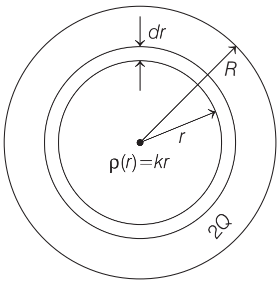
(c) $a = 2^{-1/4}R$ (d) $a = R/\sqrt{3}$

Ans. *(a)*

Key Idea Force on A is zero only when repulsion of A and B = attraction of positive charge distribution of radius a and charge A.



In given charge distribution, let r is radius of a shell of thickness dr.



Charge dQ present in shell of thickness dr = Volume of shell × Volumetric charge density

$$\Rightarrow dQ = (4\pi r^2 \times dr) \times (kr)$$
$$= 4\pi k r^3\, dr$$

Total charge in sphere is

$$2Q = \int_0^R dQ = \int_0^R 4\pi k r^3\, dr$$

$$\Rightarrow 2Q = 4\pi k \left[\frac{r^4}{4}\right]_0^R \Rightarrow k = \frac{2Q}{\pi R^4}$$

Now, using Gauss' law, electric field on the surface of sphere of radius a is

$$E\int_0^a dA = \frac{1}{\varepsilon_0}\cdot\int_0^a (kr\, 4\pi r^2\, dr)$$

$$\Rightarrow E\cdot 4\pi a^2 = \frac{1}{\varepsilon_0}\cdot 4\pi k\left(\frac{a^4}{4}\right)$$

$$\Rightarrow E = \frac{ka^2}{4\varepsilon_0} = \frac{2Q\,a^2}{4\pi\varepsilon_0 R^4}$$

Force of attraction on charge A (or B) due to this field is

$$F_1 = QE = \frac{2Q^2a^2}{4\pi\varepsilon_0 R^4}$$

Force of repulsion on charge A due to B is

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2a)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4a^2}$$

If charge A (or B) does not feel any force, then

$$F_1 = F_2$$

$$\Rightarrow \frac{2Q^2a^2}{4\pi\varepsilon_0 R^4} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4a^2}$$

$$\Rightarrow 8a^4 = R^4$$

$$\Rightarrow a = 8^{-1/4}R$$

66 Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{\frac{-2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is **[2019, 9 Jan Shift-II]**

(a) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (b) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(c) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (d) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

Ans. *(d)*

Here, volume charge density,

$$\rho(r) = \frac{A}{r^2}\cdot e^{-\frac{2r}{a}}$$

where, a and A are constant.

Let a spherical region of small element of radius r.

If Q is total charge distribution upto radius R, then

$$Q = \int_0^R \rho\cdot dV = \int_0^R \frac{A}{r^2}e^{-2r/a}(4\pi r^2\, dr)$$

(From figure, we observe $dV = A\cdot dr = 4\pi r^2\cdot dr$)

$$= 4\pi A\int_0^R e^{-2r/a}dr$$

$$= 4\pi A\left(\frac{e^{-2r/a}}{-2/a}\right)_0^R$$

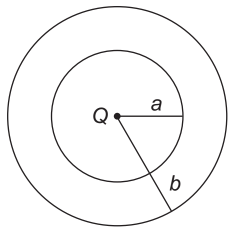
$$= 4\pi A\times\left(\frac{-a}{2}\right)(e^{-2R/a} - e^0)$$

$$= 2\pi A(-a)\,[e^{-2R/a} - 1]$$

or $Q = 2\pi a\, A(1 - e^{-2R/a})$

or $R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

67 The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is **[JEE Main 2016]**



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$

(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

Ans. *(a)*

As, Gaussian surface at distance r from centre,

$$\frac{Q + \int_a^r \frac{A}{r}4\pi r^2 dr}{\varepsilon_0} = E4\pi r^2$$

$$E\,4\pi\varepsilon_0 r^2 = Q + A\frac{4\pi}{r^2}\left(\frac{r^2 - a^2}{2}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + A\,2\pi\left(\frac{r^2 - a^2}{r^2}\right)\right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2} + A2\pi - \frac{A2\pi a^2}{r^2}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\times A\times 2\pi$$

At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is

As, $Q = 2\pi Aa^2$ i.e. $A = \frac{Q}{2\pi a^2}$

68 This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2012]**

An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement I When a charge q is taken from the centre of the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement II The electric field at a distance $r\,(r< R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II the correct explanation for Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. *(a)*

Statement I is dimensionally wrong while from Gauss' law,

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{3\varepsilon_0}$$

gives Statement II is correct.

69 The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre and a, b are constants. Then, the charge density inside the ball is **[AIEEE 2011]**

(a) $-6a\,\varepsilon_0 r$ (b) $-24\,\pi a\,\varepsilon_0$
(c) $-6a\,\varepsilon_0$ (d) $-24\pi a\,\varepsilon_0 r$

Ans. *(c)*

Electric field, $E = -\frac{d\phi}{dr} = -2ar$

By Gauss' theorem,

$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow q = -8\pi\,\varepsilon_0 a r^3$$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr} \times \frac{dr}{dV}$$

$$= (-24\pi\,\varepsilon_0 a r^2)\left(\frac{1}{4\pi r^2}\right)$$

$$= -6\varepsilon_0 a$$

70 Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r, (r < R)$ from the origin is given by **[AIEEE 2010]**

(a) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$
(c) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Ans. *(b)*

Apply Shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2 \cdot dr \cdot \rho$$

$$= 4\pi r^2 \cdot dr \cdot \rho_0\left[\frac{5}{4} - \frac{r}{R}\right]$$

$$= 4\pi\rho_0\left[\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right]$$

$$\int dq = q = 4\pi\rho_0 \int_0^r \left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$

$$q = 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

As electric field intensity,

$$E = \frac{kq}{r^2}$$

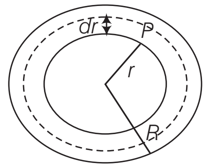
$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2} \cdot 4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

71 Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is **[AIEEE 2009]**

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$
(c) $\frac{Q r_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Q r_1^2}{3\pi\varepsilon_0 R^4}$

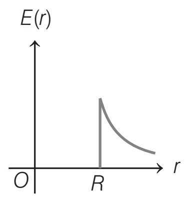
Ans. *(c)*

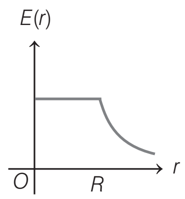


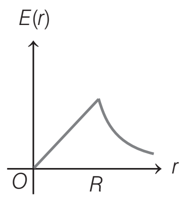
$$E 4\pi r_1^2 = \frac{\int_0^{r_1} \frac{Q}{\pi R^4} r 4\pi r^2\, dr}{\varepsilon_0}$$

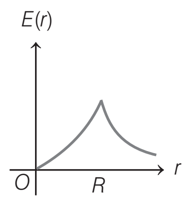
$$\Rightarrow \quad E = \frac{Q r_1^2}{4\pi\varepsilon_0 R^4}$$

72 A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < x$, where r is the distance from the centre of the shell? **[AIEEE 2008]**

(a)


(b)


(c)


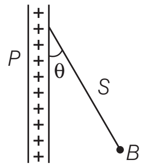
(d)


Ans. *(a)*

For uniformly charged spherical shell,

$$E = 0,\ r < R$$
$$= \frac{Q}{4\pi\varepsilon_0 r^2},\ r \geq R$$

73 A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to **[AIEEE 2005]**



(a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

Ans. *(d)*

Electric field due to a charged conducting sheet of surface charge density σ is given by $E = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$.

where, ε_0 is the permittivity in vacuum and ε_r is the relative permittivity of medium.

Here, electrostatic force on B,

$$QE = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r}$$

FBD of B is shown in figure.

In equilibrium, $T\cos\theta = mg$

and $\quad T\sin\theta = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r}$

Thus, $\quad \tan\theta = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r mg} \Rightarrow \tan\theta \propto \sigma$

74 If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[AIEEE 2003]**

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $\frac{(\phi_1 + \phi_2)}{\varepsilon_0}$

(c) $\frac{(\phi_2 - \phi_1)}{\varepsilon_0}$ (d) $(\phi_1 + \phi_2)\varepsilon_0$

Ans. *(a)*

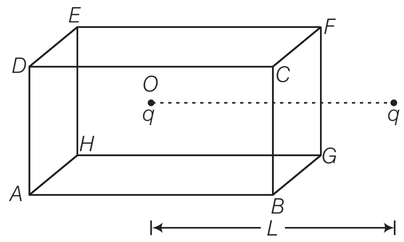
From Gauss' law, $\frac{\text{Charge enclosed}}{\varepsilon_0}$

$$= \text{Net flux}$$

$$\Rightarrow \quad \frac{q}{\varepsilon_0} = \phi_2 - \phi_1$$

or $\quad q = (\phi_2 - \phi_1)\varepsilon_0$

75 A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then, the electric flux through $ABCD$ is **[AIEEE 2002]**



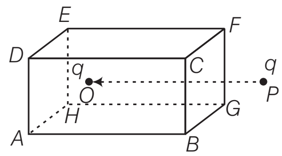
(a) $\frac{q}{4\pi\varepsilon_0 L}$ (b) $\frac{q}{2\pi\varepsilon_0 L}$

(c) $\frac{q}{2\pi\varepsilon_0 L}$ (d) None of these

Ans. *(d)*

Electric flux for any surface is defined as

$$\phi = \int \mathbf{E} \cdot d\mathbf{s}$$



As flux on the cube $= \frac{q}{\varepsilon_0}$

Flux on each face $= \frac{1}{6}\frac{q}{\varepsilon_0}$

So, flux on the face $ABCD = \frac{1}{6}\frac{q}{\varepsilon_0}$

The options (a), (c) and (d) are dimensionally incorrect, so they cannot be answers.