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Chapter

HCF AND LCM OF POLYNOMIALS AND RATIONAL EXPRESSIONS

KEY FACTS

1. The HCF of polynomials is the polynomial of highest degree and greatest numerical coefficient which divides both the polynomial exactly.

2. HCF of monomials :

- (i) Find the HCF of the numerical coefficients of all the monomials.
- (ii) Find the highest power of each of the variables common to all the monomials. Omit the variables that are not common.

(iii) The HCF of the given monomials is the product of (i) and (ii)

For example, to find the HCF of $12a^2b^4$, $15ab^2c$ and $21a^3b$. The HCF of 12, 15, 21 = 3

Highest power of variables common to the three variables = ab

$\therefore$ Reqd. HCF = $3ab$

3. HCF of polynomials :

Step 1. Factorise the polynomials.

Step 2. Find the HCF of numerical factors.

Step 3. The product of the common factors is the required HCF.

For example: HCF of $2x^2 + 2x - 4$ and $x^2 - 1$ can be found as :

$$2x^2 + 2x - 4 = 2(x^2 + x - 2) = 2(x + 2)(x - 1)$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$\therefore \text{HCF} = (x - 1)$$

4. The LCM of polynomials is the polynomial of the lowest degree and smallest numerical coefficient which is exactly divisible by the given polynomials.

5. To find the LCM of polynomials that can be easily factorised.

Step 1. Write each polynomial in factorised form.

Step 2. Include in the LCM, the factors that are common to the given polynomials and then the remaining factors that are not common.

Step 3. The LCM is the product of all the common factors and the remaining factors.

For example: LCM of $8x(x^2 - 1)$ and $6x^2(x - 1)^2$ can be found as: $8x(x^2 - 1) = 8 \times x \times (x - 1)(x + 1)$

$$6x^2(x - 1)^2 = 2 \times 3 \times x \times x \times (x - 1)(x - 1)$$

$$\therefore \text{LCM} = 8 \times 3 \times x^2 \times (x - 1)^2 \times (x + 1)$$

$$= 24x^2(x - 1)^2(x + 1)$$

6. Fractions like $\frac{x+2}{x-5}$, $\frac{x^2-4x+4}{6x^2-3x-5}$, $\frac{4}{4x^2-7}$ etc, having polynomials in the numerator or denominator or both are called algebraic fractions or **rational expressions**.
7. An algebraic expression is in the simplest form when the polynomials in the numerator and denominator do not have a common factor.
8. Addition, subtraction, multiplication and division of rational expressions is done in the same way as we do of rational numbers.

Thus if $\frac{m}{n}$ and $\frac{p}{q}$, $n \neq 0$, $q \neq 0$ are rational expressions. Then,

$$\begin{aligned} \text{(i)} \quad \frac{m}{n} + \frac{p}{q} &= \frac{mq + pn}{nq} & \text{(ii)} \quad \frac{m}{n} - \frac{p}{q} &= \frac{mq - pn}{nq} \\ \text{(iii)} \quad \frac{m}{n} \times \frac{p}{q} &= \frac{m \times p}{n \times q} = \frac{mp}{nq} & \text{(iv)} \quad \frac{m}{n} \div \frac{p}{q} &= \frac{m}{n} \times \frac{q}{p} = \frac{mq}{np} \end{aligned}$$

9. An algebraic expression is simplified by removing the brackets in the order: (i) Bar (ii) Parenthesis (iii) Curly Brackets (iv) Square Brackets and following the rule of **BODMAS**.

Solved Examples

Ex. 1. What is HCF of $8x^2y^2$, $12x^3y^2$ and $24x^4y^3z^2$?

Sol. $8x^2y^2 = 2 \times 2 \times 2 \times \underline{x \times x} \times \underline{y \times y}$

$12x^3y^2 = 2 \times 2 \times 3 \times \underline{x \times x \times x} \times \underline{y \times y}$

$24x^4y^3z^2 = 2 \times 2 \times 2 \times 3 \times \underline{x \times x \times x \times x} \times \underline{y \times y \times y} \times z \times z.$

HCF of $8x^2y^2$, $12x^3y^2$ and $24x^4y^3z^2 = 2 \times 2 \times x \times x \times y \times y = 4x^2y^2$.

Ex. 2. Find the HCF of $x^2 - 5x + 6$ and $x^2 - 9$.

Sol. $x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$= x(x-2) - 3(x-2) = (x-2)(x-3)$

$x^2 - 9 = (x-3)(x+3)$

$\therefore$ HCF of $(x^2 - 5x + 6)$ and $(x^2 - 9) = x - 3$

Ex. 3. Find the LCM of $14a^2b^3c^4$, $20ab^4c^3$ and $35a^5b^3c$.

Sol. LCM. of 14, 20 and 35 = $2 \times 5 \times 7 \times 2 = 140$

LCM of a^2 , a and $a^5 = a^5$

LCM of b^3 , b^4 and $b^2 = b^4$

LCM of c^4 , c^3 and $c = c^4$

LCM of the given monomials = $140 a^5 b^4 c^4$

2	14, 20, 35
5	7, 10, 35
7	7, 2, 7
	1, 2, 1

Ex. 4. Find the LCM of $3y + 12$, $y^2 - 16$ and $y^4 - 64y$.

Sol. $3y + 12 = 3(y + 4)$

$y^2 - 16 = (y + 4)(y - 4)$

$y^4 - 64y = y(y^3 - 64) = y(y - 4)(y^2 + 4y + 16)$

$\therefore$ LCM of given polynomials = $3y(y - 4)(y + 4)(y^2 + 4y + 16)$

Ex. 5. The HCF of two expressions is x and their LCM is $x^3 - 9x$. If one of the expressions is $x^2 + 3x$, then find the other expression.

Sol. HCF = x ,

LCM = $x^3 - 9x = x(x^2 - 9) = x(x + 3)(x - 3)$

Given expression = $x^2 + 3x = x(x + 3)$

$$\begin{aligned}\therefore \text{Other expression} &= \frac{\text{HCF} \times \text{LCM}}{\text{Given expression}} \\ &= \frac{x \times x \times (x+3) \times (x-3)}{x \times (x+3)} \\ &= x(x-3) = x^2 - 3x\end{aligned}$$

Ex. 6. If the HCF of $x^3 - 343$ and $x^2 - 9x + 14$ and $(x - 7)$, then find their LCM.

$$\begin{aligned}\text{Sol. } x^3 - 343 &= x^3 - (7)^3 = (x-7)(x^2 + 7x + 49) \\ x^2 - 9x + 14 &= x^2 - 2x - 7x + 14 \\ &= x(x-2) - 7(x-2) = (x-7)(x-2) \\ \text{HCF of the polynomials} &= (x-7) \\ \therefore \text{Reqd. LCM} &= \frac{(x^3 - 343) \times (x^2 - 9x + 14)}{\text{HCF}} = \frac{(x-7)(x^2 + 7x + 49) \times (x-7)(x-2)}{(x-7)} \\ &= (x-7)(x-2) \times (x^2 + 7x + 49)\end{aligned}$$

Ex. 7. Simplify the expression $\frac{6p^2 - 150}{p^2 - 3x - 40}$

$$\begin{aligned}\text{Sol. } \frac{6p^2 - 150}{p^2 - 3x - 40} &= \frac{6(p^2 - 25)}{p^2 - 8x + 5x - 40} \\ &= \frac{6(p-5)(p+5)}{p(p-8) + 5(p-8)} = \frac{6(p-5)(p+5)}{(p-8)(p+5)} = \frac{6(p-5)}{p-8}\end{aligned}$$

Ex. 8. Add : $\frac{a}{3xy} + \frac{2b}{6yz} + \frac{3c}{15xz}$

Sol. The least common denominator (LCD) of the given expression is $30xyz$.

$$\begin{aligned}\therefore \frac{a}{3xy} + \frac{2b}{6yz} + \frac{3c}{15xz} &= \frac{10az + 10bx + 6cy}{30xyz} \\ &= \frac{5az + 5bx + 3cy}{15xyz}\end{aligned}$$

3	3, 6, 15
	1, 2, 5

Ex. 9. Simplify : $\frac{1}{x^2 - 8x + 15} - \frac{1}{x^2 - 25}$

$$\begin{aligned}\text{Sol. } x^2 - 8x + 15 &= x^2 - 5x - 3x + 15 \\ &= x(x-5) - 3(x-5) = (x-5)(x-3) \\ x^2 - 25 &= (x-5)(x+5) \\ \therefore \frac{1}{x^2 - 8x + 15} - \frac{1}{x^2 - 25} &= \frac{1}{(x-5)(x-3)} - \frac{1}{(x-5)(x+5)} \\ &= \frac{(x+5) - (x-3)}{(x-3)(x-5)(x+5)} \quad (\text{Note the step}) \\ &= \frac{x+5-x-3}{(x-3)(x-5)(x+5)} = \frac{2}{(x-3)(x-5)(x+5)}\end{aligned}$$

Ex. 10. Simplify the expression :

$$\left[\frac{x^3 + y^3}{(x-y)^2 + 3xy} \right] \div \left[\frac{(x+y)^2 - 3xy}{x^3 - y^3} \right] \times \frac{xy}{x^2 - y^2}$$

$$\begin{aligned}
 \text{Sol. } & \left(\frac{x^3 + y^3}{(x-y)^2 + 3xy} \right) \div \left(\frac{(x+y)^2 - 3xy}{x^3 - y^3} \right) \times \frac{xy}{x^2 - y^2} \\
 &= \left(\frac{(x+y)(x^2 - xy + y^2)}{x^2 - 2xy + y^2 + 3xy} \right) \div \left(\frac{x^2 + y^2 + 2xy - 3xy}{(x-y)(x^2 + xy + y^2)} \right) \times \frac{xy}{x^2 - y^2} \\
 &= \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + xy + y^2)} \div \frac{(x^2 - xy + y^2)}{(x-y)(x^2 + xy + y^2)} \times \frac{xy}{(x+y)(x-y)} \\
 &= \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + xy + y^2)} \times \frac{(x-y)(x^2 + xy + y^2)}{(x^2 - xy + y^2)} \times \frac{xy}{(x+y)(x-y)} \\
 &= xy
 \end{aligned}$$

Question Bank-9

- HCF of the polynomials $20x^2y(x^2 - y^2)$ and $35xy^2(x - y)$ is :
 (a) $5x^2y^2(x - y)$ (b) $5xy(x - y)$
 (c) $5x^2y^2(x + y)$ (d) $5xy(x^2 - y^2)$
- HCF of $x^3 - 1$ and $x^4 + x^2 + 1$ will be
 (a) $(x - 1)$ (b) $x^2 + x + 1$
 (c) $x^2 - x + 1$ (d) $x^2 - x - 1$
- The HCF of the polynomials $x^3 - 3x^2 + x - 3$ and $x^3 - x^2 - 9x + 9$ is :
 (a) $x - 3$ (b) $x - 1$
 (c) $x^2 + 1$ (d) $(x - 1)(x - 3)$
- The LCM of the polynomials $xy + yz + zx + x^2$ and $x^2 + xy + yz + zx$ is :
 (a) $x + y$ (b) $y + z$
 (c) $(x + y)(y + z)(z + x)$ (d) $x^2 + y^2$
- The LCM of $x^2 - 10x + 16$, $x^2 - 9x + 14$ and $x^2 - 10x + 21$ is :
 (a) $(x - 2)^2(x - 3)(x - 7)^2(x - 8)$
 (b) $(x - 2)^2(x - 3)(x - 7)(x - 8)$
 (c) $(x - 2)(x - 3)(x - 7)^2(x - 8)$
 (d) $(x - 2)(x - 3)(x - 7)(x - 8)$
- The LCM of $6(x^2 + xy)$, $8(xy - y^2)$, $12(x^2 - y^2)$ and $20(x + y)^2$ is :
 (a) $120x(x + y)(x - y)$
 (b) $120xy(x + y)(x - y)$
 (c) $120xy(x + y)^2(x - y)$
 (d) $120xy(x + y)(x - y)^2$
- The HCF of $x^4 - y^4$ and $x^6 - y^6$ is :
 (a) $x^2 - y^2$ (b) $x^2 + y^2$
 (c) $x^3 + y^3$ (d) $x^3 - y^3$
- The LCM of the polynomials $x^3 + 3x^2 + 3x + 1$, $x^2 + 2x + 1$ and $x^2 - 1$ is :
 (a) $(x^2 - 1)(x + 1)^3$ (b) $(x^2 + 1)(x - 1)^2$
 (c) $(x^2 - 1)(x - 1)^2$ (d) $(x + 1)^3$
- The product of two expression is $x^3 + x^2 - 44x - 84$. If the HCF of these two expressions is $x + 6$, then their LCM will be:
 (a) $(x + 2)(x + 7)$ (b) $(x + 2)(x - 7)$
 (c) $(x - 2)(x + 7)$ (d) $(x - 2)(x - 7)$
- The HCF of $x^4 - 11x^2 + 10$, $x^2 - 5x + 4$ and $x^3 - 3x^2 + 3x - 1$ is
 (a) $x + 1$ (b) $x - 4$
 (c) $x + 2$ (d) $x - 1$
- The HCF of two polynomials $4x^2(x^2 - 3x + 2)$ and $12x(x - 2)(x^2 - 4)$ is $4x(x - 2)$. The LCM of the two polynomials is :
 (a) $12x(x^2 - 4)$
 (b) $12x^2(x^2 - 3x + 4)(x^2 - 2)$
 (c) $12x^2(x^2 - 3x + 2)(x^2 - 4)$
 (d) $12x(x^2 - 3x - 2)(x^2 - 4)$

12. The rational expression $\frac{8x^3 - 125}{4x^2 + 10x + 25}$ in its simplest form is :

- (a) $2x$ (b) 5
(c) $2x + 5$ (d) $2x - 5$

13. $\sqrt{\frac{(x^2 + 3x + 2)(x^2 + 5x + 6)}{x^2(x^2 + 4x + 3)}}$ is equal to :

- (a) $x(x + 1)$ (b) $\frac{x + 2}{x}$
(c) $\frac{x}{x + 2}$ (d) $x(x + 2)$

14. If $A = \frac{2x + 1}{2x - 1}$ and $B = \frac{2x - 1}{2x + 1}$, then $A - B$ is equal to :

- (a) $\frac{1}{4x^2 - 1}$ (b) $\frac{8x}{4x^2 - 1}$
(c) $\frac{-2}{2x^2 - 1}$ (d) $\frac{4x}{4x^2 - 1}$

15. $\frac{1}{x + 1} - \frac{1}{x - 1} - \frac{x^2}{x + 1} + \frac{x^2}{x - 1}$, when simplified is equal to :

- (a) 0 (b) 1
(c) 2 (d) -2

16. The product of the rational expressions $\frac{x^2 - y^2}{x^2 + 2xy + y^2}$ and $\frac{xy + y^2}{x^2 - xy}$ is :

- (a) xy (b) y/x
(c) x/y (d) 1

17. $\left(\frac{2x + y}{x + y} - 1\right) \div \left(1 - \frac{y}{x + y}\right)$ is equal to :

- (a) x (b) y
(c) xy (d) 1

18. $\frac{x^3 + y^3 + z^3 - 3xyz}{a^3 + b^3 + c^3 - 3abc} \times \frac{a^2 + b^2 + c^2 - ab - bc - ca}{x^2 + y^2 + z^2 - xy - yz - zx}$ equals

- (a) 1 (b) $\frac{x^2 + y^2 + z^2}{a^2 + b^2 + c^2}$
(c) $\frac{x + y + z}{a + b + c}$ (d) $\frac{xyz}{abc}$

19. What should be added to $\frac{a}{a - b} + \frac{b}{a + b}$ to get 1 ?

- (a) $\frac{-2ab}{a^2 + b^2}$ (b) $\frac{2ab}{a^2 - b^2}$
(c) $\frac{2ab}{b^2 - a^2}$ (d) $\frac{-2ab}{b^2 - a^2}$

20. Simplify $\left[\frac{1}{1 + a} + \frac{2a}{1 - a^2}\right] \times \frac{(a^2 + 4a - 5)}{(a^2 + 10a + 25)}$

- (a) $\frac{-1}{a + 1}$ (b) $\frac{1}{1 - a}$
(c) $\frac{1}{a + 5}$ (d) $\frac{-1}{a + 5}$

Answers

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (b) | 3. (a) | 4. (c) | 5. (d) | 6. (c) | 7. (a) | 8. (a) | 9. (b) | 10. (d) |
| 11. (c) | 12. (d) | 13. (b) | 14. (b) | 15. (c) | 16. (b) | 17. (d) | 18. (c) | 19. (a) | 20. (d) |

Hints and Solutions

1. (b) HCF of 20 and 35 = 5

HCF of x^2y and $xy^2 = xy$

HCF of $(x^2 - y^2)$, i.e., $(x - y)(x + y)$

and $(x - y) = (x - y)$

$\therefore$ Reqd. HCF = $5xy(x - y)$

2. (b) $x^3 - 1 = (x - 1)(x^2 + x + 1)$

$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 = (x^2 + 1) - x^2$

$= (x^2 + 1 - x)(x^2 + 1 + x)$

$\therefore$ Reqd. HCF = $x^2 + x + 1$

3. (a) $x^3 - 3x^2 + x - 3 = x^2(x - 3) + 1(x - 3)$

$= (x - 3)(x^2 + 1)$

$x^3 - x^2 - 9x + 9 = x^2(x - 1) - 9(x - 1)$

$= (x^2 - 9)(x - 1) = (x + 3)(x - 3)(x - 1)$

$\therefore$ Reqd. HCF = $(x - 3)$

4. (c) $\frac{xy + yz + zx + y^2}{(y + z)(x + y)} = \frac{x(y + z) + y(y + z)}{(y + z)(x + y)}$

$x^2 + xy + yz + zx = x(x + y) + z(y + x)$

$= (x + y)(x + z)$

$\therefore$ Reqd. LCM = $(x + y)(y + z)(x + z)$

5. (d) $x^2 - 10x + 16 = (x - 8)(x - 2)$

$x^2 - 9x + 14 = (x - 7)(x - 2)$

$x^2 - 10x + 21 = (x - 7)(x - 3)$

$\therefore$ Reqd. LCM = $(x - 2)(x - 3)(x - 7)(x - 8)$

$$\begin{array}{l|l}
 6, 8, 12, 20 & \\
 \hline
 2 & 3, 4, 6, 10 \\
 \hline
 3 & 3, 2, 3, 5 \\
 \hline
 & 1, 2, 1, 5
 \end{array}$$

$$6. (c) \quad 6(x^2 + xy) = 6x(x + y)$$

$$8(xy - y^2) = 8y(x - y)$$

$$12(x^2 - y^2) = 12(x - y)(x + y)$$

$$20(x + y)^2 = 20(x + y)(x + y)$$

$$\text{LCM of } 6, 8, 12, 20 = 120$$

$$\therefore \text{Reqd. LCM} = 120xy(x - y)(x + y)^2$$

$$7. (a) \quad x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$$

$$= (x - y)(x + y)(x^2 + y^2)$$

$$x^6 - y^6 = (x^3)^2 - (y^3)^2 = (x^3 - y^3)(x^3 + y^3)$$

$$= (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$$

$$\therefore \text{Reqd. HCF} = (x - y)(x + y) = x^2 - y^2$$

$$8. (a) \quad x^3 + 3x^2 + 3x + 1 = (x + 1)^3$$

$$x^2 + 2x + 1 = (x + 1)^2$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$\therefore \text{Reqd. LCM} = (x - 1)(x + 1)^3$$

$$= (x^2 - 1)(x + 1)^3$$

$$9. (b) \quad \text{LCM} \times \text{HCF} = \text{Product of the expressions}$$

$$\Rightarrow \text{LCM} = \frac{\text{Prod. of expressions}}{\text{HCF}}$$

$$= \frac{x^3 + x^2 - 44x - 84}{(x + 6)}$$

Performing long division, we have

$$\begin{array}{r}
 x^2 - 5x - 14 \\
 x+6 \overline{) x^3 + x^2 - 44x - 84} \\
 \underline{x^3 + 6x^2} \\
 -5x^2 - 44x \\
 \underline{-5x^2 - 30x} \\
 -14x - 84 \\
 \underline{-14x - 84} \\
 0
 \end{array}$$

$$\therefore \text{Reqd. LCM} = x^2 - 5x - 14 = (x - 7)(x + 2)$$

$$10. (d) \quad x^4 - 11x^2 + 10 = x^4 - 10x^2 - x^2 + 10$$

$$= x^2(x^2 - 10) - 1(x^2 - 10)$$

$$= (x^2 - 10)(x^2 - 1)$$

$$= (x^2 - 10)(x + 1)(x - 1)$$

$$x^2 - 5x + 4 = (x - 4)(x - 1)$$

$$x^3 - 3x^2 + 3x + 1 = (x - 1)^3$$

$$\therefore \text{Reqd. HCF} = (x - 1)$$

$$11. (c) \quad \text{LCM} = \frac{\text{product of the polynomials}}{\text{HCF}}$$

$$= \frac{4x^2(x^2 - 3x + 2) \times 12x(x - 2)(x^2 - 4)}{4x(x - 2)}$$

$$= \frac{4x^2(x - 2)(x - 1) \times 12x(x - 2)(x - 2)(x + 2)}{4x(x - 2)}$$

$$= 12x^2(x - 1)(x - 2)(x - 2)(x + 2)$$

$$= 12x^2(x^2 - 3x + 2)(x^2 - 4)$$

$$12. (d) \quad \frac{8x^3 - 125}{4x^2 + 10x + 25} = \frac{(2x)^3 - 5^3}{4x^2 + 10x + 25}$$

$$= \frac{(2x - 5)(4x^2 + 10x + 25)}{4x^2 + 10x + 25}$$

$$= 2x - 5$$

$$13. (b) \quad \sqrt{\frac{(x^2 + 3x + 2)(x^2 + 5x + 6)}{x^2(x^2 + 4x + 3)}}$$

$$= \sqrt{\frac{(x + 1)(x + 2)(x + 2)(x + 3)}{x^2(x + 1)(x + 3)}}$$

$$= \sqrt{\frac{(x + 2)^2}{x^2}} = \frac{x + 2}{x}$$

$$14. (b) \quad A - B = \frac{2x + 1}{2x - 1} - \frac{2x - 1}{2x + 1}$$

$$= \frac{(2x + 1)^2 - (2x - 1)^2}{(2x - 1)(2x + 1)}$$

$$= \frac{(4x^2 + 4x + 1) - (4x^2 - 4x + 1)}{4x^2 - 1}$$

$$= \frac{8x}{4x^2 - 1}$$

$$15. (c) \quad \frac{1}{x + 1} - \frac{1}{x - 1} - \frac{x^2}{x + 1} + \frac{x^2}{x - 1}$$

$$= \frac{(x - 1) - (x + 1) - x^2(x - 1) + x^2(x + 1)}{(x - 1)(x + 1)}$$

$$= \frac{x - 1 - x - 1 - x^3 + x^2 + x^3 + x^2}{x^2 - 1}$$

$$= \frac{2x^2 - 2}{x^2 - 1} = \frac{2(x^2 - 1)}{(x^2 - 1)} = 2$$

$$16. (b) \text{ Reqd. product} = \frac{x^2 - y^2}{x^2 + 2xy + y^2} \times \frac{xy + y^2}{x^2 - xy}$$

$$= \frac{(x+y)(x+y)}{(x+y)^2} \times \frac{y(x+y)}{x(x-y)}$$

$$= \frac{y}{x}$$

$$17. (d) \left(\frac{2x+y}{x+y} - 1 \right) \div \left(1 - \frac{y}{x+y} \right)$$

$$= \left[\frac{2x+y-(x+y)}{x+y} \right] \div \left[\frac{x+y-y}{x+y} \right]$$

$$= \frac{x}{x+y} \times \frac{x+y}{x} = 1$$

18. (c) Reqd. exp.

$$= \frac{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}$$

$$\times \frac{(a^2+b^2+c^2-ab-bc-ca)}{(x^2+y^2+z^2-xy-yz-zx)} = \frac{x+y+z}{a+b+c}$$

$$19. (c) \text{ Reqd. exp.} = 1 - \left[\frac{a}{a-b} + \frac{b}{a+b} \right]$$

$$= 1 - \left[\frac{a(a+b)+b(a-b)}{a^2-b^2} \right]$$

$$= 1 - \left[\frac{a^2+ab+ab-b^2}{a^2-b^2} \right]$$

$$= \frac{(a^2-b^2)-(a^2+2ab-b^2)}{a^2-b^2}$$

$$= \frac{-2ab}{a^2-b^2} = \frac{2ab}{b^2-a^2}$$

$$20. (d) \left(\frac{1}{1+a} + \frac{2a}{1-a^2} \right) \times \frac{(a^2+4a-5)}{a^2+10a+25}$$

$$= \left(\frac{1-a+2a}{(1-a^2)} \right) \times \frac{(a+5)(a-1)}{(a+5)^2}$$

$$= \frac{1+a}{1-a^2} \times \frac{(a+5)(a-1)}{(a+5)^2}$$

$$= \frac{(1+a)}{(1+a)(1-a)} \times \frac{-(a+5)(1-a)}{(a+5)^2} = \frac{-1}{a+5}$$

Self Assessment Sheet-9

1. If p , m and n are prime numbers, none of which is equal to the other two, what is the greatest common factor of $24pm^2n^2$, $9pmn^2$ and $36p(mn)^3$?

- (a) $3pmn$ (b) $3p^2m^2n^2$
(c) $3pmn^2$ (d) $3pmn^3$

2. The HCF of $x^5 + 2x^4 + x^3$ and $x^7 - x^5$ is

- (a) x (b) $x(x+1)$
(c) x^3 (d) $x^3(x+1)$

3. The LCM of $x^2 - 3x + 2$ and $x^3 - 2x^2 - 3x$ is :

- (a) $x(x-2)(x+3)(x^2-1)$
(b) $x(x-2)(x-3)(x^2+1)$
(c) $x(x-2)(x-3)(x^2-1)$
(d) $x(x-2)(x+3)(x^2+1)$

4. The LCM and HCF of two polynomials are respectively $(2a-5)^2(a+1)$ and $(2a-5)$. If one of the polynomials is $4a^2 - 20a + 25$, the other one is :

- (a) $4a^2 + 20a + 5$ (b) $4a^2 - 25$
(c) $2a^2 + 3a - 5$ (d) $2a^2 - 3a - 5$

5. $\frac{a+1}{a^2+5a} \times \frac{a^2-25}{a^2-a-20} \div \frac{a^2-a-2}{a^2+2a-8}$ when simplified is equal to :

- (a) 1 (b) a
(c) $\frac{1}{a}$ (d) a^2

6. The value of $\frac{x+y}{x-y} + \frac{x-y}{x+y} - \frac{2(x^2+y^2)}{x^2-y^2}$ is :

- (a) 1 (b) x
(c) y (d) 0

7. Evaluate

$$\frac{x+2}{(x+1)(2x+3)} - \frac{2x+3}{(x+1)(x+2)} + \frac{3x+5}{(2x+3)(x+2)}$$

- (a) $2x$ (b) -1
(c) 0 (d) x

8. The rational expression

$$\frac{(x^2 - xy - 12y^2)(x^2 + xy - 12y^2)}{(x^2 - 16y^2)(x^2 - 9y^2)}$$
 when simplified equals.

- (a) 1 (b) xy
(c) $(x+y)$ (d) $(x-y)$

9. $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] \div \frac{4}{x^4-1}$ when simplified is equal to :

- (a) 1 (b) 0
(c) x^2-1 (d) 2

10. The positive square root of the rational expression.

$\left[y^3 - \frac{1}{y^3} - 3\left(y - \frac{1}{y}\right) \right] \div \left(y - \frac{1}{y}\right)$ is :

- (a) $y + \frac{1}{y}$ (b) 1
(c) $y - \frac{1}{y}$ (d) 2

Answers

1. (c) 2. (d) 3. (c) 4. (d) 5. (c) 6. (d) 7. (c) 8. (a) 9. (d) 10. (c)