DPP - Daily Practice Problems

Date : ______ Start Time : ______ End Time : ______

MATHEMATICS CM10

SYLLABUS : Straight Lines and Pair of Straight Lines

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is

(a) $x + 3y = 21$ (b) $2x - 3y = 7$

(c) $x + 7y = 31$ (d) $2x + 3y = 21$

2. Two lines are given by $(x - 2y)^2 + k(x - 2y) = 0$. The value of k, so that the distance between them is 3, is :

(a) $k = 0$ (b) $k = \pm 3\sqrt{5}$

(c) $k = \pm\sqrt{5}$ (d) $k = 3$

3. A line through A $(-5, -4)$ meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C and D respectively. If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$, then the equation of the line is

(a) $2x + 3y + 22 = 0$ (b) $5x - 4y + 7 = 0$

(c) $3x - 2y + 3 = 0$ (d) None of these

4. The number of lines that are parallel to $2x + 6y + 7 = 0$ and have an intercept of length 10 between the coordinate axes is

(a) 1 (b) 2

(c) 4 (d) Infinitely many

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. The distance of the point (1, 2) from the line $x + y + 5 = 0$ measured along the line parallel to $3x - y = 7$ is equal to

(a) $4\sqrt{10}$ (b) 40

(c) $\sqrt{40}$ (d) $10\sqrt{2}$

6. If p_1, p_2 are the lengths of the normals drawn from the origin on the lines $x \cos\theta + y \sin\theta = 2a \cos 4\theta$ and $x \sec\theta + y \operatorname{cosec}\theta = 4a \cos 2\theta$

respectively, and $mp_1^2 + np_2^2 = 4a^2$. Then

(a) $m = 1, n = 1$ (b) $m = 1, n = 4$

(c) $m = 4, n = 1$ (d) $m = 1, n = -1$

7. For what value of 'p', $y^2 + xy + px^2 - x - 2y + p = 0$ represent 2 straight lines :

(a) 2 (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

8. One vertex of an equilateral triangle is (2,3) and the equation of line opposite to the vertex is $x + y = 2$, then the equation of remaining two sides are

(a) $y - 3 = (2 \pm \sqrt{3})(x - 2)$ (b) $y + 3 = (2 \pm \sqrt{3})(x + 2)$

(c) $y + 3 = ((3 \pm \sqrt{2})(x + 2)$ (d) $y - 3 = (3 \pm \sqrt{2})(x - 2)$

9. The point on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, are

(a) (3, 1), (–7, 11) (b) (3, 1), (7, 11)

(c) (–3, 1), (–7, 11) (d) (1, 3), (–7, 11)

10. The straight line $y = x - 2$ rotates about a point where it cuts the x-axis and becomes perpendicular to the straight line $ax + by + c = 0$. Then its equation is

(a) $ax + by + 2a = 0$ (b) $ax - by - 2a = 0$

(c) $bx + ay - 2b = 0$ (d) $ay - bx + 2b = 0$

11. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is

(a) 1 (b) 2

(c) 3 (d) 4

12. The slopes of the lines represented by $x^2 + 2hxy + 2y^2 = 0$ are in the ratio 1 : 2, then h equals

(a) $\pm\frac{1}{2}$ (b) $\pm\frac{3}{2}$

(c) ± 1 (d) ± 3

13. The distance of the line $2x + y = 3$ from the point (–1, 3) in the direction whose slope is 1 is

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{2\sqrt{5}}{3}$

14. The equation of the straight line, the portion of which intercepted between the coordinate axes being divided by the point (–5, 4) in the ratio 1 : 2, is

(a) $8x + 5y = 60$ (b) $8x - 5y = 60$

(c) $-8x + 5y = 60$ (d) None of these

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. The reflection of the point (4, –13) in the line $5x + y + 6 = 0$, is

(a) (–1, –14) (b) (3, 4)

(c) (1, 2) (d) (–4, 13)

16. The combined equation of the pair of lines through the point (1, 0) and parallel to the lines represented by $2x^2 - xy - y^2 = 0$ is

(a) $2x^2 - xy - y^2 - 4x - y = 0$

(b) $2x^2 - xy - y^2 - 4x + y + 2 = 0$

(c) $2x^2 + xy + y^2 - 2x + y = 0$

(d) None of these

17. P is a point on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. The coordinates of the foot of the perpendicular from P on the bisector of the angle between them are

(a) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ or $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ depending on which the point P is taken

(b) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$

(c) $\left(0, \frac{4-5\sqrt{3}}{2}\right)$

(d) $\left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

18. The distance between the parallel lines $9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$ is

(a) $\frac{2}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{10}}$

(c) $\frac{4}{\sqrt{10}}$ (d) None of these

19. Equation of the hour hand at 4 O' clock is

(a) $x - \sqrt{3}y = 0$ (b) $\sqrt{3}x - y = 0$

(c) $x + \sqrt{3}y = 0$ (d) $\sqrt{3}x + y = 0$

20. If the image of point P(2, 3) in a line L is Q(4, 5), then the image of point R(0, 0) in the same line is:

(a) (2,2) (b) (4, 5)

(c) (3, 4) (d) (7, 7)

21. The coordinates of a point which is at +3 distance from points (1, –3) of line $2x + 3y + 7 = 0$ is

(a) $\left(1 - \frac{9}{\sqrt{13}}, -3 + \frac{6}{\sqrt{13}}\right)$ (b) $\left(1 + \frac{9}{\sqrt{13}}, 1 - \frac{9}{\sqrt{13}}\right)$

(c) $\left(3 - \frac{6}{\sqrt{13}}, 3 + \frac{6}{\sqrt{13}}\right)$ (d) $\left(1 + \frac{9}{\sqrt{13}}, -3 - \frac{6}{\sqrt{13}}\right)$

22. If one of the diagonals of a square is along the line x = 2y and one of its vertices is (3, 0), then its sides through this vertex are given by the equations

(a) $y - 3x + 9 = 0,\ 3y + x - 3 = 0$

(b) $y + 3x + 9 = 0,\ 3y + x - 3 = 0$

(c) $y - 3x + 9 = 0,\ 3y - x + 3 = 0$

(d) $y - 3x + 3 = 0,\ 3y + x + 9 = 0$

23. Given a family of lines $a(2x + y + 4) + b(x - 2y - 3) = 0$, the number of lines belonging to the family at a distance $\sqrt{10}$ from P(2, –3) is

(a) 0 (b) 1

(c) 2 (d) 4

24. The line parallel to the *x*- axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(a) below the *x* - axis at a distance of $\frac{3}{2}$ from it

(b) below the *x* - axis at a distance of $\frac{2}{3}$ from it

(c) above the *x* - axis at a distance of $\frac{3}{2}$ from it

(d) above the *x* - axis at a distance of $\frac{2}{3}$ from it

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. The equation

$8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$ represents a pair of straight lines. The distance between them is

(a) $7/\sqrt{5}$ (b) $7/2\sqrt{5}$

(c) $\sqrt{7}/5$ (d) None of these

26. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

27. The equation of a straight line, which passes through the point (a, 0) and whose perpendicular distance from the point (2a, 2a) is a, is

(a) $3x - 4y - 3a = 0$ (b) $x - a = 0$

(c) both (a) and (b) (d) Neither of (a) and (b)

28. The points (1,3) and (5,1) are two opposite vertices of a rectangle. The other two vertices lie on the line y = 2x + c, then one of the remaining vertices is

(a) (4,4) (b) (2,2)

(c) (0,2) (d) (4,2)

29. (sin θ, cos θ) and (3, 2) lies on the same side of the line $x + y = 1$, then θ lies between

(a) (0, π/2) (b) (0, π)

(c) (π/4, π/2) (d) (0, π/4)

30. The perpendicular distance between the straight lines $6x + 8y + 15 = 0$ and $3x + 4y + 9 = 0$ is

(a) $\frac{3}{2}$ units (b) $\frac{3}{10}$ unit

(c) $\frac{3}{4}$ unit (d) $\frac{2}{7}$ unit

RESPONSE GRID	
	25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 10 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	38	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(c)** One vertex of square is (– 4, 5) and equation of one diagonal is $7x-y+8=0$
Diagonal of a square are perpendicular and bisect each other
Let the equation of the other diagonal be $y = mx + c$ where m is the slope of the line and c is the y-intercept.
Since this line passes through (– 4, 5)
$\therefore \quad 5=-4m+c \ldots$ (i)
Since this line is at right angle to the line
$7x-y+8=0$ or $y=7x+8$, having slope = 7,
$\therefore \quad 7\times m=-1 \quad \text{or} \quad m=\frac{-1}{7}$
Putting this value of m in equation (i) we get
$c=5-\frac{4}{7}=\frac{31}{7}$
Hence equation of the other diagonal is
$y=-\frac{1}{7}x+\frac{31}{7}$
or $\quad x+7y=31$.

2. **(b)** The lines are given by
$(x-2y)^2+k(x-2y)=0 \Rightarrow (x-2y)(x-2y+k)=0$
That is $x-2y=0$ and $x-2y+k=0$
These are parallel. The distance between the two lines
$=\left|\frac{k}{\sqrt{1^2+(-2)^2}}\right|=3 \text{ (given)} \quad \therefore |k|=3\sqrt{5} \Rightarrow k=\pm 3\sqrt{5}$

3. **(a)** The parametric equation of a line through A is

$$\frac{x+5}{\cos\theta}=\frac{y+4}{\sin\theta}=r$$

Let $AB=r_1, AC=r_2$ and $AD=r_3$

Then the coordinates of B, C, D are

$(-5,+r_i\cos\theta-4+r_i\sin\theta),\ i=1,2,3$

Now B lies on the line $x+3y+2=0$

$\therefore\ -5+r_1\cos\theta+3(-4+r_1\sin\theta)+2=0$

$$\frac{15}{r_1}=\cos\theta+3\sin\theta$$

C lies on $2x+y+4=0$

$\therefore\ 2(-5+r_2\cos\theta)+(-4+r_2\sin\theta)+4=0$

$$\Rightarrow \frac{10}{r_2}=2\cos\theta+\sin\theta$$

D lies on $x-y-5=0$

$\therefore\ -5+r_3\cos\theta+4-r_3\sin\theta-5=0 \Rightarrow \frac{6}{r_3}=\cos\theta-\sin\theta$.

From the given condition $\left(\frac{15}{r_1}\right)^2+\left(\frac{10}{r_2}\right)^2=\left(\frac{6}{r_3}\right)^2$

we get, $(\cos\theta+3\sin\theta)^2+(2\cos\theta+\sin\theta)^2=(\cos\theta-\sin\theta)^2$

$\Rightarrow (2\cos\theta+3\sin\theta)^2=0 \Rightarrow \tan\theta=-\frac{2}{3}$

$\therefore$ Equation of the line is

$$y+4=-\frac{2}{3}(x+5)\Rightarrow 2x+3y+22=0$$

4. **(b)** The equation of any line parallel to $2x+6y+7=0$ is $2x+6y+k=0$

This meets the axes at $A\left(-\frac{k}{2},0\right)$ and $B\left(0,-\frac{k}{6}\right)$

By hypothesis, AB = 10

$$\Rightarrow \sqrt{\frac{k^2}{4}+\frac{k^2}{36}}=10\Rightarrow\sqrt{\frac{10k^2}{36}}=10$$

$\Rightarrow 10k^2=3600 \Rightarrow k=\pm6\sqrt{10}$

Hence there are two lines given by

$$2x+6y\pm6\sqrt{10}=0$$

5. **(c)** The slope of the line $3x-y=7$ is $\tan\theta=3$.

or $\frac{P}{B}=\frac{3}{1} \Rightarrow H=\sqrt{9+1}=\sqrt{10}$

$$\therefore\ \sin\theta=\frac{3}{\sqrt{10}},\cos\theta=\frac{1}{\sqrt{10}}$$

The eqn of line passing through (1, 2) and parallel to $y=3x-7$ is

$$\frac{x-1}{\cos\theta}=\frac{y-2}{\sin\theta} \quad ...(i)$$

Let r be the required distance.

$\therefore$ $(1+r\cos\theta, 2+r\sin\theta)$ lies on $x+y+5=0$

$\Rightarrow 1+r\cos\theta+2+r\sin\theta+5=0$

$\Rightarrow 1+r\frac{1}{\sqrt{10}}+2+r\frac{3}{\sqrt{10}}+5=0 \Rightarrow r=2\sqrt{10}$

6. **(b)** $p_1^2=4a^2\cos^2 4\theta$

$$p_2^2=\frac{16a^2\cos^2 2\theta}{\sec^2\theta+\operatorname{cosec}^2\theta}=16a^2\cos^2 2\theta\cos^2\theta\sin^2\theta=a^2\sin^2 4\theta$$

$\therefore p_1^2+4p_2^2=4a^2$

7. **(c)** We have the equation

$y^2+xy+px^2-x-2y+p=0$

We know any general equation

$ax^2+by^2+2hxy+2gx+2fy+c=0$ ... (1)

represents two straight lines if

$abc+2fgh-af^2-bg^2-ch^2=0$ ... (2)

On comparing given equation with (1), we get

$$a=p, b=1,\ h=\frac{1}{2},\ g=-\frac{1}{2}, f=-1, c=p$$

Put these value in equation (2)

$$p\times1\times p+2\times-1\times-\frac{1}{2}\times\frac{1}{2}-p\times(-1)^2-1\times\left(-\frac{1}{2}\right)^2-p\times\left(\frac{1}{2}\right)^2=0$$

$\Rightarrow\ p^2+\frac{1}{2}-p-\frac{1}{4}-\frac{p}{4}=0 \Rightarrow p^2-\frac{5p}{4}+\frac{1}{4}=0$

$\Rightarrow\ 4p^2-5p+1=0 \Rightarrow\ (4p-1)(p-1)=0$

$\Rightarrow\ p=1,\frac{1}{4}$

8. **(a)** Since the two sides make an angle of 60° each with side $x+y=2$. Therefore equations of these sides will be

$$y-3=\frac{-1\pm\tan 60°}{1\mp(-1)\tan 60°}(x-2)=\frac{-1\pm\sqrt{3}}{1\pm\sqrt{3}}(x-2)$$

$\Rightarrow y-3=(2\pm\sqrt{3})(x-2)$

9. **(a)** Let the point (h, k) lie on a line $x+y=4$

then $h+k=4$ ...(i)

and $1=\pm\frac{4h+3k-10}{\sqrt{4^2+3^2}}\Rightarrow 4h+3k=15$ ...(ii)

and $4h+3k=5$ ...(iii)

On solving (i) and (ii); and (i) and (iii), we get the required points (3, 1) and (–7, 11).

Trick : Check with options. Obviously, points (3, 1) and (–7, 11) lie on $x+y=4$ and perpendicular distance of these points from $4x+3y=10$ is 1.

10. **(d)** Slope of the line in the new position is $\frac{b}{a}$, since it is $\perp$ to the line $ax+by+c=0$ and it cuts the x-axis at (2,0). Hence, the required line passes through (2, 0) and its slope is $\frac{b}{a}$.

Required eq. is

$$y-0=\frac{b}{a}(x-2) \Rightarrow ay=bx-2b \Rightarrow ay-bx+2b=0$$

11. (c) Let line be $y-3=m(x-2)$

y intercept is $(3-2m)$, x intercept is $\left(2-\frac{3}{m}\right)$

Area = 12

$\therefore\ 12=\frac{1}{2}\left|2-\frac{3}{m}\right||3-2m|$

$\Rightarrow 12-\frac{9}{m}-4m=+24$

$\therefore\ 4m^2+12m+9=0 \Rightarrow m=-3/2$

or $12-\frac{9}{m}-4m=-24 \Rightarrow 4m^2-36m+9=0;\ D>0$

$\Rightarrow$ There are two values of m. Hence total 3 values of m.

12. (b) We know that if m_1 and m_2 are the slopes of the lines represented by $ax^2+2hxy+by^2=0$,

then sum of slopes $= m_1+m_2=-\frac{2h}{b}$ and

product of slopes $= m_1 m_2=\frac{a}{b}$.

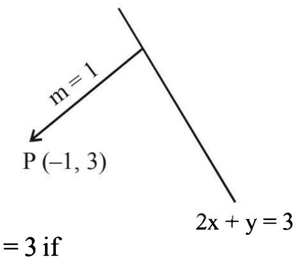
Consider the given equation which is
$x^2+2hxy+2y^2=0$
On comparing this equation with $ax^2+2hxy+by^2=0$,
we have $a=1$, $2h=2h$ and $b=2$
Let the slopes be m_1 and m_2.
Given : $m_1 : m_2=1:2$
Let $m_1=x$ and $m_2=2x$

$\therefore\ m_1+m_2=-\frac{2h}{2} \Rightarrow x+2x=-h \Rightarrow h=-3x$ ...(i)

and $m_1 m_2=\frac{a}{b} \Rightarrow x\,.\,2x=\frac{1}{2} \Rightarrow x=\pm\frac{1}{2}$ ...(ii)

$\therefore$ From eqs. (i) and (ii), we have $h=\pm\frac{3}{2}$.

13. (c) The equation of the line through (–1, 3) and having the slope 1 is $\frac{x+1}{\cos\theta}=\frac{y-3}{\sin\theta}=r$.



Any point on this line at a distance r from P (–1, 3) is

$(-1+r\cos\theta,\ 3+r\sin\theta)$

This point is on the line 2x + y = 3 if

$2(-1+r\cos\theta)+3+r\sin\theta=3$ ...(i)

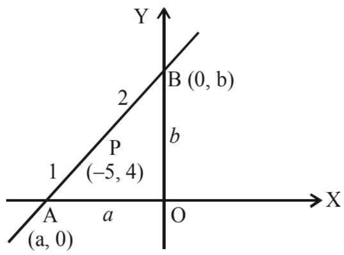
But $\tan\theta=1;\ \Rightarrow \theta=45°$

(i) becomes,

$-2+2r.\frac{1}{\sqrt{2}}+3+r.\frac{1}{\sqrt{2}}=3 \Rightarrow \frac{3r}{\sqrt{2}}=2;\ \ r=\frac{2\sqrt{2}}{3}$

Hence the required distance $=\frac{2\sqrt{2}}{3}$.

14. (c) Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$.



So, the coordinates of A and B are (a, 0) and (0, b) respectively. Since the point (–5, 4) divides AB in the ratio 1 : 2

$\therefore\ -5=\frac{1.0+2.a}{1+2}$

and $4=\frac{1.b+2.0}{2+1} \Rightarrow a=-\frac{15}{2}$ and $b=12$

So the line is $-\frac{2}{15}x+\frac{y}{12}=1$, *i.e.* $-8x+5y=60$

15. (a) Let Q(a, b) be the reflection of P(4, – 13) in the line $5x+y+6=0$

Then the mid-point $R\left(\frac{a+4}{2},\frac{b-13}{2}\right)$ lies on

$5x+y+6=0$

$\therefore\ 5\left(\frac{a+4}{2}\right)+\frac{b-13}{2}+6=0 \Rightarrow 5a+b+19=0$...(i)

Also PQ is perpendicular to $5x+y+6=0$

Therefore $\frac{b+13}{a-4}\times\left(-\frac{5}{1}\right)=-1 \Rightarrow a-5b-69=0$..(ii)

Solving (i) and (ii), we get $a=-1$, $b=-14$.

16. (b) We have the equation $2x^2-xy-y^2=0$
$\Rightarrow (2x+y)(x-y)=0$
If (h, k) be the point then remaining pair is
$(2x+y+h)(x-y+k)=0$
Where, $2x+y+h=0$ and $x-y+k=0$
It passes through the point (1, 0)
$\therefore 2\times1+0+h=0 \Rightarrow 2+h=0 \Rightarrow h=-2$
and $1-0+k=0 \Rightarrow 1+k=0 \Rightarrow k=-1$
$\therefore$ Required pair is $(2x+y-2)(x-y-1)=0$
$\Rightarrow 2x^2-2xy-2x+xy-y^2-y-2x+2y+2=0$
$\therefore\ 2x^2-xy-y^2-4x+y+2=0$

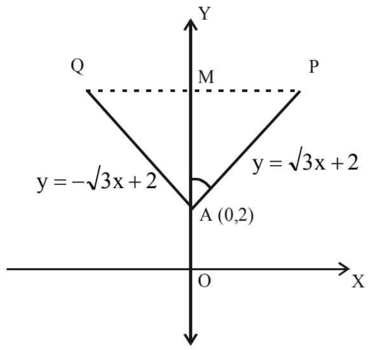
17. (b) Since $|x|=\begin{cases} x, & x\ge0 \\ -x, & x<0 \end{cases}$,

therefore the equations of two lines are

$y=\sqrt{3}x+2,\ x\ge0$ and $y=-\sqrt{3}x+2,\ x<0$

Clearly y-axis the only bisector of the angle between these two lines. There are two points P and Q on these lines at a distance of 5 units from A. Clearly M is the foot of the perpendicular from P and Q on y-axis (bisector). AM = AP $\cos 30° = \frac{5\sqrt{3}}{2}$.

Hence, coordinates of M are $\left(0, 2+\frac{5\sqrt{3}}{2}\right)$



18. (a) $9x^2-6xy+y^2+18x-6y+8=0$

$\Rightarrow ((3x)^2-2\times(3x)\times y+y^2)+6(3x-y)+8=0$

$\Rightarrow (3x-y)^2+6(3x-y)+8=0$

Let $3x-y=z$

$\therefore z^2+6z+8=0$

$\Rightarrow z^2+4z+2z+8=0$

$\Rightarrow z(z+4)+2(z+4)=0$

$\Rightarrow (z+2)(z+4)=0$

$\Rightarrow z=-2,\ z=-4$

$3x-y+2=0$ ... (i) or $3x-y+4=0$

If P_1 be the distance of line (i) from the origin, then

$$P_1=\frac{2}{\sqrt{9+1}}=\frac{2}{\sqrt{10}}$$

Also, if P_2 be the distance of line (ii) from the origin, then

$$P_2=\frac{4}{\sqrt{10}}$$

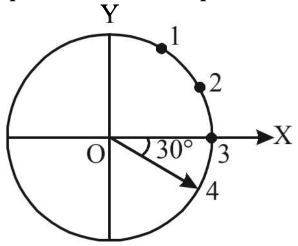
So, distance between lines

$$P=P_2-P_1=\frac{4}{\sqrt{10}}-\frac{2}{\sqrt{10}}=\frac{2}{\sqrt{10}}$$

19. (c) Since the hour, minute and second hand always pass through origin because one end of these hands is always at origin. Now at 4 O' clock, the hour hand makes 30° angle in fourth quadrant. So the equation of hour hand is

$y=mx \Rightarrow y=-\frac{1}{\sqrt{3}}x$

$\Rightarrow x+\sqrt{3}y=0$



20. (d) Mid-point of P(2, 3) and Q(4, 5) = (3, 4)

Slope of PQ = 1, Slope of the line L = – 1

Mid-point (3, 4) lies on the line L.

Equation of line L,

$y-4=-1(x-3) \Rightarrow x+y-7=0$ ...(i)

Let image of point R(0, 0) be $S(x_1, y_1)$

Mid-point of RS = $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$ lies on the line (i)

$\therefore x_1+y_1=14$ ...(ii)

Slope of RS = $\frac{y_1}{x_1}$; Since RS ⊥ line L

$\therefore \frac{y_1}{x_1}\times(-1)=-1 \ \therefore x_1=y_1$ ...(iii)

From (ii) and (iii), $x_1=y_1=7$

Hence the image of R = (7, 7)

21. (a) Slope of given line is $=-\frac{2}{3}$, $\therefore \tan\theta=-\frac{2}{3}$

Hence $90°<\theta<180°$

$\therefore \quad \sin\theta=\frac{2}{\sqrt{13}}$, $\cos\theta=-\frac{3}{\sqrt{13}}$

Equation of the line in parametric form,

$$\frac{x-1}{\left(-\frac{3}{\sqrt{13}}\right)}=\frac{y+3}{\left(\frac{2}{\sqrt{13}}\right)}=r$$

Putting $r=3$, we get the co-ordinate of desired point as

$x-1=-\frac{9}{\sqrt{13}}$, $y+3=\frac{6}{\sqrt{13}}$

or $x=1-\frac{9}{\sqrt{13}}$, $y=-3+\frac{6}{\sqrt{13}}$

22. (a) Clearly the point (3, 0) does not lie on the diagonal x = 2y. Let m be the slope of a side passing through (3, 0). Then its equation is

$y-0=m(x-3)$ ....(i)

Since the angle between a diagonal and a side of a square is $\frac{\pi}{4}$. Therefore angle between

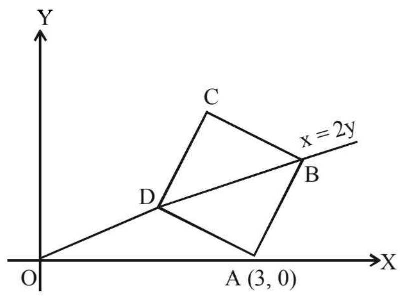
$x=2y$ & $y-0=m(x-3)$ is also $\frac{\pi}{4}$

Consequently, $\tan\frac{\pi}{4}=\pm\frac{m-\frac{1}{2}}{1+\frac{m}{2}} \Rightarrow m=3,-\frac{1}{3}$

$\therefore$ from (i), we get the required equations

$y=3(x-3) \Rightarrow y-3x+9=0$ or

$y=-\frac{1}{3}(x-3) \Rightarrow 3y+x-3=0$



23. (b) The length of perpendicular from P (2, –3) on the given family of lines

$$= \frac{a(4-3+4)+b(2+6-3)}{\sqrt{(2a+b)^2+(a-2b)^2}} = \pm\sqrt{10} \text{ (given)}$$

$$\Rightarrow 5a+5b = \pm\sqrt{10(5a^2+5b^2)}$$

$$\Rightarrow 25(a+b)^2 = 50(a^2+b^2) \Rightarrow 25(a-b)^2 = 0 \Rightarrow a = b$$

For which we get only line $3x - y + 1 = 0$

24. (a) The line passing through the intersection of lines $ax + 2by = 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

As this line is parallel to *x*-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$$\Rightarrow ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$$

$$\Rightarrow ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$$

$$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$$

$$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$$

$$y = \frac{-3(a^2+b^2)}{2(b^2+a^2)} = \frac{-3}{2}$$

So it is 3/2 units below *x*-axis.

25. (b) The distance between the parallel straight lines given by

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $2\sqrt{\dfrac{g^2 - ac}{a(a+b)}}$

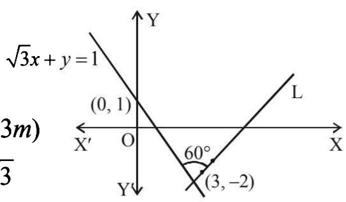
Here, $a = 8, b = 2, c = 15, g = 13$.

So, required distance

$$= 2\sqrt{\frac{169-120}{80}} = 2 \times \frac{7}{4\sqrt{5}} = \frac{7}{2\sqrt{5}}$$

26. (b) Let the slope of line L be *m*.

Then $\left|\dfrac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$



$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3}$

$\Rightarrow m = 0$ or $m = \sqrt{3}$

$\because$ L intersects *x*-axis, $\therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}(x - 3)$

or $\sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$

27. (c) Equation of line passing through (a, 0) is $y = m(x - a)$

$\Rightarrow mx - y - ma = 0$ ....(i)

Its distance from the point (2a, 2a) is

$$\left|\frac{2am - 2a - ma}{\sqrt{m^2+1}}\right| = a \quad \text{(given)}$$

$$\Rightarrow (m-2)^2 = (m^2+1) \Rightarrow m^2 - 4m + 4 = m^2 + 1$$

$$\Rightarrow 0m^2 - 4m + 3 = 0 \Rightarrow m = \frac{3}{4}, \infty$$

The required equation of lines are, from (i)

$3x - 4y - 3a = 0$ and $x - a = 0$.

28. (a) Let (1,3) and (5,1) represent vertices A and C. The middle point G(3,2) must lie on the diagonal BD, whose equation is $y = 2x + c$

$\therefore 2 = 2.3 + c \Rightarrow c = -4$

$\therefore$ equation of BD is $y = 2x - 4$

Also $GA = GB = GC = GD = \frac{1}{2}AC = \sqrt{5}$

We have to find two points along BD at distances $\pm\sqrt{5}$ from G. For this we convert equation of BD into distance form.

Slope of line BD = $\tan\theta = 2$

$\therefore \sin\theta = \dfrac{2}{\sqrt{5}}$ and $\cos\theta = \dfrac{1}{\sqrt{5}}$

$\therefore$ Distance form of the line BD through G is

$$\frac{x-3}{\frac{1}{\sqrt{5}}} = \frac{y-2}{\frac{2}{\sqrt{5}}} = r$$

Put $r = \pm\sqrt{5}$ to get the vertices of B and D as (4,4) and (2,0)

29. (d) As $(\sin\theta, \cos\theta)$ and (3, 2) lie on the same side of $x + y - 1 = 0$, they should be of same sign.

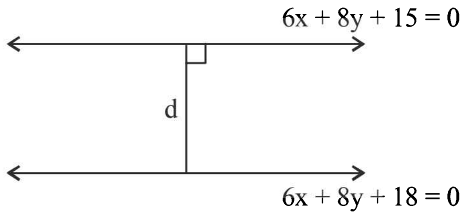
$\therefore \sin\theta + \cos\theta - 1 > 0$ as $3 + 2 - 1 > 0$

$$\Rightarrow \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) > 1$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}} \Rightarrow 0 < \theta < \frac{\pi}{4}$$

30. (b) $6x + 8y + 15 = 0$ ... (i)

and $3x + 4y + 9 = 0$ ... (ii)



Multiply equation (ii) by 2, we get

$6x + 8y + 18 = 0$

Distance between the straight lines

$$\frac{|c_2 - c_1|}{\sqrt{a^2+b^2}} = \frac{18-15}{\sqrt{(6)^2+(8)^2}} = \frac{3}{10} \text{ unit}$$

$\therefore$ Option (b) is correct.