Class 8 Maths Understanding Quadrilaterals

Introduction to Shapes

Introduction to Shapes

A **shape** is the form of an object or its external boundary, outline, or external surface, as opposed to other properties such as color, texture, or material composition. In mathematics, there are various kinds of shapes. Below are a few examples.

Plane Surface

Curved Surface

Sphere

Cone

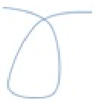
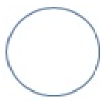
Cylinder

Class 8 Maths Understanding Quadrilaterals Curves

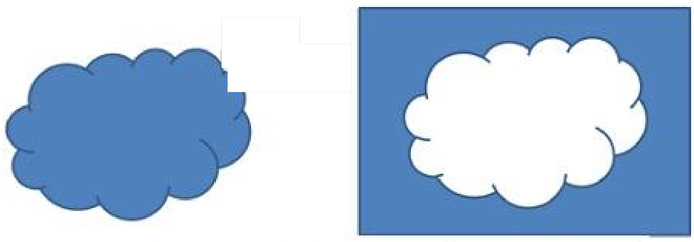
Curves

A **curve** is a line or outline which gradually deviates from being straight for some or all of its length.

- A **simple curve** does not cross itself at any point.
- A**closed curve** is a curve with no endpoints and which completely encloses an area

Curve type	Figure
Simple curve that is not closed	
Not a simple curve	
Simple closed curve	
A closed curve that is not simple	

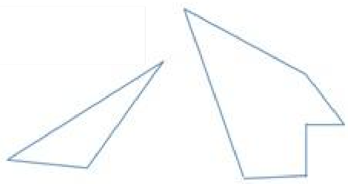
- The portion covered by a closed curve inside it is called the**interior** of a curve.
- The portion which is not covered by a closed curve inside it or the portion outside of a closed curve is called**exterior** of a curve.



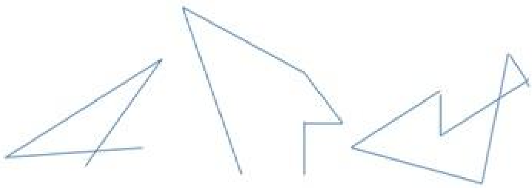
Interior and exterior of a closed curve (Blue shaded part)

Polygons

A simple closed curve made only of line segments is called a **Polygon**.



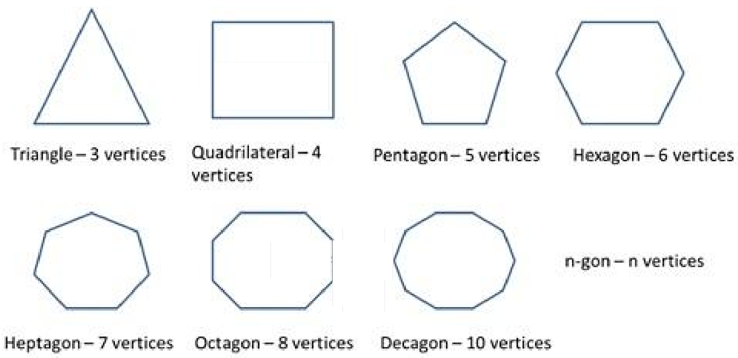
Curves that are Polygons



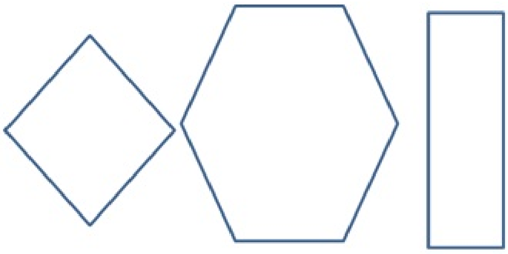
Curves that are not Polygons

Polygons are classified based on various factors as indicated below:

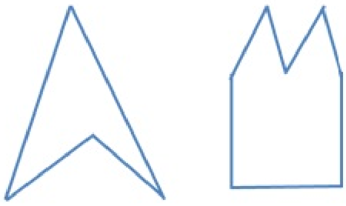
- **Based on the number of their sides or vertices**



- **Based on the portion of the diagonals in exteriors**
  - **Convex Polygons** - Those which have no portion of their diagonals in their exteriors.
  - **Concave Polygons** - Those which have at least one portion of their diagonals in their exteriors.

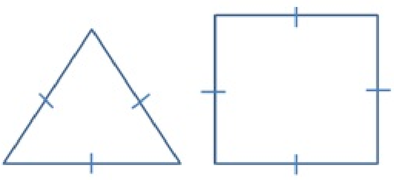


Convex Polygons

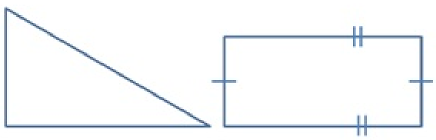


Concave Polygons

- **Based on size of and angle between the vertices**
  - **Regular Polygons:** A regular polygon is **equiangular** (angle between any two vertices is equal) and **equilateral** (length of each vertices is equal).
  - **Irregular Polygons:** An irregular polygon may/may not be equiangular (like a rectangle) but not equilateral.



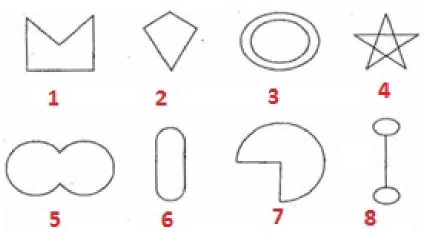
Regular Polygons (equal sides & equal angles)



Irregular Polygons (unequal sides or unequal angles)

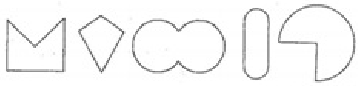
Problem: Classify the figures as

a) Simple Curve
b) Simple Closed Curve
c) Polygon
d) Convex Polygon
e) Concave Polygon

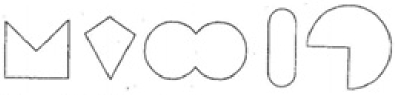


Solution:

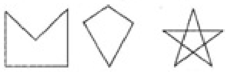
f) Simple Curve



g) Simple Closed Curve



h) Polygon

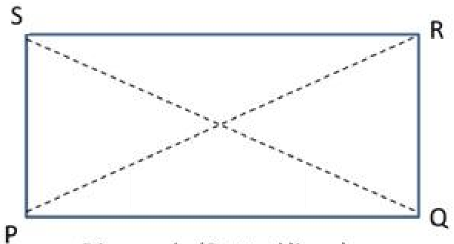


i) Convex Polygon

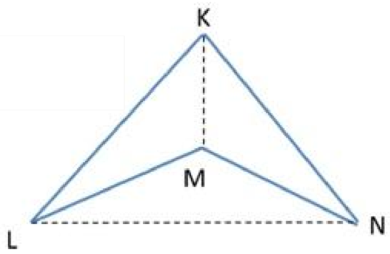
j) Concave Polygon

Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon.

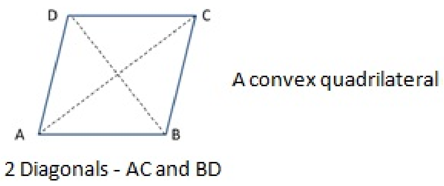


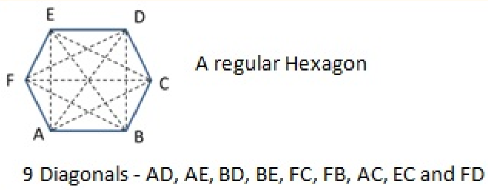
Diagonals (Dotted lines)

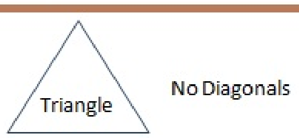


Problem: How many diagonals does each of the following have?

(a) A convex quadrilateral (b) A regular Hexagon (c) A Triangle





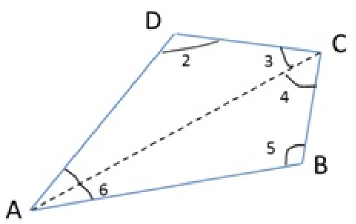


Angle sum property

For a polygon, the sum of the interior angles is always fixed regardless of the shape of the polygon.The sum is always (n-2)*180°, where 'n' is the number of sides of the polygon. A few have been given below:

Name of the Polygon	Number of Sides	Sum of interior angles
Triangle	3	(3-2)*180° = 180°
Quadrilateral	4	(4-2)*180° = 360°
n-gon	n	(n-2) *180°

Problem: What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex?



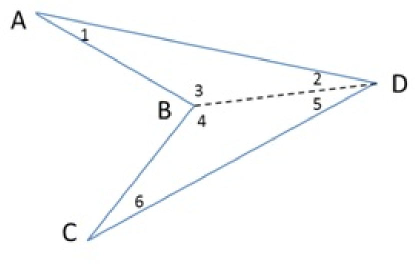
This is a convex quadrilateral. Diagonal AC divides it into 2 triangles. Using angle sum property,

∠A + ∠B + ∠C + ∠D = ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6

= (∠1 + ∠2 + ∠3) + (∠4 + ∠5 + ∠6)

= 180° + 180°

= 360°



This is a non-convex quadrilateral. Diagonal BD divides it into 2 triangles. Using angle sum property,

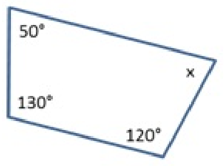
∠A + ∠B + ∠C + ∠D = ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6

= (∠1 + ∠2 + ∠3) + (∠4 + ∠5 + ∠6)

= 180° + 180°

= 360°

Problem: Find the angle measures x in the following figures:

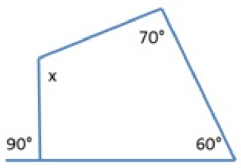


Using angle sum property,

50° + 130° + 120° + x = 360°

300° + x = 360°

x = 60°

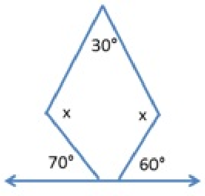


Using angle sum property,

90° + 60° + 70° + x = 360°

220° + x = 360°

x = 140°



First base interior angle = 180° - 70° = 110°

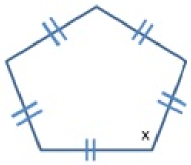
Second base interior angle = 180° - 60° = 120°

For 5 sides, angle sum of polygon = (n-2)*180° = 540°,

So, 30° + x + 110° + 120° + x = 540°

260° + 2x = 540°

x = 140°



Using angle sum property of polygon = (n-2)*180°

= (5-2)*180°

= 540°

So, x + x + x+ x+ x = 540°

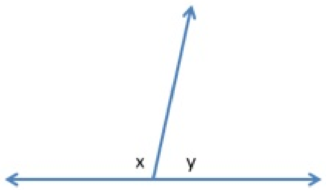
5x= 540°

x = 108°

Exterior angles sum property

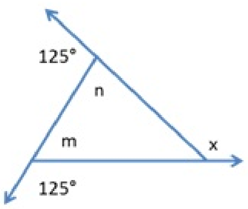
For a polygon, the sum of the exterior angles is always 360°regardless of the number of sides of the polygon.

- The sum of angles in a**linear pair** is always 180°.
- Exterior angle x° = Sum of opposite interior angles



Here the sum of the angles x + y will always be equal to 180°

Problem: Find x in the following figures:



Here 125° + m = 180° (Linear Pair)

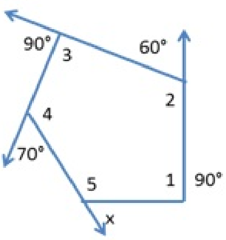
Hence, m = 55°

Here 125° + n = 180° (Linear Pair)

Hence, m = 55°

Since, Exterior angle x° = Sum of opposite interior angles

x = 55° + 55° = 110°



Sum of angles of a pentagon = (n - 2)*180°

= 540°

From the linear pair property,

∠1 + 90° = 180°,   ∠2 + 60° = 180°,   ∠3 + 90° = 180°,

∠4 + 70° = 180°,   ∠5 + x = 180°

Adding all the above,

x + (∠1 + ∠2 + ∠3 + ∠4 + ∠5) + 310° = 900°

x + 540° + 310° = 900°

x = 50°

Problem: Find the measure of each exterior angle of a regular polygon of: (a) 9 sides (b) 15 sides

Soulution: 9 sides

Sum of angles of a regular polygon

= (n - 2) * 180°

= (9 - 2) * 180 = 7 *180 = 1260°

Each interior angle

= Sum of interior angles/No. of sides

= 1260°/9 = 140°

Each exterior angle = 180° - 140° = 40°

Solution: 15 sides

Sum of angles of a regular polygon

= (n - 2) * 180°

= (15 - 2) * 180 = 13 *180 = 2340°

Each interior angle

= Sum of interior angles/No. of sides

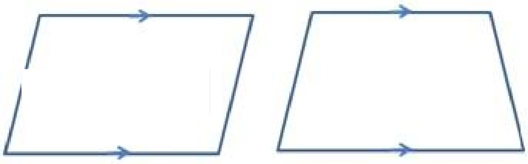
= 2340°/15 = 156°

Each exterior angle = 180° - 156° = 24°

Kinds of Quadrilaterals

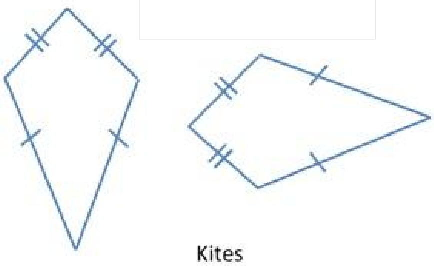
Quadrilaterals are classified based on the nature of their sides and angles.

- **Trapezium** - It has a pair of sides which are parallel.

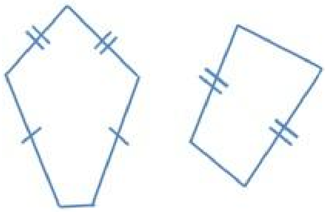


Trapezium

- **Kite** - It is a diamond shaped quadrilateral with two distinct consecutive pairs of sides of equal length.

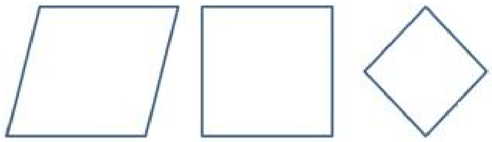


Kites

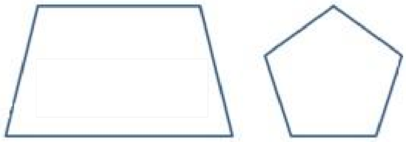


Not Kites

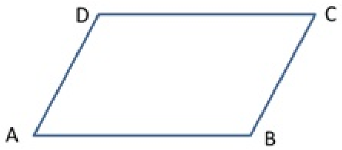
- **Parallelogram** - A parallelogram has two distinct consecutive pairs of parallel sides of equal length.
  - The opposite sides of a parallelogram are equal.
  - The opposite angles of a parallelogram are equal.
  - The adjacent angles in a parallelogram are supplementary.
  - Diagonals of a parallelogram bisect each other.



Parallelograms



Not Parallelograms



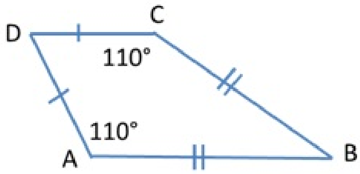
AB, CD and BC, AD are pairs of **opposite sides**.

∠A, ∠C and ∠B, ∠D are pairs of **opposite angles.**

AB-BC, BC-CD, CD-DA and DA-AB are **adjacent sides**.

∠A-∠B,∠B-∠C, ∠C-∠D and ∠D-∠A are **adjacentangles.**

Problem: Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measures



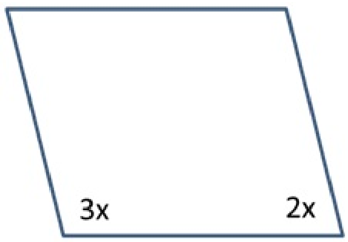
ABCD is a quadrilateral in which angles

∠A =∠C = 110°

Therefore, it could be a kite.

Problem: The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Solution:



Let two adjacent angles be 3x and 2x.

Since the adjacent angles in a parallelogram are supplementary,

3x + 2x = 180°

5x = 180°

x = 36°

Therefore, first angle = 3x = 3 * 36° = 108°

Second Angle = 2x = 2 * 36° = 72°

Problem: Two adjacent angles of a parallelogram have equal measure. Find the measure of the angles of the parallelogram.

Solution: Let each adjacent angle be x.

Since the adjacent angles in a parallelogram are supplementary,

x + x = 180°

x = 90°

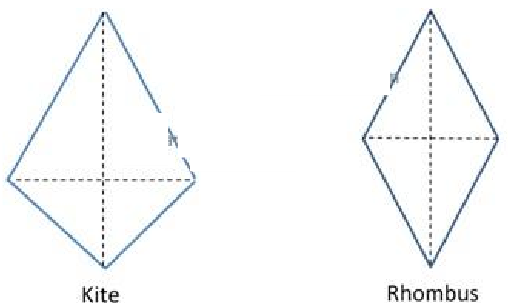
Hence, each adjacent angle is 90°

Now, the other angles would be also 90°.

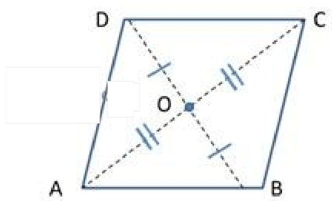
Types of Parallelograms

There are various types of parallelograms which are listed below:

- **Rhombus** - Rhombus is a kite-like shape having all sides of equal length and the opposite sides are parallel to each other.
  - Rhombus has properties of parallelogram as well as kite.



- Diagonals of a rhombus bisect each other at right angles or 90 degrees.



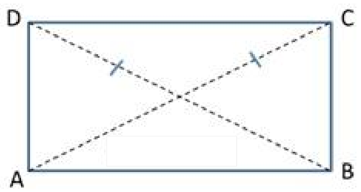
ΔAOD ≈ΔCOD (SAS congruency criterion)

m∠AOD = ***m***∠COD

∠AOD and∠COD are linear pairs so,

m∠AOD = ***m***∠COD = 90°

- **Rectangle** - A rectangle is a square with equal angles (90°).
  - Opposite sides of a rectangle are of equal length and are parallel to each other.
  - Diagonals of a rectangle are of equal length and bisect each other.



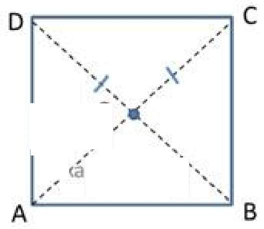
ΔABC ≈ΔABD (SAS congruency criterion)

AB = AB and AD = BC

m∠A = ***m***∠B = 90°

Hence AC = BD

- **Square** - A square is a rectangle with equal sides.
  - All sides of a square are equal and opposite sides are parallel.
  - Diagonals are of equal length and bisect each other perpendicularly.
  - All angles are 90°.



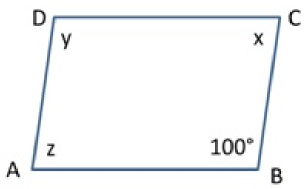
ΔAOD ≈ΔCOD (SSS congruency criterion)

m∠AOD = ***m***∠COD

∠AOD and∠COD are linear pairs so,

m∠AOD = ***m***∠COD = 90°

Problem: Find the measure of x, y, z



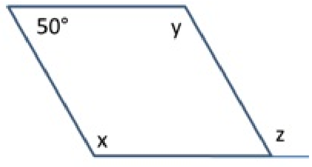
∠B + ∠C = 180° (Adjacent angles in a parallelogram are supplementary)

100° + x = 180°

x = 80°

z = x = 80° (opposite angles of a parallelogram are equal)

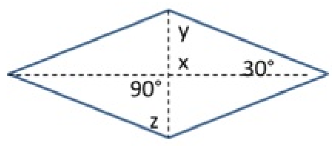
y = 100° (opposite angles of a parallelogram are equal)



x + 50° = 180° (Adjacent angles in a parallelogram are supplementary)

x = 130°

z = x = 130° (corresponding angles)

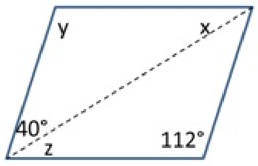


z = 80° (corresponding angles)

x + 80° = 180° (Adjacent angles in a parallelogram are supplementary)

x = 100°

y = 80° (opposite angles of a parallelogram are equal)

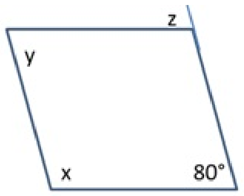


y = 112° (opposite angles of a parallelogram are equal)

y + x + 40° = 180° (Angle sum property)

x = 28°

z = 28° (Alternate angles)



x = 90° (vertically opposite angles)

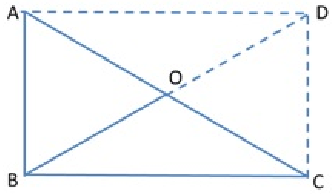
y + x + 30° = 180° (Angle sum property)

y + 90° + 30° = 180°

y = 60°

z = 60° (Alternate angles)

Problem: ABC is a right-angled triangle and O is the mid-point of the side opposite to the right angle. Explain why O is equidistant from A, B and C.



Since, two right triangles make a rectangle where O is equidistant point from A, B, C and D because O is the mid-point of the two diagonals of a rectangle.

Since AC and BD are equal diagonals and intersect at mid-point.

So, O is the equidistant from A, B, C and D.