Trigonometry

Practice set 8.1

Q. 1. In the Fig. 8.12, ∠R is the right angle of ΔPQR. Write the following ratios.

(i) sin P (ii) cos Q
(iii) tan P (iv) tan Q

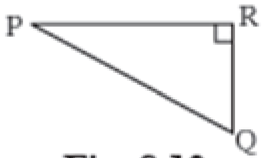


Fig. 8.12

Answer : For any right-angled triangle,

sinθ = Opposite side Side/Hypotenuse

cosθ = Adjacent sideSide/Hypotenuse

tanθ = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

cotθ = 1/tanθ

= Adjacent sideSide/Opposite side Side

secθ = 1/cosθ

= Hypotenuse/Adjacent sideSide

cosecθ = 1/sinθ

= Hypotenuse/Opposite side Side

In the given triangle let us understand, the Opposite side and Adjacent sidesides.

So for ∠ P,

Opposite side Side = QR

Adjacent sideSide = PR

So, for ∠ Q,

Opposite side Side = PR

Adjacent sideSide = QR

In general for the side Opposite side to the 90° angle is the hypotenuse.

So, for Δ PQR, hypotenuse = PQ

(i) sin P = Opposite side Side/Hypotenuse

= QR/PQ

(ii) cos Q = Adjacent sideSide/Hypotenuse

= QR/PQ

(iii) tan P = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

= QR/PR

(iv) tan Q = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

= PR/QR

Q. 2. In the right angled ΔXYZ, ∠XYZ = 90^0 and a,b,c are the lengths of the sides as shown in the figure. Write the following ratios,

(i) sin X (ii) tan Z
(iii) cos X (iv) tan X.

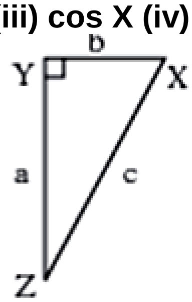


Fig. 8.13

Answer : For any right-angled triangle,

sinθ = Opposite side Side/Hypotenuse

cosθ = Adjacent Side/Hypotenuse

tanθ = sinθ/cosθ

= Opposite Side/Adjacent Side

In the given triangle let us understand, the Opposite side and Adjacent side

So for ∠ X,

Opposite Side = YZ = a

Adjacent Side = XY = b

So for ∠ Z,

Opposite Side = XY = b

Adjacent Side = YZ = a

In general for the side Opposite side to the 90° angle is the hypotenuse.

So for Δ XYZ, hypotenuse = XZ = c

(i) sin X = Opposite side Side/Hypotenuse

= YZ/XZ

= **a/c**

(ii) tan Z = sinθ/cosθ

= Opposite Side/Adjacent Side

= XY/YZ

= **b/a**

(iii) cos X= Adjacent Side/Hypotenuse

= XY/XZ

= **b/c**

(iv) tan X = sinθ/cosθ

= Opposite Side/Adjacent Side

= YZ/XY

= **a/b**

Q. 3. In right angled ΔLMN, ∠LMN =90^0, ∠L = 50^0 and ∠N = 40^0

write the following ratios.

(i) sin 50° (ii) cos 50°
(iii) tan 40° (iv) cos 40°

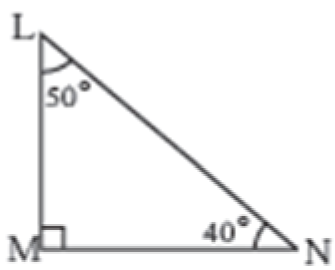


Fig. 8.14

Answer : For any right-angled triangle,

sinθ = Opposite side Side/Hypotenuse

cosθ = Adjacent sideSide/Hypotenuse

tanθ = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

cotθ = 1/tanθ

= Adjacent sideSide/Opposite side Side

secθ = 1/cosθ

= Hypotenuse/Adjacent sideSide

cosecθ = 1/sinθ

= Hypotenuse/Opposite side Side

In the given triangle let us understand, the Opposite side and Adjacent sidesides.

So for ∠ 50°,

Opposite side Side = MN

Adjacent sideSide = LM

So for ∠ 40°,

Opposite side Side = LM

Adjacent sideSide = MN

In general, for the side Opposite side to the 90° angle is the hypotenuse.

So, for Δ LMN, hypotenuse = LN

(i) sin 50° = Opposite side Side/Hypotenuse

= MN/LN

(ii) cos 50° = Adjacent sideSide/Hypotenuse

= LM/LN

(iii) tan 40° = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

= LM/MN

(iv) cos 40° = Adjacent sideSide/Hypotenuse

= MN/LN

Q. 4 In the figure 8.15 ∠PQR = 90^0, ∠PQS = 90^0, ∠PRQ = α and ∠QPS = θ Write the following trigonometric ratios.

i. sin α, cos α, tan α

ii. sin θ, cos θ, tan θ

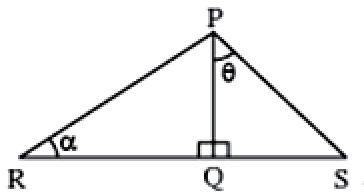


Fig. 8.15

Answer : For any right-angled triangle,

sinθ = Opposite side Side/Hypotenuse

cosθ = Adjacent sideSide/Hypotenuse

tanθ = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

cotθ = 1/tanθ

= Adjacent sideSide/Opposite side Side

secθ = 1/cosθ

= Hypotenuse/Adjacent sideSide

cosecθ = 1/sinθ

= Hypotenuse/Opposite side Side

(i) In the given triangle let us understand, the Opposite side and Adjacent sidesides.

So, for Δ PQR,

So, for ∠ α,

Opposite side Side = PQ

Adjacent sideSide = QR

In general for the side Opposite side to the 90° angle is the hypotenuse.

So, for Δ PQR, hypotenuse = PR

sin α = Opposite side Side/Hypotenuse

= PQ/PR

cos α = Adjacent sideSide/Hypotenuse

= QR/PR

tan α = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

= PQ/QR

(ii) In the given triangle let us understand, the Opposite side and Adjacent sidesides.

So for Δ PQS,

So for ∠θ,

Opposite side Side = QS

Adjacent sideSide = PQ

In general for the side Opposite side to the 90° angle is the hypotenuse.

So for Δ PQS, hypotenuse = PS

sinθ = Opposite side Side/Hypotenuse

= QS/PS

cosθ = Adjacent sideSide/Hypotenuse

= PQ/PS

tanθ = sinθ/cosθ

= Opposite side Side/Adjacent sideSide

= QS/PQ

Practice set 8.2

Q. 1. In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.

sin θ		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
cos θ	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
tan θ			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

Answer :

Sinθ	$\frac{12}{37}$	$\frac{11}{61}$	$\frac{1}{\sqrt{2}}$	½	$\frac{\sqrt{2}}{\sqrt{3}}$	$\frac{21}{29}$	$\frac{8}{17}$	$\frac{3}{5}$	$\frac{1}{3}$
Cosθ	$\frac{35}{37}$	$\frac{60}{61}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{20}{29}$	$\frac{15}{17}$	$\frac{4}{5}$	$\frac{2\sqrt{2}}{3}$
Tanθ	$\frac{12}{35}$	$\frac{11}{60}$	1	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{2}}{1}$	$\frac{21}{20}$	$\frac{8}{15}$	$\frac{3}{4}$	$\frac{1}{2\sqrt{2}}$

For first column:

Sinθ	$\frac{12}{37}$
Cosθ	$\frac{35}{37}$
Tanθ	$\frac{12}{35}$

cosθ = 35/37

Adjacent side= 35,

Hypotenuse = 37

By Pythagoras Theorem

$\text{Hypotenuse}^2 = \text{Opposite side}^2 + \text{Adjacent}^2$

$\text{Opposite side}^2 = \text{Hypotenuse}^2 - \text{Adjacent}^2$

$= 37^2 - 35^2$

= 1369 – 1225

Opposite side^2 = 144

Opposite side = 12

For second column:

Sinθ	$\frac{11}{61}$
Cosθ	$\frac{60}{61}$
Tanθ	$\frac{11}{60}$

Opposite side = 11

Hypotenuse = 61

By Pythagoras Theorem

Hypotenuse^2 = Opposite side^2 + Adjacent^2

Adjacent^2 = Hypotenuse^2 - Opposite side^2

= $61^2 - 11^2$

= 3721 – 121

Adjacent^2 = 3600

Adjacent side= 60

For third column:

Sinθ	$\frac{1}{\sqrt{2}}$
Cosθ	$\frac{1}{\sqrt{2}}$
Tanθ	1

Opposite side = 1

Adjacent side= 1

By Pythagoras Theorem

$Hypotenuse^2 = Opposite\ side^2 + Adjacent^2$

= 1 + 1

$Hypotenuse^2 = 2$

Hypotenuse = $\sqrt{2}$

For fourth column:

Sinθ	$\frac{\sqrt{2}}{\sqrt{3}}$
Cosθ	$\frac{1}{\sqrt{3}}$
Tanθ	$\frac{\sqrt{2}}{1}$

Opposite side = 1

Hypotenuse = 2

By Pythagoras Theorem

$Hypotenuse^2 = Opposite\ side^2 + Adjacent^2$

$Adjacent^2 = Hypotenuse^2 - Opposite\ side^2$

$= 2^2 - 1^2$

= 4 – 1

$Adjacent^2 = 3$

Adjacent side= $\sqrt{3}$

For fifth column:

Sinθ	$\frac{\sqrt{2}}{\sqrt{3}}$
Cosθ	$\frac{1}{\sqrt{3}}$
Tanθ	$\frac{\sqrt{2}}{1}$

Adjacent side= 1

Hypotenuse = √3

By Pythagoras Theorem

$\text{Hypotenuse}^2 = \text{Opposite side}^2 + \text{Adjacent}^2$

$\text{Opposite side}^2 = \text{Hypotenuse}^2 - \text{Adjacent}^2$

$= (\sqrt{3})^2 - 1^2$

= 3 – 1

$\text{Opposite side}^2 = 2$

Opposite side = √2

<u>For sixth column:</u>

Sinθ	$\frac{21}{29}$
Cosθ	$\frac{20}{29}$
Tanθ	$\frac{21}{20}$

Opposite side = 21

Adjacent side= 20

By Pythagoras Theorem

$\text{Hypotenuse}^2 = \text{Opposite side}^2 + \text{Adjacent}^2$

$= 21^2 + 20^2$

$Hypotenuse^2 = 841$

Hypotenuse = 29

For seventh column:

Sinθ	$\frac{8}{17}$
Cosθ	$\frac{15}{17}$
Tanθ	$\frac{8}{15}$

Opposite side = 8

Adjacent side= 15

By Pythagoras Theorem

$Hypotenuse^2 = Opposite\ side^2 + Adjacent^2$

$= 8^2 + 15^2$

$Hypotenuse^2 = 289$

Hypotenuse = 17

For eighth column:

Sinθ	$\frac{3}{5}$
Cosθ	$\frac{4}{5}$
Tanθ	$\frac{3}{4}$

Opposite side = 3

Hypotenuse = 5

By Pythagoras Theorem

$Hypotenuse^2$ = Opposite $side^2$ + $Adjacent^2$

$Adjacent^2$ = $Hypotenuse^2$ - Opposite $side^2$

$= 5^2 - 3^2$

= 25 – 9

$Adjacent^2$ = 16

Adjacent side= 4

<u>For ninth column:</u>

Sinθ	$\frac{1}{3}$
Cosθ	$\frac{2\sqrt{2}}{3}$
Tanθ	$\frac{1}{2\sqrt{2}}$

Opposite side = 1

Adjacent side= 2√2

By Pythagoras Theorem

$Hypotenuse^2$ = Opposite $side^2$ + $Adjacent^2$

$= 1^2 + (2\sqrt{2})^2$

$Hypotenuse^2$ = 9

Hypotenuse = 3

Q. 2 A. Find the values of –
$5\sin 30^0 + 3\tan 45^0$

Answer : We know,

sin 30° = ½

tan 45° = 1

⟹ 5sin 30° + 3tan 45°

⟹ $5 \times \frac{1}{2} + 3 \times 1$

⟹ 2.5 + 3

⟹ 5.5

Q. 2 B. Find the values of –

$$\frac{4}{5}\tan^2 60^\circ + 3\sin^2 60^\circ$$

Answer : We know,

tan 60° = √3

sin 60° = √3/2

⟹ $\frac{4}{5}\tan^2 60^\circ + 3\sin^2 60^\circ$

⟹ $\frac{4}{5}(\sqrt{3})^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2$

⟹ $\frac{4}{5} \times 3 + 3 \times \frac{3}{4}$

⟹ $\frac{12}{5} + \frac{9}{4}$

⟹ $\frac{48+45}{20}$

= 93/20

Q. 2 C. Find the values of –

2sin 30^0 + cos 0^0 + 3sin 90^0

Answer : We know,

sin 30° = 1/2

cos 0° = 1

sin 90° = 1

$\Rightarrow 2\sin 30° + \cos 0° + 3\sin 90°$

$\Rightarrow 2 \times \frac{1}{2} + 1 + 1$

⟹ 1 + 1 + 1

= 3

Q. 2 D. Find the values of –

$$\frac{\tan 60}{\sin 60 + \cos 60}$$

Answer : We know,

tan 60° = √3

sin 60° = √3/2

cos 60° = 1/2

$\Rightarrow \frac{\tan 60°}{\sin 60° + \cos 60°}$

$\Rightarrow \frac{\sqrt{3}}{\frac{\sqrt{3}}{2} + \frac{1}{2}}$

$\Rightarrow \frac{2\sqrt{3}}{\sqrt{3}+1}$

Q. 2 E. Find the values of –

$\cos^2 45^0 + \sin^2 30^0$

Answer : We know,

cos 45° = 1/√2

sin 30° = 1/2

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{1}{2} + \frac{1}{4}$

$\Rightarrow \frac{3}{4}$

Q. 2 F. Find the values of –

cos 60^0× cos 30^0 + sin60^0 × sin30^0

Answer : We know,

sin 30° = 1/2

sin 60° = √3/2

cos 60° = 1/2

cos 30° = √3/2

$\Rightarrow \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$\Rightarrow \frac{2\sqrt{3}}{4}$

$\Rightarrow \frac{\sqrt{3}}{2}$

Q. 3. If sinθ = 4/5 then find cosθ.

Answer : We know,

sinθ = Opposite side/Hypotenuse

Given:

sinθ = 4/5

Opposite side = 4

Hypotenuse = 5

By Pythagoras Theorem

$Hypotenuse^2 = Opposite\ side^2 + Adjacent^2$

$Adjacent^2 = Hypotenuse^2 - Opposite\ side^2$

$= 5^2 - 4^2$

= 25 – 16

= 9

$Adjacent^2 = 9$

Adjacent side= 3

cosθ = Adjacent side/Hypotenuse

= 3/5

Q. 4.

If $\cos\theta = \frac{15}{17}$ **then find sinθ**

Answer : We know,

cosθ = Adjacent side/Hypotenuse

Adjacent side = 15

Hypotenuse = 17

By Pythagoras Theorem

$Hypotenuse^2 = Opposite\ side^2 + Adjacent^2$

$Opposite\ side^2 = Hypotenuse^2 - Adjacent^2$

$= 17^2 - 15^2$

= 289 – 225

= 64

Opposite side^2 = 64

Opposite side = 8

sinθ = Opposite side /Hypotenuse

= 8/17

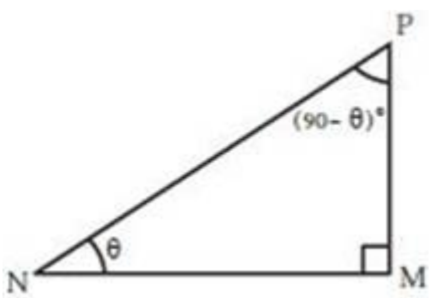
Problem set 8

Q. 1 A. Choose the correct alternative answer for following multiple choice questions.

Which of the following statements is true?
A. sin θ = cos(90-θ)

B. cos θ = tan(90-θ)
C. sin θ = tan(90-θ)
D. tan θ = tan(90-θ)

Answer : Let us consider the given triangle,



In this Δ PMN,

For ∠θ,

Opposite side = PM

Adjacent side= PN

For ∠ (90 –θ)

Opposite side = MN

Adjacent side = PM

sinθ = Opposite side/Hypotenuse

= PM/PN …………………… (i)

cos (90-θ) = Adjacent/Hypotenuse

= PM/PN ……………………. (ii)

RHS of equation (i) and (ii) are equal

∴ sinθ = cos (90-θ)

So Option A is correct.

Q. 1 B. Choose the correct alternative answer for following multiple choice questions.

Which of the following is the value of sin 90°?

A. $\frac{\sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}$

D. 1

Answer : We know that the value of sin 90° = 1

So option D is correct.

Q. 1 C. Choose the correct alternative answer for following multiple choice questions.

2tan 45^0 + cos 45^0 – sin 45^0 =?

A. 0
B. 1
C. 2
D. 3

Answer : We know that,

tan 45° = 1

We also know that

cos 45° = sin 45°

So,

⟹ 2 × 1 + cos 45° - cos 45°

= 2

So the correct option is C.

Q. 1 D. Choose the correct alternative answer for following multiple choice questions.

$$\frac{\cos 28^\circ}{\sin 62^\circ} = ?$$

A. 2
B. -1
C. 0
D. 1

Answer : We know the identity that,

sinθ = cos (90 –θ)

sin 62° = cos (90 – 62)

= cos 28°

Therefore [cos 28°/cos 28°] = 1

So option D is correct.

Q. 2. In right angled ΔTSU, TS = 5, ∠S = 90^0, SU =12 then find sin T, cos T, tan T. Similarly find sin U, cos U, tan U.

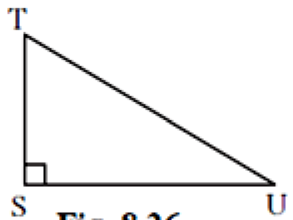


Fig. 8.26

Answer :

By applying Pythagoras theorem to given triangle we have,

$TU^2=ST^2+SU^2$

$TU^2= 5^2+12^2$

$TU^2= 25+144$

$TU^2=169$

TU=13Now,sinT $= \frac{SU}{TU} = \frac{12}{13}$

cosT $= \frac{ST}{TU} = \frac{5}{13}$

tanT $= \frac{SU}{ST} = \frac{12}{5}$

Similarly, $\sin U = \frac{5}{13}$

$\cos U = \frac{12}{13}$

$\tan U = \frac{5}{12}$

Q. 3. In right angled ΔYXZ, ∠X = 90^0, XZ = 8cm, YZ =17cm, find sin Y, cos Y, tan Y, sin Z, cos Z, tan Z.

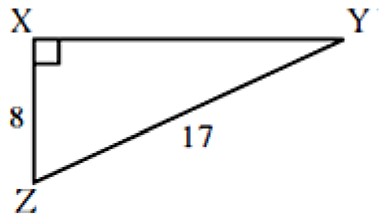


Fig. 8.27

Answer : For any right-angled triangle,

sinθ = Opposite side /Hypotenuse

cosθ = Adjacent side/Hypotenuse

tanθ = sinθ/cosθ

= Opposite side/Adjacent side

cotθ = 1/tanθ

= Adjacent side/Opposite side

secθ = 1/cosθ

= Hypotenuse/Adjacent side

cosecθ = 1/sinθ

= Hypotenuse/Opposite side

In the given triangle let us understand, the Opposite side and Adjacent sides.

So for ∠ Y,

Opposite side = XZ =8

Adjacent side= XY

So for ∠ Z,

Opposite side = XY

Adjacent side = XZ = 8

In general for the side Opposite side to the 90° angle is the hypotenuse.

So for Δ TSU,

By Pythagoras Theorem

$YZ^2 = XZ^2 + XY^2$

$XY^2 = 17^2 - 8^2$

= 289 - 64

= 225

XY = 15

(i) sin Y = Opposite side/Hypotenuse

= XZ/YZ

= 8/17

(ii) cos Y = Adjacent side/Hypotenuse

= XY/YZ

= 15/17

(iii) tan Y = sinθ/cosθ

= Opposite side/Adjacent side

= XZ/XY

= 8/15

(i) sin Z = Opposite side/Hypotenuse

= XY/YZ

= 15/17

(ii) cos Z = Adjacent side/Hypotenuse

= XZ/YZ

= 8/17

(iii) tan Z = sinθ/cosθ

= Opposite side/Adjacent side

= XZ/XY

= 8/15

Q. 4. In right angled ΔLMN, if ∠N = θ, ∠M = 90^0, cosθ = 24/25 find sinθ and tanθ Similarly, find ($\sin^2\theta$) and ($\cos^2\theta$).

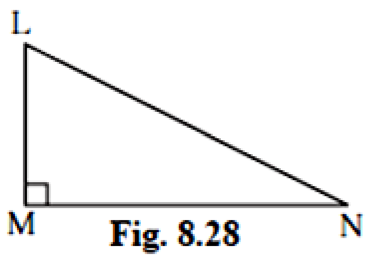


Fig. 8.28

Answer : Give:

cosθ = 24/25

cosθ = Adjacent side/Hypotenuse

Adjacent side = 24

Hypotenuse = 25

By Pythagoras Theorem

$\text{Hypotenuse}^2 = \text{Opposite side}^2 + \text{Adjacent}^2$

$\text{Opposite side}^2 = \text{Hypotenuse}^2 - \text{Adjacent}^2$

$= 25^2 - 24^2$

= 625 – 576

= 49

$\text{Opposite side}^2 = 49$

Opposite side = 7

sinθ = Opposite side/Hypotenuse

= 7/25

tanθ = sinθ/cosθ

= Opposite side/Adjacent side

= 7/24

$\sin^2\theta = (7/25)^2$

= 49/625

$\cos^2\theta = (24/25)^2$

= 576/625

Q. 5. Fill in the blanks.

i. $\sin 20^\circ = \cos \square^\circ$

ii. $\tan 30^\circ \times \tan \square^\circ = 1$

iii. $\cos 40^\circ = \sin \square^\circ$

Answer : i. We know the following identity,

sinθ = cos (90 -θ)

So sin 20° = cos (90 – 20)

∴ sin 20° = cos 70°

ii. We know that,

Let the unknown angle be θ

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\tan(30^\circ)}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\therefore \theta = 60^\circ$

iii. We know that,

$\cos\theta = \sin(90 - \theta)$

$\cos 40^\circ = \sin(90 – 40)$

$\therefore \cos 40^\circ = \sin 50^\circ$