CHAPTER-10
Vector algebra

One mark problem

1.Find the magnitude of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$. [K]

2.Find the magnitude of the vector $\vec{a} = 2\hat{\imath} - 7\hat{\jmath} - 3\hat{k}$. [K]

3.Find the magnitude of the vector $\vec{a} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$. [K]

4.Define collinear vectors. [K]

5.Define negative of a vector. [K]

6.Define a unit vector. [K]

7.Define a null vector. [K]

8.Define cointial vector. [K]

9.Define position vector of a point. [K]

10.When the two vectors are said to be equal. [K]

11.Write two different vectors having same magnitude. [U]

12.Write two different vectors having same direction. [U]

13.Find the values of x and y so that the vectors $2\hat{\imath} + 3\hat{\jmath}$ and $x\hat{\imath} + y\hat{\jmath}$ are equal. [K]

14.Find the unit vector in the direction of the $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ [U]

15.Find the unit vector in the direction of the $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ [U]

16.If $\vec{a}$ is a non zero vector of magnitude a and $\lambda\vec{a}$ is a unit vector ,find the value of λ. [K]

17.For what value of λ, the vectors $\vec{a} = 2\hat{\imath} - 3\lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ are perpendicular to each other? [U]

18.For what value of λ, is the vector $\frac{2}{3}\hat{\imath} - \lambda\hat{\jmath} + \frac{2}{3}\hat{k}$ a unit vector? [U]

19. Find the value of x for which $x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector. [U]

20.Show that the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear. [U]

21.If $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear then find a. [U]

22. Write the vector joining the points A(2,3,0) and B(-1,-2,-4). [U]

23. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively and $\vec{a} \cdot \vec{b} = \sqrt{6}$. [U]

24.If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors,then write the value of x+y+z.[U]

25.Find the direction cosines of the vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. [U]

26.Write the scalar components of the vector joining the points $A = (x_1, y_1, z_1)$ and

$B = (x_2, y_2, z_2)$. [K]

27. If vector $\overrightarrow{AB} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{OB} = 3\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$, find the position vector $\overrightarrow{OA}$. [U]

28. Find the scalar components of vector with initial point $(2,1)$ and terminal point $(-7,5)$.

29. Find the unit vector in the direction of $\vec{a}+\vec{b}$, where $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$. [U]

30. Find the position vector of a point *R* which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1 internally. [U]

31. Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, –2). [U]

32. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ [U]

33. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX , OY and OZ. [U]

Two mark problems:

1. Find the angle between the vectors $\vec{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. [U]

2. Find angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. [U]

3. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular. [U]

4. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is $60°$ and their scalar product is $\frac{1}{2}$. [U]

5. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$ [U]

6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that $\vec{a}$ and $\vec{b}$, are perpendicular. [U]

7. Find $|\vec{b}|$, if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$. [U]

8. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$. [U]

9. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. [U]

10. If two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$, find $|\vec{a} - \vec{b}|$. [U]

11. If $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{k}$, then find $|2\vec{b} \times \vec{a}|$ [U]

12. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. [U]

13. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. [U]

14.Find the area of the parallelogram whose adjacent sides determine by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$.[U]

15.Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a}=-2\hat{i}-5\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}-\hat{k}$.[U]

16.Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$, $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector.Find the angle between $\vec{a}$ and $\vec{b}$
[U]

17.Find λ and μ, if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$.
[U]

18.If $\vec{a}.\vec{a}$ =0 and $\vec{a}.\vec{b}$ =0, then what can be concluded about the vector $\vec{b}$?
[U]

19. Show that the points A(-2,3,5), B(1,2,3) and C(7,0, -1) are collinear.
[U]

20. Show that the points A(1,2,7), B(2,6,3) and C(3,10, -1) are collinear.
[U]

21.Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$,for any two nonzero vectors $\vec{a}$ and $\vec{b}$.
[U]

22.If either vector $\vec{a}$ =0 or $\vec{b}$ =0 ,then $\vec{a}.\vec{b}$ =0.But the converse need not be true.Justify your answer with an example.
[U]

23.Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$
[U]

24.Find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5, 5)
[U]

25.The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$ .Find the unit vector parallel to its diagonal.
[U]

26.Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+5\hat{k}$.
[U]

27.Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b}$ =8 and $\vec{b}=2\hat{i}+6\hat{j}+3\hat{k}$
[U]

Three mark problems:

1.Show that the position vector of the point P, which divides the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio $m:n$ is $\frac{m\vec{b}+n\vec{a}}{m+n}$.
[A]

2.Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$. [U]

3.Find λ, if the vectors $\vec{a}=\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}-\hat{k}$ and $\vec{c}=\lambda\hat{i}+7\hat{j}+3\hat{k}$ are coplanar.
[U]

4.Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.
[U]

5.Find λ, such that the four points A (3, 2, 1), B (4, λ, 5), C (4, 2, –2) and D (6, 5, –1) are coplanar.
[U]

6.Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

[U]

7.Prove that $[\vec{a}\ \ \vec{b}\ \ \vec{c}+\vec{d}]=[\vec{a}\ \ \vec{b}\ \ \vec{c}]+[\vec{a}\ \ \vec{b}\ \ \vec{d}]$
[U]

8.Prove that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$
[U]

9.Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar.
[U]

10.If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.
[U]

11.Show that the points $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$ are the vertices of a right angled triangle
[U]

12.If the vertices A, B and C of a triangle are $(1,2,3)$, $(-1,0,0)$ and $(0,1,2)$ respectively,then find the $\angle ABC$.[U]

13. Three vectors $\vec{\mathbf{a}},\vec{\mathbf{b}}\&\vec{\mathbf{c}}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, . Evaluate the quantity $\boldsymbol{\mu}=\vec{\mathbf{a}}.\vec{\mathbf{b}}+\vec{\mathbf{b}}.\vec{\mathbf{c}}+\vec{\mathbf{c}}.\vec{\mathbf{a}}$, if $|\vec{\mathbf{a}}|=\mathbf{1},|\vec{\mathbf{b}}|=\mathbf{4}\ \&|\vec{\mathbf{c}}|=\mathbf{2}$.
[U]

14. Find the vector of magnitude 5 units and parallel to the resultant of the vectors $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\imath}-2\hat{\jmath}+\hat{k}$
[U]

15. If $\vec{a},\vec{b}$ & $\vec{c}$ are three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$ & each vector is orthogonal to sum of the other two vectors then find $|\vec{a}+\vec{b}+\vec{c}|$.
[U]

16.If $\hat{\imath}+\hat{\jmath}+\hat{k}$, $2\hat{\imath}+5\hat{\jmath}$, $3\hat{\imath}+2\hat{\jmath}-3\hat{k}$ & $\hat{\imath}-6\hat{\jmath}-\hat{k}$ are the position vectors of points A,B,C & D respectively then find the cosine angle between $\overrightarrow{AB}$ & $\overrightarrow{CD}$.
[U]

17.If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ & $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ such that $\vec{a}$ is perpendicular to $(\lambda\vec{b} + \vec{c})$ then find λ.
[U]

18.The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ then find the unit vector parallel to its diagonal.Also find area of the parallelogram. [U]

19.The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ & $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ. [U]

20. If $\hat{\imath}, \hat{\jmath}, \hat{k}$ are mutually perpendicular unit vectors , $\vec{\alpha} = 3\hat{\imath} - \hat{\jmath}$, $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. [U]

21.If $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ & $\vec{c} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$ then find the unit vector perpendicular to both $\vec{a} + \vec{b}$ & $\vec{b} + \vec{c}$. [U]

22. If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ & $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ & $\vec{b}$ and $\vec{c}.\vec{d} = 15$. [U]

23.If $\vec{a}$,$\vec{b}$ & $\vec{c}$ are mutually perpendicular vectors of equal magnitudes then prove that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$,$\vec{b}$ & $\vec{c}$. [U]