Chapter 13

PERMUTATION AND COMBINATION

1. FUNDAMENTAL PRINCIPLES OF COUNTING

1.1 Fundamental Principle of Multiplication

If an event can occur in m different ways following which another event can occur in n different ways following which another event can occur in p different ways. Then the total number of ways of simultaneous happening of all these events in a definite order is $m \times n \times p$.

1.2 Fundamental Principle of Addition

If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in $(m + n)$ ways.

2. SOME BASIC ARRANGEMENTS AND SELECTIONS

2.1 Combinations

Each of the different selections made by taking some or all of a number of distinct objects or items, irrespective of their arrangements or order in which they are placed, is called a combination.

2.2 Permutations

Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a permutation.

NOTES :

1. Let r and n be positive integers such that $l \le r \le n$. Then, the number of all permutations of n distinct items or objects taken r at a time, is

$$^nP_r = {}^nC_r \times r\,!$$

Proof : Total ways $= n(n-1)(n-2)\ldots(n-\overline{r-1})$

$$= \frac{n(n-1)(n-2)\ldots(n-\overline{r-1})(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$$= {}^nP_r.$$

So, the total no. of arrangements (permutations) of n-distinct items, taking r at a time is ^nP_r or $P(n, r)$.

2. The number of all permutations (arrangements) of n distinct objects taken all at a time is n!.

3. The number of ways of selecting r items or objects from a group of n distinct items or objects, is

$$\frac{n!}{(n-r)!r!} = {}^nC_r.$$

3. GEOMETRIC APPLICATIONS OF ^nC_r

(i) Out of n non-concurrent and non-parallel straight lines, points of intersection are $^nC_2$.

(ii) Out of 'n' points the number of straight lines are (when no three are collinear) $^nC_2$.

(iii) If out of n points m are collinear, then No. of straight lines = $^nC_2 - {^mC_2} + 1$

(iv) In a polygon total number of diagonals out of n points (no three are collinear) = $^nC_2 - n = \frac{n(n-3)}{2}$.

(v) Number of triangles formed from n points is $^nC_3$. (when no three points are collinear)

(vi) Number of triangles out of n points in which m are collinear, is $^nC_3 - {^mC_3}$.

(vii) Number of triangles that can be formed out of n points (when none of the side is common to the sides of polygon), is $^nC_3 - {^nC_1} - {^nC_1} \cdot {^{n-4}C_1}$

(viii) Number of parallelograms in two systems of parallel lines (when 1st set contains m parallel lines and 2nd set contains n parallel lines), is = $^nC_2 \times {^mC_2}$

(ix) Number of squares in two system of perpendicular parallel lines (when 1st set contains m equally spaced parallel lines and 2nd set contains n same spaced parallel lines)

$$= \sum_{r=1}^{m-1} (m-r)(n-r); (m < n)$$

(x) The maximum number of parts into which a plane is cut by n lines is $\frac{n^2+n+2}{2}$

4. PERMUTATIONS UNDER CERTAIN CONDITIONS

The number of all permutations (arrangements) of n different objects taken r at a time :

(i) When a particular object is to be always included in each arrangement, is $^{n-1}C_{r-1} \times r!$.

(ii) When a particular object is never taken in each arrangement, is $^{n-1}C_r \times r!$.

5. DIVISION OF OBJECTS INTO GROUPS

5.1 Division of items into groups of unequal sizes

1. The number of ways in which (m + n) distinct items can be divided into two unequal groups containing m and n items, is $\frac{(m+n)!}{m!n!}$.

2. The number of ways in which (m+ n+ p) items can be divided into unequal groups containing m, n, p items, is

$$^{m+n+p}C_m \cdot {^{n+p}C_m} = \frac{(m+n+p)!}{m!n!p!}.$$

3. The number of ways to distribute (m + n+ p) items among 3 persons in the groups containing m, n and p items

= (No. of ways to divide) × (No. of groups)!

$$= \frac{(m+n+p)!}{m!n!p!} \times 3!.$$

5.2 Division of Objects into groups of equal size

The number of ways in which mn different objects can be divided equally into m groups, each containing n objects and the order of the groups is not important, is

$$\left(\frac{(mn)!}{(n!)^m}\right)\frac{1}{m!}$$

The number of ways in which mn different items can be divided equally into m groups, each containing n objects and the order of groups is important, is

$$\left(\frac{(mn)!}{(n!)^m}\times\frac{1}{m!}\right)m! = \frac{(mn)!}{(n!)^m}$$

6. PERMUTATIONS OF ALIKE OBJECTS

1. The number of mutually distinguishable permutations of n things, taken all at a time, of which p are alike of one kind, q alike of second kind such that p + q = n, is

$$\frac{n!}{p!q!}$$

2. The number of permutations of n things, of which p are alike of one kind, q are alike of second kind and remaining all are distinct, is $\frac{n!}{p!q!}$. Here $p + q \neq n$

3. The number of permutations of n things, of which p_1 are alike of one kind; p_2 are alike of second kind; p_3 are alike of third kind; ..... ; p_r are alike of r^{th} kind such that $p_1 + p_2 + ... + p_r = n$, is $\frac{n!}{p_1!p_2!p_3!....p_r!}$.

4. Suppose there are r things to be arranged, allowing repetitions. Let further p_1, p_2, ..., p_r be the integers such that the first object occurs exactly p_1 times, the second occurs exactly p_2 times subject, etc. Then the total number of permutations of these r objects to the above condition, is

$$\frac{(p_1 + p_2 + ... + p_r)!}{p!p_2!p_3!....p_r!}.$$

7. DISTRIBUTION OF ALIKE OBJECTS

(i) The total number of ways of dividing n identical items among r persons, each one of whom, can receive 0, 1, 2, or more items ($\leq$ n), is ${}^{n+r-1}C_{r-1}$.

OR

The total number of ways of dividing n identical objects into r groups, if blank groups are allowed, is ${}^{n+r-1}C_{r-1}$.

(ii) The total number of ways of dividing n identical items among r persons, each of whom, receives at least one item is ${}^{n-1}C_{r-1}$.

OR

The number of ways in which n identical items can be divided into r groups such that blank groups are not allowed, is ${}^{n-1}C_{r-1}$.

(iii) The number of ways in which n identical items can be divided into r groups so that no group contains less than k items and more than m (m < k) is

The coefficient of x^n in the expansion of

$(x^m + x^{m+1} + ... x^k)^r$

8. NO. OF INTEGRAL SOLUTIONS OF LINEAR EQUATIONS AND INEQUATIONS

Consider the eqn. $x_1 + x_2 + x_3 + x_4 + ... + x_r = n$ ...(i)

where $x_1, x_2, ..., x_r$ and n are non–negative integers.

This equation may be interpreted as that n identical objects are to be divided into r groups.

1. The total no. of non-negative integral solutions of the equation $x_1 + x_2 + ...+ x_r = n$ is ${}^{n+r-1}C_{r-1}$.

2. The total number of solutions of the same equation in the set N of natural numbers is ${}^{n-1}C_{r-1}$.

3. In order to solve inequations of the form

$$x_1 + x_2 + \ldots + x_m \le n$$

we introduce a dummy (artificial) variable x_{m+1} such that $x_1 + x_2 + \ldots + x_m + x_{m+1} = n$, where $x_{m+1} \ge 0$.

The no. of solutions of this equation are same as the no. of solutions of in Eq. (i).

9. CIRCULAR PERMUTATIONS

1. The number of circular permutations of n distinct objects is $(n-1)!$.

2. If anti-clockwise and clockwise order of arrangements are not distinct then the number of circular permutations of n distinct items is $1/2 \{(n-1)!\}$

   e.g., arrangements of beads in a necklace, arrangements of flowers in a garland etc.

10. SELECTION OF ONE OR MORE OBJECTS

1. The number of ways of selecting one or more items from a group of n distinct items is $2^n - 1$.

Proof : Out of n items, 1 item can be selected in ${}^nC_1$ ways; 2 items can be selected in ${}^nC_2$ ways; 3 items can be selected in ${}^nC_3$ ways and so on......

Hence, the required number of ways

$$= {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n$$

$$= ({}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n) - {}^nC_0$$

$$= 2^n - 1.$$

2. The number of ways of selecting r items out of n identical items is 1 .

3. The total number of ways of selecting zero or more items from a group of n identical items is $(n + 1)$.

4. The total number of selections of some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind, is

$[(p + 1)(q + 1)(r + 1)] - 1$.

5. The total number of ways of selecting one or more items from p identical items of one kind; q identical items of second kind; r identical items of third kind and n different items, is $(p + 1)(q + 1)(r + 1) 2^n - 1$

11. THE NUMBER OF DIVISORS AND THE SUM OF THE DIVISORS OF A GIVEN NATURAL NUMBER

Let $N = p_1^{n_1} . p_2^{n_2} . p_3^{n_3} \ldots . p_k^{n_k}$ ...(1)

where $p_1, p_2, \ldots, p_k$ are distinct prime numbers and $n_1, n_2, \ldots, n_k$ are positive integers.

1. Total number of divisors of $N = (n_1 + 1)(n_2 + 1) \ldots (n_k + 1)$.

2. This includes 1 and n as divisors. Therefore, number of divisors other than 1 and n, is

$$(n_1 + 1)(n_2 + 1)(n_3 + 1) \ldots (n_k + 1) - 2.$$

3. The sum of all divisors of (1) is given by

$$= \left\{\frac{p_1^{n_1+1} - 1}{p_1 - 1}\right\}\left\{\frac{p_2^{n_2+1} - 1}{p_2 - 1}\right\}\left\{\frac{p_3^{n_3+1} - 1}{p_3 - 1}\right\} \ldots \left\{\frac{p_k^{n_k+1} - 1}{p_k - 1}\right\}.$$

12. DEARRANGEMENTS

If n distinct objects are arranged in a row, then the no. of ways in which they can be dearranged so that none of them occupies its original place, is

$$n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-....+(-1)^{n}\frac{1}{n!}\right\}$$

and it is denoted by D (n).

If r $(0 \le r \le n)$ objects occupy the places assigned to them i.e., their original places and none of the remaining (n - r) objects occupies its original places, then the no. of such ways, is

$$D(n-r) = {}^{n}C_{r} \,.\, D(n-r)$$

$$= {}^{n}C_{r} \,.\, (n-r)\,!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....+(-1)^{n-r}\frac{1}{(n-r)!}\right\}.$$

13. SOME IMPORTANT RESULT OF PERMUTATION

13.1 Let X and Y be non-empty finite sets, $|X| = m$ and $|Y| = n$. Then

1. The number of functions from Y into X is m^n.
2. The number of injections (one-one functions) from Y into X is zero it $m < n$, and ${}^{m}C_{n}.n!$ $(={}^{m}P_{n})$ if $m \ge n$.
3. The number of bijections of Y onto X is zero if $m \ne n$, and m! if $m = n$.

13.2 For any positive integers m and r such that $m \ge r$, let $\alpha_m(r)$ be the number of surjections of an m-element set onto an r-element set. Then $\sum_{s=1}^{r} {}^{r}C_{s}\alpha_{m}(s) = r^{m}$

13.3 Let n be a positive integer and $n = p_1^{\alpha 1} p_2^{\alpha 2} ... p_k^{\alpha k}$ be a prime decomposition of n. Then the number of distinct ordered pairs of positive integers (p, q), such that the least common multiple of p and q is n, is $(2\alpha_1 + 1)(2\alpha_2 + 1) ... (2\alpha_k + 1)$

13.4 For any positive integer r, let d_r be the number of derangements of an r-element set. Then

$$1+\sum_{r=1}^{n} {}^{n}C_{r}d_{r} = n!$$

for any integer $n > 0$ or

$$\sum_{r=0}^{n} {}^{n}C_{r}d_{r} = n!\left(where\ d_{0} = 1\right)$$

and $$d_{0} = n! - \sum_{r=0}^{n-1} {}^{n}C_{r}\ d_{r}$$

SOLVED EXAMPLES

Example – 1

Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

Sol. Since a signal may consist of either 2 flags, 3 flags, 4 flags or 5 flags. Therefore,

Total number of signals = Number of 2 flags signals

+ Number of 3 flags signals

+ Number of 4 flags signals

+ Number of 5 flags signals

$= 5 \times 4 + 5 \times 4 \times 3 + 5 \times 4 \times 3 \times 2 + 5 \times 4 \times 3 \times 2 \times 1$

$= 20 + 60 + 120 + 120 = 320$

Example – 2

Find the total number of ways of answering 5 objective type questions, each question having 4 choices.

Sol. Since each question can be answered in 4 ways. So, the **total number of ways** of answering 5 questions is $4 \times 4 \times 4 \times 4 \times 4 = 4^5$.

Example – 3

How many three-digit numbers more than 600 can be formed by using the digits 2, 3, 4, 6, 7.

Sol. Clearly, repetition of digits is allowed. Since a three-digit number greater than 600 will have 6 or 7 at hundred's place. So, hundred's place can be filled in 2 ways. Each of the ten's and one's place can be filled in 5 ways.

Hence, total number of required numbers $= 2 \times 5 \times 5 = 50$.

Example – 4

How many numbers divisible by 5 and lying between 4000 and 5000 can be formed from the digits 4, 5, 6, 7 and 8.

Sol. Clearly, a number between 4000 and 5000 must have 4 at thousand's place. Since the number is divisible by 5 it must have 5 at unit's place. Now, each of the remaining places (viz. hundred's and ten's) can be filled in 5 ways.

Hence, total number of required numbers $= 1 \times 5 \times 5 \times 1 = 25$.

Example – 5

The flag of a newly formed forum is in the form of three blocks, each to be coloured differently. If there are six different colours on the whole to choose from, how many such designs are possible ?

Sol. Since there are six colours to choose from, therefore, first block can be coloured in 6 ways. After choosing first block second and third can be choosen in 5 and 4 ways respectively.

Hence, by the fundamental principle of multiplication, the number of flag-designs is $6 \times 5 \times 4 = 120$.

Example – 6

Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, when

(i) the repetition of the letters is not allowed.

(ii) the repetition of the letters is allowed.

Sol. (i) The total number of words is same as the number of ways of filling in 4 vacant places by the 4 letters. The first place can be filled in 4 different ways by any one of the 4 letters R, O, S, E. Since the repetition of letters is not allowed. Therefore, the second, third and fourth place can be filled in by any one of the remaining 3, 2, 1 different ways respectively.

Thus, by the fundamental principle of counting the required number of ways is $4 \times 3 \times 2 \times 1 = 24$.

Hence, required number of words = 24.

(ii) If the repetition of the letters is allowed, then each of the 4 vacant places can be filled in succession in 4 different ways.

Hence, required number of words $= 4 \times 4 \times 4 \times 4 = 256$.

Example – 7

How many numbers are there between 100 and 1000 such that at least one of their digits is 7 ?

Sol. Clearly, a number between 100 and 1000 has 3-digits

$\therefore$ Total no. of 3-digit nos having atleast one of their digits as 7

= (3digit nos) - (3-digit no. in which 7 does not appear)

Total number of 3-digit number $= 9\times10\times10 = 900$.

Total no. of 3-digit no. in which 7 does not appear at all : We have to form 3-digit nos by using the digits 0 to 9, except 7.

So, hundred's place can be filled in 8 ways and each of the ten's and one's place can be filled in 9 ways.

So, required ways $= 8\times9\times9 = 648$

Hence, total number of 3-digit numbers having at least one of their digits as 7 is $900 - 648 = 252$.

Example – 8

Find the exponent of 2 in 50 ! ?

Sol. $$E_2(50!) = \left[\frac{50}{2}\right] + \left[\frac{50}{2^2}\right] + \left[\frac{50}{2^3}\right] + \left[\frac{50}{2^4}\right] + \left[\frac{50}{2^5}\right]$$

$$= 25 + 12 + 6 + 3 + 1 = 47.$$

Example – 9

Find the number of triangles obtained by joining 10 points on a plane if ?

(i) no three of which are collinear

(ii) four points are collinear

Sol. (i) Since no three point are collinear, any three non-collinear points can be selected to form a triangle.

Number of triangles required = ${}^{10}C_3 = 120$

(ii) If four points are collinear

Required no. of triangles = ${}^{10}C_3 - {}^{4}C_3 = 120 - 4 = 116$

(because selection of 3 collinear point does not make a triangle.)

Example – 10

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 7 blue balls if each selection consists of 3 balls of each colour.

Sol. The number of ways of selecting 9 balls from 6 red balls, 5 white balls and 7 blue balls containing 3 balls of each colour = ${}^{6}C_3 \times {}^{5}C_3 \times {}^{7}C_3$

$$= \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{7!}{3!(7-3)!} = 7000$$

Example – 11

In how many ways can a team of 3 boys and 2 girls be selected from 6 boys and 5 girls ?

Sol. Required number of ways.

$= {}^{6}C_3 \times {}^{5}C_2$

$$= \frac{6!}{3!(6-3)!} \times \frac{5!}{2!(5-2)!} = 20 \times 10 = 200$$

Example – 12

Among 22 cricket players, there are 3 wicketkeepers and 6 bowlers. In how many ways can a team of 11 players be chosen so as to include exactly one wicket keeper and atleast 4 bowlers ?

Sol. We have to choose 11 players which include exactly 1 wicket keeper and atleast 4 bowlers.

Combinations include 1 wicket keeper – 4 bowlers,

1 wicket keeper – 5 bowlers and 1 wicket keeper – 6 bowlers

Total number of combinations.

$= {}^{3}C_1 \times {}^{6}C_4 \times {}^{13}C_6 + {}^{3}C_1 \times {}^{6}C_5 \times {}^{13}C_5 + {}^{3}C_1 \times {}^{6}C_6 \times {}^{13}C_4$

$= 77220 + 23166 + 2145 = 102531$

Example – 13

In how many ways can 5 students be selected out of 11 students if

(i) 2 particular students are included ?

(ii) 2 particular students are not included ?

Sol. There are 11 students, we have to select 5 students.

(i) 2 particular students are included then reqd no. of ways

$$= {}^{11-2}C_{5-2} = {}^{9}C_3 = \frac{9!}{3!\,6!} = \frac{9\times8\times7\times6!}{3\times2\times6!} = 84$$

(ii) 2 particular students are not included then reqd no. of ways

$$ {}^{11-2}C_5 = {}^{9}C_5 = \frac{9!}{5!\,4!}$$

$$= \frac{9\times8\times7\times6}{4\times3\times2} = 9\times7\times2 = 126$$

Example – 14

How many different signals can be made by 5 flags from 8 flags of different colours ?

Sol. The total number of signals is the number of arrangements of 8 flags by taking 5 flags at a time.

Hence, required number of signals = ${}^8C_5 \times 5! = 6720$

Example – 15

How many different signals can be given using any number of flags from 5 flags of different colours ?

Sol. The signals can be made by using at a time one or two or three or four or five flags.

Hence, by the fundamental principle of addition,

Total number of signals = ${}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5$

$= 5 + 20 + 60 + 120 + 120 = 325$

Example – 16

How many 4-letter words, with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed ?

Sol. There are 10 letters in the word 'LOGARITHMS'.

So, the number of 4 - letter words = ${}^{10}C_4 \times 4! = {}^{10}P_4 = 5040$

Example – 17

In how many ways can a cricket team be selected from a group of 25 players containing 10 batsmen 8 bowlers, 5 all –rounders and 2 wicketkeepers ? Assume that the team of 11 players requires 5 batsmen, 3 all–rounders. 2–bowlers and 1 wicketkeepeer.

Sol. Divide the selection of team into four operation.

I : Selection of batsman can be done (5 from 10) in ${}^{10}C_5$ ways.

II : Selection of bowlers can be done (2 from 8) in ${}^8C_2$ ways.

III : Selection of all–rounders can be done (3 from 5) in ${}^5C_3$ ways.

IV: Selection of wicketkeeper can be done (1 from 2) in ${}^2C_1$ ways.

⇒ the team can be selected in = ${}^{10}C_5 \times {}^8C_2 \times {}^5C_3 \times {}^2C_1$ ways =

$$\frac{10! \times 8 \times 7 \times 10 \times 2}{5!\,5!\,2!} = 141120.$$

Example – 18

A mixed doubles tennis game is to be arranged from 5 married couples. In how many ways the game be arranged if no husband and wife pair is included in the same game ?

Sol. To arrange the game we have to do the following operations.

(i) Select two men from 5 men in ${}^5C_2$ ways.

(ii) Select two women from 3 women excluding the wives of the men already selected. This can be done in ${}^3C_2$ ways.

(iii) Arrange the 4 selected persons in two teams. If the selected men are M_1 and M_2 and the selected women are W_1 and W_2, this can be done in 2 ways :

M_1W_1, play against M_2W_2

M_2W_1 play against M_1W_2

Hence the number of ways to arrange the game

$= {}^5C_2\, {}^3C_2\,(2) = 10 \times 3 \times 2 = 60$

Example – 19

In how many ways can 7 departments be divided among 3 ministers such that every minister gets at least one and atmost 4 departments to control ?

Sol. The ways in which we can divide 7 departments among 3 ministers such that each minister gets atleast 1 and atmost 4.

S.No.	M_1	M_2	M_3
1	4	2	1
2	2	2	3
3	3	3	1

Note : If we have a case (2, 2, 3), then there is no need to make cases (3, 2, 2) or (2, 3, 2) because we will include them whem we apply distribution formula to distribute ways of division among ministers.

Case I : We divide 7 departements among 3 ministers in number 4, 2, 1 i.e. unequal division. As any minister can get 4 departments, any can get 2 any can get 1 department, we should apply distribution formula.

we get :

Number of ways to divide and distribute department in number 4, 2, 1

$$= \left[\frac{\lfloor 7}{\lfloor 4\,\lfloor 2\,\lfloor 1}\right] \times 3! = 630 \qquad ....(i)$$

Case II : It is 'equal as well as unequal' division. As any minister can get any number of departments, we use complete distribution formula.

we get :

Number of ways to divide departments in number 2, 2, 3,

$$= \left[\frac{\underline{|7}}{\underline{|2}\,\underline{|2}\,\underline{|3}}\;\frac{1}{\underline{|2}}\right]\times \underline{|3}! = 630 \qquad ....(ii)$$

Case III : It is also 'equal as well as unequal' division. As any minister can get any number of departments, we use complete distribution of formula.

we get :

Number of ways to divide and distribute in number 3, 3, 1

$$= \left[\frac{\underline{|7}}{(\underline{|3})^2\,(\underline{|1})\,\underline{|2}}\;\frac{1}{\underline{|2}}\right]\times \underline{|3}! = 420 \qquad ....(iii)$$

Combining (i), (ii) and (iii), we get number of ways to divide 7 departments among 3 minister

$= 630 + 630 + 420 = 1680$ ways.

Example – 20

Find the sum of all five–digit numbers that can be formed using digits 1, 2, 3, 4, 5 if repetition is not allowed ?

Sol. There are 5! = 120 five digit numbers and there are 5 digits. Hence by symmetry or otherwise we can see that each digit will appear in any place

(unit's or ten's or . . . . . .) $\frac{5!}{5}$ times.

$\Rightarrow$ X = sum of digits in any place

$$\Rightarrow \quad X = \frac{5!}{5}\times 5 + \frac{5!}{5}\times 4 + \frac{5!}{5}\times 3 + \frac{5!}{5}\times 2 + \frac{5!}{5}\times 1$$

$$\Rightarrow \quad X = \frac{5!}{5}\times(5+4+3+2+1) = \frac{5!}{5}(15)$$

$\Rightarrow$ the sum of all numbers

$= X + 10X + 100X + 1000X + 10000X$

$= X(1 + 10 + 100 + 1000 + 10000)$

$$= \frac{5!}{5}(15)(1+10+100+1000+10000)$$

$= 24(15)(11111) = 3999960$

Example – 21

Find the number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated.

Sol. Considering 4 particular flowers as one group of flower, we have five flowers (one group of flowers and remaining four flowers) which can be strung to form a garland in $\frac{4!}{2}$ ways. But 4 particular flowers can be arranged themselves in 4! ways. Thus, the required number of ways $= \frac{4!\times 4!}{2} = 288$.

Example – 22

In how many ways 6 letters can be placed in 6 envelopes such that

(a) No letter is placed in its corresponding envelope.

(b) at least 4 letters are placed in correct envelopes.

(c) at most 3 letters are placed in wrong envelopes.

Sol. **(a)** Using dearrangement theorem.

Number of ways to place 6 letters in 6 envelopes such that all are placed in wrong envelopes.

$$= \underline{|6}\left[1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + ... + \frac{1}{\underline{|6}}\right]$$

$= 360 - 120 + 30 - 6 + 1 = 265$

(b) Number of ways to place letters such that at least 4 letters are placed in correct envelopes

= 4 letters are placed in correct envelopes and 2 are in wrong + 5 letters are placed in correct envelopes and 1 in wrong + All 6 letters are placed in correct envelopes

$= {}^6C_4 \times 1 + 0$ (not possible to place 1 in wrong envelope) + 1

$$= \frac{6\times 5}{2} + 1 = 16$$

(c) Number of ways to place 6 letters in 6 envelopes such that at most 3 letters are placed in wrong envelopes = 0 letter is wrong envelope and 6 in correct + 1 letter in wrong envelope and 5 in correct 2 letters in wrong envelopes and 4 are in correct + 3 letters in wrong envelopes and 3 in correct = 1 + 0 (not possible to place 1 in wrong envelope) +

$${}^6C_4 \times 1 + {}^6C_3\ \underline{|3}\left[1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}}\right]$$

$$= 1 + \frac{6\times 5}{2} + \frac{6\times 5\times 4}{6}\left(\frac{\underline{|3}}{\underline{|2}} - \frac{\underline{|3}}{\underline{|3}}\right)$$

$= 1 + 15 + 20 \times 2 = 56.$

Example – 23

How many different words can be formed with the letters of the word EQUATION so that

(i) the words begin with E ?

(ii) the words begin with E and end with N ?

Sol. Clearly, the given word contains 8 letters out of which 5 are vowels and 3 consonants.

(i) Since all words must begin with E. So, we fix E at the first place. So, total number of words = $^7P_7 = 7!$

(ii) Since all words must begin with E and end with N. So, we fix E at the first place and N at the last place.

Hence, the required number of words = $^6P_6 = 6!$

Example – 24

In how many ways 5 boys and 3 girls can be seated in a row so that no two girls are together ?

Sol. The 5 boys can be seated in a row in $^5P_5 = 5!$ ways. In each of these arrangements 6 places are created, shown by the cross-marks, as given below :

$\times B \times B \times B \times B \times B \times$

Since no two girls are to sit together, so we may arrange 3 girls in 6 places. This can be done in $^6P_3$ ways i.e. 3 girls can be seated in $^6P_3$ ways.

Hence, the total number of seating arrangements

$= {}^5P_5 \times {}^6P_3 = 5! \times 6 \times 5 \times 4 = 14400.$

Example – 25

Find the number of ways in which 5 boys and 5 girls be seated in a row so that

(i) No two girls may sit together.

(ii) All the girls sit together and all the boys sit together.

(iii) All the girls are never together.

Sol. (i) 5 boys can be seated in a row in $^5P_5 = 5!$ ways. Now, in the 6 gaps 5 girls can be arranged in $^6P_5$ ways.

Hence, the number of ways in which no two girls sit together

$= 5! \times {}^6P_5 = 5! \times 6!$

(ii) The two groups of girls and boys can be arranged in 2! ways. 5 girls can be arranged among themselves in 5! ways. Similarly, 5 boys can be arranged among themselves in 5! ways. Hence, by the fundamental principle of counting, the total number of requisite seating arrangements $= 2!(5! \times 5!) = 2(5!)^2$.

(iii) The total number of ways in which all the girls are never together

= Total number of arrangements –

Total number of arrangements in which all the girls are always together

$= 10! - 2(5!)^2$

Example – 26

Five boys and five girls form a line with the boys and girls alternating. Find the number of ways of making the line.

Sol. 5 boys can be arranged in a line in $^5P_5 = 5!$ ways. Since the boys and girls are alternating. So, corresponding each of the 5! ways of arrangements of 5 boys we obtain 5 places marked by cross as shown below :

(i) $B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times$ (ii) $\times B_1 \times B_2 \times B_3 \times B_4 \times B_5$.

Clearly, 5 girls can be arranged ir 5 places marked by cross in $(5! + 5!)$ ways.

Hence, the total number of ways of making the line

$= 5! \times (5! + 5!) = 2(5!)^2$.

Example – 27

How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word 'EQUATION' so that the two consonants occur together ?

Sol. There are 5 vowels and 3 consonants in the word 'EQUATION' 3 vowels out of 5 and 2 consonants out of 3 can be chosen in $^5C_3 \times {}^3C_2$ ways. As consonants occur together, Considering 2 consonants as one letter, we have 4 letters which can be arranged in 4! ways. But two consonants can be put together in 2! ways. Therefore, 5 letters in each group can be arranged in 4! × 2! ways.

The required no. of words = $({}^5C_3 \times {}^3C_2) \times 4! \times 2! = 1440$.

Example – 28

How many words with or without meaning, each 2 of vowels and 3 consonants can be formed from the letters of the word DAUGHTER ?

Sol. There are 3 vowels and 5 consonants in the word DAUGHTER out of which 2 vowels and 3 consonants can be chosen in ${}^3C_2 \times {}^5C_3$ ways. These selected five letters can now be arranged in 5! ways.

Hence, required number of words

$= {}^3C_2 \times {}^5C_3 \times 5!$

$= 3 \times 10 \times 120 = 3600$

Example – 29

(i) How many different words can be formed with the letters of the word HARYANA ?

(ii) How many of these begin with H and end with N ?

(iii) In how many of these H and N are together ?

Sol. (i) There are 7 letters in which 3 are alike

So, total number of words $= \dfrac{7!}{3!1!1!1!1!} = \dfrac{7!}{3!} = 840$.

(ii) After fixing H in first place and N in last place, we have 5 letters out of which three are alike

So, total number of words $= \dfrac{5!}{3!} = 20$.

(iii) If H and N together we have 6 letters out of which 3 are alike. These 6 letters can be arranged in $\dfrac{6!}{3!}$ ways. But H and N can be arranged amongst themselves in 2! ways.

Hence, the requisite number of words $= \dfrac{6!}{3!} \times 2! = 120 \times 2 = 240$.

Example – 30

How many different words can be formed by using all the letters of the word 'ALLAHABAD' ?

(i) In how many of them vowels occupy the even positions?

(ii) In how many of them both L do not come together ?

Sol. There are 9 letters in the word 'ALLAHABAD' out of which 4 are A's, 2 are L's and the rest are all distinct.

So, the requisite number of words $= \dfrac{9!}{4!\,2!} = 7560$.

(i) Four A's will occupy four even places in 1 way. Now, we are left with 5 places and 5 letters, of which two are alike (2 L's) and other distinct, can be arranged in $\dfrac{5!}{2!}$ ways.

Total no. of reqd. words $= \dfrac{5!}{2!} \times \dfrac{4!}{4!} = \dfrac{5!}{2!} = 60$.

(ii) The no. of words in which both L come together $= \dfrac{8!}{4!} = 1680$.

Hence, the no. of words in which both L do not come together

= Total no. of words – No. of words in which both L come together

$= 7560 - 1680 = 5880$.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

The fundamental principle of counting

1. There are 4 letter boxes in a post office. In how many ways can a man post 8 distinct letters ?

(a) 4×8 (b) 8^4

(c) 4^8 (d) P (8, 4)

2. In an examination there are three multiple choice questions and each question has 4 choices out of which only one is correct. If all the questions are compulsory, then number of ways in which a student can fail to get all answers correct, is

(a) 11 (b) 12

(c) 27 (d) 63

3. Every one of the 5 available lamps can be switched on to illuminate certain Hall. The total number of ways in which the hall can be illuminated, is :

(a) 32 (b) 31

(c) 5 (d) 5!

4. A new flag is to be designed with six vertical strips using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent strips have the same colour is

(a) 12×81 (b) 16×192

(c) 20×125 (d) 24×216

5. 4 buses run between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are

(a) 12 (b) 16

(c) 4 (d) 8

Arrangement & selection for different objects

6. How many numbers lying between 500 and 600 can be formed with the help of the digits 1, 2, 3, 4, 5, 6 when the digits are not to be repeated

(a) 20 (b) 40

(c) 60 (d) 80

7. The number of 3 digit odd numbers, that can be formed by using the digits 1, 2, 3, 4, 5, 6 when the repetition is allowed, is

(a) 60 (b) 108

(c) 36 (d) 30

8. The number of all four digit numbers is equal to

(a) 9999 (b) 9000

(c) 10^4 (d) none of these

9. The number of all four digits numbers with distinct digits is

(a) $9 \times 10 \times 10 \times 10$ (b) ${}^{10}P_4$

(c) $9 \times {}^{9}P_3$ (d) none of these

10. The number of even numbers that can be formed by using the digits 1, 2, 3, 4 and 5 taken all at a time (without repetition) is

(a) 120 (b) 48

(c) 1250 (d) none of these

11. The number of all three digit numbers having no digit as 5 is

(a) 252 (b) 225

(c) 648 (d) none of these

12. All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :

(a) 5 (b) 325

(c) 345 (d) 365

13. How many of the 900 three digit numbers have at least one even digit ?

(a) 775 (b) 875

(c) 450 (d) 750

14. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :

(a) 144 (b) 72

(c) 288 (d) 720

15. An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is:

(a) $72(\underline{|7})$ (b) $18(\underline{|7})$

(c) $40(\underline{|7})$ (d) $36(\underline{|7})$

16. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do not occupy odd places, is :

(a) 160 (b) 120

(c) 60 (d) 48

17. Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval:

(a) [8, 9] (b) [10, 12)

(c) (11, 13] (d) (14, 17)

18. On the occasion of Deepawali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is

(a) ${}^{20}C_2$ (b) $2.{}^{20}C_2$

(c) $2.{}^{20}P_2$ (d) None of these

19. In a touring cricket team, there are 16 players in all including 5 bowlers and 2 wicket-keepers. How many teams of 11 players from these, can be chosen, so as to include three bowlers and one wicket-keeper

(a) 650 (b) 720

(c) 750 (d) 800

20. Three couples (husband and wife) decide to form a committee of three members. The number of different committee that can be formed in which no couple finds a place is :

(a) 60 (b) 12

(c) 27 (d) 8

21. 5 Indian and 5 American couples meet at a party and shake hands. If no wife shakes hands with her own husband and no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :

(a) 95 (b) 110

(c) 135 (d) 150

22. A rack has 5 different pairs of shoes. The number of ways in which 4 shoes can be chosen from it so that there will be no complete pair is

(a) 1920 (b) 200

(c) 110 (d) 80

23. In how many ways can six boys and five girls stand in a row if all the girls are to stand together but the boys cannot all stand together ?

(a) 172,800 (b) 432,000

(c) 86,400 (d) None of these

24. There are five different green dyes, four different blue dyes and three different red dyes. The total number of combinations of dyes that can be chosen taking at least one green and one blue dye is

(a) 3255 (b) 2^{12}

(c) 3720 (d) none of these

25. Given 6 different toys of red colour, 5 different toys of blue colour and 4 different toys of green colour. Combination of toys that can be chosen taking at least one red and one blue toys are :

(a) 31258 (b) 31248

(c) 31268 (d) None of these

26. A father with 8 children takes them 3 at a time to the Zoological gardens, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is

(a) 336 (b) 112

(c) 56 (d) None of these

27. In how many ways can two balls of the same colour be selected out of 4 distinct black and 3 distinct white balls

(a) 5 (b) 6

(c) 9 (d) 8

28. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(a) 602 (b) 603

(c) 600 (d) 601

29. If the letters of the word 'MOTHER' are written in all possible orders and these words are written out as in a dictionary, find the rank of the word 'MOTHER'.

(a) 307 (b) 308

(c) 309 (d) 120

30. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary then the rank of the word RANDOM is

(a) 614 (b) 615

(c) 613 (d) 616

31. If all permutations of the letters of the word AGAIN are arranged as in dictionary, then fiftieth word is

(a) NAAGI (b) NAGAI

(c) NAAIG (d) NAIAG

32. If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is :

(a) 15^{th} (b) 16^{th}

(c) 17^{th} (d) 18^{th}

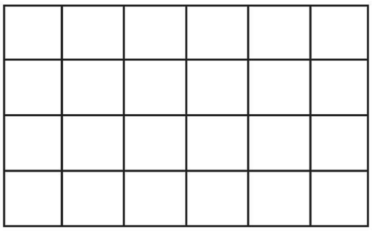
33. The letters of word "RADHIKA" are permuted and arranged in alphabetical order as in English dictionary. The number of words the appear before the word "RADHIKA" is :

(a) 2193 (b) 2195

(c) 2119 (d) 2192

Geometrical counting problems

34. Number of rectangles in figure shown which are not squares is :



(a) 159 (b) 160

(c) 161 (d) None of these

35. There are n points on a circle. The number of straight lines formed by joining them is equal to

(a) ${}^{n}C_2$ (b) ${}^{n}P_2$

(c) ${}^{n}C_2 - 1$ (d) none of these

36. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(a) 7 (b) 5

(c) 10 (d) 8

37. There are 10 points in a plane, out of these 6 are collinear. If N is the number of traingles formed by joining these points, then

(a) $N > 190$ (b) $N \le 100$

(c) $100 < N \le 140$ (d) $140 < N \le 190$

38. Number of diagonals of a convex hexagon is

(a) 3 (b) 6

(c) 9 (d) 12

39. The interior angles of a regular polygon measure 150° each. The number of diagonals of the polygon is

(a) 35 (b) 44

(c) 54 (d) 78

40. Out of 10 points in a plane 6 are in a straight line. The number of triangles formed by joining these points are

(a) 100 (b) 150

(c) 120 (d) None of these

41. The number of straight lines that can be formed by joining 20 points no three of which are in the same straight line except 4 of them which are in the same line

(a) 183 (b) 186

(c) 197 (d) 185

42. There are *n* distinct points on the circumference of a circle. The number of pentagons that can be formed with these points as vertices is equal to the number of possible triangles. Then the value of *n* is

(a) 7 (b) 8

(c) 15 (d) 30

Arrangement & selection of like objects

43. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?

(a) $7.{}^{6}C_4.{}^{8}C_4$ (b) $8.{}^{6}C_4.{}^{7}C_4$

(c) $6.7.{}^{8}C_4$ (d) $6.8.{}^{7}C_4$

44. The number of all possible different arrangements of the word "BANANA" is

(a) $\underline{|6}$ (b) $\underline{|6} \times \underline{|2} \times \underline{|3}$

(c) $\dfrac{\underline{|6}}{\underline{|2}\,\underline{|3}}$ (d) none of these

45. The total number of ways of arranging the letters AAAA BBB CC D E F in a row such that letters C are separated from one another is

(a) 2772000 (b) 1386000

(c) 4158000 (d) none of these

46. A library has 'a' copies of one book, 'b' copies of each of two books, 'c' copies of each of three books, and single copy each of 'd' books. The total number of ways in which these books can be arranged in a row is

(a) $\dfrac{(a+b+c+d)!}{a!b!c!}$ (b) $\dfrac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$

(c) $\dfrac{(a+2b+3c+d)!}{a!b!c!}$ (d) none of these

47. A question paper on mathematics consists of twelve questions divided into three parts. A, B and C, each containing four questions. In how many ways can an examinee answer five questions, selecting atleast one from each part.

(a) 624 (b) 208

(c) 2304 (d) none of these

48. Number of ways in which 4 boys and 2 girls (all are of different heights) can be arranged in a line so that boys as well as girls among themselves are in decreasing order of height (from left to right), is :

(a) 1 (b) 6!

(c) 15 (d) None of these

49. The total number of selections of atleast one fruit which can be made from 3 bananas, 4 apples and 2 oranges is

(a) 39 (b) 315

(c) 512 (d) none of these

50. The total number of different combinations of one or more letters which can be made from the letters of the word 'MISSISSIPPI' is

(a) 150 (b) 148

(c) 149 (d) None of these

Distribution of different objects

51. The set S = {1, 2, 3,..., 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(a) $12!/3!(4!)^3$ (b) $12!/3!(3!)^4$

(c) $12!/(4!)^3$ (d) $12!/(3!)^4$

52. In an election three districts are to be canvassed by 2, 3 and 5 men respectively. If 10 men volunteer, the number of ways they can be alloted to the different districts is :

(a) $\frac{10!}{2!\,3!\,5!}$ (b) $\frac{10!}{2!\,5!}$

(c) $\frac{10!}{(2!)^2\,5!}$ (d) $\frac{10!}{(2!)^2\,3!\,5!}$

53. The number of ways to give 16 different things to three persons A, B, C so that B gets 1 more than A and C gets 2 more than B, is :

(a) $\frac{16!}{4!5!7!}$ (b) 4!5!7!

(c) $\frac{16!}{3!5!8!}$ (d) 3!5!8!

54. The number of ways in which 52 cards can be divided into 4 sets, three of them having 17 cards each and the fourth one having just one card

(a) $\frac{52!}{(17!)^3}$ (b) $\frac{52!}{(17!)^3 3!}$

(c) $\frac{51!}{(17!)^3}$ (d) $\frac{51!}{(17!)^3 3!}$

55. The number of ways in which 12 balls can be divided between two friends, one receiving 8 and the other 4, is

(a) $\frac{12!}{8!4!}$ (b) $\frac{12!2!}{8!4!}$

(c) $\frac{12!}{8!4!2!}$ (d) none of these

Distribution of alike objects

56. The number of ways of distributing 8 identical balls in 3 distinct boxes, so that none of the boxes is empty, is

(a) 5 (b) 21

(c) 3^8 (d) ${}^8C_3$

57. The total number of ways in which 11 identical apples can be distributed among 6 children is that at least one apple is given to each child

(a) 252 (b) 462

(c) 42 (d) none of these

58. If a,b,c,d are odd natural numbers such that $a + b + c + d = 20$, then the number of values of (a, b, c, d) is :

(a) 165 (b) 455

(c) 310 (d) 255

59. Number of ways in which 25 identical balls can be distributed among Ram, shyam, Sunder and Ghanshyam such that at least 1, 2, 3, and 4 balls are given to Ram, Shyam, Sunder and Ghanshyam respectively, is :

(a) ${}^{18}C_4$ (b) ${}^{28}C_3$

(c) ${}^{24}C_3$ (d) ${}^{18}C_3$

60. The total number of ways in which n^2 number of identical balls can be put in n numbered boxed (1, 2, 3, ... n) such that i^{th} box contains at least i number of balls, is :

(a) ${}^{n^2}C_{n-1}$ (b) ${}^{n^2-1}C_{n-1}$

(c) ${}^{\frac{n^2+n-2}{2}}C_{n-1}$ (d) None of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is : **(2015/Online Set–1)**

(a) 1880 (b) 1120
(c) 1240 (d) 1960

2. If all the worlds (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **(2016)**

(a) 59^{th} (b) 52^{nd}
(c) 58^{th} (d) 46^{th}

3. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is : **(2016/Online Set–1)**

(a) $\frac{11!}{(2!)^3}$ (b) 110
(c) 56 (d) 59

4. The value of $\sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$ is equal to : **(2016/Online Set–1)**

(a) 560 (b) 680
(c) 1240 (d) 1085

5. If $\frac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11,$ then n satisfies the equation : **(2016/Online Set–2)**

(a) $n^2 + 3n - 108 = 0$ (b) $n^2 + 5n - 84 = 0$
(c) $n^2 + 2n - 80 = 0$ (d) $n^2 + n - 110 = 0$

6. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party is : **(2017)**

(a) 485 (b) 468
(c) 469 (d) 484

7. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is : **(2017/Online Set–1)**

(a) 44^{th} (b) 45^{th}
(c) 46^{th} (d) 47^{th}

8. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is : **(2017/Online Set–2)**

(a) $5 \times 6!$ (b) $6 \times 6!$
(c) $7!$ (d) $5 \times 7!$

9. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : **(2018)**

(a) at least 750 but less than 1000
(b) at least 1000
(c) less than 500
(d) at least 500 but less than 750

10. n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is : **(2018/Online Set–1)**

(a) 6 (b) 7
(c) 8 (d) 9

11. The number of four letter words that can be formed using the letters of the word **BARRACK** is : **(2018/Online Set–2)**

(a) 120 (b) 144
(c) 264 (d) 270

12. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4 (repetition of digits is not allowed) and are multiple of 3 is : **(2018/Online Set–3)**

(a) 24 (b) 30

(c) 36 (d) 48

13. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **(8-04-2019/Shift-1)**

14. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is: **(8-04-2019/Shift-2)**

(a) 288 (b) 360

(c) 306 (d) 310

15. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then m = n = k, then k is **(9-04-2019/Shift-1)**

16. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is: **(9-04-2019/Shift-2)**

(a) 157 (b) 262

(c) 225 (d) 190

17. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is: **(10-04-2019/Shift-1)**

18. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is **(10-4-2019/Shift-2)**

(a) 170 (b) 180

(c) 210 (d) 190

19. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to : **(12-04-2019/Shift-2)**

(a) 28 (b) 27

(c) 25 (d) 24

20. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: **(9-01-2019/Shift-1)**

(a) 500 (b) 200

(c) 300 (d) 350

21. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1,3, 7, 9 (repetition of digits allowed) is equal to: **(9-01-2019/Shift-2)**

(a) 374 (b) 372

(c) 375 (d) 250

22. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is : **(11-01-2019/Shift-2)**

(a) $6^5 \times (15)!$ (b) $5! \times 6!$

(c) $(15)! \times 6!$ (d) $5^6 \times 15$

23. Consider three boxes, each containing 10 balls labelled 1, 2, ... , 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is: **(12-01-2019/Shift-1)**

24. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is **(12-01-2019/Shift-2)**

(a) 12 (b) 11

(c) 9 (d) 7

25. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is .... **(2-9-2020/Shift-1)**

26. Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is : **(2-09-2020/Shift-2)**

(a) 201 (b) 199
(c) 101 (d) 200

27. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - \text{up to } 51^{th} \text{ term})$ $+ (1! - 2! + 3! - \ldots\ldots$ up to 51th term) is equal to : **(3-09-2020/Shift-1)**

(a) 1 – 51 (51)! (b) 1 + (52)!
(c) 1! (d) 1 + (51)!

28. The total number of 3-digit numbers, whose sum of digits is 10, is .... **(3-09-2020/Shift-2)**

29. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to : **(4-09-2020/Shift-1)**

(a) ${}^{51}C_7 - {}^{30}C_7$ (b) ${}^{51}C_7 + {}^{30}C_7$
(c) ${}^{50}C_7 - {}^{30}C_7$ (d) ${}^{50}C_6 - {}^{30}C_6$

30. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ______ **(4-9-2020/Shift-2)**

31. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is **(5-09-2020/Shift-1)**

32. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is: **(5-09-2020/Shift-2)**

(a) 2250 (b) 2255
(c) 1500 (d) 3000

33. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated? **(6-09-2020/Shift-1)**

(a) 2! 3! 4! (b) $(3!)^3.(4!)$
(c) $3! (4!)^3$ (d) $(3!)^2.(4!)$

34. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____ **(6-09-2020/Shift-2)**

35. Total number of six-digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is **(7-01-2020/Shift-1)**

(a) 5^6 (b) $\frac{1}{2}(6!)$
(c) 6! (d) $\frac{5}{2}(6!)$

36. The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, where k is an integer, is : **(7-01-2020/Shift-2)**

(a) 4 (b) 6
(c) 2 (d) 3

37. If a, b and c are the greatest values of ${}^{19}C_p, {}^{20}C_q, {}^{21}C_r$ respectively, then: **(8-01-2020/Shift-1)**

(a) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (b) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$
(c) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (d) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

38. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at most three of them are red is____. **(8-01-2020/Shift-1)**

39. The number of 4 letter words (with or without meaning) that can be made from the eleven letters of the word "EXAMINATION" is **(8-01-2020/Shift-2)**

40. If the number of five digit numbers with distinct digits and 2 at the 10th place is $336k$, then k is equal to: **(9-01-2020/Shift-1)**

(a) 8 (b) 7
(c) 4 (d) 6

41. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _________. **(20-07-2021/Shift-1)**

42. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$, where $\alpha > 0$. Then then value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ______? **(20-07-2021/Shift-2)**

43. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to______ ? **(25-07-2021/Shift-1)**

44. Let n be a non-negative integer. Then the number of divisors of the form "$4n+1$" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to _____.

(27-07-2021/Shift-2)

45. Let $A = \{0,1,2,3,4,5,6,7\}$. Then the number of bijective functions $f: A \to A$ such that $f(1)+f(2) = 3 - f(3)$ is equal to ______. **(22-07-2021/Shift-2)**

46. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______.

(22-07-2021/Shift-2)

47. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to **(25-07-2021/Shift-2)**

(a) 3 (b) 1

(c) 4 (d) 2

48. Let $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \{\pm3,\pm2,\pm1,0\}\right\}$ Define $f: M \to Z$, as $f(A) = \det(A)$ for all $A \in M$, where Z is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to _______ ?

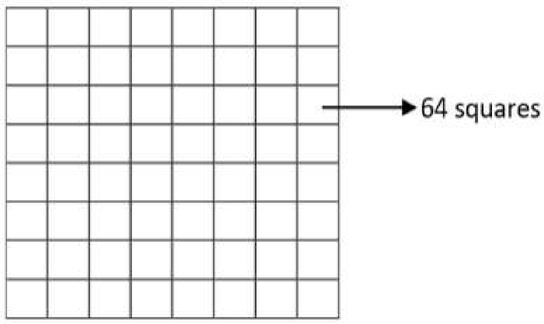
(25-07-2021/Shift-1)

49. Let $P_1, P_2, ..., P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, p_j, P_k such that $i+j+k \neq 15$, is **(01-09-2021/Shift-2)**

(a) 12 (b) 419

(c) 455 (d) 443

50. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is



(01-09-2021/Shift-2)

(a) $\frac{1}{18}$ (b) $\frac{1}{7}$

(c) $\frac{1}{9}$ (d) $\frac{2}{7}$

51. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is _______

(01-09-2021/Shift-2)

52. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is______.

(27-08-2021/Shift-1)

53. If ${}^1P_1 + 2.{}^2P_2 + 3.{}^3P_3 + ...15.{}^{15}P_{15} = {}^qp_r - s,\ 0 \le s \le 1$, then ${}^{q+s}C_{r-s}$ is equal to ______. **(26-08-2021/Shift-1)**

54. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ______. **(26-08-2021/Shift-1)**

55. Let $S=\{1,2,3,4,5,6,9\}$. Then the number of elements in the set $T=\{A\subseteq S: A\neq\phi$ and the sum of all the elements of A is not a multiple of 3$\}$ is ______. **(27-08-2021/Shift-2)**

56. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is ______ ? **(31-08-2021/Shift-1)**

57. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is______ **(31-08-2021/Shift-2)**

58. A natural number has prime factorization given by $n=2^x3^y5^z$, where y and z are such that $y+z=5$ and $y^{-1}+z^{-1}=\frac{5}{6}, y>z$. Then the number of odd divisors of n, including 1, is: **(26-02-2021/Shift-2)**

(a) 6 (b) 6x

(c) 11 (d) 12

59. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is _____ **(26-02-2021/Shift-2)**

60. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is : **(26-02-2021/Shift-1)**

(a) 35 (b) 82

(c) 77 (d) 42

61. The total number of positive integral solutions (x,y,z) such that $xyz=24$ is: **(25-02-2021/Shift-1)**

(a) 45 (b) 36

(c) 24 (d) 30

62. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is _____. **(25-02-2021/Shift-1)**

63. The total number of two digit numbers 'n', such that 3^n+7^n is a multiple of 10, is: **(25-02-2021/Shift-2)**

64. The students $S_1, S_2,....,S_{10}$ are to be divided into 3 groups. A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is ____. **(24-02-2021/Shift-2)**

65. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways the committee can be formed is **(24-02-2021/Shift-1)**

(a) 575 (b) 1050

(c) 1625 (d) 560

66. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is : **(18-03-2021/Shift-1)**

(a) 122234 (b) 22264

(c) 122664 (d) 26664

67. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is .... **(18-03-2021/Shift-1)**

68. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let ? be the number of triangles having these points from different sides as vertices and ? be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta-\alpha)$ is equal to : **(16-03-2021/Shift-2)**

(a) 795 (b) 1173

(c) 1890 (d) 717

69. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to : **(17-03-2021/Shift-1)**

(a) 4 (b) 5

(c) 2 (d) 6

70. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to : **(17-03-2021/Shift-2)**

(a) 360 (b) 364

(c) 240 (d) 333

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. Total 5–digit numbers divisible by 4 that can be formed using 0, 1, 2, 3, 4, 5, when the repetition of digits is allowed is

(a) 1250 (b) 875

(c) 1620 (d) 1000

2. The number of 4–digit numbers that can be made with the digits 1, 2, 3, 4 and 5 in which at least two digits are identical, is :

(a) $4^5 - 5!$ (b) 505

(c) 600 (d) None of these

3. How many numbers greater than 1000, but not greater than 4000 can be formed with the digits 0, 1, 2, 3, 4, repetition of digits being allowed :

(a) 374 (b) 375

(c) 376 (d) None of these

4. The letters of the word SURITI are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word SURITI is :

(a) 236 (b) 245

(c) 307 (d) 315

5. An ice cream parlour has ice creams in eight different varieties. Number of ways of choosing 3 ice creams taking atleast two ice creams of the same variety, is :

(a) 56 (b) 64

(c) 100 (d) none of these

(Assume that ice creams of the same variety are identical and available in unlimited supply)

6. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin and none is left over, then the number of ways in which the division may be made is :

(a) 420 (b) 630

(c) 710 (d) none of these

7. Four couples (husband and wife) decide to form a committee of four members. The number of different committee that can be formed in which no couple finds a place is :

(a) 10 (b) 12

(c) 14 (d) 16

8. A variable name in a certain computer language must be either an alphabet or a alphabet followed by a decimal digit. Total number of different variable names that can exist in that language is equal to :

(a) 280 (b) 290

(c) 286 (d) 296

9. Every one of the 10 available lamps can be switched on to illuminate certain Hall. The total number of ways in which the hall can be illuminated, is :

(a) 55 (b) 1023

(c) 2^{10} (d) 10!

10. The number of all possible selections of one or more questions from 10 given questions, each question having an alternative is :

(a) 3^{10} (b) $2^{10} - 1$

(c) $3^{10} - 1$ (d) 2^{10}

11. There are 20 questions in a questions paper. If no two students solve the same combination of questions but solve equal number of questions then the maximum number of students who appeared in the examination is :

(a) ${}^{20}C_9$ (b) ${}^{20}C_{11}$

(c) ${}^{20}C_{10}$ (d) None of these

12. The number of numbers divisible by 3 that can be formed by four different even digits is :

(a) 18 (b) 36

(c) 20 (d) None of these

13. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is 1, 16, 31, etc.) This process is continued until a number is reached which has already been marked, then unmarked numbers are

(a) 200 (b) 400

(c) 600 (d) 800

14. The number of times of the digits 3 will be written when listing the integer from 1 to 1000 is :

(a) 269 (b) 300

(c) 271 (d) 302

15. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$

(c) ${}^{52}C_4$ (d) None of these

16. There are n concurrent lines and another line parallel to one of them. The number of different triangles that will be formed by the (n + 1) lines, is

(a) $\frac{(n-1)n}{2}$ (b) $\frac{(n-1)(n-2)}{2}$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{(n+1)(n+2)}{2}$

17. The sides AB, BC and CA of a triangle ABC have a, b and c interior points on them respectively, then find the number of triangles that can be constructed using these interior points as vertices.

(a) ${}^{a+b+c}C_3$

(b) ${}^{a+b+c}C_3 - ({}^{a}C_3 + {}^{b}C_3 + {}^{c}C_3)$

(c) ${}^{a+b+c+3}C_3$

(d) None of these

18. There are 12 books on Algebra and Calculus in our library, the books of the same subject being different. If the number of selections each of which consists of 3 books on each topic is greatest then the number of books of Algebra and Calculus in the library are respectively :

(a) 3 and 9 (b) 4 and 8

(c) 5 and 7 (d) 6 and 6

19. A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is

(a) 41 (b) 36

(c) 47 (d) 76

20. An ice cream parlour has ice creams in eight different varieties. Number of ways of choosing 3 ice creams taking atleast two ice creams of the same variety, is :

(a) 56 (b) 64

(c) 100 (d) none of these

21. A bag contains 2 Apples, 3 Oranges and 4 Bananas. The number of ways in which 3 fruits can be selected if atleast one banana is always in the combination (Assume fruit of same species to be alike) is :

(a) 6 (b) 10

(c) 29 (d) 7

22. The number of ways in which the sum of upper faces of four distinct dices can be six.

(a) 10 (b) 4

(c) 6 (d) 7

23. If the letters of the word MOTHER are arranged in all possible orders and these words are written as in a dictionary, then the rank of the word MOTHER will be

(a) 240 (b) 261

(c) 308 (d) 309

24. There are 12 books on Algebra and Calculus in our library, the books of the same subject being different. If the number of selections each of which consists of 3 books on each topic is greatest then the number of books of Algebra and Calculus in the library are respectively :

(a) 3 and 9 (b) 4 and 8

(c) 5 and 7 (d) 6 and 6

25. If 3n different things can be equally distributed among 3 persons in k ways then the number of ways to divide the 3n things in 3 equal groups is :

(a) $k \times 3!$ (b) $\frac{k}{3!}$

(c) 3k (d) None of these

26. The number of ways in which the sum of upper faces of four distinct dices can be six.

(a) 10 (b) 4

(c) 6 (d) 7

27. The number of subsets of the set $A = \{a_1, a_2, \ldots, a_n\}$ which contain even number of elements is

(a) 2^{n-1} (b) $2^n - 1$

(c) $2^n - 2$ (d) 2^n

28. The number of divisors of $2^3 \cdot 3^3 \cdot 5^3 \cdot 7^5$ of the form $4n + 1, n \in N$ is :

(a) 46 (b) 47

(c) 96 (d) 94

29. There are n different books and p copies of each in a library. The number of ways in which one or more books can be selected is :

(a) $p^n + 1$ (b) $(p + 1)^n - 1$

(c) $(p + 1)^n - p$ (d) p^n

30. Let $p, q \in \{1,2,3,4\}$. The number of equations of the form $px^2 + qx + 1 = 0$ having real roots must be

(a) 15 (b) 9

(c) 7 (d) 8

31. The number of ways in which n different prizes can be distributed amongst m (< n) persons if each is entitled to receive at most n – 1 prizes, is :

(a) $n^m - n$ (b) m^n

(c) $m^n - m$ (d) None of these

32. Two classrooms A and B having capacity of 25 and (n – 25) seats respectively. A_n denotes the number of possible seating arrangments of room 'A', when 'n' students are to be seated in these rooms, starting from room 'A' which is to be filled up full to its capacity.

If $A_n - A_{n-1} = 25!\,({}^{49}C_{25})$ then 'n' equals

(a) 50 (b) 48

(c) 49 (d) 51

33. The number of ways in which we can choose 3 squares of unit area on a chess board such that one of the squares has its two sides common to other two squares

(a) 290 (b) 292

(c) 294 (d) 296

34. A teacher takes 3 children from her class to the zoo at a time as often as she can, but does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 more than a particular child goes to the zoo. The number of children in her class is :

(a) 12 (b) 10

(c) 60 (d) None of these

35. Two lines intersect at O. Points $A_1, A_2 \ldots, A_n$ are taken on one of them and $B_1, B_2, \ldots, B_n$ on the other, the number of triangle that can be drawn with the help of these (2n + 1) points is :

(a) n (b) n^2

(c) n^3 (d) n^4

36. The total number of six digit numbers $x_1\, x_2\, x_3\, x_4\, x_5\, x_6$ having the property $x_1 < x_2 \le x_3 < x_4 < x_5 \le x_6$, is equal to:

(a) ${}^{11}C_6$ (b) ${}^{16}C_2$

(c) ${}^{17}C_2$ (d) ${}^{18}C_2$

37. Find number of arangements of 4 letters taken from the word EXAMINATION.

(a) 2454 (b) 2500

(c) 2544 (d) None of these

38. The minimum marks required for clearing a certain screening paper is 210 out of 300. The screening paper consists of '3' sections each of Physics, Chemistry, and Maths. Each section has 100 as maximum marks. Assuming there is no negative marking and marks obtained in each section are integers, the number of ways in which a student can qualify the examinatin is (Assuming no cut–off limit) :

(a) ${}^{210}C_3 - {}^{90}C_3$ (b) ${}^{93}C_3$

(c) ${}^{213}C_3$ (d) $(210)^3$

39. There are 100 different books in a shelf. Number of ways in which 3 books can be selected so that no two of which are neighbours is :

(a) ${}^{100}C_3 - 98$ (b) ${}^{97}C_3$

(c) ${}^{96}C_3$ (d) ${}^{98}C_3$

Objective Questions II [One or more than one correct option]

40. The number of ways in which 10 candidates $A_1, A_2, \ldots, A_{10}$ can be ranked so that A_1 is always before A_2 is :

(a) $\dfrac{10!}{2}$ (b) $8! \times {}^{10}C_2$

(c) ${}^{10}P_2$ (d) ${}^{10}C_2$

41. If P (n, n) denotes the number of permutations of n different things taken all at a time then P (n, n) is also identical to

(a) r ! P (n, n – r) (b) (n – r) . P (n, r)

(c) n . P (n – 1, n – 1) (d) P (n, n – 1)

where $0 \le r \le n$

42. Which of the following statements are correct ?

(a) Number of words that can be formed with 6 only of the letters of the word "CENTRIFUGAL" if each word must contain all the vowels is 3 . 7 !

(b) There are 15 balls of which some are white and the rest are black. If the number of ways in which the balls can be arranged in a row, is maximum then the number of white balls must be equal to 7 or 8. Assume balls of the same colour to be alike.

(c) There are 12 things, 4 alike of one kind, 5 alike and of another kind and the rest are all different. The total number of combinations is 240.

(d) Number of selections that can be made of 6 letters from the word "COMMITTEE" is 35.

43. Identify the correct statement(s)

(a) Number of zeroes standing at the end of $\underline{|125}$ is 30.

(b) A telegraph has 10 arms and each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$.

(c) Number of numbers greater than 4 lacs which can be formed by using only the digit 0, 2, 2, 4, 4 and 5 is 90.

(d) In a table tennis tournament, every player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100.

44. There are 10 questions, each question is either True or False. Number of different sequences of incorrect answers is also equal to

(a) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both Heads and Tails are present.

(b) Number of ways in which a multiple choice question containing 10 alternatives with one or more than one correct alternatives, can be answered.

(c) Number of ways in which it is possible to draw a sum of money with 10 coins of different denominations taken some or all at a time.

(d) Number of different selections of 10 indistinguishable things takes some or all at a time.

45. The continued product, 2 . 6 . 10 . 14 .... to n factors is equal to :

(a) ${}^{2n}C_n$

(b) ${}^{2n}P_n$

(c) $(n+1)(n+2)(n+3)\ldots\ldots(n+n)$

(d) none of these

46. The number of ways of distributing 10 different books among 4 students ($S_1 - S_4$) such that S_1 and S_2 get 2 books each and S_3 and S_4 get 3 books each is :

(a) 12600 (b) 25200

(c) ${}^{10}C_4$ (d) $\frac{10!}{2!2!3!3!}$

47. The combinatorial coefficient ${}^{n-1}C_p$ denotes

(a) the number of ways in which n things of which p are alike and rest different can be arranged in a circle.

(b) the number of ways in which p different things can be selected out of n different thing if a particular thing is always excluded.

(c) number of ways in which n alike balls can be distributed in p different boxes so that no box remains empty and each box can hold any number of balls.

(d) the number of ways in which (n – 2) white balls and p black balls can be arranged in a line if black balls are separated, balls are all alike except for the colour.

48. The maximum number of permutations of 2n letters in which there are only a's and b's, taken all at a time is given by :

(a) ${}^{2n}C_n$

(b) $\frac{2}{1}\cdot\frac{6}{2}\cdot\frac{10}{3}\ldots\frac{4n-6}{n-1}\cdot\frac{4n-2}{n}$

(c) $\frac{n+1}{1}\cdot\frac{n+2}{2}\cdot\frac{n+3}{3}\cdot\frac{n+4}{4}\ldots\frac{2n-1}{n-1}\cdot\frac{2n}{n}$

(d) $\frac{2^n[1.3.5\ldots(2n-3)(2n-1)]}{n!}$

49. The combinatorial coefficient C(n, r) is equal to

(a) number of possible subsets of r members from a set of n distinct members.

(b) number of possible binary messages of length n with exactly r 1's.

(c) number of non decreasing 2–D paths from the lattice point (0, 0) to (r, n)

(d) number of ways of selecting r things out of n different things when a particular thing is always included plus the number of ways of selecting 'r' things out of n, when a particular thing is always excluded

50. Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3, ... n is :

(a) $\left(\frac{n-1}{2}\right)^2$ if n is odd (b) $\frac{n(n-2)}{4}$ if n is odd

(c) $\frac{(n-1)^2}{4}$ if n is odd (d) $\frac{n(n-2)}{4}$ if n is even

51. The number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which

(a) 5 persons are allotted 3 different residential flats so that each person is alloted at most one flat and no two persons are alloted the same flat.

(b) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction.

(c) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy.

(d) 3 mathematics professors are assigned five different lectures to be delivered, so that each professor gets at least one lecture.

Numerical Value Type Questions

52. Number of ways in which 12 identical coins can be distributed in 6 different purses, if not more than 3 and not less than 1 coin goes in each purse is ....

53. The number of non negative integral solution of the inequation $x + y + z + w \leq 7$ is ....

54. 10 identical balls are to be distributed in 5 different boxes kept in a row and labled A, B, C, D and E. Find the number of ways in which the balls can be distributed in the boxes if no two adjacent boxes remain empty.

55. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is.... .

56. In how many ways it is possible to select six letters, including at least one vowel from the letters of the word "F L A B E L L I F O R M". (It is a picnic spot in U. S. A.)

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For each question, choose the option corresponding to the correct matching.

57.

	Column–I		Column–II
(A)	Four different movies are running in a town. Ten students go to watch these four movies. The number of ways in which every movie is watched by atleast one student, is (Assume each way differs only by number of students watching a movie)	**(P)**	11
(B)	Consider 8 vertices of a regular octagon and its centre. If T denotes the number of triangles and S denotes the number of straight lines that can be formed with these 9 points then the value of (T – S) equals	**(Q)**	36
(C)	In an examination, 5 children were found to have their mobiles in their pocket. The Invigilator fired them and took their mobiles in his possession. Towards the end of the test, Invigilator randomly returned their mobiles. The number of ways in which at most two children did not get their own moblies is	**(R)**	52
(D)	The product of the digits of 3214 is 24. The number of 4 digit natural numbers such that the product of their digits is 12, is	**(S)**	60
(E)	The number of ways in which a mixed double tennis game can be arranged from amongst 5 married couple if no husband and wife plays in the same game, is	**(T)**	84

The correct matching is :

(a) (A–R); (B–T); (C–P); (D–Q); (E–S)

(b) (A–T); (B–R); (C–P); (D–Q); (E–S)

(c) (A–P); (B–T); (C–R); (D–Q); (E–S)

(d) (A–S); (B–Q); (C–R); (D–T); (E–P)

58. Match the Column

Column–I	Column–II
(A) Number of increasing permutations of m symbols are there from the n set numbers $\{a_1, a_2, ..., a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < ... a_{n-1} < a_n$ is	**(P)** n^m
(B) There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys	**(Q)** ${}^mC_n$
(C) Number of ways in which n red balls and (m – 1) green balls can be arranged in a line, so tha no two red balls are together, is (balls of the same colour are alike)	**(R)** ${}^nC_m$
(D) Number of ways in which 'm' different toys can be distributed in 'n' children if every child may receive any number of toys, is	**(S)** m^n

The correct matching is :

(a) (A–R); (B –S); (C–Q); (D–P)

(b) (A–S); (B –R); (C–Q); (D–P)

(c) (A–Q); (B –S); (C–R); (D–P)

(d) (A–P); (B –Q); (C–S); (D–R)

59. 5 balls are to be placed in 3 boxes. Each box can hold all the 5 balls. Number of ways in which the balls can be placed so that no box remains empty, if :

Column–I	Column–II
(A) balls are identical but boxes are different	**(P)** 2
(B) balls are different but boxes are identical	**(Q)** 25
(C) balls as well as boxes are identical	**(R)** 50
(D) balls as well as boxes are identical but boxes are kept in a row	**(S)** 6

You may note that two or more entries of column–I can match with only entry of column–II

The correct matching is :

(a) (A–S); (B–Q); (C–P); (D–S)

(b) (A–Q); (B–S); (C–P); (D–S)

(c) (A–P); (B–Q); (C–S); (D–S)

(d) (A–Q); (B–P); (C–S); (D–S)

Text

60. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find the values of n and r.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. The number of arrangements of the letters of the word BANANA in which the two N′s do not appear adjacently, is **(2002)**

(a) 40 (b) 60

(c) 80 (d) 100

2. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2s^4t^2$, then the number of ordered pairs (p, q) is **(2006)**

(a) 252 (b) 254

(c) 225 (d) 224

3. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is **(2007)**

(a) 360 (b) 192

(c) 96 (d) 48

4. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only, is **(2009)**

(a) 55 (b) 66

(c) 77 (d) 88

5. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets atleast one ball is **(2012)**

(a) 75 (b) 150

(c) 210 (d) 243

6. Six cards and six envelopes are numbered 1,2,3,4,5,6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is **(2014)**

(a) 264 (b) 265

(c) 53 (d) 67

7. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is **(2016)**

(a) 380 (b) 320

(c) 260 (d) 95

8. Let $S = \{1, 2, 3, \ldots, 9\}$. For k = 1, 2,…..,5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$ **(2017)**

(a) 125 (b) 210

(c) 252 (d) 126

Objective Questions II [One or more than one correct option]

9. For non-negative integers s and r, let

$$\binom{s}{r} = \begin{cases} \dfrac{s!}{r!(s-r)!} & if\ r \le s, \\ 0 & if\ r > s \end{cases}$$

For positive integers m and n, let

$$g(m,n) = \sum_{p=0}^{m+n} \frac{f(m,n,p)}{\binom{n+p}{p}}$$ where for any nonnegative integer p,

$$f(m,n,p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}$$

Then which of the following statements is/are TRUE? **(2020)**

(a) $g(m,n) = g(n,m)$ for all positive integers m,n

(b) $g(m,n+1) = g(m+1,n)$ for all positive integers m,n

(c) $g(2m,2n) = 2g(m,n)$ for all positive integers m,n

(d) $g(2m,2n) = (g(m,n))^2$ for all positive integers m,n

10. Let $S_1 = \{(i, j, k) : i, j, k \in \{1, 2, ..., 10\}\}$,

$S_2 = \{(i, j) : 1 \le i < j+2 \le 10, i, j \in \{1, 2, ..., 10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \le i < j < k < l, i, j, k, l \in \{1, 2, ..., 10\}\}$

And $S_4 = \{(i, j, k, l) : i, j, k$ and l are distinct elements in $\{1, 2, ..., 10\}\}$.

If the total number of elements in the set S_r is n_r, $r = 1, 2, 3, 4$. then which of the following statement is (are) TRUE? **(2021)**

(a) $n_1 = 1000$ (b) $n_2 = 44$

(c) $n_3 = 220$ (d) $\frac{n_4}{12} = 420$

Numerical Value Type Questions

11. Let *n* be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let *m* be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is **(2015)**

12. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$ **(2017)**

13. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is _______. **(2018)**

14. Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the numbers of ways of distributing the hats such that the person seated in adjacent seats get different coloured hats is **(2019)**

15. An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that no two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is _____. **(2020)**

16. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____ . **(2020)**

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

17. Consider all possible permutations of the letters of the word ENDEANOEL. **(2008)**

(A)	The number of permutations contianing the word ENDEA, is	**(P)**	$5!$
(B)	The number of permutations in which the letter E occurs in the first and the last positions, is	**(Q)**	$2 \times 5!$
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	**(R)**	$7 \times 5!$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions, is	**(S)**	$21 \times 5!$

The correct matching is :

(a) (A-P; B-S; C-Q; D-Q)

(b) (A-S; B-P; C-Q; D-Q)

(c) (A-Q; B-S; C-Q; D-P)

(d) (A-S; B-Q; C-P; D-Q)

18. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are NOT in the committee together. **(2018)**

Column A	Column B
(A) The value of α_1 is	**(P)** 136
(B) The value of α_2 is	**(Q)** 189
(C) The value of α_3 is	**(R)** 192
(D) The value of α_4 is	**(S)** 200
	(T) 381
	(U) 461

The correct matching is :

(a) (A–S; B–U; C–T; D–Q)
(b) (A–U; B–S; C–T; D–Q)
(c) (A–T; B–U; C–S; D–Q)
(d) (A–Q; B–T; C–U; D–S)

Using the following passage, solve Q.19 and Q.20

Passage

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = The number of such n-digit integers ending with digit 1 and c_n = The number of such n-digit integers ending with digit 0. **(2012)**

19. Which of the following is correct ?

(a) $a_{17} = a_{16} + a_{15}$
(b) $c_{17} \neq c_{16} + c_{15}$
(c) $b_{17} \neq b_{16} + c_{16}$
(d) $a_{17} = c_{17} + b_{16}$

20. The value of b_6 is

(a) 7
(b) 8
(c) 9
(d) 11

Answer Key

CHAPTER -13 | PERMUTATION AND COMBINATION

EXERCISE - 1 :
BASIC OBJECTIVE QUESTIONS

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)
6. (a)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (c)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)
16. (b)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (d)
21. (c)	**22.** (d)	**23.** (b)	**24.** (c)	**25.** (b)
26. (c)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (a)
31. (c)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (a)
36. (b)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (a)
41. (d)	**42.** (b)	**43.** (a)	**44.** (c)	**45.** (b)
46. (b)	**47.** (a)	**48.** (c)	**49.** (d)	**50.** (c)
51. (c)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (b)
56. (b)	**57.** (a)	**58.** (a)	**59.** (d)	**60.** (c)

EXERCISE - 2 :
PREVIOUS YEAR JEE MAIN QUESTIONS

1. (c)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (a)
6. (a)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (b)	**13.** (180)	**14.** (d)	**15.** (78)
16. (d)	**17.** (60)	**18.** (a)	**19.** (c)	**20.** (c)
21. (a)	**22.** (c)	**23.** (120)	**24.** (a)	**25.** (309)
26. (a)	**27.** (b)	**28.** (54)	**29.** (a)	**30.** (135)
31. (240)	**32.** (a)	**33.** (b)	**34.** (120)	**35.** (d)
36. (a)	**37.** (a)	**38.** (490)	**39.** (2454)	**40.** (a)
41. (777)	**42.** (9)	**43.** (238)	**44.** (924)	**45.** (720)
46. (96)	**47.** (d)	**48.** (16)	**49.** (d)	**50.** (a)
51. (77)	**52.** (100)	**53.** (136)	**54.** (52)	**55.** (80)
56. (576)	**57.** (5143)	**58.** (d)	**59.** (1000)	**60.** (c)
61. (d)	**62.** (32)	**63.** (45)	**64.** (31650)	**65.** (c)
66. (d)	**67.** (300)	**68.** (d)	**69.** (a)	**70.** (d)

ANSWER KEY

CHAPTER -13 | PERMUTATION AND COMBINATION

EXERCISE - 3 :
ADVANCED OBJECTIVE QUESTIONS

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (b)
6. (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (c)
11. (c)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)
16. (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (d)	**24.** (d)	**25.** (b)
26. (a)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (c)
31. (c)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (c)
36. (a)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (a,b)
41. (a,c,d)	**42.** (a,b,d)	**43.** (b,c)	**44.** (b,c,d)	
45. (b,c)	**46.** (b,d)	**47.** (b,d)	**48.** (a,b,c,d)	
49. (a,b,d)		**50.** (a,d)	**51.** (b,c,d)	**52.** (141)
53. (330)	**54.** (771)	**55.** (9)	**56.** (296)	**57.** (b)
58. (a)	**59.** (a)	**60.** (n = 9 and r = 3)		

EXERCISE - 4 :
PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)
6. (c)	**7.** (a)	**8.** (d)	**9.** (a,b,d)	
10. (a,b,d)	**11.** (5)	**12.** (5)	**13.** (625)	
14. (30)	**15.** (495)	**16.** (1080)		
17. (a)	**18.** (a)	**19.** (a)	**20.** (b)	