NURTURE COURSE

COMPOUND ANGLES

CONTENTS

COMPOUND ANGLES

JEE (Main) Syllabus :

Trigonometric Identities

JEE (Advanced) Syllabus :

Trigonometric functions, their periodicity and graphs, addition and subtraction formulae, formulae involving multiple and sub-multiple angles.

TRIGONOMETRIC RATIOS & IDENTITIES

1. INTRODUCTION TO TRIGONOMETRY :

The word 'trigonometry' is derived from the Greek words 'trigon' and 'metron' and it means 'measuring the sides of a triangle'. The subject was originally developed to solve geometric problems involving triangles. It was studied by sea captains for navigation, surveyor to map out the new lands, by engineers and others. Currently, trigonometry is used in many areas such as the science of seismology, designing electric circuits, describing the state of an atom, predicting the heights of tides in the ocean, analysing a musical tone and in many other areas.

(a) **Measurement of angles :** Commonly two systems of measurement of angles are used.

(i) **Sexagesimal or English System :** Here 1 right angle = 90° (degrees)

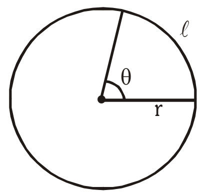
1° = 60' (minutes)

1' = 60" (seconds)

(ii) **Circular system :** Here an angle is measured in radians. One radian corresponds to the angle subtended by an arc of length 'r ' at the centre of the circle of radius r. It is a constant quantity and does not depend upon the radius of the circle.

(b) Relation between the these systems : $\frac{D}{90}=\frac{R}{\pi/2}$

(c) If θ is the angle subtended at the centre of a circle of radius 'r',



by an arc of length 'ℓ' then $\frac{\ell}{r}=\theta$.

Note that here ℓ, r are in the same units and θ is always in radians.

Illustration 1 : If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

Solution : Let r_1 and r_2 be the radii of the given circles and let their arcs of same length 's' subtend angles of 60° and 75° at their centres.

Now, $60°=\left(60\times\frac{\pi}{180}\right)^c=\left(\frac{\pi}{3}\right)^c$ and $75°=\left(75\times\frac{\pi}{180}\right)^c=\left(\frac{5\pi}{12}\right)^c$

$\therefore\quad \frac{\pi}{3}=\frac{s}{r_1}$ and $\frac{5\pi}{12}=\frac{s}{r_2}$

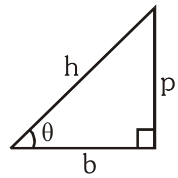
$\Rightarrow\quad \frac{\pi}{3}r_1=s$ and $\frac{5\pi}{12}r_2=s \Rightarrow \frac{\pi}{3}r_1=\frac{5\pi}{12}r_2 \Rightarrow 4r_1=5r_2 \Rightarrow \quad r_1 : r_2 = 5 : 4$ **Ans.**

Do yourself - 1 :

(i) The radius of a circle is 30 cm. Find the length of an arc of this circle if the length of the chord of the arc is 30 cm.

2. T-RATIOS (or Trigonometric functions) :

In a right angle triangle



$\sin\theta = \frac{p}{h}$; $\cos\theta = \frac{b}{h}$; $\tan\theta = \frac{p}{b}$; $\operatorname{cosec}\theta = \frac{h}{p}$; $\sec\theta = \frac{h}{b}$ and $\cot\theta = \frac{b}{p}$

'p' is perpendicular ; 'b' is base and 'h' is hypotenuse.

Note : The quantity by which the cosine falls short of unity i.e. $1 - \cos\theta$, is called the versed sine θ of θ and also by which the sine falls short of unity i.e. $1 - \sin\theta$ is called the coversed sine of θ.

3. BASIC TRIGONOMETRIC IDENTITIES :

(1) $\sin\theta . \operatorname{cosec}\theta = 1$

(2) $\cos\theta . \sec\theta = 1$

(3) $\tan\theta . \cot\theta = 1$

(4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(5) $\sin^2\theta + \cos^2\theta = 1$ or $\sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta = 1 - \sin^2\theta$

(6) $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta = 1 + \tan^2\theta$ or $\tan^2\theta = \sec^2\theta - 1$

(7) $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

(8) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ or $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ or $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

(9) $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

(10) Expressing trigonometrical ratio in terms of each other :

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}\ \theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec\ \theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}}{\sqrt{\operatorname{cosec}\ \theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan\ \theta}}{\tan\theta}$	$\sqrt{1+\cot\ \theta}$	$\frac{\sec\theta}{\sqrt{\sec\ \theta-1}}$	cosec

Illustration 2 : If $\sin\theta + \sin^2\theta = 1$, then prove that $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1 = 0$

Solution : Given that $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$

L.H.S. $= \cos^6\theta(\cos^2\theta + 1)^3 - 1 = \sin^3\theta(1 + \sin\theta)^3 - 1 = (\sin\theta + \sin^2\theta)^3 - 1 = 1 - 1 = 0$

Illustration 3 : $4(\sin^6\theta + \cos^6\theta) - 6(\sin^4\theta + \cos^4\theta)$ is equal to

(A) 0 (B) 1 (C) –2 (D) none of these

Solution : $4[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)] - 6[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$= 4[1 - 3\sin^2\theta\cos^2\theta] - 6[1 - 2\sin^2\theta\cos^2\theta]$

$= 4 - 12\sin^2\theta\cos^2\theta - 6 + 12\sin^2\theta\cos^2\theta = -2$ **Ans.(C)**

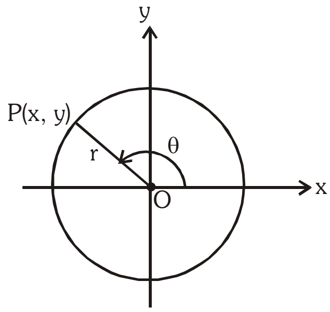
Do yourself - 2 :

(i) If $\cot\theta = \frac{4}{3}$, then find the value of $\sin\theta$, $\cos\theta$ and $\text{cosec}\theta$ in first quadrant.

(ii) If $\sin\theta + \text{cosec}\theta = 2$, then find the value of $\sin^8\theta + \text{cosec}^8\theta$

4. NEW DEFINITION OF T-RATIOS :

By using rectangular coordinates the definitions of trigonometric functions can be extended to angles of any size in the following way (see diagram). A point P is taken with coordinates (x, y). The radius vector OP has length r and the angle θ is taken as the directed angle measured anticlockwise from the x-axis. The three main trigonometric functions are then defined in terms of r and the coordinates x and y.



$\sin\theta = y/r$,

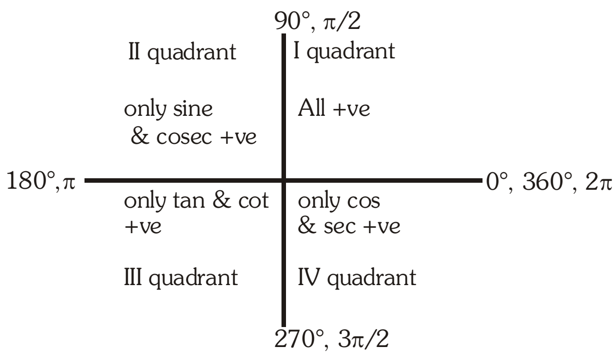
$\cos\theta = x/r$

$\tan\theta = y/x$,

(The other function are reciprocals of these)

This can give negative values of the trigonometric functions.

5. SIGNS OF TRIGONOMETRIC FUNCTIONS IN DIFFERENT QUADRANTS :



6. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

(a) $\sin(2n\pi + \theta) = \sin\theta$, $\cos(2n\pi + \theta) = \cos\theta$, where $n \in I$

(b)

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$
$\sin(90° - \theta) = \cos\theta$	$\cos(90° - \theta) = \sin\theta$
$\sin(90° + \theta) = \cos\theta$	$\cos(90° + \theta) = -\sin\theta$
$\sin(180° - \theta) = \sin\theta$	$\cos(180° - \theta) = -\cos\theta$
$\sin(180° + \theta) = -\sin\theta$	$\cos(180° + \theta) = -\cos\theta$
$\sin(270° - \theta) = -\cos\theta$	$\cos(270° - \theta) = -\sin\theta$
$\sin(270° + \theta) = -\cos\theta$	$\cos(270° + \theta) = \sin\theta$
$\sin(360° - \theta) = -\sin\theta$	$\cos(360° - \theta) = \cos\theta$
$\sin(360° + \theta) = \sin\theta$	$\cos(360° + \theta) = \cos\theta$

7. VALUES OF T-RATIOS OF SOME STANDARD ANGLES :

Angles / T-ratio	0°	30°	45°	60°	90°	180°	270°
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	N.D.	0	N.D.
$\cot\theta$	N.D.	$\sqrt{3}$	1	$1/\sqrt{3}$	0	N.D.	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D.	–1	N.D.
$\text{cosec}\theta$	N.D.	2	$\sqrt{2}$	$2/\sqrt{3}$	1	N.D.	–1

N.D. → Not Defined

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in I$

(b) $\sin(2n+1)\frac{\pi}{2} = (-1)^n$; $\cos(2n+1)\frac{\pi}{2} = 0$ where $n \in I$

Illustration 4 : If $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ then θ is equal to -

(A) 30° (B) 150° (C) 210° (D) none of these

Solution : Let us first find out θ lying between 0 and 360°.

Since $\sin\theta = -\frac{1}{2} \Rightarrow \theta = 210°$ or $330°$ and $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$ or $210°$

Hence, $\theta = 210°$ or $\frac{7\pi}{6}$ is the value satisfying both. **Ans. (C)**

Do yourself - 3 :

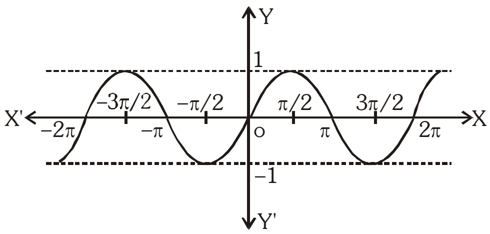
(i) If $\cos\theta = -\frac{1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, then find the value of $4\tan^2\theta - 3\text{cosec}^2\theta$.

(ii) Prove that : (a) $\cos 570° \sin 510° + \sin(-330°)\cos(-390°) = 0$

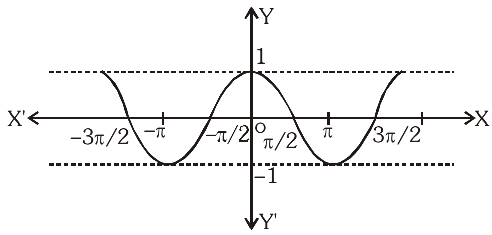
(b) $\tan\frac{11\pi}{3} - 2\sin\frac{9\pi}{3} - \frac{3}{4}\text{cosec}^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6} = \frac{3-2\sqrt{3}}{2}$

8. GRAPH OF TRIGONOMETRIC FUNCTIONS :

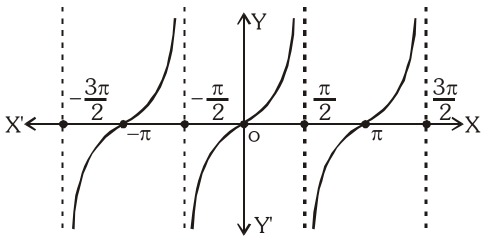
(i) y = sinx



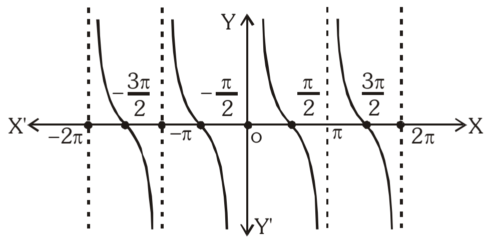
(ii) y = cosx



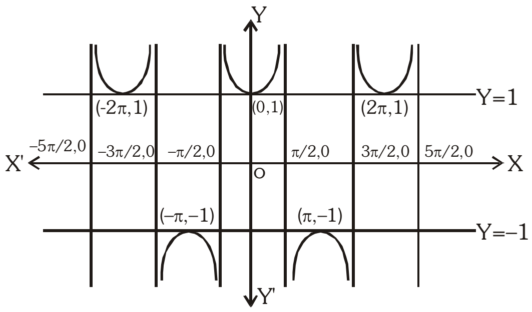
(iii) y = tanx



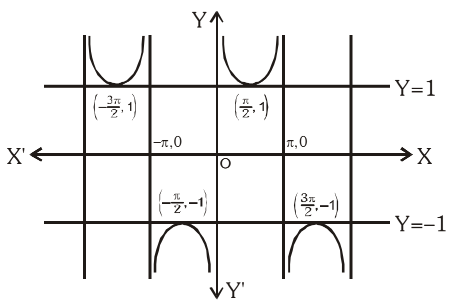
(iv) y = cotx



(v) y = secx



(vi) y = cosecx



9. DOMAINS, RANGES AND PERIODICITY OF TRIGONOMETRIC FUNCTIONS :

T-Ratio	Domain	Range	Period
sin x	R	[–1,1]	2π
cos x	R	[–1,1]	2π
tan x	R–{(2n+1)π/2 ; n∈I}	R	π
cot x	R–{nπ : n ∈ I}	R	π
sec x	R– {(2n+1) π/2 : n ∈ I}	(–∞,–1] ∪[1,∞)	2π
cosec x	R– {nπ : n ∈ I}	(–∞,–1] ∪[1,∞)	2π

10. TRIGONOMETRIC RATIOS OF THE SUM & DIFFERENCE OF TWO ANGLES :

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$. **(ii)** $\sin(A-B) = \sin A\cos B - \cos A\sin B$.

(iii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$ **(iv)** $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$ **(vi)** $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

(vii) $\cot(A+B) = \dfrac{\cot B\cot A - 1}{\cot B + \cot A}$ **(viii)** $\cot(A-B) = \dfrac{\cot B\cot A + 1}{\cot B - \cot A}$

Some more results :

(i) $\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B) = \cos^2 B - \cos^2 A$.

(ii) $\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$.

Illustration 5 : Prove that $\sqrt{3}$ cosec20° – sec20° = 4.

Solution :

$$\text{L.H.S.} = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ.\cos 20^\circ}$$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{2\sin 20^\circ\cos 20^\circ} = \frac{4(\sin 60.\cos 20^\circ - \cos 60^\circ.\sin 20^\circ)}{\sin 40^\circ}$$

$$= 4.\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = 4.\frac{\sin 40^\circ}{\sin 40^\circ} = 4 = \text{R.H.S.}$$

Illustration 6 : Prove that tan70° = cot70° + 2cot40° **.**

Solution :

$$\text{L.H.S.} = \tan 70^\circ = \tan(20^\circ + 50^\circ) = \frac{\tan 20^\circ + \tan 50^\circ}{1 - \tan 20^\circ\tan 50^\circ}$$

or tan70° – tan20° tan50° tan70° = tan20° + tan50°

or tan70° = tan70° tan50° tan20° + tan20° + tan50° = 2 tan 50° + tan20°

= cot70° + 2cot40° = R.H.S.

Do yourself - 4 :

(i) If $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{9}{41}$, $0 < A \,\&\, B < \dfrac{\pi}{2}$, then find the value of the following :

(a) sin(A + B) (b) sin(A – B) (c) cos(A + B) (d) cos(A – B)

(ii) If x + y = 45°, then prove that :

(a) (1 + tanx)(1 + tany) = 2 (b) (cotx – 1)(coty – 1) = 2

(Remember these results)

11. FORMULAE TO TRANSFORM THE PRODUCT INTO SUM OR DIFFERENCE :

(i) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$. **(ii)** $2\cos A\sin B = \sin(A+B) - \sin(A-B)$.

(iii) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$ **(iv)** $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Illustration 7: If sin2A = λ sin2B, then prove that $\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$.

Solution : Given sin2A = λ sin2B

$$\Rightarrow \frac{\sin 2A}{\sin 2B} = \frac{\lambda}{1}$$

Applying componendo & dividendo,

$$\frac{\sin 2A + \sin 2B}{\sin 2B - \sin 2A} = \frac{\lambda+1}{1-\lambda}$$

$$\Rightarrow \frac{2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)}{2\cos\left(\frac{2B+2A}{2}\right)\sin\left(\frac{2B-2A}{2}\right)} = \frac{\lambda+1}{1-\lambda}$$

$$\Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin\{-(A-B)\}} = \frac{\lambda+1}{1-\lambda} \quad \Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\times -\sin(A-B)} = \frac{\lambda+1}{-(\lambda-1)}$$

$$\Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin(A-B)} = \frac{\lambda+1}{\lambda-1} \quad \Rightarrow \tan(A+B)\cot(A-B) = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

12. FORMULAE TO TRANSFORM SUM OR DIFFERENCE INTO PRODUCT :

(i) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ii) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(iii) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(iv) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

Illustration 8 : $\frac{\sin 5\theta + \sin 2\theta - \sin\theta}{\cos 5\theta + 2\cos 3\theta + 2\cos^2\theta + \cos\theta}$ is equal to -

(A) tan θ (B) cos θ (C) cot θ (D) none of these

Solution : L.H.S.= $\frac{2\sin 2\theta\cos 3\theta + \sin 2\theta}{2\cos 3\theta.\cos 2\theta + 2\cos 3\theta + 2\cos^2\theta} = \frac{\sin 2\theta[2\cos 3\theta + 1]}{2[\cos 3\theta(\cos 2\theta + 1) + (\cos^2\theta)]}$

$$= \frac{\sin 2\theta[2\cos 3\theta + 1]}{2[\cos 3\theta(2\cos^2\theta) + \cos^2\theta]} = \frac{\sin 2\theta(2\cos 3\theta + 1)}{2\cos^2\theta(2\cos 3\theta + 1)} = \tan\theta$$

Ans. (A)

Illustration 9: Show that sin12°.sin48°.sin54° = 1/8

Solution : L.H.S. $= \frac{1}{2}[\cos 36° - \cos 60°]\sin 54° = \frac{1}{2}\left[\cos 36° \sin 54° - \frac{1}{2}\sin 54°\right]$

$$= \frac{1}{4}[2\cos 36° \sin 54° - \sin 54°] = \frac{1}{4}[\sin 90° + \sin 18° - \sin 54°]$$

$$= \frac{1}{4}[1 - (\sin 54° - \sin 18°)] = \frac{1}{4}[1 - 2\sin 18° \cos 36°]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 18°}{\cos 18°}\cos 18° \cos 36°\right] = \frac{1}{4}\left[1 - \frac{\sin 36° \cos 36°}{\cos 18°}\right]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 36° \cos 36°}{2\cos 18°}\right] = \frac{1}{4}\left[1 - \frac{\sin 72°}{2\sin 72°}\right] = \frac{1}{4}\left[1 - \frac{1}{2}\right] = \frac{1}{8} = \text{R.H.S.}$$

Do yourself - 5 :

(i) Simplify $\frac{\sin 75° - \sin 15°}{\cos 75° + \cos 15°}$

(ii) Prove that

(a) (sin3A + sinA)sinA + (cos3A – cosA)cosA = 0

(b) $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$

(c) $\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$

13. TRIGONOMETRIC RATIOS OF SUM OF MORE THAN TWO ANGLES :

(i) sin (A+B+C) = sinAcosBcosC + sinBcosAcosC + sinCcosAcosB – sinAsinBsinC

$= \Sigma \sin A \cos B \cos C - \Pi \sin A$

$= \cos A \cos B \cos C\,[\tan A + \tan B + \tan C - \tan A \tan B \tan C]$

(ii) cos (A+B+C) = cosA cosB cosC – sinA sinB cosC – sinA cosB sinC – cosA sinB sinC

$= \Pi \cos A - \Sigma \sin A \sin B \cos C$

$= \cos A \cos B \cos C\,[1 - \tan A \tan B - \tan B \tan C - \tan C \tan A]$

(iii) $\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}\ \ \frac{S_1 - S_3}{1 - S_2}$

14. TRIGONOMETRIC RATIOS OF MULTIPLE ANGLES :

(a) Trigonometrical ratios of an angle 2θ in terms of the angle θ :

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $1 + \cos 2\theta = 2\cos^2\theta$ **(iv)** $1 - \cos 2\theta = 2\sin^2\theta$

(v) $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta} = \dfrac{\sin 2\theta}{1+\cos 2\theta}$ **(vi)** $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Illustration 10: Prove that : $\dfrac{2\cos 2A+1}{2\cos 2A-1} = \tan(60°+A)\tan(60°-A)$.

Solution : R.H.S. = $\tan(60° + A)\tan(60° - A)$

$$= \left(\frac{\tan 60°+\tan A}{1-\tan 60°\tan A}\right)\left(\frac{\tan 60°-\tan A}{1+\tan 60°\tan A}\right) = \left(\frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A}\right)\left(\frac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}\right)$$

$$= \frac{3-\tan^2 A}{1-3\tan^2 A} = \frac{3-\dfrac{\sin^2 A}{\cos^2 A}}{1-3\dfrac{\sin^2 A}{\cos^2 A}} = \frac{3\cos^2 A-\sin^2 A}{\cos^2 A-3\sin^2 A} == \frac{2\cos^2 A+\cos^2 A-2\sin^2 A+\sin^2 A}{2\cos^2 A-2\sin^2 A-\sin^2 A-\cos^2 A}$$

$$= \frac{2(\cos^2 A-\sin^2 A)+\cos^2 A+\sin^2 A}{2(\cos^2 A-\sin^2 A)-(\sin^2 A+\cos^2 A)} = \frac{2\cos 2A+1}{2\cos 2A-1} = \text{L.H.S.}$$

Do yourself - 6 :

(i) Prove that :

(a) $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$ (b) $\dfrac{1+\sin 2\theta+\cos 2\theta}{1+\sin 2\theta-\cos 2\theta} = \cot\theta$

(b) Trigonometrical ratios of an angle 3θ in terms of the angle θ :

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **(ii)** $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(iii) $\tan 3\theta = \dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

Illustration 11: Prove that : tanA + tan(60° + A) + tan(120° + A) = 3tan3A

Solution : L.H.S. = tanA + tan(60° + A) + tan(120° + A)

= tanA + tan(60° + A) + tan{180° –(60° – A)}

= tanA + tan(60° + A) – tan(60° – A) [∵ tan(180° – θ) = –tanθ]

$$= \tan A + \frac{\tan 60° + \tan A}{1 - \tan 60° \tan A} - \frac{\tan 60° - \tan A}{1 + \tan 60° \tan A} = \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A + 3\tan A + \sqrt{3}\tan^2 A - \sqrt{3} + \tan A + 3\tan A - \sqrt{3}\tan^2 A}{(1 - \sqrt{3}\tan A)(1 + \sqrt{3}\tan A)}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A} = \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A} = 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right) = 3\tan 3A = \text{R.H.S.}$$

Do yourself - 7 :

(i) Prove that :

(a) $\cot\theta \cot(60° - \theta)\cot(60° + \theta) = \cot 3\theta$ (b) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(c) $\sin 4\theta = 4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta$

15. TRIGONOMETRIC RATIOS OF SUB MULTIPLE ANGLES :

Since the trigonometric relations are true for all values of angle θ, they will be true if instead of θ be substitute $\frac{\theta}{2}$

(i) $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \dfrac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

(ii) $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2} = \dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

(iii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

(iv) $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

(v) $\tan\frac{\theta}{2} = \dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1 + \cos\theta}$

(vi) $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

(vii) $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

(viii) $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

(ix) $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

(x) $2\sin\frac{\theta}{2} = \pm\sqrt{1+\sin\theta} \pm \sqrt{1-\sin\theta}$

(xi) $2\cos\frac{\theta}{2} = \pm\sqrt{1+\sin\theta} \mp \sqrt{1-\sin\theta}$

(xii) $\tan\frac{\theta}{2} = \frac{\pm\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

for **(vii)** to **(xii)**, we decide the sign of ratio according to value of θ.

Illustration 12: $\sin 67\frac{1}{2}^\circ + \cos 67\frac{1}{2}^\circ$ is equal to

(A) $\frac{1}{2}\sqrt{4+2\sqrt{2}}$ (B) $\frac{1}{2}\sqrt{4-2\sqrt{2}}$ (C) $\frac{1}{4}\left(\sqrt{4+2\sqrt{2}}\right)$ (D) $\frac{1}{4}\left(\sqrt{4-2\sqrt{2}}\right)$

Solution : $\sin 67\frac{1}{2}^\circ + \cos 67\frac{1}{2}^\circ = \sqrt{1+\sin 135^\circ} = \sqrt{1+\frac{1}{\sqrt{2}}}$ (using $\cos A + \sin A = \sqrt{1+\sin 2A}$)

$= \frac{1}{2}\sqrt{4+2\sqrt{2}}$

Ans.(A)

Do yourself - 8 :

(i) Find the value of

(a) $\sin\frac{\pi}{8}$ (b) $\cos\frac{\pi}{8}$ (c) $\tan\frac{\pi}{8}$

16. TRIGONOMETRIC RATIOS OF SOME STANDARD ANGLES :

(i) $\sin 18^\circ = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4} = \cos 72^\circ = \cos\frac{2\pi}{5}$

(ii) $\cos 36^\circ = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4} = \sin 54^\circ = \sin\frac{3\pi}{10}$

(iii) $\sin 72^\circ = \sin\frac{2\pi}{5} = \frac{\sqrt{10+2\sqrt{5}}}{4} = \cos 18^\circ = \cos\frac{\pi}{10}$

(iv) $\sin 36^\circ = \sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ = \cos\frac{3\pi}{10}$

(v) $\sin 15^\circ = \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ = \cos\frac{5\pi}{12}$

(vi) $\cos 15^\circ = \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ = \sin\frac{5\pi}{12}$

(vii) $\tan 15° = \tan\frac{\pi}{12} = 2-\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \cot 75° = \cot\frac{5\pi}{12}$

(viii) $\tan 75° = \tan\frac{5\pi}{12} = 2+\sqrt{3} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \cot 15° = \cot\frac{\pi}{12}$

(ix) $\tan(22.5°) = \tan\frac{\pi}{8} = \sqrt{2}-1 = \cot(67.5°) = \cot\frac{3\pi}{8}$

(x) $\tan(67.5°) = \tan\frac{3\pi}{8} = \sqrt{2}+1 = \cot(22.5°) = \cot\frac{\pi}{8}$

Illustration 13 : Evaluate sin78° – sin66° – sin42° + sin6°.

Solution : The expression $= (\sin 78° - \sin 42°) - (\sin 66° - \sin 6°) = 2\cos(60°)\sin(18°) - 2\cos 36°.\sin 30°$

$$= \sin 18° - \cos 36° = \left(\frac{\sqrt{5}-1}{4}\right) - \left(\frac{\sqrt{5}+1}{4}\right) = -\frac{1}{2}$$

Do yourself - 9 :

(i) Find the value of

(a) $\sin\frac{\pi}{10} + \sin\frac{13\pi}{10}$ (b) $\cos^2 48° - \sin^2 12°$

17. CONDITIONAL **TRIGONOMETRIC IDENTITIES :**

If A + B + C = 180°, then

(i) tan A + tan B + tan C = tan A tan B tan C

(ii) cot A cot B + cot B cot C + cot C cot A = 1

(iii) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(iv) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

(v) sin 2A + sin 2B + sin 2C = 4 sinA sinB sinC

(vi) cos 2A + cos 2B + cos 2C = –1–4 cosA cosB cosC

(vii) sin A + sin B + sin C = 4 $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(viii) cos A + cos B + cos C = 1 + 4 $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Illustration 14 : In any triangle ABC, sin A – cos B = cos C, then angle B is

(A) π/2 (B) π/3 (C) π/4 (D) π/6

Solution : We have , sin A – cos B = cos C

sin A = cos B + cos C

$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{\pi-A}{2}\right)\cos\left(\frac{B-C}{2}\right) \qquad \because \; A+B+C=\pi$

$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow \quad \cos\frac{A}{2} = \cos\frac{B-C}{2}$ or $A = B - C$; But $A + B + C = \pi$

Therefore $2B = \pi \Rightarrow B = \pi/2$ **Ans.(A)**

Illustration 15: If $A + B + C = \frac{3\pi}{2}$, then cos 2A + cos 2B + cos2C is equal to-

(A) 1 – 4cosA cosB cosC (B) 4 sinA sin B sinC

(C) 1 + 2cosA cosB cosC (D) 1 – 4 sinA sinB sinC

Solution : $\cos 2A + \cos 2B + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C$

$= 2\cos\left(\frac{3\pi}{2} - C\right)\cos(A-B) + \cos 2C \qquad \because \; A+B+C = \frac{3\pi}{2}$

$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C = 1 - 2\sin C\,[\cos(A-B) + \sin C]$

$= 1 - 2\sin C\left[\cos(A-B) + \sin\left(\frac{3\pi}{2} - (A+B)\right)\right]$

$= 1 - 2\sin C\,[\cos(A-B) - \cos(A+B)] = 1 - 4\sin A\sin B\sin C$

Ans.(D)

Do yourself - 10 :

(i) If ABCD is a cyclic quadrilateral, then find the value of sinA + sinB – sinC – sinD

(ii) If $A + B + C = \frac{\pi}{2}$, then find the value of tanA tanB + tanBtanC + tanC tanA

18. MAXIMUM & MINIMUM VALUES OF TRIGONOMETRIC EXPRESSIONS :

(i) $a\cos\theta + b\sin\theta$ will always lie in the interval $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$ i.e. the maximum and minimum values are $\sqrt{a^2+b^2}, -\sqrt{a^2+b^2}$ respectively.

(ii) Minimum value of $a^2\tan^2\theta + b^2\cot^2\theta = 2ab$ where $a, b > 0$

(iii) $-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)} \le a\cos(\alpha+\theta) + b\cos(\beta+\theta) \le \sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$ where α and β areknown angles.

(iv) In case a quadratic in $\sin\theta$ & $\cos\theta$ is given then the maximum or minimum values can be obtained by making perfect square.

Illustration 16: Prove that : $-4 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 10$, for all values of θ.

Solution : We have, $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) = 5\cos\theta + 3\cos\theta\cos\frac{\pi}{3} - 3\sin\theta\sin\frac{\pi}{3} = \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta$

Since, $-\sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$

$\Rightarrow \quad -7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$

$\Rightarrow \quad -7 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) \le 7$ for all θ.

$\Rightarrow \quad -7 + 3 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 7 + 3$ for all θ.

$\Rightarrow \quad -4 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 10$ for all θ.

Illustration 17: Find the maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$ -

(A) 1 (B) 2 (C) 3 (D) 4

Solution : We have $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$

$= 1 + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) + \sqrt{2}(\cos\theta + \sin\theta) = 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)(\cos\theta + \sin\theta)$

$= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) . \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$

$\therefore$ maximum value $= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) . \sqrt{2} = 4$ **Ans. (D)**

Do yourself - 11 :

(i) Find maximum and minimum value of $5\cos\theta + 3\sin\left(\theta + \frac{\pi}{6}\right)$ for all real values of θ.

(ii) Find the minimum value of $\cos\theta + \cos 2\theta$ for all real values of θ.

(iii) Find maximum and minimum value of $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$.

19. IMPORTANT RESULTS :

(i) $\sin\theta\sin(60° - \theta)\sin(60° + \theta) = \frac{1}{4}\sin 3\theta$

(ii) $\cos\theta . \cos(60° - \theta)\cos(60° + \theta) = \frac{1}{4}\cos 3\theta$

(iii) $\tan\theta\tan(60° - \theta)\tan(60° + \theta) = \tan 3\theta$

(iv) $\cot\theta\cot(60° - \theta)\cot(60° + \theta) = \cot 3\theta$

(v) (a) $\sin^2\theta + \sin^2(60° + \theta) + \sin^2(60° - \theta) = \frac{3}{2}$

(b) $\cos^2\theta + \cos^2(60° + \theta) + \cos^2(60° - \theta) = \frac{3}{2}$

(c) $\tan\theta + \tan(60° + \theta) + \tan(120° + \theta) = 3\tan 3\theta$

(vi) (a) If tan A + tan B + tan C = tan A tan B tan C, then A + B + C = nπ, n ∈ I

(b) If tan A tan B + tan B tan C + tan C tan A = 1, then $A + B + C = (2n + 1)\frac{\pi}{2}, n \in I$

(vii) $\cos\theta\cos 2\theta\cos 4\theta \ldots \cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

(viii) (a) cotA – tanA = 2cot2A (b) cotA + tanA = 2cosec2A

(ix) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots \sin(\alpha + \overline{n-1}\,\beta) = \frac{\sin\left\{\alpha + \left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$

(x) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha + 2\beta) + \ldots + \cos(\alpha + \overline{n-1}\,\beta) = \frac{\cos\left\{\alpha + \left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$

Do yourself - 12 :

(i) Evaluate $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots\ldots\ldots$ to n terms

Miscellaneous Illustration :

Illustration 18: Prove that

$\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Solution : We know $\tan\theta = \cot\theta - 2\cot 2\theta$ .....(i)

Putting $\theta = \alpha, 2\alpha, 2^2\alpha$, ...in (i), we get

$\tan\alpha = (\cot\alpha - 2\cot 2\alpha)$

$2(\tan 2\alpha) = 2(\cot 2\alpha - 2\cot 2^2\alpha)$

$2^2(\tan 2^2\alpha) = 2^2(\cot 2^2\alpha - 2\cot 2^3\alpha)$

....

$2^{n-1}(\tan 2^{n-1}\alpha) = 2^{n-1}(\cot 2^{n-1}\alpha - 2\cot 2^n\alpha)$

Adding,

$\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha = \cot\alpha - 2^n\cot 2^n\alpha$

∴ $\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Illustration 19: If A,B,C and D are angles of a quadrilateral and $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\sin\frac{D}{2}=\frac{1}{4}$, prove that $A = B = C = D = \pi/2$.

Solution :

$$\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right)\left(2\sin\frac{C}{2}\sin\frac{D}{2}\right)=1$$

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right\}\left\{\cos\left(\frac{C-D}{2}\right)-\cos\left(\frac{C+D}{2}\right)\right\}=1$$

Since, A + B = 2π – (C + D), the above equation becomes,

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right\}\left\{\cos\left(\frac{C-D}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right\}=1$$

$$\Rightarrow \cos^2\left(\frac{A+B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{C-D}{2}\right)\right\}+1-\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{C-D}{2}\right)=0$$

This is a quadratic equation in $\cos\left(\frac{A+B}{2}\right)$ which has real roots.

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{C-D}{2}\right)\right\}^2-4\left\{1-\cos\left(\frac{A-B}{2}\right).\cos\left(\frac{C-D}{2}\right)\right\}\geq 0$$

$$\left(\cos\frac{A-B}{2}+\cos\frac{C-D}{2}\right)^2\geq 4$$

$\Rightarrow \cos\frac{A-B}{2}+\cos\frac{C-D}{2}\geq 2$, Now both $\cos\frac{A-B}{2}$ and $\cos\frac{C-D}{2}\leq 1$

$$\Rightarrow \cos\frac{A-B}{2}=1\ \&\ \cos\frac{C-D}{2}=1$$

$$\Rightarrow \frac{A-B}{2}=0=\frac{C-D}{2}$$

⇒ A = B, C = D.

Similarly A = C, B = D ⇒ A = B = C = D = π/2

ANSWERS FOR DO YOURSELF

1 :	**(i)**	10π cm	**2 :**	**(i)**	$\frac{3}{5},\frac{4}{5},\frac{5}{3}$	**(ii)**	2	
3 :	**(i)**	8	**4 :**	**(i)**	(a) $\frac{187}{205}$	(b) $\frac{-133}{205}$	(c) $\frac{-84}{205}$	(d) $\frac{156}{205}$
5 :	**(i)**	$\frac{1}{\sqrt{3}}$	**8:**	**(i)**	(a) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$	(b) $\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$	(c) $\sqrt{2}-1$	
9:	**(i)**	(a) $-\frac{1}{2}$ (b) $\frac{\sqrt{5}+1}{8}$	**10 :**	**(i)**	0	**(ii)**	1	
11 :	**(i)**	7 & –7	**(ii)**	$-\frac{9}{8}$	**(iii)** $4+\sqrt{10}$ & $4-\sqrt{10}$	**12 :**	**(i)**	0

EXERCISE (O-1)

1. If sin x + $\sin^2$ x = 1, then the value of $\cos^2x + \cos^4x$ is -

(A) 0 (B) 2 (C) 1 (D) 3

2. $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4\theta) + 1$ is equal to -

(A) 2 (B) 0 (C) 4 (D) 6

3. If $\tan A = -\frac{1}{2}$ and $\tan B = -\frac{1}{3}$, (where A,B > 0), then A + B can be

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

4. $\cos^2 48° - \sin^2 12°$ is equal to -

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\frac{\sqrt{5}+1}{8}$ (C) $\frac{\sqrt{3}-1}{4}$ (D) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

5. The expression $\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta}$ is equals -

(A) tan θ (B) tan 2θ (C) sin 2θ (D) cos2θ

6. If 3 sin α = 5 sin β, then $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}} =$

(A) 1 (B) 2 (C) 3 (D) 4

7. $\frac{\sin(A-C)+2\sin A+\sin(A+C)}{\sin(B-C)+2\sin B+\sin(B+C)}$ is equal to -

(A) tan A (B) $\frac{\sin A}{\sin B}$ (C) $\frac{\cos A}{\cos B}$ (D) $\frac{\sin C}{\cos B}$

8. $\frac{1+\sin 2\theta+\cos 2\theta}{1+\sin 2\theta-\cos 2\theta} =$

(A) $\frac{1}{2}\tan\theta$ (B) $\frac{1}{2}\cot\theta$ (C) tan θ (D) cot θ

9. If A = tan 6º tan 42º and B = cot 66º cot 78º, then -

(A) A = 2B (B) A = 1/3B (C) A = B (D) 3A = 2B

10. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, then xy + yz + zx =

(A) –1 (B) 0 (C) 1 (D) 2

11. If $\tan\alpha = (1+2^{-x})^{-1}$, $\tan\beta = (1+2^{x+1})^{-1}$, then α + β =

(A) π/6 (B) π/4 (C) π/3 (D) π/2

12. If tan A + tan B + tan C = tan A. tan B. tan C, then -

(A) A,B,C must be angles of a triangle

(B) the sum of any two of A,B,C is equal to the third

(C) A+B+C must be n integral multiple of π

(D) None of these

13. The value of sin10º + sin20º + sin30º+....+ sin 360º is equal to -

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) 2

14. The number of real solutions of the equation $\sin(e^x) = 2^x + 2^{-x}$ is -

(A) 1 (B) 0 (C) 2 (D) Infinite

15. If $f(x) = \frac{\sin 3x}{\sin x}$, $x \neq n\pi$, then the range of values of f(x) for real values of x is -

(A) [–1,3] (B) (–∞,–1] (C) (3, + ∞) (D) [–1,3)

16. If cos x + cos y + cos α = 0 and sin x + sin y + sin α = 0, then $\cot\left(\frac{x+y}{2}\right) =$

(A) sin α (B) cos α (C) cot α (D) 2 sin α

17. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is :-

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1

18. Maximum and minimum value of $2\sin^2\theta - 3\sin\theta + 2$ is -

(A) $\frac{1}{4}, -\frac{7}{4}$ (B) $\frac{1}{4}, \frac{21}{4}$ (C) $\frac{21}{4}, -\frac{3}{4}$ (D) $7, \frac{7}{8}$

19. For θ ∈ (0, π/2), the maximum value of $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$ is attained at θ =

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

20. Minimum value of the expression $\cos^2\theta - (6\sin\theta\cos\theta) + 3\sin^2\theta + 2$, is -

(A) $4+\sqrt{10}$ (B) $4-\sqrt{10}$ (C) 0 (D) 4

EXERCISE (O-2)

1. If $x + y = 3 - \cos 4\theta$ and $x - y = 4\sin 2\theta$ then

(A) $x^4 + y^4 = 9$ (B) $\sqrt{x}+\sqrt{y}=16$ (C) $x^3 + y^3 = 2(x^2 + y^2)$ (D) $\sqrt{x}+\sqrt{y}=2$

2. If $\tan B = \frac{n\sin A\cos A}{1-n\cos^2 A}$ then tan(A + B) equals

(A) $\frac{\sin A}{(1-n)\cos A}$ (B) $\frac{(n-1)\cos A}{\sin A}$ (C) $\frac{\sin A}{(n-1)\cos A}$ (D) $\frac{\sin A}{(n+1)\cos A}$

3. If $A = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and $B = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$ then $\sqrt{A^2+B^2}$ is equal to

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{3}$

4. If $\cos\alpha = \frac{2\cos\beta - 1}{2-\cos\beta}$ then $\tan\frac{\alpha}{2}\cdot\cot\frac{\beta}{2}$ has the value equal to {where α, β ∈ (0, π)}

(A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\sqrt{3}$

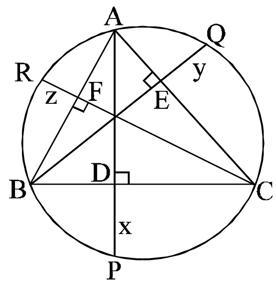
5. If $\frac{\cos 3x}{\cos x} = \frac{1}{3}$ for some angle x, $0 \le x \le \frac{\pi}{2}$, then the value of $\frac{\sin 3x}{\sin x}$ for some x, is

(A) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) 1 (D) $\frac{2}{3}$

6. If $\frac{5\pi}{2} < x < 3\pi$, then the value of the expression $\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}$ is

(A) $-\cot\frac{x}{2}$ (B) $\cot\frac{x}{2}$ (C) $\tan\frac{x}{2}$ (D) $-\tan\frac{x}{2}$

7. As shown in the figure AD is the altitude on BC and AD produced meets the circumcircle of ΔABC at P where DP = x. Similarly EQ = y and FR = z. If a, b, c respectively denotes the sides BC, CA and AB then $\frac{a}{2x}+\frac{b}{2y}+\frac{c}{2z}$ has the value equal to



(A) tanA + tanB + tanC (B) cotA + cotB + cotC

(C) cosA + cosB + cosC (D) cosecA + cosecB + cosecC

8. The exact value of $\frac{96 \sin 80^\circ \sin 65^\circ \sin 35^\circ}{\sin 20^\circ + \sin 50^\circ + \sin 110^\circ}$ is equal to

(A) 12 (B) 24 (C) –12 (D) 48

9. The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$ is equal to :

(A) $\cot 3x$ (B) $\tan 3x$ (C) $3\tan 3x$ (D) $\frac{3-9\tan^2 x}{3\tan x - \tan^3 x}$

10. The value of $\operatorname{cosec}\frac{\pi}{18} - \sqrt{3}\sec\frac{\pi}{18}$ is a

(A) surd (B) rational which is not integral

(C) negative integer (D) natural number

11. If tan x + tan y = 25 and cot x + cot y = 30, then the value of tan(x + y) is

(A) 150 (B) 200 (C) 250 (D) 100

12. If $\cos(\alpha+\beta) + \sin(\alpha-\beta) = 0$ and $2010\tan\beta + 1 = 0$, then $\tan\alpha$ is equal to

(A) 1 (B) –1 (C) 2010 (D) $\frac{1}{2010}$

13. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(A) $\frac{\pi}{3}$ and $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ and $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ and $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ and $\frac{3\pi}{10}$

14. The value of $\cot 7\frac{1}{2}^\circ + \tan 67\frac{1}{2}^\circ - \cot 67\frac{1}{2}^\circ - \tan 7\frac{1}{2}^\circ$ is :

(A) a rational number (B) irrational number

(C) $2(3+2\sqrt{3})$ (D) $2(3-\sqrt{3})$

15. If m and n are positive integers satisfying

$$1+\cos 2\theta+\cos 4\theta+\cos 6\theta+\cos 8\theta+\cos 10\theta=\frac{\cos m\theta\cdot\sin n\theta}{\sin\theta}$$ then $(m+n)$ is equal to

(A) 9 (B) 10 (C) 11 (D) 12

Paragraph for Question Nos. 16 to 18

Consider the polynomial $P(x) = (x-\cos 36°)(x-\cos 84°)(x-\cos 156°)$

16. The coefficient of x^2 is

(A) 0 (B) 1 (C) $-\frac{1}{2}$ (D) $\frac{\sqrt{5}-1}{2}$

17. The coefficient of x is

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $-\frac{3}{4}$ (D) zero

18. The absolute term in P(x) has the value equal to

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\frac{\sqrt{5}-1}{16}$ (C) $\frac{\sqrt{5}+1}{16}$ (D) $\frac{1}{16}$

Multiple Objective Type :

19. Let $y=\frac{\cos x+\cos 2x+\cos 3x+\cos 4x+\cos 5x+\cos 6x+\cos 7x}{\sin x+\sin 2x+\sin 3x+\sin 4x+\sin 5x+\sin 6x+\sin 7x}$, then which of the following hold good?

(A) The value of y when $x = \pi/8$ is not defined.
(B) The value of y when $x = \pi/16$ is 1.
(C) The value of y when $x = \pi/32$ is $\sqrt{2}-1$.
(D) The value of y when $x = \pi/48$ is $2+\sqrt{3}$.

20. Two parallel chords are drawn on the same side of the centre of a circle of radius R . It is found that they subtend an angle of θ and 2 θ at the centre of the circle . The perpendicular distance between the chords is

(A) $2R\sin\frac{3\theta}{2}\sin\frac{\theta}{2}$
(B) $\left(1-\cos\frac{\theta}{2}\right)\left(1+2\cos\frac{\theta}{2}\right)R$
(C) $\left(1+\cos\frac{\theta}{2}\right)\left(1-2\cos\frac{\theta}{2}\right)R$
(D) $2R\sin\frac{3\theta}{4}\sin\frac{\theta}{4}$

EXERCISE (S-1)

1. Prove that : $\cos^2\alpha+\cos^2(\alpha+\beta)-2\cos\alpha\cos\beta\cos(\alpha+\beta)=\sin^2\beta$

2. Prove that : $\cos 2\alpha=2\sin^2\beta+4\cos(\alpha+\beta)\sin\alpha\sin\beta+\cos 2(\alpha+\beta)$

3. Prove that : $\tan\alpha+2\tan 2\alpha+4\tan 4\alpha+8\cot 8\alpha=\cot\alpha$.

4. Prove that : (a) $\tan 20°\,.\tan 40°\,.\tan 60°\,.\tan 80°=3$

(b) $\tan 9°-\tan 27°-\tan 63°+\tan 81°=4$. (c) $\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}=\frac{3}{2}$

5. If $X=\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$, $Y=\cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$, then prove that $\frac{X}{Y}-\frac{Y}{X}=2\tan 2\theta$.

6. If $m\tan(\theta-30°)=n\tan(\theta+120°)$, show that $\cos 2\theta=\dfrac{m+n}{2(m-n)}$.

7. If $\cos(\alpha+\beta)=\dfrac{4}{5}$; $\sin(\alpha-\beta)=\dfrac{5}{13}$ & α,β lie between 0 & $\dfrac{\pi}{4}$, then find the value of $\tan 2\alpha$.

8. If the value of the expression sin25°. sin35°.sin85° can be expressed as $\dfrac{\sqrt{a}+\sqrt{b}}{c}$, where $a,b,c\in N$ and are in their lowest form, find the value of $(a+b+c)$.

9. Prove that $(4\cos^2 9°-3)(4\cos^2 27°-3)=\tan 9°$.

10. If $A+B+C=\pi$, prove that $\sum\left(\dfrac{\tan A}{\tan B.\tan C}\right)=\sum(\tan A)-2\sum(\cot A)$.

11. If $\alpha+\beta=\gamma$, prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1+2\cos\alpha\cos\beta\cos\gamma$.

12. Let $P(k)=\left(1+\cos\dfrac{\pi}{4k}\right)\left(1+\cos\dfrac{(2k-1)\pi}{4k}\right)\left(1+\cos\dfrac{(2k+1)\pi}{4k}\right)\left(1+\cos\dfrac{(4k-1)\pi}{4k}\right)$, then find the value of **(a)** P(5) and **(b)** P(6).

13. Calculate without using trigonometric tables :

(a) $4\cos 20°-\sqrt{3}\cot 20°$ (b) $\dfrac{2\cos 40°-\cos 20°}{\sin 20°}$

(c) $\cos^6\dfrac{\pi}{16}+\cos^6\dfrac{3\pi}{16}+\cos^6\dfrac{5\pi}{16}+\cos^6\dfrac{7\pi}{16}$ (d) $\tan 10°-\tan 50°+\tan 70°$

14. Given that $(1+\tan 1°)(1+\tan 2°)....(1+\tan 45°)=2^n$, find n.

15. In a right angled triangle, acute angles A and B satisfy

$$\tan A+\tan B+\tan^2 A+\tan^2 B+\tan^3 A+\tan^3 B=70$$

find the angle A and B in radians.

EXERCISE (S-2)

1. (a) If $y=10\cos^2 x-6\sin x\cos x+2\sin^2 x$, then find the greatest & least value of y.

(b) If $y=1+2\sin x+3\cos^2 x$, find the maximum & minimum values of y $\forall\ x\in R$.

(c) If $y=9\sec^2 x+16\operatorname{cosec}^2 x$, find the minimum value of y for all permissible value of x.

(d) If $a\le 3\cos\left(\theta+\dfrac{\pi}{3}\right)+5\cos\theta+3\le b$, find a and b, where a is the minimum value & b is the maximum value.

2. Let $k=1°$, then prove that $\displaystyle\sum_{n=0}^{88}\frac{1}{\cos nk.\cos(n+1)k}=\frac{\cos k}{\sin^2 k}$

3. If $A+B+C=\pi$; prove that $\tan^2\dfrac{A}{2}+\tan^2\dfrac{B}{2}+\tan^2\dfrac{C}{2}\ge 1$.

4. (a) If $4\sin x.\cos y+2\sin x+2\cos y+1=0$ where $x,y\in[0,2\pi]$ find the largest possible value of the sum $(x+y)$.

(b) If M and m denotes maximum and minimum value of $\sqrt{49\cos^2\theta+\sin^2\theta}+\sqrt{49\sin^2\theta+\cos^2\theta}$ then find the value of $(M+m)$.

5. Determine the smallest positive value of x (in degrees) for which

$\tan(x + 100°) = \tan(x + 50°) \tan x \tan(x - 50°)$.

6. Find the positive integers p,q,r,s satisfying $\tan\frac{\pi}{24} = \left(\sqrt{p} - \sqrt{q}\right)\left(\sqrt{r} - s\right)$.

7. If the product (sin 1°) (sin 3°) (sin 5°) (sin 7°)..........(sin 89°) = $\frac{1}{2^n}$, then find the value of n.

8. If $f(\theta) = \sum_{n=1}^{6} \text{cosec}\left(\theta + \frac{(n-1)\pi}{4}\right)\text{cosec}\left(\theta + \frac{n\pi}{4}\right)$, where $0 < \theta < \frac{\pi}{2}$, then find the minimum value of f(θ).

9. Let $x_1 = \prod_{r=1}^{5} \cos\frac{r\pi}{11}$ and $x_2 = \sum_{r=1}^{5} \cos\frac{r\pi}{11}$, then show that $x_1 . x_2 = \frac{1}{64}\left(\text{cosec}\frac{\pi}{22} - 1\right)$, where Π denotes the continued product.

10. If x and y are real number such that $x^2 + 2xy - y^2 = 6$, find the minimum value of $(x^2 + y^2)^2$.

11. Find the exact value of $\tan^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{5\pi}{16} + \tan^2\frac{7\pi}{16}$

12. If 'θ' is eliminated from the equations $\cos\theta - \sin\theta = b$ and $\cos 3\theta + \sin 3\theta = a$, find the eliminant.

13. Given that $3 \sin x + 4 \cos x = 5$ where $x \in (0,\pi/2)$. Find the value of $2 \sin x + \cos x + 4 \tan x$.

EXERCISE (JM)

1. If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then tan x is- **[AIEEE-2006]**

(1) $(4-\sqrt{7})/3$ (2) $-(4+\sqrt{7})/3$ (3) $(1+\sqrt{7})/4$ (4) $(1-\sqrt{7})/4$

2. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[AIEEE-2010]**

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$ (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

3. If $A = \sin^2 x + \cos^4 x$, then for all real x :- **[AIEEE-2011]**

(1) $1 \le A \le 2$ (2) $\frac{3}{4} \le A \le \frac{13}{16}$ (3) $\frac{3}{4} \le A \le 1$ (4) $\frac{13}{16} \le A \le 1$

4. In a ΔPQR, if 3 sinP + 4 cosQ = 6 and 4 sinQ + 3 cos P = 1, then the angle R is equal to : **[AIEEE-2012]**

(1) $\frac{3\pi}{4}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

5. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as **[JEE-MAIN 2013]**

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

6. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to **[JEE-MAIN 2013]**

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$ (3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

7. Let $f_K(x) = \frac{1}{k}\left(\sin^k x+\cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals :

[JEE-MAIN 2014]

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

8. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is :- **[JEE-MAIN 2017]**

(1) $-\frac{7}{9}$ (2) $-\frac{3}{5}$ (3) $\frac{1}{3}$ (4) $\frac{2}{9}$

EXERCISE (JA)

1. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}, t_4 = (\cot\theta)^{\cot\theta}$, then -

[JEE 06,3M,–1M]

(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_4 > t_3 > t_1 > t_2$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

One or more than one is/are correct : [Q.5(a) & (b)]

2. **(a)** If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then **[JEE 2009, 4 + 4]**

(A) $\tan^2 x = \frac{2}{3}$ (B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$ (D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

(b) For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\text{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\text{cosec}\left(\theta+\frac{m\pi}{4}\right) = 4\sqrt{2}$ is (are) -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

3. **(a)** The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

(b) Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$ where k > 0, then the value of [k] is -

[**Note :** [k] denotes the largest integer less than or equal to k] **[JEE 2010, 3+3]**

4. Let $P=\{\theta:\sin\theta-\cos\theta=\sqrt{2}\cos\theta\}$ and $Q=\{\theta:\sin\theta+\cos\theta=\sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P\subset Q$ and $Q-P\neq\varnothing$ (B) $Q\not\subset P$

(C) $P\not\subset Q$ (D) $P = Q$ **[JEE 2011,3]**

5. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **[JEE(Advanced)-2016, 3(–1)]**

(A) $3-\sqrt{3}$ (B) $2(3-\sqrt{3})$ (C) $2(\sqrt{3}-1)$ (D) $2(2+\sqrt{3})$

ANSWER KEY

EXERCISE (O-1)

1. C **2.** B **3.** D **4.** B **5.** B **6.** D **7.** B **8.** D
9. C **10.** B **11.** B **12.** C **13.** A **14.** B **15.** D **16.** C
17. B **18.** D **19.** A **20.** B

EXERCISE (O-2)

1. D **2.** A **3.** B **4.** D **5.** A **6.** D **7.** A **8.** B
9. D **10.** D **11.** A **12.** B **13.** B **14.** B **15.** C **16.** A
17. C **18.** B **19.** B,D **20.** B,D

EXERCISE (S-1)

7. $\frac{56}{33}$ **8.** 24 **12.** (a) $\frac{3-\sqrt{5}}{32}$; (b) $\frac{2-\sqrt{3}}{16}$ **13.** (a) –1, (b) $\sqrt{3}$, (c) $\frac{5}{4}$, (d) $\sqrt{3}$

14. n = 23 **15.** $\frac{\pi}{12}$ and $\frac{5\pi}{12}$

EXERCISE (S-2)

1. (a) $y_{max}=11$, $y_{min}=1$; (b) $y_{max}=\frac{13}{3}$, $y_{min}=-1$; (c) 49; (d) a = –4 & b = 10 **4.** (a) $\frac{23\pi}{6}$ (b) 18

5. x = 30° **6.** p = 3, q = 2; r = 2; s = 1 **7.** $\frac{89}{2}$ **8.** $2\sqrt{2}$ **10.** 18

11. 28 **12.** $a = 3b - 2b^3$ **13.** 5

EXERCISE (JM)

1. 2 **2.** 2 **3.** 3 **4.** 3 **5.** 2 **6.** 1 **7.** 4 **8.** 1

EXERCISE (JA)

1. B **2.** **(a)** A, B; **(b)** C,D **3.** (a) 2; (b) k = 3 **4.** D **5.** C

Important Notes