5.ARITHMETIC PROGRESSIONS

1. If a, (a - 2) and 3a are in AP, then the Value of a is:
(a) -3
(b) -2
(c) 3
(d) 2

2. What is the common difference of AP in which $a_{21} - a_7 = 84$?

3. Calculate the common difference of AP: $\frac{1}{2b}, \frac{1-6b}{2b}, \frac{1-12b}{2b}, \dots \dots$

4. Which is the first negative term of the AP: 35, 30, 25, 20...?
(a) 7th Term
(b) 5th Term
(c) 9th Term
(d) 11th Term

5. Find the next term of the arithmetic progression: $\sqrt{12}, \sqrt{27}, \sqrt{48}$,......
(a) $\sqrt{75}$
(b) $\sqrt{60}$
(c) $\sqrt{80}$
(d) $\sqrt{90}$

6. For what value of 'k' will k + 9, 2k – 1 & 2k + 7 are the consecutive terms of an AP?

7. How many terms of the AP 27, 24, 21, ... should be taken so that their sum is zero?

8. Find the sum of first 8 multiples of 3.

9. In an AP, if $S_5 + S_7 = 167$ & $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first 'n' terms.

10. The first & the last terms of an AP are 7 & 49 respectively. If sum of all its terms is 420, find its common difference.

11. Find the number of natural numbers between 101 & 999 which are divisible by both 2 & 5.

12. Find the sum of n terms of the series $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\dots\dots\dots$

13. If the sum of the first n terms of an AP is $\frac{1}{2}(3n^2+7n)$, then find its nth term. Hence write its 20th term.

14. If Sn denotes the sum of first n terms of an AP. Prove that $S12 = 3(S8 - S4)$.

15. If the sum of first p terms of an A.P. is the same as the first q terms (where $p \neq q$), then show that the sum of first (p + q) terms is zero.

16. The ratio of the sums of first m & first n terms of an AP is $m^2 : n^2$. Show that the ratio of its mth & nth terms is $(2m - 1) : (2n - 1)$.

17. The sum of three numbers in an AP is 12 & sum of their cubes is 288. Find the numbers.

18. The sum of four consecutive numbers in an AP. Is 32 & the ratio of the product of the first & the last term to the product of two middle terms is 7:15. Find the numbers.

HINTS

1. Since a, a-2 and 3a are in AP

$\therefore a-2-a=3a-(a-2)$

$\Rightarrow 2(a-2)=a+3a$

$\Rightarrow 2a-4=4a$

$\Rightarrow 2a=-4$

$\Rightarrow a=-2$

2. 2. Let the common difference of an A.P. be d.

Then,

a18=a1+17×d

a14=a1+13×d

Solving the two equations,

a18−a14=a1+17d−a1−13d

⇒a18−a14=4d

Substituting 4d=32,

⇒d=8

3. D=a_2-a_1

ie,d=1-6b/2b - 1/2b

1-6b-1/2b

-6b/2b

-3 is the answer

4. Here, a = 18, d = $-\frac{5}{2}$

a_n = a + (n - 1) d

=> - 47 = 18 + (n - 1) $-\frac{5}{2}$

=>5n/2 = 18 + 47 + 5/2 = 67.5

Hence, it is 27th term

5. write √12 as 2√3, √27 as 3√3, √48 as 4√3.

So this forms a AP with common difference =√3

next term will be 5√3=75

6. Let,

k + 9 = a

2k - 1 = b

2k + 7 = c

To be in AP,

a + c = 2b

(k + 9) + (2k + 7) = 2(2k - 1)

k + 9 + 2k + 7 = 4k - 2

3k + 16 = 4k - 2

3k - 4k = - 2 - 16

- k = - 18

k = 18

For k = 18, the terms k+9 , 2k - 1 , 2k + 7 are in AP

7. Let first term be a=27

And common difference be d=−3

According to question, sum is zero,

⟹n/2[2a+(n−1)d]=0

⟹[54+(n−1)(−3)]=0

⟹n=19

Hence, 19 terms of AP should be taken to make sum zero.

8. First 8 multiples of 3-

3,6,9,.....upto 8 terms

The above series is in A.P. where,

First term (a)=3

Common difference (d)=3

No. of terms (n)=8

Sum of terms (Sn)=?

As we know that, in an A.P.,

$S_n=n/2[2a+(n-1)d]$

$\therefore S_8=8/2[2\times3+(8-1)\times3]$

$\Rightarrow S_8=4\times(6+21)$

$\Rightarrow S_8=4\times27=108$

9. Let the first term is a and the common difference is d

By using $S_n=n/2[2a+(n-1)d]$ we have,

$S_5=5/2[2a+(5-1)d]$

$=5/2[2a+4d]$

$S_7=7/2[2a+(7-1)d]=7/2[2a+6d]$

Given: $S_7+S_5=167$

$\therefore 5/2[2a+4d]+7/2[2a+6d]=167$

$\Rightarrow 10a+20d+14a+42d=334$

$\Rightarrow 24a+62d=334 \quad ...(1)$

$S_{10}=10/2[2a+(10-1)d]=5(2a+9d)$

Given: $S_{10}=235$

So $5(2a+9d)=235$

$\Rightarrow 2a+9d=47 \quad ...(2)$

Multiply equation (2) by 12, we get

$24a+108d=564....(3)$

Subtracting equation (3) from (1), we get

$-46d=-230$

$\therefore d=5$

Substing the value of d=5 in equation (1) we get

$2a+9(5)=47$ or $2a=2$

$\therefore a=1$

Then A.P is 1,6,11,16,21,⋯

10. a=7 l=49 Sn=420

$S_n=n/2[a+l]$

So $420\times2=n[7+49]$

n=15

$l=a+(n-1)d$

$\Rightarrow 49=7+14\,d$

$\Rightarrow 7=1+2\,d \Rightarrow 2d=6$

$\Rightarrow d=3$

11. The list of numbers between 101 and 999 that are divisible by 2 and 5 are:

110,120,130,...990

The numbers are in A.P, with first term, $a=110$, common difference, $d=10$

Last term, $a_n=990$

We know that, $a_n=a+(n-1)d$

$990=110+(n-1)10$

$\Rightarrow 990-110=10n-10$

$\Rightarrow 880+10=10n$

$\Rightarrow 890=10n$

$\Rightarrow n=89$

Therefore, the number of terms between 101 and 999 that are divisible by 2 and 5 are 89.

12. (4 + 4 + 4 + 4 + 4 + ... upto n terms) + (-1/n - 2/n - 3/n - ....upto n terms)

= 4 (1+1+1+1.......... upto n terms) - 1/n (1 + 2 + 3 +4 ....upto n terms)

13. $S_n=1/2(3n^2+7n)$

$S_1=1/2(3+7)=5$

$S_2=1/2(3*4+7*2)=26/2=13$

We know

$S_1=a_1=5$

$S_2=a_1+a_2=13$

$S_2-s_1=a_1+a_2-a_1$

$13-5=a_2$

$a_2=8$

We know $d=a_2-a_1$

$d=8-5=3$

nth term of AP $=a_n=5+(n-1)3$

$a_n= 2+3n$

Therefore 20th term =

$a_{20}= 2+3(20)=62$

Hence 20th term of AP is 62

14. let a is the first term of Ap and d is the common difference

$S_n=n/2 \{2a+(n-1) d$

now $S_{12}=12/2 \{2a+(12-1) d\}=12a+66d$

$S_8=8/2 \{2a+7d\}=8a+28d$

$S_4=4/2 \{2a+3d\}=4a+6d$

$LHS=S_{12}=12a+66d$

$RHS=3 (S_8-S_4)=3 (8a+28d-4a-6d)=12a+66d$

LHS =RHS

15. $S_p=S_q$

$\Rightarrow p/2(2a+(p-1)d)=q/2(2a+(q-1)d)$

$\Rightarrow p(2a+(p-1)d)=q(2a+(q-1)d)$

$\Rightarrow 2ap+p^2d-pd=2aq+q^2d-qd$

$\Rightarrow 2a(p-q)+(p+q)(p-q)d-d(p-q)=0$

$\Rightarrow (p-q)[2a+(p+q)d-d]=0$

$\Rightarrow 2a+(p+q)d-d=0$

$\Rightarrow 2a+((p+q)-1)d=0$

$\Rightarrow$ Sp+q=0

16. (HINT)Let Sm and Sn be the sum of the first m and first n terms of the AP respectively. Let, a be the first term and d be a common difference

$Sn/Sm=n^2/m^2$

17. Hera 3a=12

a=4

Also $(a-d)^3+a^3+(a+d)^3=288$,

or $^{3a3}+6ad^2=288$

$24d^2=288-3\times64=96$

$d^2=4$

$d=\pm2$

Hence the numbers are 2,4,6 or 6,4,2

18. Let the four consecutive numbers in AP be (a−3d),(a−d),(a+d) and (a+3d)

So, according to the question.

a−3d+a−d+a+d+a+3d=32

4a=32

a=32/4

a=8......(1)

Now, (a−3d)(a+3d)/(a−d)(a+d)=7/15

$15(a^2-9d^2)=7(a^2-d^2)$

$15a^2-135d^2=7a^2-7d^2$

$15a^2-7a^2=135d^2-7d^2$

$8a^2=128d^2$

Putting the value of a=8 in above we get.

$8(8)^2=128d^2$

$128d^2=512$

$d^2=512/128$

$d^2=4$

d=2

So, the four consecutive numbers are

8−(3×2)

8−6=2

8−2=6

8+2=10

8+(3×2)

8+6=14

Four consecutive numbers are 2,6,10and14.