Boolean Algebra

Q.1. Show that in a Boolean algebra, the zero element 0 and the unit element 1 are unique.

Solution : 1

Let two zero elements in a Boolean algebra, B, be 0 and 0′.

As, 0 is an additive identity, then by definition,

$a + 0 = a$ for all $a \in B$.

Therefore, $0' + 0 = 0'$ --------- (i) [taking $a = 0'$]

Also as 0′ is an additive identity, then by definition,

$a + 0' = a$ for all $a \in B$,

Therefore, $0 + 0' = 0$ --------- (ii) [taking $a = 0$]

However, by commutativity, we have $0' + 0 = 0 + 0' \Rightarrow 0' = 0$ [using (i) and (ii)]

Hence, we see zero element in a Boolean algebra is unique.

Let 1 and 1′ be two unit elements in B. Then by definition

$a.1 = a$ and $a.1' = a$ for all $a \in B$.

Therefore, $1'.1 = 1'$ and $1.1' = 1$

However by commutativity, we have $1'.1 = 1.1'$.

Hence, $1' = 1$.

Hence, unit element in B is unique.

Q.2. Prove the following :

i. If $x + y = 0$, then $x = 0 = y$,
ii. $x.y' = 0$ if and only if $x.y = x$,
iii. $x = 0$ if and only if $y = x.y' + x'.y$ for all y.

Solution : 2

i. We have, x + y = 0
   Now x = x.(x + y) [By absorption law]
   = x.0 [given, x + y = 0]
   = 0.
   Similarly, y = y. (y + x) = y.(x + y) = y.0 = 0.

ii. Let us suppose, x.y′ = 0 => (x.y′)′ = 0′ => x′ + y = 1 [using x′ + y = 1]
   Now x = x.1 = x.(x′ + y)
   = x.x′ + x.y
   = 0 + x.y
   = x.y.
   Then we have, x.y′ = (x.y).y′ = x.(y.y′) = x.0 = 0

iii. Let us suppose that x = 0
   Then x.y′ + x′.y = 0.y′ + 0′.y
   = 0 + 1.y
   = 0 + y = y for all y.
   Now suppose, y = x.y′ + x′.y for all y.
   It means it is true for y = 0.
   Therefore, 0 = x.0′ + x′.0 = x.1 + x′.0 = x + 0 = x.
   Hence, x = 0.

Q.3. If x + y = x + z and x′ + y = x′ + z , then prove that y = z.

Solution : 3

We have, x + y = x + z ------------ (i)

and x′ + y = x′ + z ----------- (ii)

Now, y = y + 0 = y + x.x′ [As, x.x′ = 0]

= (y + x).(y + x′) [by distributive law]

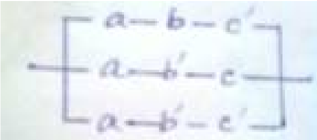
= (x + y).(x′ + y) [by commutative law]

= (x + z).(x′ + z) [using (i0 and (ii)]

= x.x′ + z [by distributive law]

= 0 + z = z.

Q.4. Write the Boolean expression for the following switching circuit :



Fig

Simplify the expression. Construct the switching circuit for the simplified expression.

Solution : 4

The Boolean expression for the given switching circuit is :

abc′ + ab′c + ab′c′= abc′ + ab′(c + c′)

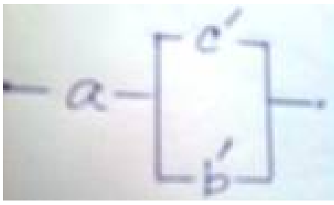
= abc′ + ab′ [As, c + c′ = 1]

= a (bc′ + b′)

= a (b + b′)(c′ + b′)

= a (c′ + b′).

The simplified switching circuit is :



Fig

Q.5. If a, b are elements of Boolean algebra, prove that : (a + b)′ = a′ b′.

Solution : 5

(a + b)′. a′ b′= a a′ b′ + b a′ b′

= 0. b′+ a′. 0 [As, aa′ = 0; bb′ = 0]

= 0 + 0 = 0 [identity for +]

and (a + b)′ + a′ b′ = (a + b + a′).(a + b + b′)

= (1 + b).(a + 1) [As, a + a′ = 1, b + b′ = 1]

= 1.1 = 1 [identity for .]

Therefore, (a + b) and a′ b′ are complement of each other.

Hence, (a + b)′ = a′ b′.

Q.6. P, Q and R represent switches in ‘on’ position and P′, Q′ and R′ represent switches in ‘off’ positions. Construct a switching circuit representing the polynomial
PR + Q(Q′ + R) (P + QR).
Use Boolean algebra to prove that the above circuit can be simplified to an expression in which, when P and R are ‘on’ or Q and R are ‘on’, the light is on. Construct an equivalent circuit.

Solution : 6

Fig. [sol 3(b)/page 359, 10 yrs.]

PR + Q(Q′ + R)(P + QR) = PR + (Q.Q′ + QR)(P + QR)

= PR + (0 + QR)(P + QR) [As, QQ′ = 0]

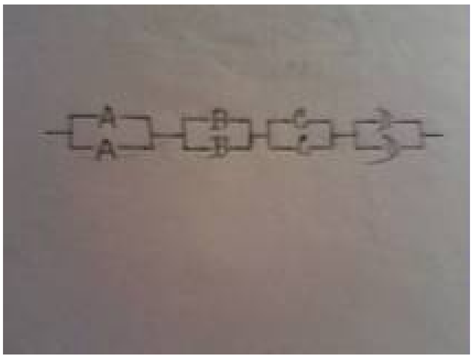
= PR + PQR + QR [As, QQRR = 0]

= PR + (P + 1)QR

= PR + QR [As, P + 1 = 1]

= (P + Q)R

Q.7. (i) Write down the Boolean Expression corresponding to the switching circuit given below :



Fig

(ii) Simplify the expression and construct the switching circuit for the simplified expression.

Solution : 7

i. (A + A)(B + D)(C + A)(C + D)

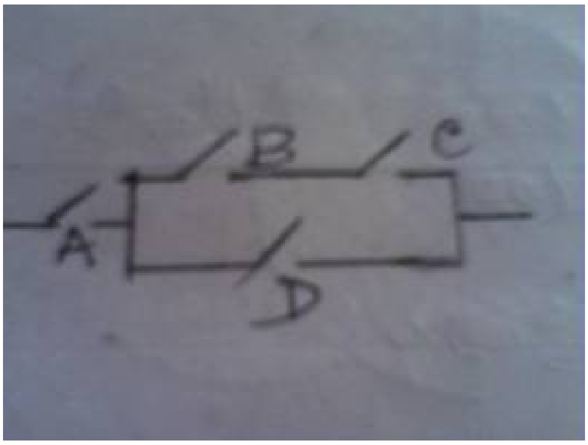
=> A(C + A)(B + D)(C + D)

=> (AC + A)(BC + D)

=> A(C + 1)(BC + D) [Distributive Law]

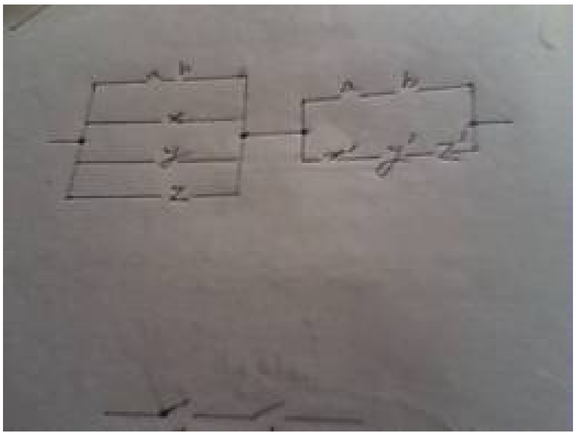
=> A.(BC + D) [C + 1 = 1.]

ii.



Fig

Q.8. Write the Boolean expression for the following given circuit :



Fig

Using the laws of Boolean Algebra , simplify the circuit and construct an equivalent switching circuit.

Solution : 8

Given : (a b + x + y + z).(a b + x′ y′ z′)

=> a b. a b + a b. x′ y′ z′ + x. a b + x. x′ y′ z′ + y. a b + y. x′ y′ z′ + z. a b + z. x′ y′ z′
=> a b + a b. x′ y′ z′ + x. a b + y. a b

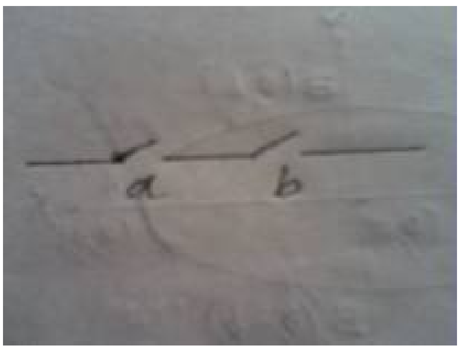
+ z. a b [As a. a′ = 0]

=> a b (1 + x′ y′ z′) + x. a b + y. a b + z. a b [1 + a = 1]

=> a b. 1 + x. a b + y. a b + z. a b

=> a b. (1 + x + y + z)

=> a b . 1 = a b.



Fig

Q.9. If a, b, c are elements of Boolean Algebra, prove that :

a b + c(a′ + b′) = a b + c.

Solution : 9

a b + c (a′ + b′)

=> (a b + c).(a b + a′ + b′) [As, a + b c = (a + b).(a + c)]

=> (a b + c).[(a + a′).(b + a′) + b′] [As, a + a′ = 1]

=> (a b + c).[1.(b + a′) + b′]

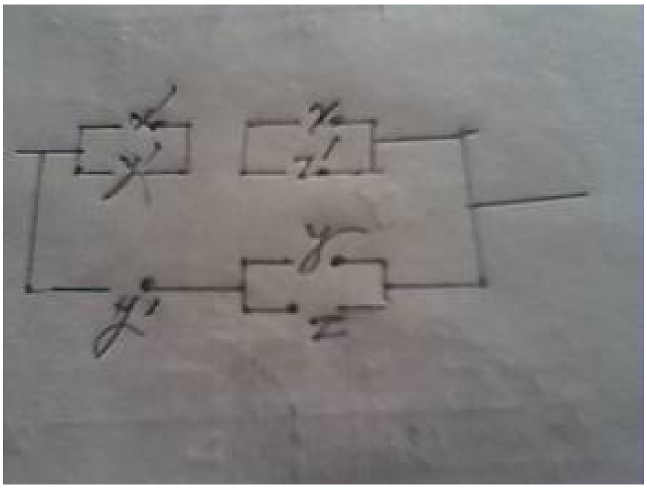
=> (a b + c).[b + b′ + a]

=> (a b + c).[1 + a′]

=> (a b + c).1 = a b + c.

Q.10. If x, y and z represent three switches in an on position and x′, y′ and z′ represent the three switches in an off position. Construct a switching circuit representing the polynomial
(x′ + y′)(x + z′) + y′(y + z)
Using the law of Boolean Algebra , show that the above polynomial is equivalent to
x′ z′ + y′
and construct an equivalent switching circuit.

Solution : 10



Fig

Given : $(x' + y')(x + z') + y'(y + z)$

=> $x' x + x' z' + y' x + y' z' + y' y + y' z$

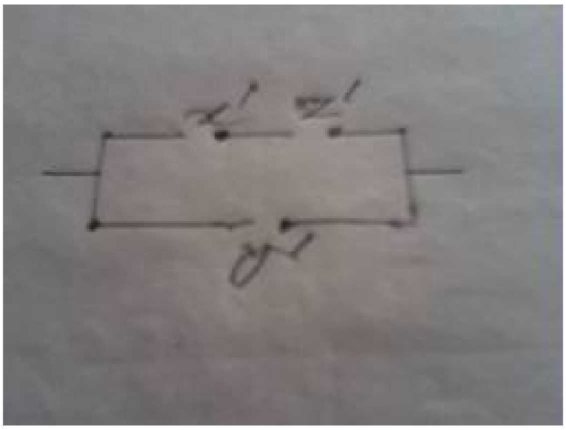
=> $x' z' + y' x + y' z' + y' z$ [As, $x' x = 0$, $y' y = 0$]

=> $x' z' + y' x + y'(z' + z)$

=> $x' z' + y' x + y'.1$ [As, $z' + z = 1$]

=> $x' z' + y'(x + 1)$

=> $x' z' + y'$. [As, $x + 1 = 1$]



Fig

Q.11. Prove that every element in a Boolean Algebra has a unique inverse.

Solution : 11

Let a' and a'' be two inverses of a.

Then $a' = a'.1$

$= a'.(a + a'')$ [$1 = a + a''$]

$= a'. a + a'. a''$

$= 0 + a' .a''$ [$a'. a = 0$]

$= a. a'' + a'. a''$ [$a. a'' = 0$]

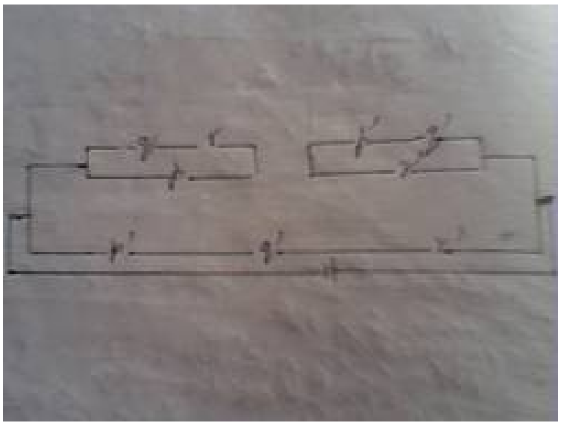
$= (a + a').a''$

$= 1.a''$ [$a + a' = 1$]

$= a''$.

Thus we see that a' and a'' are the same. Hence there is an unique inverse.

Q.12. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 12

Given : $(q\, r + p).(p'\, q' + r') + p'\, q'\, r'$

$\Rightarrow q\, r.\ p'\, q' + q\, r\, r' + p\, p'\, q' + p\, r' + p'\, q'\, r'$

$\Rightarrow (q\, q')(r\, p') + q(r\, r') + (p\, p')\, q' + p\, r' + p'\, q'\, r'$

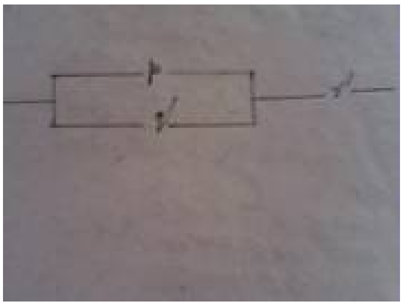
$\Rightarrow p\, r' + p;\ q'\, r'$ [As, $q\, q' = r\, r' = p\, p' = 0$]

$\Rightarrow (p + p'\, q').r'$

$\Rightarrow (p + p')(p + q')r'$

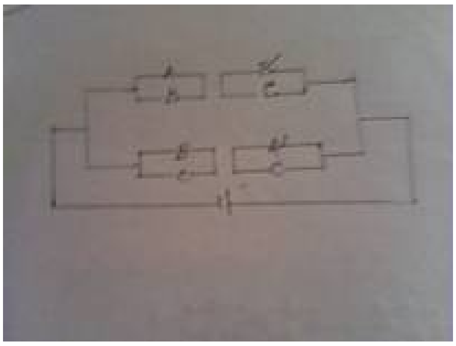
$\Rightarrow 1.(p + q')r'$ [$p + p' = 1$]

$\Rightarrow (p + q')r'$



Fig

Q.13. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 13

Given : (A + B)(B′ + C) + (B + C)(A′ + C)

= AB′ + AC + BB′ + BC + BA′ + BC + CA′ + CC

= AB′ + AC + BC + BA′ + CA′ + C [BB′ = 0, BC + BC = BC, CC = C]

= AB′ + AC + BC + CA′ + C + BA′

= AB′ + (A + B + A′+ 1)C + BA′

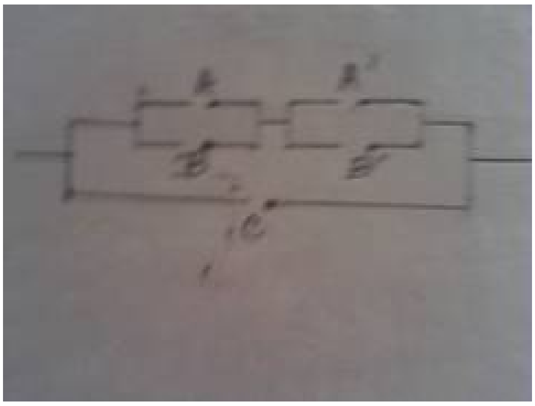
= AB′ + C + BA′ [B + B′ = 0, B + 1 = 1]

= AB′ + BA′ + C

= (AB′ + B)(AB′ + A′) + C

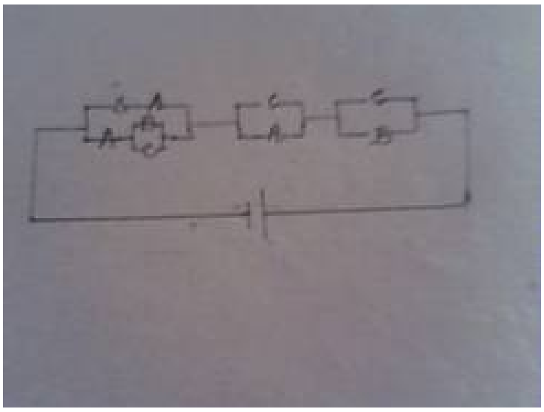
= (A + B)(B′ + B).(A + A′) (B′ + A′) + C

= (A + B)(B′ + A′) + C.



Fig

Q.14. Write the Boolean function corresponding to the following switching circuit network:



Fig

Use the Laws of Boolean Algebra to simplify the circuit. Construct the network for the simplified circuit.

Solution : 14

Given : [CA + A.(B + C)].(C + A)(C + B)

= (CA + AB + AC).(C + CB + AC + AB) [C.C = C]

= (AC + AB).[C(1 + B) + AC + AB] [1 + B = 1]

= (AC + AB).(C + AC + AB)

= (AC + AB).[(A + 1)C + AB]

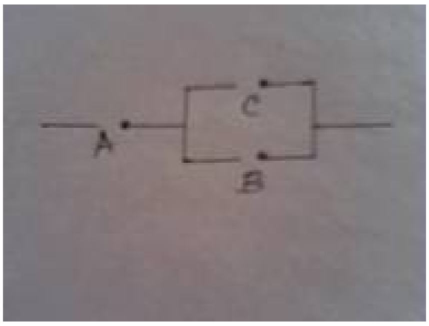
= (AC + AB).(C + AB)

= AC + ACB + ABC + AB [ABC + ABC = ABC]

= AC(1 + B) + AB(C + 1)

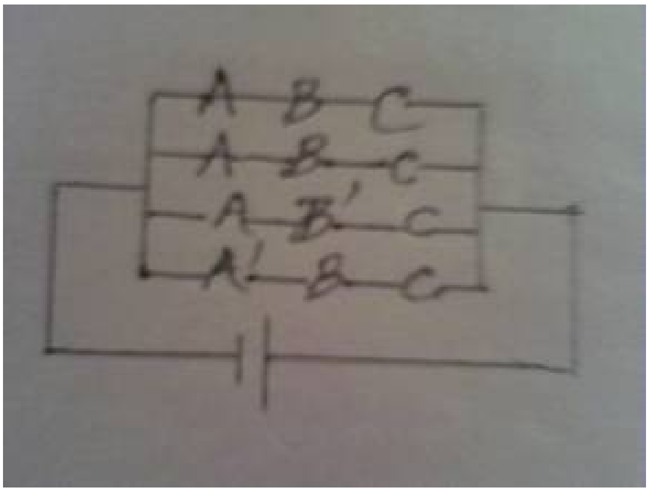
= AC + AB [1 + B = 1 + C = 1]

= A.(C + B)



Fig

Q.15. Write the statement for the following switching circuit :



Fig

Simplify the statement. Construct the switching circuit for the simplified form.

Solution : 15

The Boolean function is :

ABC + ABC′ + AB′C + A′BC

= AB(C + C′) + AB′C + A′BC [C + C′ = 1]

= AB.1 + AB′C + A′BC

= (AB + AB′)(AB + C) + A′BC

= A(B + B′)(AB + C) + A′BC

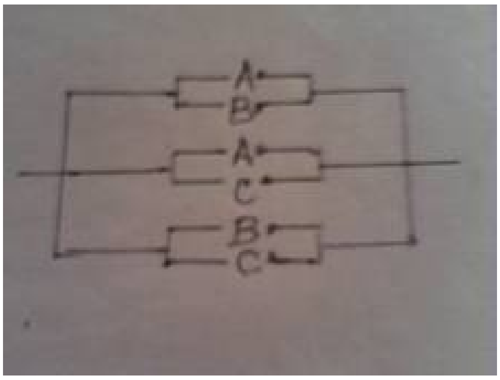
= A(AB + C) + A′BC

= AB + AC + A′BC

= AB + (A + A′B)C

= AB + (A + A′)(A + B)C [A + A′ = 1]

= AC + AC + BC .



Fig

Q.16. Prove that the current will flow through the network represented by the function :

[AB(A′B + AB′)]′

Irrespective of whether x and y are closed or open.

Solution : 16

Given : [AB(A′B + AB′)]′ = [AB.A′B + AB.AB′]′

= [B(AA′)B + A(BB′)A]′

= [0 + 0]′

= [0]′= 1.

Hence, the current will flow.