Polynomials

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1. Apply division algorithm to find the quotient q (x) and remainder r (x) an dividing f (x) by g (x), where $f(x)=x^3-6x^2+11x-6,\ g(x)=x^2+x+1$

Ans. $f(x)=g(x)\times q(x)+r(x)$

$$
\begin{array}{r|l}
 & x-7 \\
\hline
x^2+x+1 & x^3-6x^2+11x-6 \\
 & \underline{\underset{-}{x^3}\ \underset{-}{+x^2}\ \underset{-}{+x}} \\
 & -7x^2+10x-6 \\
 & \underline{\underset{+}{\mp}7x^2\ \underset{+}{\mp}7x\ \underset{+}{\mp}7} \\
 & -17x+1
\end{array}
$$

$\therefore\ (x^3-6x^2+11x-6)=x^2+2x+1(x-7)+(17x+1)$

2. If two zeros of the polynomial $x^4-6x^3-26x^2+138x-35$ **are** $2\pm\sqrt{3}$ **, find the other zeros.**

Ans. Two zeros are $2\pm\sqrt{3}$

$\therefore\ Sum$ of zeros $=4$

and product of the zeros $=1$

$\therefore\ (x^2-4x+1)$ is the factor of $x^4-6x^3-26x^2+138x-35$

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & \underline{\underset{-}{x^4}\ \underset{+}{\mp}4x^3\ \underset{-}{+}x^2} \\
 & -2x^3-27x^2+138x-35 \\
 & \underline{\underset{+}{\mp}2x^3\ \underset{-}{+}8x^2\ \underset{+}{\mp}2x} \\
 & -35x^2+140x-35 \\
 & \underline{\underset{+}{\mp}35x^2\ \underset{-}{+}140x\ \underset{+}{\mp}35} \\
 & 0
\end{array}
$$

Now,

$$x^2-2x-35$$
$$=x^2-7x+5x-35$$
$$=x(x-7)+5(x-7)$$
$$=(x-5)(x-7)$$
$\therefore$ Zeros are

$x=7$ and $x=-5$

$\therefore$ Other two zeros are 7 *and* -5

3. What must be subtracted from the polynomial $f(x)=x^4+2x^3-13x^2-12x+21$ so that the resulting polynomial is exactly divisible by $g(x)=x^2-4x+3$?

Ans.

$$\begin{array}{r|l}
 & x^2+6x+8 \\
\hline
x^2-4x+3 & x^4+2x^3-13x^2-12x+21 \\
 & \underline{\underset{-}{x^4}\ \underset{+}{-}4x^3\ \underset{-}{+}\ 3x^2} \\
 & \quad 6x^3-16x^2-12x+21 \\
 & \quad \underline{\underset{-}{6x^3}\ \underset{+}{-}14x^2\ \underset{-}{+}18x} \\
 & \qquad 8x^2-30x+21 \\
 & \qquad \underline{\underset{-}{8x^2}\ \underset{+}{-}32x\ \underset{-}{+}24} \\
 & \qquad\qquad 2x-3
\end{array}$$

We must be subtract (2x – 3) to become a factor.

4. What must be added to $6x^5+5x^4+11x^3-3x^2+x+5$ so that it may be exactly divisible by $3x^2-2x+4$?

Ans.

$$
\begin{array}{r}
x^2+6x+8 \\
3x^2-2x+4\overline{\smash{)}6x^5+5x^4+11x^3-3x^2+x+5} \\
\underline{\underset{-}{}6x^5\underset{+}{-}4x^4\underset{-}{+}8x^3} \\
9x^4+3x^3-3x^2+x+5 \\
\underline{\underset{-}{}9x^4\underset{+}{-}6x^3\underset{-}{+}12x^2} \\
9x^3-15x^2+x+5 \\
\underline{\underset{-}{}9x^3\underset{+}{-}6x^2\underset{-}{+}12x} \\
-9x^2-11x+5 \\
\underline{\underset{+}{-}9x^2\underset{-}{+}6x\underset{+}{-}12} \\
-17x+17
\end{array}
$$

So we must be added $\left(3x^2-2x+4\right)-\left(-17x+17\right)$

$=3x^2-2x+4+17x-17$

$=3x^2+15x-13$

5. Find all the zeros of the polynomial f (x) = $2x^4-3x^3-3x^2+6x-2$, if being given that two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Ans. $\sqrt{2}$ *and* $-\sqrt{2}$ are the zeros.

$\therefore\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$ is the factor of the given polynomial.

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & \underline{2x^4 \qquad \mp 4x^2} \\
 & -3x^3 + x^2 + 6x - 2 \\
 & \underline{\mp 3x^3 \qquad \pm 6x} \\
 & x^2 - 2 \\
 & \underline{x^2 \mp 2} \\
 & 0
\end{array}
$$

$$
\begin{aligned}
q(x) &= 2x^2 - 3x + 1 \\
&= 2x^2 - 2x - x + 1 \\
&= 2x(x-1) - 1(x-1) \\
&= (2x-1)(x-1)
\end{aligned}
$$

$\therefore$ other two zero's are

$x = 1$ and $x = \frac{1}{2}$

6. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g (x) the quotient and the remainder were (x – 2) and -2x + 4 respectively, find g (x).

Ans. $p(x) = q(x) \times g(x) + r(x)$

$$
\begin{aligned}
g(x) &= \frac{p(x) - r(x)}{q(x)} \\
&= \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}
\end{aligned}
$$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & \underline{x^3 \mp 2x^2} \\
 & -x^2 + 3x - 2 \\
 & \underline{\mp x^2 \pm 2x} \\
 & x - 2 \\
 & \underline{x \mp 2} \\
 & 0
\end{array}
$$

$g(x) = x^2 - x + 1$

7. Find all zeros of f(x) = $2x^3 - 7x^2 + 3x + 6$ if its two zeros are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Ans. $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$

Two zeros are $\pm\sqrt{\frac{3}{2}}$

$\therefore \left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right) = \frac{1}{2}(2x^2-3)$

$\therefore$ $(2x^2-3)$ is the factor of $f(x)$.

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
2x^2-3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\
 & \underline{2x^4 \qquad \mp 3x^2} \\
 & -2x^3 - 4x^2 + 3x + 6 \\
 & \underline{\mp 2x^3 \qquad \pm 3x} \\
 & -4x^2 + 6 \\
 & \underline{\mp 4x^2 \pm 6} \\
 & 0
\end{array}
$$

$g(x) = x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x-2) + 1(x-2)$

$= (x+1)(x-2)$

$\therefore$ other two zeros are

$x+1=0$ or $x=-1$

and $x-2=0$ or $x=2$

$\therefore$ other two zeros are -1 and 2

8. Obtain all zeros of the polynomial f (x) = $2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1.

Ans. $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, two zeros are -2 *and* -1

$\therefore$ $(x+2)$ and $(x+1)$ are the factors of $f(x)$.

$\therefore$ $(x+2)(x+1) = x^2 + 3x + 2$

$$
\begin{array}{r|l}
 & \quad 2x^2-5x-3 \\
\hline
x^2+3x+2 & 2x^4+x^3-14x^2-19x-6 \\
 & \underset{-}{}2x^4 \underset{-}{+}6x^3 \underset{-}{+}4x^2 \\
\hline
 & \qquad -5x^3-18x^2-19x-6 \\
 & \qquad \mp 5x^3 \mp 15x^2 \mp 10x \\
\hline
 & \qquad\qquad -3x^2-9x-6 \\
 & \qquad\qquad \mp 3x^2 \mp 9x \mp 6 \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

$$
\begin{aligned}
&Now\ 2x^2-5x-3 \\
&= 2x^2-6x+x-3 \\
&= 2x(x-3)+1(x-3) \\
&= (x-3)(2x+1)
\end{aligned}
$$

$\therefore$ *zeros* are

$x-3=0$

$\Rightarrow\ x=3$

and $\ 2x+1=0$

$\Rightarrow x=-\dfrac{1}{2}$

other two zeros are 3 and $-\dfrac{1}{2}$

9. Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$. If two of its zeros are $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$

Ans. $3x^4+6x^3-2x^2-10x-5$

zeros are $\pm\sqrt{\dfrac{5}{3}}$

$\left(x+\sqrt{\dfrac{5}{3}}\right)\left(x-\sqrt{\dfrac{5}{3}}\right)$ is the factor given polynomial i.e. $\left(x^2-\dfrac{5}{3}\right)\dfrac{1}{3}\left(3x^2-5\right)$

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & -\,3x^4 \quad \mp 5x^2 \\
\hline
 & 6x^3+3x^2-10x-5 \\
 & -\,6x^3 \quad \mp 10x \\
\hline
 & 3x^2 \quad -5 \\
 & -\,3x^2 \quad \mp 5 \\
\hline
 & 0
\end{array}
$$

$Now\ x^2+x+1$

$\Rightarrow\ (x+1)^2$

$\therefore$ other zeros are

$(x+1)=0$ and $x+1=0$

$x=-1$ and $x=-1$

$\therefore$ other two zeros -1 and -1

10. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $(x + a)$, find '*k*' and '*a*'.

Ans.

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-25x+10 \\
 & \underline{x^4-2x^3+kx^2} \\
 & -4x^3+(16-k)x^2-25x+10 \\
 & \underline{-4x^3+8x^2-4kx} \\
 & (8-k)x^2+(4k-25)x+10 \\
 & \underline{(8-k)x^2-(16-2k)x+(8k-k^2)} \\
 & (2k-9)x+(k^2-8k+10)
\end{array}
$$

but remainder is $(x+a)$

$\therefore$ equating the cofficient of x and constant term.

so $2k\text{-}8k+10=a$

$\Rightarrow\ 25-40+10=a$

$\Rightarrow\ -5=a$

$\therefore\ k=5$ and $a=-5$

11. Find the value of '*k*' for which the polynomial $x^4+10x^3+25x^2+15x+k$ is exactly divisible by (x + 7).

Ans. $p(x)=x^4+10x^3+25x^2+15x+k$

$\therefore\ (x+7)$ is the factor.

$\therefore\ p(-7)=0$

$or\ (-7)^4+10(-7)^3+25(-7)^2+15(-7)+k=0$

$2401-3430+1225-105+k=0$

$k=91$

12. If α and β are the zeros of the polynomial f (x) = x^2+px+q, find polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

Ans. $f(x)=x^2+px+q,$ if α and β are zeros

$\therefore\ \alpha+\beta=-p\ and\ \alpha\beta=q$

If zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$=(-p)^2-4q$

$(\alpha-\beta)^2=-p^2-4q$

Now sum of zeros

$(\alpha+\beta)^2+(\alpha-\beta)^2=(-p)^2+(p^2-4q)$

$=2p^2-4q$

Pr oduct of zeros

$(\alpha+\beta)^2(\alpha-\beta)^2=(-p)^2+(p^2-4q)$

$=4p^4-4p^2q$

$\therefore$ required polynomial is

$x^2-(\text{sum of zeros})\,x+$ product of zeros

$=x^2-(2p^2-4q)x+4p^4-4p^2q$

$=x^2-2p^2x-4qx+p^4-4p^2q$