Continuity

1. DEFINITION

If the graph of a function has no break or jump, then it is said to be continuous function. A function which is not continuous is called a discontinuous function.

2. CONTINUITY OF A FUNCTION AT A POINT

A Function $f(x)$ is said to be continuous at some point x=a of its domain if

$$\lim_{x\to a} f(x) = f(a)$$

i.e., If $\lim_{x\to a-0} f(x) = \lim_{x\to a+0} f(x) = f(a)$

i.e., If $f(a-0) = f(a+0) = f(a)$

i.e., If {LHL at x=a } = {RHL at x = a} = { value of the function at x = a } .

3. CONTINUITY FROM LEFT AND RIGHT

Function f(x) is said to be

(i) Left Continuous at x=a if $\lim_{x\to a-0} f(x) = f(a)$ i.e . f(a – 0) = f(a)

(ii) Right Continuous at x=a if $\lim_{x\to a+0} f(x) = f(a)$ i.e. f (a + 0) = f (a)

Thus a function f(x) is continuous at a point x= a if it is left continuous as well as right continuous at x=a.

4. CONTINUITY IN AN INTERVAL

(1) A function f(x) is continuous in an open interval (a, b) if it is continuous at every point of the interval.

(2) A function f(x) is continuous in a closed interval [a, b] if it is

- (i) continuous in (a, b)
- (ii) right continuous at x=a
- (iii) left continuous at x=b

5. CONTINUOUS FUNCTIONS

A function is said to be continuous function if it is continuous at every point in its domain. Following are examples of some continuous functions:

(i) $f(x) = x$ (Identify function)

(ii) $f(x) = c$ (Constant function)

(iii) $f(x) = a_0x^n + a_1x^{n-1} + ... + a^n$ (Polynomial function)

(iv) $f(x) = \sin x, \cos x$ (Trigonometric function)

(v) $f(x) = a^x$, e^x , e^{-x} (Expoential function)

(vi) $f(x) = \log x$ (Logarithmic function)

(vii) $f(x) = \sinh x, \cosh x, \tanh x$ (Hyperbolic function)

(viii) $f(x) = |x|, x + |x|, x - |x|, x|x|$ (Absolute value functions)

6. DISCONTINUOUS FUNCTIONS

A function is said to be a discontinuous function if it is discontinuous at atleast one point in its domain. Following are examples of some discontinuous functions:

No.	Functions	Points of discontinuity
(i)	$[x]$	Every Integers
(ii)	$x - [x]$	Every Integers
(iii)	$\frac{1}{x}$	$x = 0$
(iv)	$\tan x$, $\sec x$	$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, ...$
(v)	$\cot x$, $\operatorname{cosec} x$	$x = 0, \pm\pi, \pm 2\pi, ...$
(vi)	$\sin\frac{1}{x}$, $\cos\frac{1}{x}$	$x = 0$
(vii)	$e^{1/x}$	$x = 0$
(viii)	$\coth x$, $\operatorname{cosech} x$	$x = 0$

7. PROPERTIES OF CONTINUOUS FUNCTIONS

The sum, difference, product, quotient (if $Dr \neq 0$) and composite of two continuous functions are always continuous functions. Thus if f(x) and g(x) are continuous functions then following are also continuous functions:

(i) $f(x) + g(x)$

(ii) $f(x) - g(x)$

(iii) $f(x).g(x)$

(iv) $\lambda f(x)$, where λ is a constant

(v) $\frac{f(x)}{g(x)}$, if $g(x) \neq 0$

(vi) $f[g(x)]$

8. IMPORTANT POINT

The discontinuity of a function f(x) at x = a can arise in two ways

(i) If $\lim_{x \to a^-} f(x)$ exist but $\neq$ f(a) or $\lim_{x \to a^+} f(x)$ exist but $\neq$ f(a) , then the function f(x) is said to have a removable discontinuty.

(ii) The function f(x) is said to have an unremovable discontinuity when $\lim_{x \to a} f(x)$ does not exist.

i.e. $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

Differentiability at a point

Let f(x) be a ral valued function defined on an open interval (a, b) and let $c \in$ (a, b). Then f(x) is said to be differentiable or derivable at x = c, iff $\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}$ exists finitely.

This limit is called the derivative or differential coefficient of the function f(x) at x = c, and is denoted by $f'(c)$ or

$$Df(c) \text{ or } \left\{\frac{d}{dx}f(x)\right\}_{x=c}$$

$$\lim_{x\to c^-}\frac{f(x)-f(c)}{-h}=\lim_{h\to 0}\frac{f(c+h)-f(c)}{-h}$$

is called the left hand derivative of f(x) at x = c and is denoted by $f'(c^-)$ or $Lf'(c)$ while.

$$\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}\text{or}\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

is called the right hand derivative off (x) at x = c and is denoted by $f'(c^+)$ or $Rf'(c)$.

Thus, f(x) is differentiable at x = c $\Leftrightarrow$ $Lf'(c) = Rf'(c)$.

If $Lf'(c) \neq Rf'(c)$ we say that f(x) is not differentiable at x = c.

Differentiability in a set

A fuction f(x) defined on an open interval (a, b) is said tobe differentiable or derivable inopen interval (a, b) if it is differentiable at each point of (a, b)