4.Quadratic Equations

QUADRATIC EQUATIONS

The polynomial of degree two is called quadratic polynomial and equation corresponding to a quadratic polynomial P(x) is called a quadratic equation in variable x.

Thus, $P(x) = ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ is known as the standard form of quadratic equation.

There are two types of quadratic equation.
(i) **Complete quadratic equation :** The equation $ax^2 + bx + c$ 0 where $a \neq 0$, $b \neq 0$,$c \neq 0$
(ii) **Pure quadratic equation :** An equation in the form of $ax^2 = 0$, $a \neq 0$, $b = 0$, $c = 0$

ZERO OF A QUADRATIC POLYNOMIAL

The value of x for which the polynomial becomes zero is called zero of a polynomial

For instance,

1 is zero of the polynomial x^2 — 2x + 1 because it become zero at x = 1.

SOLUTION OF A QUADRATIC EQUATION BY

FACTORISATION

A real number x is called a root of the quadratic equation $ax^2 + bx + c = 0$, a 0 if $a\alpha^2 + b\alpha + c = 0$.In this case, we say $x = \alpha$ is a solution of the quadratic equation.

NOTE:

1. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
2. Roots of quadratic equation $ax^2 + bx + c = 0$ can be found by factorizing it into two linear factors and equating each factor to zero.

SOLUTION OF A QUADRATIC EQUATION BY COMPLETING THE SQUARE

By adding and subtracting a suitable constant, we club the x^2 and x terms in the quadratic equation so that they become complete square, and solve for x.

In fact, we can convert any quadratic equation to the form $(x + a)^2 — b^2 = 0$ and then we can easily find its roots.

DISCRIMINANT

The expression b^2 — 4ac is called the discriminant of the quadratic equation.

SOLUTION OF A QUADRATIC EQUATION BY DISCRIMINANT METHOD

Let quadratic equation is $ax^2 + bx + c = 0$

Step 1. Find $D = b^2$ — 4ac.

Step 2.

(i) If $D > 0$, roots are given by

$x = -b + \sqrt{D} / 2a , -b – \sqrt{D} / 2a$

(ii) If $D = 0$ equation has equal roots and root is given by $x = -b / 2a$.

(iii) If $D < 0$, equation has no real roots.

ROOTS OF THE QUADRATIC EQUATION

Let the quadratic equation be $ax^2 + bx + c = 0$ ($a \neq 0$).

Thus, if b^2 — 4ac ≥ 0, then the roots of the quadratic

—b ± $\sqrt{b^2}$ — 4ac / 2a equation are given by

QUADRATIC FORMULA

—b ± $\sqrt{b^2}$ — 4ac / 2a is known as the quadratic formula

which is useful for finding the roots of a quadratic equation.

NATURE OF ROOTS

(i) If b^2 — 4ac > 0, then the roots are **real and distinct.**

(ii) If b^2 — 4ac = 0, the roots are **real and equal or coincident.**

(iii) If b^2 — 4ac <0, the roots are not **real (imaginary roots)**

FORMATION OF QUADRATIC EQUATION WHEN TWO ROOTS ARE GIVEN

If α and β are two roots of equation then the required quadratic equation can be formed as x^2 — $(\alpha + \beta)x + \alpha\beta = 0$

NOTE :

Let α and β be two roots of the quadratic equation ($ax^2 + bx + c = 0$ then

Sum of Roots: – the coefficient of x / the coefficient t of $x^2 \Rightarrow \alpha + \beta = -b / a$

Product of Roots :

$\alpha\beta$ = constant term / the coefficient t of $x^2 \Rightarrow \alpha\beta = c / a$

METHOD OF SOLVING WORD PROBLEMS

Step 1: Translating the word problem into Mathematics form (symbolic form) according to the given condition

Step 2 : Form the word problem into Quadratic equations and solve them.