# **Chapter : 18. SOLUTION OF TRIANGLES**

# Exercise : 18A

#### **Question: 1**

In any  $\triangle ABC$ , prov

#### Solution:

Left hand side,

a(b cos C - c cos B) = ab cos C - ac cos B =  $ab \frac{a^2 + b^2 - c^2}{2ab} - ac \frac{a^2 + c^2 - b^2}{2ac} [As, \cos C = \frac{a^2 + b^2 - c^2}{2ab} \& \cos B = \frac{a^2 + c^2 - b^2}{2ac}]$ =  $\frac{a^2 + b^2 - c^2}{2} - \frac{a^2 + c^2 - b^2}{2}$ =  $\frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2}$ =  $\frac{2(b^2 - c^2)}{2}$ =  $b^2 - c^2$ 

= Right hand side. [Proved]

## **Question: 2**

In any  $\triangle ABC$ , prov

#### Solution:

Left hand side,

ac cos B – bc cos A

$$= \operatorname{ac} \frac{a^{2} + c^{2} - b^{2}}{2ac} - \operatorname{bc} \frac{b^{2} + c^{2} - a^{2}}{2bc} [As, \cos B = \frac{a^{2} + c^{2} - b^{2}}{2ac} \& \cos A = \frac{b^{2} + c^{2} - a^{2}}{2bc}]$$

$$= \frac{a^{2} + c^{2} - b^{2}}{2} - \frac{b^{2} + c^{2} - a^{2}}{2}$$

$$= \frac{a^{2} + c^{2} - b^{2} - b^{2} - c^{2} + a^{2}}{2}$$

$$= \frac{2(a^{2} - b^{2})}{2}$$

$$= a^{2} - b^{2}$$

= Right hand side. [Proved]

#### **Question: 13**

In any  $\triangle ABC$ , prov

## Solution:

Need to prove:  $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{(a^2+b^2+c^2)}{2abc}$ 

Left hand side

$$= \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$

$$= \frac{b^2 + c^2 - a^2}{2abc} + \frac{c^2 + a^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc}$$

$$= \frac{b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2}{2abc}$$

$$= \frac{a^2 + b^2 + c^2}{2abc}$$

= Right hand side. [Proved]

#### **Question: 4**

In any  $\triangle ABC$ , prov

# Solution:

Need to prove:  $\frac{c-b\cos A}{b-\cos A} = \frac{\cos B}{\cos C}$ 

.

Left hand side

$$= \frac{c - b \cos A}{b - c \cos A}$$

$$= \frac{c - b \frac{b^2 + c^2 - a^2}{2bc}}{b - c \frac{b^2 + c^2 - a^2}{2bc}}$$

$$= \frac{\frac{2c^2 - b^2 - c^2 + a^2}{2c}}{\frac{2b^2 - b^2 - c^2 + a^2}{2b}}$$

$$= \frac{\frac{c^2 + a^2 - b^2}{2c}}{\frac{b^2 + a^2 - c^2}{2b}}$$

 $= \frac{\frac{2ac}{b^2 + a^2 - c^2}}{\frac{b^2 + a^2 - c^2}{2ab}}$ [Multiplying the numerator and denominator by  $\frac{1}{a}$ ]

$$=\frac{\cos B}{\cos C}$$

= Right hand side. [Proved]

#### **Question: 5**

In any  $\triangle ABC$ , prov

#### Solution:

Need to prove: 2(bc cos A + ca cos B + ab cos C) =  $(a^2 + b^2 + c^2)$ 

Left hand side

 $2(bc \cos A + ca \cos B + ab \cos C)$ 

 $2(bc\frac{b^2 + c^2 - a^2}{2bc} + ca\frac{c^2 + a^2 - b^2}{2ca} + ab\frac{a^2 + b^2 - c^2}{2ab})$  $b^{2} + c^{2} - a^{2} + c^{2} + a^{2} - b^{2} + a^{2} + b^{2} - c^{2}$  $a^2 + b^2 + c^2$ 

Right hand side. [Proved]

#### **Question: 6**

In any  $\triangle ABC$ , prov

### Solution:

Need to prove:  $4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$ 

Right hand side

=  $4(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2})$  $= 4(bc\frac{s(s-a)}{bc} + ca\frac{s(s-b)}{ca} + ab\frac{s(s-c)}{ab})$ , where s is half of perimeter of triangle. = 4(s(s - a) + s(s - b) + s(s - c)) $=4(3s^2 - s(a + b + c))$ We know, 2s = a + b + cSo,  $4(3(\frac{a+b+c}{2})^2 - \frac{(a+b+c)^2}{2})$  $= 4(3\frac{(a+b+c)^2}{4} - \frac{(a+b+c)^2}{2})$  $= 4(\frac{3(a+b+c)^2 - 2(a+b+c)}{4})$  $= 3(a + b + c)^{2} - 2(a + b + c)^{2}$  $= (a + b + c)^2$ = Right hand side. [Proved] **Question: 7** In any  $\triangle ABC$ , prov Solution: Need to prove:  $a \sin A - b \sin B = c \sin (A - B)$ Left hand side, = a sin A - b sin B = (b cosC + c cosB) sinA - (c cosA + a cosC) sinB  $= b \cos C \sin A + c \cos B \sin A - c \cos A \sin B - a \cos C \sin B$ = c(sinA cosB - cosA sinB) + cosC(b sinA - a sinB)We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius. Therefore, = c(sinA cosB - cosA sinB) + cosC(2R sinB sinA - 2R sinA sinB)= c(si nA cosB - cosA sinB) $= c \sin (A - B)$ = Right hand side. [Proved] **Question: 8** 

In any  $\triangle ABC$ , prov

Need to prove:  $a^2 \sin (B - C) = (b^2 - c^2) \sin A$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius. Therefore,  $a = 2R \sin A ---- (a)$ Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ From Right hand side,  $= (b^2 - c^2) \sin A$  $= \{(2R \sin B)^2 - (2R \sin C)^2\} \sin A$  $=4R^2(sin^2B - sin^2C)sinA$ We know,  $\sin^2 B - \sin^2 C = \sin(B + C)\sin(B - C)$ So,  $=4R^{2}(\sin(B + C)\sin(B - C))\sin A$ =  $4R^2(\sin(\pi - A)\sin(B - C))\sin A [As, A + B + C = \pi]$ =  $4R^2(\sin A \sin(B - C))\sin A [As, \sin(\pi - \theta) = \sin \theta]$  $=4R^2\sin^2A\sin(B-C)$  $= a^2 sin(B - C)$  [From (a)] = Left hand side. [Proved]

# **Question: 9**

In any  $\triangle ABC$ , prov

## Solution:

Need to prove:  $\frac{\sin(A-B)}{\sin(A+B)} = \frac{(a^2-b^2)}{c^2}$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

 $a = 2R \sin A ---- (a)$ 

Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ 

From Right hand side,

$$= \frac{a^{2} - b^{2}}{c^{2}}$$

$$= \frac{4R^{2} \sin^{2} A - 4R^{2} \sin^{2} B}{4R^{2} \sin^{2} C}$$

$$= \frac{4R2 (\sin^{2} A - \sin^{2} B)}{4R^{2} \sin^{2} C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^{2} C}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^{2} (\pi - (A + B))}$$

$$= \frac{\sin(A + B) \sin(A - B)}{\sin^{2} (A + B)}$$

 $=\frac{\sin(A-B)}{\sin(A+B)}$ 

= Left hand side. [Proved]

#### **Question: 10**

In any  $\triangle ABC$ , prov

### Solution:

Need to prove:  $\frac{(b-c)}{a} \cos \frac{A}{2} = \sin \frac{(B-C)}{2}$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

 $a = 2R \sin A ---- (a)$ 

Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ 

From Left hand side,

$$= \frac{2R \sin B - 2R \sin C}{2R \sin A} \cos \frac{A}{2}$$
$$= \frac{2 \cos(\frac{B+C}{2}) \sin(\frac{B-C}{2})}{\sin A} \cos \frac{A}{2}$$
$$= \frac{2 \sin(\frac{B-C}{2}) \cos(\frac{\pi}{2} - \frac{A}{2})}{\sin A} \cos \frac{A}{2}$$
$$= \frac{2 \cos^2 \frac{A}{2} \sin(\frac{B-C}{2})}{\sin A}$$
$$= \frac{\sin A \sin(\frac{B-C}{2})}{\sin A}$$
$$= \sin \frac{B-C}{A}$$

= Right hand side. [Proved]

## **Question: 11**

In any  $\triangle ABC$ , prov

# Solution:

Need to prove:  $\frac{(a+b)}{c} \sin \frac{C}{2} = \cos \frac{(A-B)}{2}$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

 $a = 2R \sin A \cdots (a)$ 

Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ 

Now, 
$$\frac{a+b}{c} = \frac{2R(\sin A + \sin B)}{2R \sin C} = \frac{\sin A + \sin B}{\sin C}$$
  
Therefore,  $\frac{c}{a+b} = \frac{\sin C}{\sin A + \sin B} = \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}$   
 $\Rightarrow \frac{c}{a+b} = \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\sin(\frac{\pi}{2} - \frac{C}{2}) \cos \frac{A-B}{2}}$ 

$$\Rightarrow \frac{c}{a+b} = \frac{\sin\frac{C}{2}\cos\frac{C}{2}}{\cos\frac{C}{2}\cos\frac{A-B}{2}}$$
$$\Rightarrow \frac{c}{a+b} = \frac{\sin\frac{C}{2}}{\cos\frac{A-B}{2}}$$
$$\Rightarrow \frac{a+b}{c}\sin\frac{C}{2} = \cos\frac{A-B}{2} \text{ [Proved]}$$

## **Question: 12**

In any  $\triangle ABC$ , prov

#### Solution:

Need to prove:  $\frac{(b+c)}{a}$ .  $\cos \frac{(B+C)}{2} = \cos \frac{(B-C)}{2}$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

a = 2R sinA ---- (a)

Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ 

Now, 
$$\frac{a}{b+c} = \frac{2RsinA}{2RsinB+2RsinC} = \frac{sinA}{sinB+sinC}$$
  

$$\Rightarrow \frac{a}{b+c} = \frac{2sin\frac{A}{2}cos\frac{A}{2}}{2sin\frac{B+C}{2}cos\frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{sin\frac{A}{2}cos\frac{A}{2}}{sin(\frac{\pi}{2}-2)cos\frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{sin\frac{A}{2}cos\frac{A}{2}}{cos\frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{sin\frac{A}{2}}{cos\frac{B-C}{2}}$$

$$\Rightarrow \frac{a}{b+c} = \frac{cos(\frac{\pi}{2}-\frac{A}{2})}{cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{a}{b+c} = \frac{cos(\frac{\pi}{2}-\frac{A}{2})}{cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{a}{b+c} = \frac{cos(\frac{B+C}{2})}{cos(\frac{B-C}{2})}$$

$$\Rightarrow \frac{b+c}{a} = cos(\frac{B+C}{2}) = cos\frac{B-C}{2} [Proved]$$

### **Question: 13**

In any  $\triangle ABC$ , prov

#### Solution:

Need to prove:  $a^{2}(\cos^{2}B - \cos^{2}C) + b^{2}(\cos^{2}C - \cos^{2}A) + c^{2}(\cos^{2}A - \cos^{2}B) = 0$ 

From left hand side,

$$= a^{2}(\cos^{2}B - \cos^{2}C) + b^{2}(\cos^{2}C - \cos^{2}A) + c^{2}(\cos^{2}A - \cos^{2}B)$$
  
=  $a^{2}((1 - \sin^{2}B) - (1 - \sin^{2}C)) + b^{2}((1 - \sin^{2}C) - (1 - \sin^{2}A)) + c^{2}((1 - \sin^{2}A) - (1 - \sin^{2}B))$   
=  $a^{2}(-\sin^{2}B + \sin^{2}C) + b^{2}(-\sin^{2}C + \sin^{2}A) + c^{2}(-\sin^{2}A + \sin^{2}B)$ 

We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius. Therefore,  $a = 2R \sin A \cdots$  (a) Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ So,  $= 4R^2[\sin^2A(-\sin^2B + \sin^2C) + \sin^2B(-\sin^2C + \sin^2A) + \sin^2C(-\sin^2A + \sin^2B)$   $= 4R^2[-\sin^2A\sin^2B + \sin^2A\sin^2C - \sin^2B\sin^2C + \sin^2A\sin^2B - \sin^2A\sin^2C + \sin^2B\sin^2C ]$   $= 4R^2[0]$ = 0 [Proved]

# **Question: 14**

In any  $\triangle ABC$ , prov

#### Solution:

Need to prove:  $\frac{(\cos^2 B - \cos^2 C)}{b+c} + \frac{(\cos^2 C - \cos^2 A)}{c+a} + \frac{(\cos^2 A - \cos^2 B)}{a+b} = 0$ We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

 $a = 2R \sin A ---- (a)$ 

Similarly,  $b = 2R \sin B$  and  $c = 2R \sin C$ 

From left hand side,

$$= \frac{(\cos^2 B - \cos^2 C)}{b + c} + \frac{(\cos^2 C - \cos^2 A)}{c + a} + \frac{(\cos^2 A - \cos^2 B)}{a + b}$$
$$= \frac{(1 - \sin^2 B - 1 + \sin^2 C)}{b + c} + \frac{(1 - \sin^2 C - 1 + \sin^2 A)}{c + a} + \frac{(1 - \sin^2 A - 1 + \sin^2 B)}{a + b}$$
$$\sin^2 C - \sin^2 B - \sin^2 A - \sin^2 C - \sin^2 B - \sin^2 A$$

$$=\frac{\sin^2 C - \sin^2 B}{b+c} + \frac{\sin^2 A - \sin^2 C}{c+a} + \frac{\sin^2 B - \sin^2 A}{a+b}$$

Now,

$$= \frac{\sin^2 C - \sin^2 B}{2R(\sin B + \sin C)} + \frac{\sin^2 A - \sin^2 C}{2R(\sin C + \sin A)} + \frac{\sin^2 B - \sin^2 A}{2R(\sin A + \sin B)}$$
$$= \frac{1}{2R} \left[ \frac{(\sin B + \sin C)(\sin C - \sin B)}{\sin B + \sin C} + \frac{(\sin A + \sin C)(\sin A - \sin C)}{\sin A + \sin C} + \frac{(\sin A + \sin B)(\sin B - \sin A)}{\sin A + \sin B} \right]$$
$$= \frac{1}{2R} \left[ \sin C - \sin B + \sin A - \sin C + \sin B - \sin A \right]$$
$$= 0 \text{ [Proved]}$$

#### **Question: 15**

In any  $\triangle ABC$ , prov

Need to prove: 
$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

Left hand side,

$$= \frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$$
$$= \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2}$$
$$= \frac{1}{a^2} - \frac{1}{b^2} + 2(\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2})$$

We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

$$\frac{\sin^2 B}{b^2} - \frac{\sin^2 A}{a^2} = \frac{1}{4R^2} - \frac{1}{4R^2} = 0$$

Hence,

 $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$  [Proved]

## **Question: 16**

In any  $\triangle ABC$ , prov

#### Solution:

Need to prove:  $(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$ 

We know,

$$\tan A = \frac{abc}{R} \frac{1}{b^2 + c^2 - a^2} \dots (a)$$
  
Similarly,  $\tan B = \frac{abc}{R} \frac{1}{c^2 + a^2 - b^2}$  and  $\tan C = \frac{abc}{R} \frac{1}{a^2 + b^2 - c^2}$ 

Therefore,

$$(b^2 + c^2 - a^2) \tan A = \frac{abc}{R} [from (a)]$$

Similarly,

$$(c^{2} + a^{2} - b^{2})\tan B = \frac{abc}{R}$$
 and  $(a^{2} + b^{2} - c^{2})\tan C = \frac{abc}{R}$ 

Hence we can conclude comparing above equations,

 $(c^2 - a^2 + b^2) \tan A = (a^2 - b^2 + c^2) \tan B = (b^2 - c^2 + a^2) \tan C$ 

### [Proved]

## **Question: 17**

If in a  $\triangle ABC$ ,

# Solution:

Given:  $\angle C = 90^0$ 

Need to prove:  $sin(A - B) = \frac{(a^2 - b^2)}{(a^2 + b^2)}$ 

Here,  $\angle C = 90^0$ ; sinC = 1

So, it is a Right-angled triangle.

And also,  $a^2 + b^2 = c^2$ 

Now,

$$\frac{a^2 + b^2}{a^2 - b^2} \sin(A - B) = \frac{c^2}{a^2 - b^2} \sin(A - B)$$

We know that,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where R is the circumradius.

Therefore,

$$= \frac{4R^{2} \sin^{2} C}{4R^{2} \sin^{2} A - 4R^{2} \sin^{2} B} \sin(A - B) = \frac{\sin(A - B)}{\sin^{2} A - \sin^{2} B} [As, \sin C = 1]$$

$$= \frac{\sin(A - B)}{(\sin A + \sin B)(\sin A - \sin B)} = \frac{\sin(A - B)}{[2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}][2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}]}$$

$$= \frac{\sin(A - B)}{2 \sin \frac{A + B}{2} \cos \frac{A + B}{2} \cdot 2 \sin \frac{A - B}{2} \cos \frac{A - B}{2}} = \frac{\sin(A - B)}{\sin(A + B) \sin(A - B)}$$

$$= \frac{1}{\sin(A + B)}$$

$$= \frac{1}{\sin(\pi - C)} = \frac{1}{\sin C} = 1$$

Therefore,

$$\Rightarrow \frac{a^2 + b^2}{a^2 - b^2} \sin(A - B) = 1$$
$$\Rightarrow \sin(A - B) = \frac{a^2 - b^2}{a^2 + b^2} [Proved]$$

#### **Question: 18**

In a  $\triangle ABC$ , if

### Solution:

Given:  $\frac{\cos A}{a} = \frac{\cos B}{b}$ 

Need to prove:  $\triangle ABC$  is isosceles.

 $\frac{\cos A}{a} = \frac{\cos B}{b}$   $\Rightarrow \frac{\sqrt{1-\sin^2 A}}{a} = \frac{\sqrt{1-\sin^2 B}}{b}$   $\Rightarrow \frac{1-\sin^2 A}{a^2} = \frac{1-\sin^2 B}{b^2} \text{ [Squaring both sides]}$   $\Rightarrow \frac{1}{a^2} - \frac{\sin^2 A}{a^2} = \frac{1}{b^2} - \frac{\sin^2 B}{b^2}$ We know,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ Therefore,  $\frac{\sin^2 A}{a^2} = \frac{\sin^2 B}{b^2}$ So,  $\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}$ 

That means a and b sides are of same length. Therefore, the triangle is isosceles. [Proved]

# **Question: 19**

In a  $\triangle ABC$ , if

#### Solution:

Given:  $\sin^2 A + \sin^2 B = \sin^2 C$ 

Need to prove: The triangle is right-angled

$$\sin^{2}A + \sin^{2}B = \sin^{2}C$$
We know,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ 
So,
$$\sin^{2}A + \sin^{2}B = \sin^{2}C$$

$$\frac{a^{2}}{4R^{2}} + \frac{b^{2}}{4R^{2}} = \frac{c^{2}}{4R^{2}}$$

$$a^{2} + b^{2} = c^{2}$$

This is one of the properties of right angled triangle. And it is satisfied here. Hence, the triangle is right angled. [Proved]

## **Question: 20**

Solve the triangl

### Solution:

Given: a = 2 cm, b = 1 cm and c =  $\sqrt{3}$ cm

Perimeter =  $a + b + c = 3 + \sqrt{3}$  cm

Area = 
$$\sqrt{s(s-a)(s-b)(s-c)}$$
  
=  $\sqrt{\frac{3+\sqrt{3}}{2}}(\frac{3+\sqrt{3}}{2}-2)(\frac{3+\sqrt{3}}{2}-1)(\frac{3+\sqrt{3}}{2}-\sqrt{3})$   
=  $\sqrt{\frac{3+\sqrt{3}}{2}}\cdot\frac{\sqrt{3}-1}{2}\cdot\frac{\sqrt{3}+1}{2}\cdot\frac{3-\sqrt{3}}{2}$   
=  $\sqrt{\frac{(9-3)(3-1)}{16}}$   
=  $\sqrt{\frac{12}{16}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \text{ cm}^2 \text{ [Proved]}$ 

## **Question: 21**

In a  $\triangle ABC$ , if a =

#### Solution:

Given: a = 3 cm, b = 5 cm and c = 7 cm

Need to find: cos A, cos B, cos C

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 7^2 - 3^2}{2.5.7} = \frac{65}{70} = \frac{13}{14}$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{7^2 + 3^2 - 5^2}{2.7.3} = \frac{33}{42}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3^2 + 5^2 - 7^2}{2.3.5} = \frac{-15}{30} = -\frac{1}{2}$$

# **Question: 22**

If the angles of

Given: Angles of a triangle are in the ratio 1 : 2 : 3Need to prove: Its corresponding sides are in the ratio  $1:\sqrt{3}:2$ Let the angles are x , 2x , 3x Therefore, x + 2x + 3x =  $180^{\circ}$  $6x = 180^{\circ}$  $x = 30^{\circ}$ So, the angles are  $30^{\circ}$  ,  $60^{\circ}$  ,  $90^{\circ}$ So, the ratio of the corresponding sides are:  $= \sin 30^{\circ} : \sin 60^{\circ} : \sin 90^{\circ}$  $= \frac{1}{2}:\frac{\sqrt{3}}{2}:1$ 

# Exercise : 18B

# **Question: 1**

Two boats leave a

 $= 1: \sqrt{3}: 2$  [Proved]

Solution:



Both the boats starts from A and boat 1 reaches at B and boat 2 reaches at C.

Here, AB = 60Km and AC = 50Km

So, the net distance between ta boats is:

 $\left| \overrightarrow{BC} \right| = \left| \overrightarrow{AC} - \overrightarrow{AB} \right|$ 

 $= \sqrt{60^2 + 50^2 - 2.60.50.\cos 60^0}$ 

 $=\sqrt{3600+2500-3000}$ 

= 55.67Km

# **Question: 2**

A town B is 12 km



Distance from A to B is  $=\sqrt{12^2 + 18^2} = \sqrt{468} = 21.63$ Km

Let, bearing from A to B is  $\theta$ .

So, 
$$\tan \theta = \frac{18}{12} = \frac{3}{2}$$
  
 $\theta = \tan^{-1}(\frac{3}{2}) = 56.31^{0} = 56^{0}20$ 

# **Question: 3**

At the foot of a

## Solution:

After ascending 1 km towards the mountain up an incline of  $30^0$ , the elevation changes to  $60^0$ 

So, according to the figure given,  $AB = AF x \sin 30^0 = (1 x 0.5) = 0.5$  Km.

At point A the elevation changes to  $60^0$ .

In this figure,  $\triangle ABF \cong \triangle ACS$ 

Comparing these triangles, we get AB = AC = 0.5Km

Now,  $CS = AC \times tan60^0 = (0.5 \times 1.73) = 0.865 Km$ 

Therefore, the total height of the mountain is = CS + DC

= CS + BA

= (0.865 + 0.5) Km

= 1.365 Km