

QUICK REVISION TEST

MATRIX

1 Match the following Trigonometric ratios with the equations whose one of the roots is given

Column I

A. $\cos 20^\circ$

B. $\sin 10^\circ$

C. $\tan 15^\circ$

D. $\sin 6^\circ$

Column II

P. $x^3 - 3x^2 - 3x + 1 = 0$

Q. $32x^5 - 40x^3 + 10x - 1 = 0$

R. $8x^3 - 6x - 1 = 0$

S. $8x^3 - 6x + 1 = 0$

Answer:

A-R; B-S; C-P; D-Q;

Solution: A) $A = 20^\circ \Rightarrow 3A = 60^\circ \Rightarrow \cos 3A = \frac{1}{2}$

B) $A = 20^\circ \Rightarrow 3A = 60^\circ \Rightarrow 8x^3 - 6x - 1 = 0$ where $x = \cos 20^\circ$

C) $A = 10^\circ \Rightarrow \sin 3A = \frac{1}{2} \Rightarrow 8x^3 - 6x + 1 = 0$ where $x = \sin 10^\circ$

D) $A = 6^\circ \Rightarrow \sin 5A = \frac{1}{2} \Rightarrow 32x^5 - 40x^3 + 10x - 1 = 0$ where
 $x = \sin 6^\circ$

2. Match the following

Column I

A. The maximum value of $\cos(2A + \theta) + \cos(2B + \theta)$
 $(\theta \in R \text{ and } A, B \text{ are constants})$

B. Maximum value of $\cos 2A + \cos 2B$ ($A, B \in \left(0, \frac{\pi}{2}\right)$, $A + B$ is constant)

C. Minimum value of $\sec 2A + \sec 2B$ ($A, B \in \left(0, \frac{\pi}{4}\right)$, $A + B$ is constant)

D. Minimum value of $\sqrt{\tan \theta + \cot \theta - 2 \cos(2A + 2B)}$ ($\theta \in R$ A, B are constants)

Column II

P. $2 \sin(A + B)$

Q. $2 \sec(A + B)$

R. $2 \cos(A + B)$

S. $2 \cos(A - B)$

Answer: A-S; B-R; C-Q; D-P;

Solution:

$$A) \cos(2A+\theta) + \cos(2B+\theta) = 2\cos(A+B+\theta)\cos(A-B) \leq 2\cos(A-B)$$

$$B) \cos 2A + \cos 2B = 2\cos(A+B)\cos(A-B) \leq 2\cos(A+B)$$

$$C) y = \sec x \text{ always concave up} \quad \therefore \frac{\sec 2A + \sec 2B}{2} \geq \sec(A+B)$$

$$D) \sqrt{\tan\theta + \cot\theta - 2\cos(2A+2B)} = \sqrt{(\tan\theta - \sqrt{\cot\theta})^2 + 4\sin^2(A+B)} \geq 2\sin(A+B)$$

3 Match the followig

Column I

Column II

A. If $\tan\theta$ is the G.M. between $\sin\theta$ and $\cos\theta$

$$\text{then } 2 - 4\sin^2\theta + 3\sin^4\theta - \sin^6\theta =$$

P. 1

$$B. \sqrt{3}\cot 20^\circ - 4\cos 20^\circ =$$

Q. 0

$$C. \cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 76^\circ \cot 16^\circ =$$

R. 3

$$D. \sum_{r=1}^9 \sin^2\left(\frac{r\pi}{18}\right) =$$

S. 5

Answer: A-P; B-P; C-R; D-S;

$$\text{Solution: (A) } \tan^2\theta = \sin\theta \cos\theta \Rightarrow \sin\theta = \cos^3\theta \therefore (1 - \sin^2\theta) + (1 - 3\sin^2\theta) + 3\sin^4\theta - \sin^6\theta \\ = \cos^2\theta + (1 - \sin^2\theta)^3 = \cos^2\theta + \cos^6\theta = \cos^2\theta + \sin^2\theta = 1$$

$$(B) \sin 40^\circ = \sin(60^\circ - 20^\circ)$$

$$2\sin 20^\circ \cos 20^\circ = \frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ$$

$$4\cos 20^\circ = \sqrt{3}\cot 20^\circ - 1$$

$$(C) \frac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ} = \frac{3\sin 76^\circ \sin 16^\circ + \cos 76^\circ \cos 16^\circ}{\sin(76^\circ + 16^\circ)}$$

$$= \frac{2\sin 76^\circ \sin 16^\circ + \cos(76^\circ - 16^\circ)}{\sin(76^\circ + 16^\circ)}$$

$$= \frac{\cos 60^\circ - \cos 92^\circ + \cos 60^\circ}{\sin 92^\circ} = \frac{1 - \cos 92^\circ}{\sin 92^\circ} = \tan 46^\circ = \cot 44^\circ$$

$$(D) \sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{2\pi}{18}\right) + \dots + \sin^2\left(\frac{\pi}{2}\right) = 5$$

4 Match column I with column II

Column I

A. Number of solutions of $|\log |x|| = |\sin x|$ in $[-\pi, \pi]$ is

B. Number of solutions of $e^{\sin x} = \frac{1}{3}$ in $[-\pi, \pi]$ is

C. Number of solutions of $\tan x = \tan^{-1} x$ in $(0, 2\pi)$ is

D. Number of solutions of $\sin^4 x = 1 + \tan^4 x$ is

Column II

P. 0

Q. 1

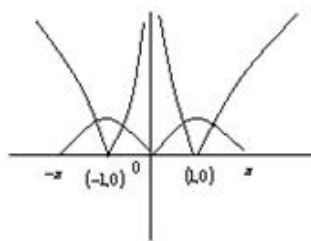
R. 2

S. 3

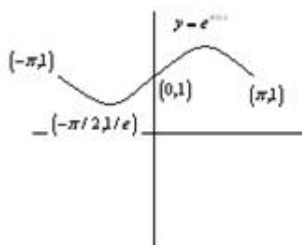
T. 4

Answer: A-T; B-P; C-Q; D-P;

Solution: (A) From graph we can see no. of solution is 4



$$(B) \quad x = \frac{-\pi}{2} \Rightarrow e^{-\sin x} = \frac{1}{e} \Rightarrow x = \frac{\pi}{2} \Rightarrow e^{\sin x} = e$$



$\frac{1}{e} > \frac{1}{3}$ So, $y = e^{\sin x}$ can not intersect $y = \frac{1}{3}$

(C) $\tan x = \tan^{-1} x$ has solution in $\left(\pi, \frac{3\pi}{2}\right)$

This can be seen from the graph.

(D) $\sin^4 x \leq 1, 1 + \tan^4 x \geq 1$

RHS = 1 for $x = 0$ where LHS = 1 for $x = \pm \frac{\pi}{2}$

So, there is no solution.

5 Let $a = (\sin^{-1} x)^{\sin^{-1} x}$; $b = (\sin^{-1} x)^{\cos^{-1} x}$; $c = (\cos^{-1} x)^{\sin^{-1} x}$; $d = (\cos^{-1} x)^{\cos^{-1} x}$ then match the column I with column II

Column I

A. $x \in (0, \cos 1)$

B. $x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$

C. $x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$

D. $x \in (\sin 1, 1)$

Column II

P. $a > b > d > c$

Q. $d > c > a > b$

R. $b > a > d > c$

S. $a < b < d < c$

T. $c > d > a > b$

Answer: A-Q; B-T; C-R; D-P;

Solution: (A) $x \in (0, \cos 1) \Rightarrow \cos^{-1} x > \sin^{-1} x$

Also $\cos^{-1} x > 1$ & $\sin^{-1} x < 1$

$\therefore (\cos^{-1} x)^{\cos^{-1} x}$ is greatest & $(\sin^{-1} x)^{\cos^{-1} x}$ is least And $(\sin^{-1} x)^{\sin^{-1} x} < (\cos^{-1} x)^{\sin^{-1} x}$

$\therefore d > c > a > b$

(B) $x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right) \Rightarrow \cos 1 < x < \frac{1}{\sqrt{2}} \Rightarrow \cos^{-1} x > \sin^{-1} x$ & $\cos^{-1} x < 1, \sin^{-1} x < 1$

So, $(\cos^{-1} x)^{\sin^{-1} x}$ is greatest & $(\sin^{-1} x)^{\cos^{-1} x}$ is least

Also $(\cos^{-1} x)^{\cos^{-1} x} > (\sin^{-1} x)^{\sin^{-1} x} \therefore c > d > a > b$

(C) For $\frac{1}{\sqrt{2}} < x < \sin 1$

We have $1 > \sin^{-1} x > \cos^{-1} x$

so, $(\sin^{-1} x)^{\cos^{-1} x}$ is greatest & $(\cos^{-1} x)^{\sin^{-1} x}$ is least

And $(\sin^{-1} x)^{\sin^{-1} x} > (\cos^{-1} x)^{\cos^{-1} x}$

Hence $b > a > d > c$

(D) $\sin 1 < x < 1$

$\Rightarrow \sin^{-1} x > 1 > \cos^{-1} x$

$\Rightarrow a$ is greatest & c is the least and between b, d greater is b .

$\therefore a > b > d > c$.S

6 Volume of parallelopiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq.units.

Column I

A. Volume of parallelopiped formed by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is

B. Volume of tetrahedron formed by vectors $\vec{a}, \vec{b}$ and $\vec{c}$

C. Volume of parallelopiped formed by vectors $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$

D. Volume of parallelopiped formed by vectors $\vec{a} - \vec{b}, \vec{b} - \vec{c}$ and $\vec{c} - \vec{a}$

Column II

P. 0 sq. units

Q. 12 sq. units

R. 6 sq.units

S. 1 sq.units

Answer: A-R; B-S; C-Q; D-P;

Solution: $[\vec{a} \vec{b} \vec{c}] = \pm 6$

$$\text{a) } V = \left| \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} \right| = 6$$

$$\text{b) } V = \frac{1}{6} \left| \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} \right| = 1$$

$$\text{c) } V = \left| 2 \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} \right| = 12$$

$$\text{d) } V = 0$$

7 Match the following

Column I

Column II

A. The coordinates of a point on the line $x = 4y + 5, z = 3y - 6$ at a distance 3 from the point $(5, 3, -6)$ is

P. $(-1, -2, 0)$

B. The plane containing the lines $\frac{x-2}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-4}{1} = \frac{y-2}{4} = \frac{z-3}{7}$ passes through

Q. $(5, 0, -6)$

C. A line passes through two points $A(2, -3, -1)$ and $B(8, -1, 2)$. The coordinates of a point on this line farthest to the origin at a distance of 14 units from A is

R. $(2, 5, 7)$

D. The coordinates of the foot of the perpendicular from the point $(3, -1, 11)$ on the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

S. $(14, 1, 5)$

Answer: A-Q; B-P; C-S; D-R;

Solution: (a) $\frac{x-5}{4} = y = \frac{z+6}{3}$

$(5, 0, -6)$ lie on the line and 3 units from $(5, 3, -6)$

Let $P = (4t + 5, t, 3t - 6)$

$A = (5, 3, -6)$

$PA = 3 \Rightarrow t = 0$

$\therefore P = (5, 0, -6) \Rightarrow a \rightarrow q$

(b) Plane equation

$$\begin{vmatrix} x-2 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$$(x-2)(7) - (y+3)14 + (z+5)7 = 0$$

$$x - 2 - 2y - 6 + z + 5 = 0$$

$$x - 2y + z - 3 = 0$$

$$\Rightarrow b \rightarrow p$$

(c) Equation of the line is

$$\frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3} = t$$

$$\text{Let } P = (6t+2, 2t-3, 3t-1)$$

$$PA = 14 \Rightarrow \sqrt{36t^2 + 4t^2 + 9t^2} = 14$$

$$\Rightarrow 7t = \pm 14 \Rightarrow t = \pm 2$$

$$\therefore P = (14, 1, 5) \text{ or } (-10, -7, -7)$$

$$(d) A = (3, -1, 11) \text{ foot } F(2t, 3t+2, 4t+3)$$

$$AF \perp L \Rightarrow t = 1$$

$$\therefore F(2, 5, 7)$$

$$d \rightarrow r$$

8 Match the following

Column I

A. In a triangle ABC, if $\cos A + \cos B + \cos C = \frac{9}{5}$ Then $\frac{R}{r}$ is

B. In a triangle ABC, if $r_1 = 2r_2 = 3r_3$ then $a:b$ is

C. If $b = 3, c = 2, A = 120^\circ$ then length of bisector of angle A is

D. The ratio of circum radii of a triangle to its pedal triangle is

Column II

P. $\frac{6}{5}$

Q. $\frac{4}{5}$

R. $\frac{5}{4}$

S. a root of $4x^2 - 13x + 10 = 0$

Answer: A-R,S; B-R,S; C-P; D-S;

Solution: A. $1 + \frac{r}{R} = \frac{9}{5} \Rightarrow \frac{r}{R} = \frac{4}{5} \Rightarrow \frac{R}{r} = \frac{5}{4}$

B. $\frac{s-a}{1} = \frac{s-b}{2} = \frac{s-c}{3} \Rightarrow a:b = 5:4$

C. $\frac{2bc}{b+c} \cos \frac{A}{2} = \frac{2 \cdot 3 \cdot 2}{3+2} \cdot \cos 60^\circ = \frac{6}{5}$

D. 2:1

9 Match the following

Column I

A. Locus of point z satisfying $\operatorname{Re}(z^2) = \operatorname{Re}(z + \bar{z})$

B. Locus of z satisfying $|z - z_1| + |z - z_2| = \lambda, \lambda \in R^+$ and $\lambda > |z_1 - z_2|$

C. Locus of z satisfying $\left| \frac{z - z_1}{z - z_2} \right| = k (k \neq 1, > 0)$

D. Locus of Z satisfying $|z + \bar{z}| + |z - \bar{z}| = 2$

Answer: A-Q; B-P; C-S; D-R;

Solution: Conceptual

Column II

P. Ellipse

Q. Hyperbola with eccentricity $\sqrt{2}$

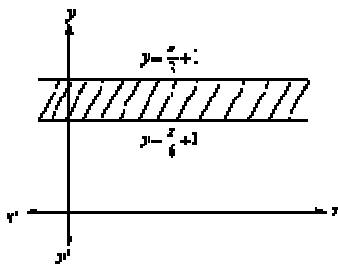
R. A square

S. A circle

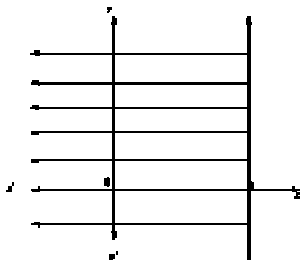
10 Match the column – I (in which arg and diagram is given) with column – II (in which corresponding locus is given)

Column I

A.



B.

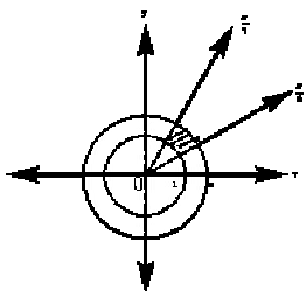


Column II

P. $\frac{\pi}{6} < \arg Z < \frac{\pi}{3}, 2 < |z| < 4$

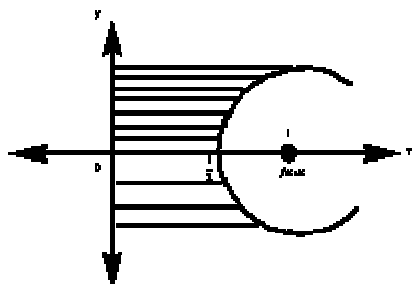
Q. $z + \bar{z} < 4$

C.



$$\text{R. } \frac{\pi}{6} < \text{Im}(z-i) < \frac{\pi}{3}$$

D.



$$\text{S. } \left| \arg \bar{z} + \frac{\pi}{4} \right| < \frac{\pi}{12}, 4 < z \bar{z} < 16$$

$$\text{T. } \text{Re}(z) < |z-1|$$

Answer:

A-R; B-Q; C-PS; D-T;

Solution: Let $Z = x+iy$

$$\text{A) Given } \frac{\pi}{6} + 1 < y < \frac{\pi}{3} + 1 \Rightarrow \frac{\pi}{6} < y-1 < \frac{\pi}{3} \Rightarrow \frac{\pi}{6} < \text{Im}(Z-i) < \frac{\pi}{3}$$

$$\text{B) Given } y < 2 \Rightarrow 2y < 4 \Rightarrow z + \bar{z} < 4$$

$$\text{C) } \frac{\pi}{6} < \arg(z) < \frac{\pi}{3} \text{ and } 2 < |z| < 4$$

$$\Rightarrow -\frac{\pi}{3} < -\arg z < -\frac{\pi}{6} \text{ and } 4 < |z|^2 < 16 \Rightarrow \frac{\pi}{4} - \frac{\pi}{3} < \frac{\pi}{4} + \arg \bar{z} < \frac{\pi}{4} - \frac{\pi}{6} \text{ and } 4 < z \bar{z} < 16$$

$$\Rightarrow \left| \arg \bar{z} + \frac{\pi}{4} \right| < \frac{\pi}{12} \text{ and } 4 < z \bar{z} < 16$$

$$\text{D) Equation of parabola is } y^2 = 4\left(\frac{1}{2}\right)\left(x - \frac{1}{2}\right) \Rightarrow y^2 = 2x - 1$$

$$z = (x, y) \text{ lies out sides of } y^2 = 2x - 1 \Rightarrow y^2 > 2x - 1 \Rightarrow y^2 - 2x + 1 > 0 \Rightarrow x^2 - 2x + 1 + y^2 > x^2$$

$$\sqrt{(x-1)^2 + y^2} > (x-1) \Rightarrow |z-1| > \text{Re}(z)$$

- 11 Given $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$ and $f(1) = 2$. If in $\triangle ABC$, $a = f(3)$, $b = f(1) + f(3)$, $c = f(2) + f(3)$. Also r and R are the inradius and circumradius of $\triangle ABC$. Now match the following

Column I

$$\text{A. } \frac{R}{r} \geq$$

Column II

$$\text{P. } \frac{16}{9}$$

B. $\tan^2 A \geq$

Q. $\frac{7}{9}$

C. $\sec^2 A \leq$

R. $\frac{16}{7}$

D. $r \leq$

S. $\sqrt{7}$

Answer:

A-P,Q,R; B-Q; C-P,R,S; D-S;

Solution: $f(2) = f(1) \times f(1) = 4$, $f(3) = f(1) \times f(2) = 8$

$$\therefore a = 8, b = 10, c = 12 \Rightarrow \Delta = 15\sqrt{7} \text{ and } R = \frac{16}{\sqrt{7}} \Rightarrow r = \frac{\Delta}{S} = \sqrt{7}$$

$$\cos A = \frac{3}{4} \Rightarrow \sec^2 A = \frac{16}{9} \text{ and } \tan^2 A = \frac{7}{9}$$

12 Match following Column – I with Column –II

Column I

A. Locus of the point 'z' satisfying the equation $\operatorname{Re}(z^2) = \operatorname{Re}(z + \bar{z})$ is

B. Locus of the point 'z' satisfying the equation $|z - z_1| + |z - z_2| = \lambda$, $\lambda \in \mathbb{R}^+$ and z_1, z_2 are two fixed complex numbers such that $\lambda > |z_1 - z_2|$ is

C. Locus of the point 'z' satisfying the equation $\left| \frac{2z - i}{z + 1} \right| = m$, where $i = \sqrt{-1}$, $m \in \mathbb{R}^+$ is

D. If $|\bar{z}| = 25$ then the locus of the points represented by $-1 + 75\bar{z}$ is

Column II

P. a straight line

Q. a circle

R. an ellipse

S. a hyperbola

Answer: A-S; B-R; C-P,Q; D-Q;

Solution: a) Let $z = x + iy \Rightarrow x^2 - y^2 = 2x \Rightarrow z$ lies on a hyperbola

b) $|z - z_1| + |z - z_2| = \lambda$ and $\lambda > |z_1 - z_2| \Rightarrow z$ lies on an ellipse

c) $\frac{|z - i/2|}{|z + 1|} = \frac{m}{2}$, if $m = 2 \Rightarrow z$ lies on a straight line if $m \neq 2 \Rightarrow z$ lies on a circle

d) $|z| = 25$, let $-1 + 75\bar{z} = \omega \Rightarrow |\omega + 1| = 75|z| = 75 \times 25 \Rightarrow \omega$ lies on a circle

13 Match the following Column-I, Column-II

Column I

A. If the line $3x + ay - 20 = 0$ cuts the circle $x^2 + y^2 = 25$ at real, distinct or coincident points and $a \in [k, \infty)$ then $[k]$ is $[.]$ denotes G.I.F

B. If the point $(2, k)$ lies out side the circles $x^2 + y^2 + x - 2y - 14 = 0$ and $x^2 + y^2 = 13$ then least positive integral value of k is

Column II

P. 1

Q. 2

C. No. of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + 24 = 0$ is **R. 3**

D. The lines given by $x^2 - 3xy - 3x + 9y = 0$ are normals of a circle which touches the circle $x^2 + y^2 - 6x + 6y + 17 = 0$ externally then radius is **S. 4**

T. 5

Answer: A-Q; B-T; C-S; D-R;

Solution: A) $r \geq d \Rightarrow 5 \geq \frac{|20|}{\sqrt{9+a^2}} \Rightarrow \sqrt{9+a^2} \geq 4$

$$a^2 \geq 7 \Rightarrow |a| \in [\sqrt{7}, \infty] \Rightarrow [K] = 2$$

B) $S_{11} > 0$

$$4 + k^2 + 2 - 2k - 14 > 0 \Rightarrow k^2 - 2k - 8 > 0 \Rightarrow (k-4)(K+2) > 0$$

$$k \in (-\infty, 2) \cup (4, \infty) \Rightarrow S_{11} > 0 \Rightarrow 4 + K^2 > 13 \Rightarrow k^2 > 9$$

$$K \in (-\infty, -3) \cup (4, \infty) \Rightarrow \therefore K \in (-\infty, -3) \cup (4, \infty) \Rightarrow \therefore K = 5$$

$$C) C_1 = (0,0) r_1 = 2 \Rightarrow C_2 = (3,4) r_2 = 1 \Rightarrow C_1 C_2 = 5$$

$$C_1 C_2 > r_2 + r_1$$

Circles are touches internally

No. common tangents = 4

$$D) x(x-3y) - 3(x-3y) = 0$$

Consider a circle with center $(-1, 8)$, radius 2 units

Consider another circle with center $(2, 4)$ radius 3 units

No. of common tangents = 3

$$(x-3)(x-3y) = 0$$

$$\left. \begin{array}{l} x = 3 \\ y = \frac{1}{3}x \end{array} \right\} \text{centre of circle}$$

$$C_1 = (3,1) r_1 = r \Rightarrow C_2 = (3,-3) r_2 = 1$$

$$C_1 C_2 = r_1 + r_2 \Rightarrow 4 = r + 1 \Rightarrow r = 3$$

- 14** From the point $P(4, -4)$ tangents PA and PB are drawn to the circle $x^2 + y^2 - 6x + 2y + 5 = 0$ (C is centre of the circle)

Column I

A. Length of AB

B. Tangents of angle between PA and PC

C. Area of triangle PAB

D. Absolute difference of slope of PA and PB

Column II

P. $5/2$

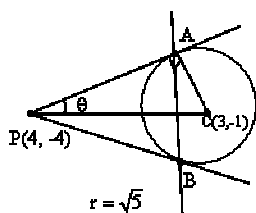
Q. $\sqrt{10}$

R. 1

S. $3/4$

Answer: A-Q; B-R; C-P; D-P;

Solution:



A - Q; B - R; C - P; D - P

Chord of contact of P is $S_1 = 0$, $x - 3y - 11 = 0$

Length of AB = $2\sqrt{r^2 - d^2} = \sqrt{10}$

$$\sin \theta = \frac{AC}{PC} = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}} \Rightarrow \tan \theta = 1$$

$$\text{Area of } \triangle PAB = \frac{1}{2} \times \sqrt{10} \times \frac{5}{\sqrt{10}} = \frac{5}{2}$$

Let slope be m

Tangent line be $mx - y = 4m + 4$, $r = d$

$$\sqrt{5} = \frac{|3m + 1 - 4m - 4|}{\sqrt{1 + m^2}} \Rightarrow 2m^2 - 3m - 2 = 0 \quad m = \frac{-1}{2}, 2 \Rightarrow |m_1 - m_2| = \frac{5}{2}$$

15 Match the following

Column I

A. A ray of light is traveled along the line $2x - y + 1 = 0$ meets the line $x + y = 0$ and then reflects. The equation of the reflected ray is

B. A(1, 2), B(-1, 5) are two vertices of $\triangle ABC$. The third vertex C lies on $2x + y = 2$. The locus of centroid of $\triangle ABC$ is

C. Equation of the line making an angle 45° with $3x - y + 7 = 0$ and passing through (1, 1) is

D. A right angled triangle ABC where $\angle C = \frac{\pi}{2}$ with $AB : BC : CA = 5 : 4 : 3$ moves such that A and B are always on X & Y axes. The locus of C is

Column II

P. $3x - 4y = 0$

Q. $3x + 4y = 0$

R. $x - 2y + 1 = 0$

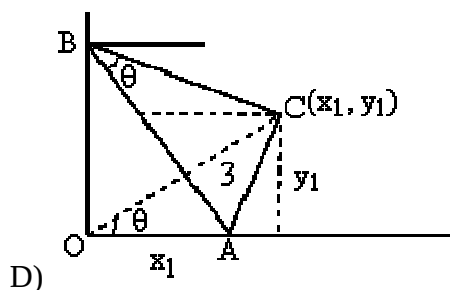
S. $2x + y = 3$

Answer: A-R; B-S; C-RS; D-PQ;

Solution: A) $\frac{x+y}{2} = \frac{2x-y+1}{2}$ i.e. $x - 2y + 1 = 0$

B) $(3x_1, 3y_1, -7)$ lies on $2x + y = 2 \Rightarrow 6x + 3y = 9 \Rightarrow 2x + y = 3$

C) $m_1 = 3$, $m_2 = \frac{3-1}{1+3} = \frac{1}{2}$ or -2. the lines are $x - 2y + 1 = 0$ & $2x + y = 3$



D)

$\angle AOB = \angle ACB = 90^\circ \Rightarrow OACB$ is cyclic

If $\angle AOC = \theta$, then $\angle ABC = \theta$ (angle in the same segment)

$$\Rightarrow \tan \theta = \frac{3}{4} = \frac{|y_1|}{|x_1|}$$

Locus is $3x - 4y = 0$ or $3x + 4y = 0$

16 Match the following

Column I

A. The family of lines $x(2a + 3b) + y(3a + 2b) = 2(b - a)$ passes through the point for all values of a and b then the point is

B. If $9a^2 + 4b^2 - c^2 + 12ab = 0$,

then the family of lines $ax + by + c = 0$ are concurrent at

C. If $12a^2 - 6b^2 - c^2 + ab + 5bc + ac = 0$

then the family of lines $ax + by + c = 0$ is concurrent at

D. If $3a + 2b - c = 0$ then the family of lines $ax + by + c = 0$ is concurrent at

Column II

P. $(3, -2)$

Q. $(3, 2)$

R. $(-4, -3)$

S. $(+2, -2)$

T. $(-3, -2)$

Answer: A-S; B-Q,T; C-P,R; D-T;

Solution: A) $a(2x + 3y + 2) + b(3x + 2y - 2) = 0$ Solve: $(2, -2)$

$$B) 9a^2 + 4b^2 - c^2 + 12ab = 0 = D(3a + 2b)^2 - c^2 = 0 \quad c = \pm(3a + 2b)$$

$$\left. \begin{aligned} 3a + 2b + c &= 0 \\ 3a + 2b - c &= 0 \end{aligned} \right\} = D(3, 2), (-3, 2)$$

$$C) 12a^2 + (b + c)a - (6b^2 + c^2 - 5bc) = 0$$

$$a = -\frac{(b + c) \pm \sqrt{(b + c)^2 + 48(6b^2 + c^2 - 5bc)}}{24} \Rightarrow 3a - 2b + c = 0, 4a + 3b - c = 0$$

$$D) 3a + 2b - c = 0 \Rightarrow \begin{aligned} ax + by + c &= 0 \Rightarrow ax + by + 3a + 2b = 0 \\ x &= -3, y = -2 \quad a(x + 3) + b(y + 2) = 0 \end{aligned}$$

17 Given two vectors $\vec{a} = -\vec{i} + 2\vec{j} + 2\vec{k}$, $\vec{b} = -2\vec{i} + \vec{j} + 2\vec{k}$

Column I

- A. A vector coplanar with $\vec{a}$ and $\vec{b}$
 B. A vector which is perpendicular to both $\vec{a}$ and $\vec{b}$
 C. A vector which is equally inclined to $\vec{a}$ and $\vec{b}$
 D. A vector which forms a Triangle with $\vec{a}$ and $\vec{b}$

Column II

- P. $-3\vec{i} + 3\vec{j} + \vec{k}$
 Q. $2\vec{i} - 2\vec{j} + 3\vec{k}$
 R. $\vec{i} + \vec{j}$
 S. $\vec{i} - \vec{j} + 5\vec{k}$

Answer: A-R; B-Q; C-P,Q,S; D-R;

Solution: A) Vector $\vec{i} + \vec{j}$ is coplanar with $\vec{a}$ and $\vec{b}$

B) $\vec{a} \times \vec{b} = 2\vec{i} - 2\vec{j} + 3\vec{k}$

C) $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$ then $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$ for $\vec{c} = \vec{i} - \vec{j} + 5\vec{k}$

D) Vector is forms Δ le with $\vec{a}, \vec{b} \Rightarrow \vec{c} = \vec{a} + \vec{b} = -3\vec{i} + 3\vec{j} + 4\vec{k}$

18 The Parabola $y^2 = 4ax$ has a chord AB joining points A($at_1^2, 2at_1$) and B ($at_2^2, 2at_2$)

Column I

- A. If normal chord AB subtends 90° at origin then
 B. If AB is a focal chord then
 C. If AB subtends 90° at point (0 , 0) then
 D. If AB is inclined at 45° to the axis of parabola then

Column II

- P. $t_2 = -t_1 + 2$
 Q. $t_2 = \frac{-4}{t_1}$
 R. $t_2 = \frac{-1}{t_1}$
 S. $t_1^2 = 2$

Answer: A-S; B-R; C-Q; D-P;

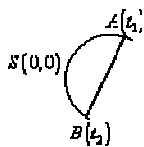
Solution: Slop of the normal at t is $-t$. Slop of the chord joining t_1 and t_2 is $\frac{2}{t_1 + t_2}$

A) $t_2 = -t_1 - \frac{2}{t_1}$, and $\frac{2}{t_1} \cdot \frac{2}{t_2} = -1 \Rightarrow t_1^2 = 2$

B) $t_1 t_2 = -1$

C) $\frac{2}{t_1} \cdot \frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$

D) $\frac{2}{t_1 + t_2} = \tan 45^\circ \Rightarrow t_1 + t_2 = 2$



- 19 The vertices of a triangle are $A(-10,8)$, $B(14,8)$ & $C(-10,26)$. Let G, I, O, S be the centroid, incentre, orthocenter, circumcentre respectively of $\triangle ABC$

Column I

- A. The inradius r is
B. The circumradius R is
C. The area of $\triangle IGO$ is
D. The area of $\triangle SGI$ is

Column II

- P. 15
Q. 12
R. 6
S. 3

Answer: A-R; B-P; C-R; D-S;

Solution: (A) $\triangle ABC = 216, S = \frac{18+24+30}{2} = 36, r = \frac{\Delta}{S} = \frac{216}{36} = 6$

(B) $R = \frac{30}{2} = 15$

(C) $I = (-4,14), G = (-2,14), O = A$

$\triangle IGO = \frac{1}{2} \begin{vmatrix} -4 & 14 & 1 \\ -10 & 8 & 1 \\ -2 & 14 & 1 \end{vmatrix} = 6$

(D) $S = (2,17), \triangle SGI = 3$

- 20 Match the following

Column I

- A. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle centre at $(2,1)$ and radius r then the solution of the equation $[x] \leq r - 1$
B. If the straight line $y = mx$ touches or lies outside the Circle $x^2 + y^2 - 20y + 90 = 0$. Then the solution of $[x] = |m|$ is
C. $\overline{AB}$ is a chord of the circle $x^2 + y^2 = 25$ and the tangents at A and B intersect at ' C '. If $(2, 2\sqrt{3})$ be the midpoint of $\overline{AB}$ And ' I ' is the area of the quadrilateral $OACB$. Then the Solution of the equations $[x] = \left[\frac{I}{8} \right]$ is
D. A circle ' C ' of radius unity touches both the co-ordinate axes lies in first quadrant. If another circle ' C_1 ' also touches both axes in first quadrant and cuts ' C ' orthogonally and if ' r ' is radius of ' C_1 ' then the solution of the equation $[x] = [r]$ is

Column II

- P. $2/3$
Q. 2
R. $-1/2$
S. $16/5$
T. 0

Answer: A-PQRT; B-PQST; C-Q; D-PST;

Solution: $A \rightarrow r = 3$

$$[x] \leq 2 \rightarrow x \in (-\infty, 3)$$

$$B \rightarrow |m| \leq 3, [x] = 0, 1, 2, 3$$

$$C \rightarrow l = \frac{75}{4} \rightarrow [x] = 2$$

$$D \rightarrow r = 2 \pm \sqrt{3}$$

21 Match the following

Column I

Column II

A. If $\sqrt{3}bx + ay = 2ab$ Touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a part whose

P. 0

Eccentric angle is θ then $\cos ec\theta$

B. If e_k is the eccentricity of $(x-3)(y+2) = k^2$ then $e_2 - e_3 =$

Q. 1

C. If $x^2 + y^2 = a^2$ is drawn without intersecting the Curve $xy=9$ then integral values of 'a' are

R. 2

D. If $xy = 1 + \sin^2 \theta$ is a family of rectangular hyperbolas And 'K' is the area of the triangle formed by any tangent With co-ordinate axes then K can be equal to

S. 3

T. 4

Answer: A-R; B-P; C-PQRST; D-RST;

Solution: $A \rightarrow \theta = \frac{\pi}{6}$

$B \rightarrow$ for any k it is rectangular hyperbola

$$C \rightarrow a = 0, 1, 2, 3, 4$$

$D \rightarrow$ compare with $xy = c^2$

$$\text{Tangent at } (x_1, y_1) \text{ is } \frac{x}{x_1} + \frac{y}{y_1} = 2 \Rightarrow K = 2c^2 = 2(1 + \sin^2 \theta)$$

22 A function is defined as $f : \{x_1, x_2, x_3, x_4, x_5, x_6\} \rightarrow \{y_1, y_2, y_3\}$

Column I

Column II

A. Number of onto functions

P. is divisible by 9

B. Number of functions in which $f(x_i) \neq y_i$

Q. is divisible by 5

C. Number of invertible functions

R. is divisible by 4

D. Number of many one functions

S. is divisible by 3

T. is a prime

Answer: A-P,Q,R,S; B-P,R,S; C-P,Q,R,S; D-P,S;

Solution: A) $3^6 - {}^3C_1 2^6 + {}^3C_2 1^6 = 540$

B) x_1, x_2, x_3 can be assigned in 2^3 ways and x_4, x_5, x_6 can be in 3^3 ways

$\therefore$ Total no of ways = $2^3 \times 3 = 216$

C) Number of invertible functions is zero, since it is not possible to have one-one functions.

D) All functions are many one functions = 729

23 Match the following Column-I with Column-II

Column I

Column II

A. The number of cubes with the six faces numbered 1 to 6 can be made, if the sum of the number in each pair of opposite faces is 7 equals

P. 2

B. A man is expected to vote for atleast one of three positions mayor, secretary, attorney. The number of ways he can vote if there are 3 candidates each for three positions is $9k$ where k is

Q. 3

C. The number of ways in which 4 married couple can be seated at a round table if no husband and wife as well as no two men are to seat together is $3k$ where k is

R. 4

D. The sum of all numbers of the form $\frac{\angle 12}{\angle a \angle b \angle c}$ where $a, b, c \in W$ satisfy $a + b + c = 12$, is 3^{3k} where k is

S. 7

T. 1

Answer: A-P; B-S; C-R; D-R;

Solution: A) 1,2,3 have same vertex; 1,2,4 have the same vertex 2 cases are possible

B) ${}^3C_1 \cdot {}^3C_1 + {}^3C_2 \cdot {}^3C_1 + {}^3C_3 \cdot {}^3C_1 \cdot {}^3C_1 = 9 + 27 + 27 = 63 \therefore k = 7$

C) First ladies be seated in $\angle 3$ ways

Then men can be placed in two ways

D) $(1 + x + y)^{12} = \sum \frac{\angle 12}{\angle a \angle b \angle c} x^a y^b$ where $a + b + c = 12 \therefore \frac{\angle 12}{\angle a \angle b \angle c} = 3^{12} \therefore k = 4$

24 Match the following Column-I with Column-II

Column I

Column II

A. Number of straight lines joining any two of 10 points of which four points are collinear

P. 30

- B.** Maximum number of points of intersection of 10 straight lines in the plane **Q.** 60
- C.** Maximum number of points of intersection of six circles in the plane **R.** 40
- D.** Maximum number of points of intersection of six parabolas **S.** 45

Answer: A-R; B-S; C-P; D-Q;

Solution: A) ${}^{10}C_2 - {}^4C_2 + 1 = 45 - 6 + 1 = 40$

B) $1 \times {}^{10}C_2 = 45$

C) $2 \times {}^6C_2 = 30$

D) ${}^6C_2 \times 4 = 60$

25 Match the following Column-I with Column-II

Column I

Column II

A. If $|A| = 2$, then $|2A^{-1}| =$ (where A is of order 3)

P. 1

B. If $|A| = \frac{1}{8}$ then $|adj(adj\ 2A)| =$ (where A is of order 3)

Q. 4

C. If $(A+B)^2 = A^2 + B^2$, and $|A| = 2$, then $|B| =$ (where A and B are of odd order)

R. 24

D. $|A_{2 \times 2}| = 2$, $|B_{3 \times 3}| = 3$ and $|C_{4 \times 4}| = 4$, then $|ABC|$ is equal to

S. 0

T. None

Answer: A-Q; B-P; C-S; D-T;

Solution: A) $|A| = 2 \Rightarrow |2A^{-1}| = \frac{2^3}{|A|} = 4$

B) $|adj(adj(2A))| = |2A|^4 = 2^{12} |A|^4 = \frac{2^{12}}{2^{12}} = 1$

C) $(A+B)^2 = A^2 + B^2 \Rightarrow AB + BA = 0 \Rightarrow |AB| = |-BA| = -|BA| = -|AB| \Rightarrow |AB| = 0$
 $\Rightarrow |B| = 0$

D) Product ABC is not defined

26 A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructing by replacing the elements of P. A subset Q of A is chosen at random. The probability that

Column I

A. $P \cap Q = \phi$

B. $n(P \cap Q) = 1$

C. $P \cup Q = A$

D. $n(P) = n(Q)$

Column II

P. $\frac{n \cdot 3^{n-1}}{4^n}$

Q. $\left(\frac{3}{4}\right)^n$

R. ${}^{2n}C_n \mid 4^n$

S. $\frac{9}{2} \frac{n(n-1)}{4^n}$

Answer: A-Q; B-P; C-Q; D-R;

Solution:

A) $n(S) = 4^n, P \cap Q \Rightarrow n(E) = {}^nC_0 2^n + {}^nC_1 2^{n-1} + \dots + {}^nC_n$

$\Rightarrow P(E) = \left(\frac{3}{4}\right)^n = (2+1)^n = 3^n$

B) $n(P \cap Q) = 1 \Rightarrow n(E) = {}^nC_1 \cdot (2+1)^{n-1} = n \cdot 3^{n-1} \Rightarrow P(E) = \frac{n \cdot 3^{n-1}}{4^n}$

C) $P \cup Q = A \Rightarrow n(E) = {}^nC_0 \cdot 1^0 + {}^nC_1 \cdot 2^1 + {}^nC_2 \cdot 2^2 + \dots + {}^nC_n \cdot 2^n$

$P(E) = \left(\frac{3}{4}\right)^n = (1+2)^n = 3^n$

D) $n(P) = n(Q) \Rightarrow ({}^nC_0)^2 + ({}^nC_1)^2 + \dots + ({}^nC_n)^2 = {}^{2n}C_n$

$P(E) = {}^{2n}C_n \mid 4^n$

27 Let $f(x) = \begin{cases} a(x^2 - x) + b & \text{if } x < 1 \\ x - 1 & \text{if } 1 \leq x \leq 3 \\ cx^2 - dx + 2 & \text{if } x > 3 \end{cases}$ be a differentiable function $\forall x \in R$ then

Column I

A. $a =$

B. $b =$

Column II

P. 0

Q. $\frac{1}{3}$

C. $c =$

R. $\frac{1}{2}$

D. $d =$

S. 1

Answer: A-S; B-P; C-Q; D-S;

Solution:

f is diff. $\Rightarrow f$ is cont. $\Rightarrow f(1) = b = 0$

$[a(2x-1)] = 1 \Rightarrow a = 1$

$2 = f(3) = 9c - 3d + 2 \Rightarrow 3d = 9c \Rightarrow d = 3c$

$1 = [2cx - d] \Rightarrow 6c - d = 1$

$x = 3 \Rightarrow 3c = 1 \Rightarrow c = \frac{1}{3} \Rightarrow d = 1$

28 Column 1 gives functions and column 2 gives the nature of the functions

Column I

A. $f : [0, \infty) \rightarrow [0, \infty), f(x) = \frac{x}{1+x}$

B. $f : \mathbb{R} \rightarrow \{0\} \rightarrow \mathbb{R}, f(x) = x - \frac{1}{x}$

C. $f : \mathbb{R} \rightarrow \{0\} \rightarrow \mathbb{R}, f(x) = x + \frac{1}{x}$

D. $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + \sin x$

Column II

P. one - one onto

Q. one - one but not onto

R. onto but not one - one

S. neither one - one nor onto

Answer: A-Q; B-R; C-S; D-P;

Solution: Match the items of Column I with those of Column II.

Column I

a) $f : [0, \alpha) \rightarrow [0, \alpha), f(x) = \frac{x}{1+x}$

b) $f : \mathbb{R} \rightarrow \{0\} \rightarrow \mathbb{R}, f(x) = x - \frac{1}{x}$

c) $f : \mathbb{R} \rightarrow \{0\} \rightarrow \mathbb{R}, f(x) = x + \frac{1}{x}$

d) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + \sin x$

Column II

p) one - one onto

q) one - one but not onto

r) onto but not one - one

s) neither one - one nor onto

29 A function is defined as $f : \{a_1, a_2, a_3, a_4, a_5, a_6\} \rightarrow \{b_1, b_2, b_3\}$

Column I

A. Number of surjective functions

B. Number of functions in which $f(a_i) \neq b_i$

C. Number of invertible functions

D. Number of many one functions

Column II

P. divisible by 9

Q. divisible by 5

R. divisible by

S. divisible by 3

T. not possible

Answer: A-PQRS; B-PRS; C-PQRS; D-PS;

Solution: Match the items of Column I with those of Column II.

A function is defined as $f : \{a_1 a_2 a_3 a_4 a_5 a_6\} \rightarrow \{b_1 b_2 b_3\}$

Column I

a) Number of surjective functions

b) Number of functions in which $f(a_i) \neq b_i$

c) Number of invertible functions

d) Number of many one functions

Column II

p) divisible by 9

q) divisible by 5

r) divisible by 4

s) divisible by 3

t) not divisible

30 Match The Following

Column I

A. A non-differentiable point of $f(x) = [x] + [1-x], x \in (-1, 3)$

B. The derivative of $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ w.r.t $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ at $x = 0$

C. An integral value of 'a' such that $f(x) = x^3 - ax^2 + x + 1$ possesses at every

Column II

P. 2

Q. 0

R. 4

point on it, a tangent making acute angle with positive direction of x-axis is

D. Let $f(x) = \begin{cases} x+3, & x \in [1, 2] \\ 6-x, & x \in [4, 5] \end{cases}$ then the number of points of intersection of $y = f(x)$ and $y = f^{-1}(x)$ is **S. 3**

T. 1

Answer: A-PQT; B-R; C-QT; D-P;

Solution: A) $f(x) = [x] + 1 + [-x], x \in (-1, 3)$

Not diff at 0, 1, 2

B) Let
$$\left. \begin{aligned} y &= \tan^{-1} \left(\frac{2 \sin \theta \cos \theta}{\cos 2\theta} \right) = 2 \sin^{-1} \theta \\ z &= \tan^{-1} \left(\frac{\sec \phi - 1}{\tan \phi} \right) = \frac{1}{2} \cdot \tan^{-1} x \end{aligned} \right\} \frac{dy}{dx} = \frac{\frac{2}{\sqrt{1-x^2}}}{\frac{1}{2} \left(\frac{1}{1+x^2} \right)} = 4 \text{ (at } x = 0 \text{)}$$

C) $\frac{dy}{dx} > 0 \Rightarrow 3x^2 - 2ax + 1 > 0 \quad \forall x \in R$

$\Delta < 0 \Rightarrow 4(a^2 - 3) < 0 \Rightarrow a \in (-\sqrt{3}, \sqrt{3}) \Rightarrow a = -1, 0, 1$

D) $f(x) = \begin{cases} x+3, & x \in [1, 2] \\ 6-x, & x \in [4, 5] \end{cases}$

$f^{-1}(x) = \begin{cases} x-3, & x \in [4, 5] \\ 6-x, & x \in [1, 2] \end{cases}$

Then $f(x) = f^{-1}(x) \Rightarrow x+3 = 6-x$ or $6-x = x-3$

$$\Rightarrow \left. \begin{aligned} x &= \frac{3}{2} \\ y &= \frac{9}{2} \end{aligned} \right\} \text{ or } \left. \begin{aligned} x &= \frac{9}{2} \\ y &= \frac{3}{2} \end{aligned} \right\}$$

$\left(\frac{3}{2}, \frac{9}{2} \right) \text{ or } \left(\frac{9}{2}, \frac{3}{2} \right)$

No. of points of intersection = 2

31

Let $f(x) = |x| + |x^2 - 1| + 1$ and if

$G_1 = \{\text{set of points of local maxima}\}$

$G_2 = \{\text{set of points of local minima}\}$

$G_3 = \{\text{set of non-differentiable points}\}$

$G_4 = \{\text{set of points where derivative zero}\}$

Then match the following

Column I

A. $n(G_1)$

B. $n(G_2)$

C. $n(G_3)$

D. $n(G_4)$

Column II

P. 5

Q. 4

R. 3

S. 2

Answer: A-S; B-R; C-R; D-S;

$$\text{Solution: } f(x) = \begin{cases} x^2 - x & , -\infty < x < -1 \\ -x^2 - x + 2, & -1 \leq x < 0 \\ -x^2 + x + 2, & 0 \leq x < 1 \\ x^2 + x & , 1 \leq x < \infty \end{cases} \Rightarrow f^1(x) = \begin{cases} 2x - 1 & , -\infty < x < -1 \\ -2x - 1, & -1 < x < 0 \\ -2x + 1, & 0 < x < 1 \\ 2x + 1 & , 1 < x < \infty \end{cases}$$

	L.D	R.D		
-1	-3	1	not diff	local min
0	-1	1	not diff	local min
1	-1	3	not diff	local min

$$\text{Also } f^1(x) = 0 \Rightarrow x = \frac{-1}{2}, \quad x = \frac{1}{2}$$

(local max) (local max)

32 For $0 < x < 1$, match the following

Column I

A. $\int \frac{dx}{(1-\sqrt{x})\sqrt{1-x}}$

B. $\int \frac{dx}{(1+\sqrt{x})\sqrt{1-x}}$

C. $\int \frac{dx}{(1-\sqrt{x})\sqrt{x-x^2}}$

D. $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$

Column II

P. $2\left(\frac{\sqrt{x}+1}{\sqrt{1-x}}\right) + c$

Q. $2\left(\frac{1-\sqrt{x}}{\sqrt{1-x}} + \sin^{-1}\sqrt{x}\right) + c$

R. $2\left(\frac{\sqrt{x}-1}{\sqrt{1-x}}\right) + c$

S. $2\left(\frac{1+\sqrt{x}}{\sqrt{1-x}} - \sin^{-1}\sqrt{x}\right) + c$

Answer: A-R; B-Q; C-P; D-R;

Solution: Put $n = \sin^2 \theta$

33 Match the following Column-I with Column-II

Column I

Column II

A. $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \sqrt{\cot x}}$

P. $\frac{\pi}{2}$

B. $\int_{-1}^1 \frac{d}{dx} \left(\tan^{-1} \frac{1}{x} \right) dx$

Q. $-\frac{\pi}{4}$

C. $\int_{-\pi}^{\pi} \frac{\cos^2 x dx}{1 + a^x}$

R. $\frac{\pi}{4}$

D. If $\int \frac{dx}{1 + \sin x} = \tan \left(a + \frac{x}{2} \right) + b$

S. $-\frac{\pi}{2}$

Answer: A-R; B-S; C-P; D-Q;

Solution: B) $I = \int_{-1}^1 \frac{-1}{1+x^2} = -2 \int_0^1 \tan^{-1} x = -2 \left(\frac{\pi}{4} \right) = -\frac{\pi}{2}$

C) $2I = 2 \int_0^{\pi} \cos^2 x dx \Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{\pi}{2}$

34 $f(x) = x^2 + 4x + 3$ then

Column I

Column II

A. No. of non differentiable points of $|f(x)|$

P. 3

B. No. of non differentiable points of $f(|x|)$

Q. 1

C. No. of non differentiable points of $|f(|x|)|$

R. 2

D. The minimum value of $|f(|x|)|$ is

S. 5

Answer: A-R; B-Q; C-Q; D-P;

Solution: $f(x) = x^2 + 4x + 3$

A) $|f(x)|$ graph

No. of non differentiable points two

B) $f(|x|) = f(x), x \geq 0 = f(-x) \quad x < 0$

No. of non differentiable point one

C) $|f(|x|)| = f(|x|)$ graphs are equal $\therefore$ No. of non differentiable points one

D) Mini value of $|f(|x|)|$ is 3

35 Match the following Column-I with Column-II

Column I

Column II

A. If the subnormal at any point on the curve $y = 3^{1-k} \cdot x^k$ is of constant length then k is equal to

P. 1

B. If k be the slope of common tangent to the curves $y^2 = 4x$ and $x^2 = 32y$ then k is equal to

Q. 2

C. The value of k for which the area of the triangle included between the axes and any tangent to the curve $x^k y = 1$ is constant is

R. -1/2

D. If $kx^2 - 6y^2 = 1$ and $9x^2 + 6y^2 = 1$ intersect orthogonally then -9/k equal to

S. 1/2

Answer: A-S; B-R; C-P; D-Q;

Solution: A) $y = 3^{1-k} \cdot x^k$

$$m = \frac{dy}{dx} = k \cdot 3^{1-k} \cdot x^{k-1}$$

$$S.N = \lambda$$

$$k \cdot 3^{2-2k} \cdot x^{2k-1} = \lambda \Rightarrow k = \frac{1}{2}$$

$$\text{B) Slope of common tangent} = -\left(\frac{a}{b}\right)^{1/3} = -\frac{1}{2}$$

$$\text{C) } y = \frac{1}{x^k}$$

$$\text{Slope } m = \frac{-k}{x_1^{k+1}}$$

$$\text{Tangent line } kx + x_1^{k+1}y = (k+1)x_1$$

$$\text{Area of the triangle} = \frac{(k+1)^2}{ak} x_1^{1-k}$$

$$1-k=0 \Rightarrow k=1$$

$$\text{D) } \frac{-9}{k} = 2$$