Polynomials

Exercise 3:

Solution 1:

$$
\begin{aligned}
& p(x) = x^3 + kx^2 - 4x + 5 \\
& \Rightarrow p(3) = (3)^3 + k(3)^2 - 4(3) + 5 \\
& \qquad\quad = 27 + 9k - 12 + 5 \\
& \qquad\quad = 9k + 20 \\
& \text{Now, } p(3) = 0 \\
& \Rightarrow 9k + 20 = 0 \\
& \Rightarrow 9k = -20 \\
& \Rightarrow k = -\frac{20}{9}
\end{aligned}
$$

Solution 2(1):

$$
\begin{array}{r|l}
 & x^3 + 7x - 13 \\
\hline
x + 2 & x^4 + 2x^3 + 7x^2 + x - 5 \\
 & x^4 +_{(-)} 2x^3 \\
\hline
 & \qquad 7x^2 + x - 5 \\
 & \quad {}_{(-)}7x^2 +_{(-)} 14x \\
\hline
 & \qquad\qquad -13x - 5 \\
 & \qquad\quad -_{(+)}13x -_{(+)}26 \\
\hline
 & \qquad\qquad\qquad 21
\end{array}
$$

Quotient (Q): $x^3 + 7x - 13$

Remainder (R): 21

t>

Solution 2(2):

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 29 \\
\hline
x + 2 & 2x^3 - 5x^2 + 11x + 19 \\
 & {}_{(-)}2x^3 +_{(-)} 4x^2 \\
\hline
 & \qquad -9x^2 + 11x + 19 \\
 & \qquad -_{(+)}9x^2 -_{(+)}18x \\
\hline
 & \qquad\qquad 29x + 19 \\
 & \qquad\quad {}_{(-)}29x +_{(-)}58 \\
\hline
 & \qquad\qquad\qquad -39
\end{array}
$$

Quotient (Q): $2x^2 - 9x + 29$

Remainder (R): -39

Solution 2(3):

$$\begin{array}{r|l} & 5x^2 - x + 10 \\ x + 2 & 5x^3 + 9x^2 + 8x + 20 \\ & \underline{{}_{(-)}5x^3 + {}_{(-)}10x^2} \\ & -x^2 + 8x + 20 \\ & \underline{-{}_{(+)}x^2 - {}_{(+)}2x} \\ & 10x + 20 \\ & \underline{{}_{(-)}10x + {}_{(-)}20} \\ & 0 \end{array}$$

Quotient (Q): $5x^2 - x + 10$

Remainder (R): 0

Solution 3:

If we divide the total number of chocolates that the student had ($x^4 - 3x^3 + 5x^2 + 8x + 5$) by the number of chocolates received by each friend ($x^2 - 1$), the quotient will give the number of friends and the remainder will give the number of chocolates left over for his teacher.

$$\begin{array}{r|l} & x^2 - 3x + 6 \\ x^2 - 1 & x^4 - 3x^3 + 5x^2 + 8x + 5 \\ & \underline{{}_{(-)}x^4 \qquad - {}_{(+)}x^2} \\ & -3x^3 + 6x^2 + 8x + 5 \\ & \underline{-{}_{(+)}3x^3 \qquad + {}_{(-)}3x} \\ & 6x^2 + 5x + 5 \\ & \underline{{}_{(-)}6x^2 \qquad - {}_{(+)}6} \\ & 5x + 11 \end{array}$$

Thus, the number of friends = $x^2 - 3x + 6$

The number of chocolates left over for his teacher = $5x + 11$.

Given that 26 chocolates are left with him.

$\therefore 5x + 11 = 26$

$\therefore 5x = 15$

$\therefore x = 3$

The total number of chocolates the student had

$= x^4 - 3x^3 + 5x^2 + 8x + 5$

$= (3)^4 - 3(3)^3 + 5(3)^2 + 8(3) + 5$

$= 81 - 81 + 45 + 24 + 5$

$= 74$

Thus, the student had 74 chocolates.

The number of chocolates received by each friend = $x^2 - 1$

$= (3)^2 - 1$

$= 9 - 1$

$= 8$

Thus, each friend received 8 chocolates.

The number of friends

$= x^2 – 3x + 6$

$= (3)^2 – 3(3) + 6$

$= 9 – 9 + 6$

$= 6$

Thus, the boy has 6 friends.**E**

Solution 4:

If we divide the total sum collected Rs$(2x^3 + x^2 – 5x – 3)$ by the contribution received from each student Rs.$(2x + 3)$, the quotient will give the number of students and the remainder must be zero.

$$
\begin{array}{r|l}
 & x^2 - x - 1 \\
\hline
2x + 3 & 2x^3 + x^2 - 5x - 3 \\
 & 2x^3 + 3x^2 \\
\hline
 & \quad -2x^2 - 5x - 3 \\
 & \quad -2x^2 - 3x \\
\hline
 & \qquad -2x - 3 \\
 & \qquad -2x - 3 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Thus, the number of students in the class is $(x^2 - x - 1)$.

Solution 5:

$$\text{Second polynomial} = \frac{\text{Product of two polynomials}}{\text{First polynomial}}$$

$$= \frac{x^4 - 3x^3 + 8x^2 - 9x + 15}{x^2 - 3x + 5}$$

$$
\begin{array}{r|l}
 & x^2 + 3 \\
\hline
x^2 - 3x + 5 & x^4 - 3x^3 + 8x^2 - 9x + 15 \\
 & {}_{(-)}x^4 -_{(+)} 3x^3 +_{(-)} 5x^2 \\
\hline
 & \qquad 3x^2 - 9x + 15 \\
 & \qquad {}_{(-)}3x^2 -_{(+)} 9x +_{(-)} 15 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$\therefore$ The second polynomial is $(x^2 + 3)$.

Solution 6:

Since $(x – 4)$ is a factor of $x^3 – 6x^2 + 4x + 16$, split the term other than the first and the last to obtain the other factor.

$x^3 – 6x^2 + 4x + 16$

$= \underline{x^3 – 4x^2} – \underline{2x^2 + 8x} – \underline{4x + 16}$

$= x^2(x – 4) – 2x(x – 4) – 4(x – 4)$

$= (x – 4)(x^2 – 2x – 4)$

Thus, the other factor is $(x^2 – 2x – 4)$.

Solution 7:

$(107)^2$

$= (100 + 7)^2$

$= (100)^2 + 2(100)(7) + (7)^2$

$= 10000 + 1400 + 49$

$= 11449$

Solution 8:

If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

Let a = (-7), b = 12 and c = (-5)

∴ a + b + c = (-7) + 12 + (-5) = -7 + 12 + -5 = 0

Now, $a^3 + b^3 + c^3 = 3abc$

∴ $(-7)^3 + (12)^3 + (-5)^3$

= 3(-7)(12)(-5)

= (-21)(-60)

= 1260

Solution 9:

$4x^2 + 9y^2 + 25z^2 + 12xy - 30yz - 20zx$

$= (2x)^2 + (3y)^2 + (-5z)^2 + 2(2x)(3y) + 2(3y)(-5z) + 2(-5z)(2x)$

$= (2x + 3y - 5z)^2$

Solution 10(1) :

$$
\begin{array}{r l}
 & 2x^2 + 5x + 19 \\
x - 2 \,\big) & 2x^3 + x^2 + 9x + 17 \\
 & \underline{{}_{(-)}2x^3 - {}_{(+)}4x^2} \\
 & 5x^2 + 9x + 17 \\
 & \underline{{}_{(-)}5x^2 - {}_{(+)}10x} \\
 & 19x + 17 \\
 & \underline{{}_{(-)}19x - {}_{(+)}38} \\
 & 55
\end{array}
$$

Quotient (Q): $2x^2 + 5x + 19$

Remainder (R): 55

Solution 10(2) :

$$
\begin{array}{r l}
 & x^4 - x^3 + x^2 - x + 1 \\
x + 1 \,\big) & x^5 + 1 \\
 & \underline{{}_{(-)}x^5 + {}_{(-)}x^4} \\
 & -x^4 + 1 \\
 & \underline{-{}_{(+)}x^4 - {}_{(+)}x^3} \\
 & x^3 + 1 \\
 & \underline{{}_{(-)}x^3 + {}_{(-)}x^2} \\
 & -x^2 + 1 \\
 & \underline{-{}_{(+)}x^2 - {}_{(+)}x} \\
 & x + 1 \\
 & \underline{{}_{(-)}x + {}_{(-)}1} \\
 & 0
\end{array}
$$

Quotient(Q) : $x^4 - x^3 + x^2 - x + 1$

Remainder(R) : 0

Solution 10(3) :

$$
\begin{array}{r|l}
 & 3x^3 + 10x^2 + 4x + 9 \\
\hline
x - 1 & 3x^4 + 7x^3 - 6x^2 + 5x - 9 \\
 & _{(-)}3x^4 - _{(+)}3x^3 \\
\cline{2-2}
 & 10x^3 - 6x^2 + 5x - 9 \\
 & _{(-)}10x^3 - _{(+)}10x^2 \\
\cline{2-2}
 & 4x^2 + 5x - 9 \\
 & _{(-)}4x^2 - _{(+)}4x \\
\cline{2-2}
 & 9x - 9 \\
 & _{(-)}9x - _{(+)}9 \\
\cline{2-2}
 & 0
\end{array}
$$

Quotient (Q): $3x^3 + 10x^2 + 4x + 9$

Remainder (R): 0

Solution 10(4) :

$$
\begin{array}{r|l}
 & 7x - 18 \\
\hline
x^2 + x + 3 & 7x^3 - 11x^2 + 3x - 49 \\
 & _{(-)}7x^3 +_{(-)}7x^2 + _{(-)}21x \\
\cline{2-2}
 & -18x^2 - 18x - 49 \\
 & _{(+)}-18x^2 - _{(+)}18x - _{(+)}54 \\
\cline{2-2}
 & 5
\end{array}
$$

Quotient (Q): $7x - 18$

Remainder (R): 5

Solution 11 :

$a + b + c = 6$

$\therefore (a + b + c)^2 = (6)^2$

$\therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 36$

$\therefore 60 + 2(ab + bc + ca) = 36$

$\therefore 2(ab + bc + ca) = 36 - 60$

$\therefore 2(ab + bc + ca) = -24$

$\therefore ab + bc + ca = -12$

$a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

$= (6)[(60) - (-12)]$

$= 6(60 + 12)$

$= 6(72)$

$= 432$

Solution 12(1) :

a. $(x - 3)$

If $p(3) = 0$, then factor of $p(x)$ is $(x - 3)$.

Solution 12(2) :

b. 13

Remainder $= p(2) = (2)^3 + 2(2)^2 - 6(2) + 9 = 13$

If $x^3 + 2x^2 - 6x + 9$ is divided by $(x - 2)$, then 13 is the remainder

Solution 12(3) :

d. 5

The degree of the polynomial $x^5 + 3x^3 – 7x^2 + 9x + 11$ is 5.

Solution 12(4) :

d. -18

$(x – 2)$ is a factor of $p(x) = 3x^4 – 2x^3 + 7x^2 – 21x + k$.

$\therefore$ p(2) = 0

$\therefore 3(2)^4 – 2(2)^3 + 7(2)^2 – 21(2) + k = 0$

$\therefore 48 – 16 + 28 – 42 + k = 0$

$\therefore 18 + k = 0$

$\therefore k = -18$

Solution 12(5) :

b. $\frac{3}{7}$

$p(x) = 0$

$\Rightarrow 7x - 3 = 0$

$\Rightarrow 7x = 3$

$\Rightarrow x = \frac{3}{7}$

$\therefore \frac{3}{7}$ is the zero of $7x - 3$.

Solution 12(6) :

b. 2

When polynomial p(x) is divided by x + 1, the remainder is p(-1).

$\therefore$ Remainder $= p(-1) = (-1)^2 + 6(-1) + 7 = 2$

Solution 12(7) :

d. (y + 3)(y + 7)

$y^2 + 10y + 21$

$= y^2 + 3y + 7y + 21$

$= y(y + 3) + 7(y + 3)$

$= (y + 3)(y + 7)$

Factors of $y^2 + 10y + 21$ are (y + 3)(y + 7).

Solution 12(8) :

c. 26

$a^3 – b^3 = (a – b)^3 + 3ab(a – b) = (2)^3 + 3(3)(2) = 26$

Solution 12(9) :

d. 0

$a^3 + b^3 + c^3 – 3abc$

$= a^3 + a^3 + a^3 – 3a(a)(a)$

$= 3a^3 – 3a^3$

$= 0$

Solution 12(10) :

c. $x^2 + x - 6$

The last term of the dividend is (-18) and the last term of the divisor is (3). Hence the last term of the quotient must be $\frac{(-18)}{3} = (-6)$.

The correct option is $x^2 + x - 6$.

Solution 12(11) :

b. 1

Remainder = $p(-3) = (-3)^3 + 28 = (-27) + 28 = 1$

Solution 12(12) :

c. 12

Opposite of the remainder should be added to make the polynomial divisible by $x - 4$.

Remainder = $p(4) = (4)^3 - 76 = 64 - 76 = -12$

Thus, 12 should be added.

Solution 12(13) :

c. $5x + 7y$

$25x^2 - 49y^2 = (5x)^2 - (7y)^2 = (5x + 7y)(5x - 7y)$

Solution 12(14) :

b. 1

$p(0) = -6 \neq 0$, $p(1) = 0$

$p(2) = 8 \neq 0$ and $p(3) = 24 \neq 0$.

Hence, 1 is a zero of p(x).

Solution 12(15) :

c. $x^2 - 4x + 16$

$$\text{Number of textbooks} = \frac{x^3 + 64}{x + 4} = \frac{(x + 4)(x^2 - 4x + 16)}{(x + 4)}$$
$$= x^2 - 4x + 16$$

Solution 12(16) :

c. $64x^3 - 343y^3 - 336x^2y + 588xy^2$

$(4x - 7y)^3$

$=(4x)^3 - (7y)^3 - 3(4x)(7y)(4x - 7y)$

$= 64x^3 - 343y^3 - 84xy(4x - 7y)$

$= 64x^3 - 343y^3 - 336x^2y + 588xy^2$

Exercise 3.1:

Solution 1:

1. $p(x) = 3x^7 - 6x^5 + 4x^3 - x^2 - 5$

   The degree of the polynomial is 7.
2. $p(x) = x^{100} - (x^{10})^{20} + 3x^{50} + x^{25} + x^5 - 7$

   $\therefore p(x) = x^{100} - x^{200} + 3x^{50} + x^{25} + x^5 - 7$

The degree of polynomial is 200.

3. $p(x) = 7x – 3x^2 + 4x^3 – x^4$

   Thus, the degree of the polynomial is 4.

4. $p(x) = 3.14x^2 + 1.57x + 1$

   The degree of the polynomial is 2.

Solution 2:

1. $p(x) = 4x^3 + 3x^2 + 2x + 1$

   In the polynomial p(x), the coefficient of x^3 is 4.

2. $p(x) = x^2 + 2x + 1$

   In the polynomial p(x) the term with x^3 is absent.

   Hence, the coefficient of x^3 in the polynomial p(x) is 0.

3. $p(x) = x^2 - \sqrt{3}x^3 + 4x^7 + 6$

   In the polynomial p(x), the coefficient of x^3 is $-\sqrt{3}$.

Solution 3:

1. $(x) = x^2 + 27$

   The degree of the polynomial p(x) is 2.

   Hence the given polynomial is a quadratic polynomial.

2. $p(x) = 2010x + 2009$

   The degree of the polynomial p(x) is 1.

   Hence the given polynomial is a linear polynomial.

3. $p(x) = 4x^2 + 7x^3 + 3$

   The degree of the polynomial p(x) is 3.

   Hence the given polynomial is a cubic polynomial.

4. $p(x) = (x – 1)(x + 1) = x^2 – 1$

   The degree of the polynomial p(x) is 2.

   Hence, the given polynomial is a quadratic polynomial.

Solution 4:

1. $p(x) = x^7 + 10x^5 + 4x^3 + 3x + 1$

   The index in each term of the given algebraic expression is a whole number.

   Thus, p(x) is a polynomial.

(2) $p(x) = x^{\frac{-5}{2}} + 10x + 4$

The index of x in the first term of the given algebraic expression is $\frac{-5}{2}$, which is not a whole number.

Thus, p(x) is not a polynomial.

(3) $p(x) = x + \frac{1}{x} = x + x^{-1}$

The index in second term of the given algebraic expression is -1 which is not a whole number.

Thus, p(x) is not a polynomial.

Solution 5:

$8x^{10}$ is a monomial of degree 10.

$4x^{20} + 9x^{15}$ is a binomial of degree 20.

$\sqrt{2}y^{25} - 7y^{10} + 5$ is a trinomial of degree 25.

Exercise 3.2:

Solution 1:

Given, $p(x) = x^3 - x$
If $x = 3$,
$(3) = 3^3 - 3 = 27 - 3 = 24 \neq 0$
Hence, 3 is not a zero of $p(x) = x^3 - x$.

If $x = 0$,
$p(0) = 0^3 - 0 = 0 - 0 = 0$
Hence, 0 is a zero of $p(x) = x^3 - x$.

Solution 2:

$p(x) = x^4 + 2x^3 - x + 5$
At $x = 2$,
$p(2) = (2)^4 + 2(2)^3 - 2 + 5 = 16 + 16 - 2 + 5$
$\therefore p(2) = 35$
$p(x) = 3x^3 - 5x^2 + 6x - 9$

At $x = 0$,
$p(0) = 3(0)^3 - 5(0)^2 + 6(0) - 9 = 0 - 0 + 0 - 9$
$\therefore p(0) = -9$
At $x = -1$
$p(-1)$
$= 3(-1)^3 - 5(-1)^2 + 6(-1) - 9$
$= 3(-1) - 5(1) + 6(-1) - 9$
$= -3 - 5 - 6 - 9$
$\therefore p(-1) = -23$

$p(x) = 5x^3 + 11x^2 + 10$
At $x = -2$,
$p(-2)$
$= 5(-2)^3 + 11(-2)^2 + 10$
$= 5(-8) + 11(4) + 10$
$= -40 + 44 + 10$
$\therefore p(-2) = 14$

1. $p(x) = x^4 + 2x^3 - x + 5$
   At $x = 2$,
   $p(2) = (2)^4 + 2(2)^3 - 2 + 5 = 16 + 16 - 2 + 5$
   $\therefore p(2) = 35$
2. $p(x) = 3x^3 - 5x^2 + 6x - 9$
   At $x = 0$,
   $p(0) = 3(0)^3 - 5(0)^2 + 6(0) - 9 = 0 - 0 + 0 - 9$
   $\therefore p(0) = -9$
   At $x = -1$
   $p(-1)$
   $= 3(-1)^3 - 5(-1)^2 + 6(-1) - 9$
   $= 3(-1) - 5(1) + 6(-1) - 9$
   $= -3 - 5 - 6 - 9$

$\therefore p(-1) = -23$

3. $p(x) = 5x^3 + 11x^2 + 10$

   At $x = -2$,

   $p(-2)$

   $= 5(-2)^3 + 11(-2)^2 + 10$

   $= 5(-8) + 11(4) + 10$

   $= -40 + 44 + 10$

   $\therefore p(-2) = 14$

Solution 3:

1. $p(x) = x^7$

   $\therefore p(0) = 0^7 = 0$

   $\therefore p(1) = (1)^7 = 1$

   $\therefore p(2) = (2)^7 = 128$

2. $p(x) = (x – 1)(x + 3)$

   $\therefore p(0) = (0 – 1)(0 + 3) = -1 \times 3 = -3$

   $\therefore p(1) = (1 – 1)(1 + 3) = 0 \times 4 = 0$

   $\therefore p(2) = (2 – 1)(2 + 3) = 1 \times 5 = 5$

3. $p(x) = x^2 – 2x$

   $\therefore p(0) = (0)^2 – 2(0) = 0 – 0 = 0$

   $\therefore p(1) = (1)^2 – 2(1) = 1 – 2 = -1$

   $\therefore p(2) = (2)^2 – 2(2) = 4 – 4 = 0$

Solution 4:

(1) Let $p(x) = 3x + 2 = 0$

$\therefore 3x + 2 = 0$

$\therefore 3x = -2$

$\therefore x = -\frac{2}{3}$

Therefore, $-\frac{2}{3}$ is the only zero of the polynomial $p(x)=3x + 2$.

(2) Let $p(x) = 5x - 3 = 0$

$\therefore 5x - 3 = 0$

$\therefore 5x = 3$

$\therefore x = \frac{3}{5}$

Therefore, $\frac{3}{5}$ is the only zero of the polynomial $p(x)=5x - 3$.

(3) Let $p(x) = 3 = 0$

Here, $p(x) = 3$ is a constant polynomial.

A constant polynomial does not have any zero.

Therefore, it is impossible to have a zero of polynomial $p(x) = 3$.

Exercise 3.3:

Solution 1(1):

$$
\begin{array}{r l}
 & x^4 + x^3 + x^2 + x + 1 \\
x - 1 \big) & \overline{x^5 - 1} \\
 & \underline{{}_{(-)}x^5 - {}_{(+)}x^4} \\
 & x^4 - 1 \\
 & \underline{{}_{(-)}x^4 - {}_{(+)}x^3} \\
 & x^3 - 1 \\
 & \underline{{}_{(-)}x^3 - {}_{(+)}x^2} \\
 & x^2 - 1 \\
 & \underline{{}_{(-)}x^2 - {}_{(+)}x} \\
 & x - 1 \\
 & \underline{{}_{(-)}x - {}_{(+)}1} \\
 & 0
\end{array}
$$

Quotient (Q): $x^4 + x^3 + x^2 + x + 1$

Remainder (R): 0

Solution 1(2):

$$
\begin{array}{r l}
 & x^3 + 5x^2 + 2x + 1 \\
x - 1 \big) & \overline{x^4 + 4x^3 - 3x^2 - x + 1} \\
 & \underline{{}_{(-)}x^4 - {}_{(+)}x^3} \\
 & 5x^3 - 3x^2 - x + 1 \\
 & \underline{{}_{(-)}5x^3 - {}_{(+)}5x^2} \\
 & 2x^2 - x + 1 \\
 & \underline{{}_{(-)}2x^2 - {}_{(+)}2x} \\
 & x + 1 \\
 & \underline{{}_{(-)}x - {}_{(+)}1} \\
 & 2
\end{array}
$$

Quotient (Q): $x^3 + 5x^2 + 2x + 1$

Remainder (R): 2

Solution 2(1):

$$
\begin{array}{r l}
 & 2t^3 - 5t^2 - 18t + 45 \\
t - 1 \big) & \overline{2t^4 - 7t^3 - 13t^2 + 63t - 45} \\
 & \underline{{}_{(-)}2t^4 - {}_{(+)}2t^3} \\
 & -5t^3 - 13t^2 + 63t - 45 \\
 & \underline{-{}_{(+)}5t^3 + {}_{(-)}5t^2} \\
 & -18t^2 + 63t - 45 \\
 & \underline{-{}_{(+)}18t^2 + {}_{(-)}18t} \\
 & 45t - 45 \\
 & \underline{{}_{(-)}45t - {}_{(+)}45} \\
 & 0
\end{array}
$$

Quotient (Q): $2t^3 - 5t^2 - 18t + 45$

Remainder (R): 0

Solution 2(2):

$$
\begin{array}{l}
\qquad\qquad 2t^3 - t^2 - 16t + 15 \\
t - 3 \,\overline{\big)\, 2t^4 - 7t^3 - 13t^2 + 63t - 45} \\
\quad {}_{(-)}2t^4 - {}_{(+)}6t^3 \\
\hline
\qquad - t^3 - 13t^2 + 63t - 45 \\
\qquad - {}_{(+)}t^3 + {}_{(-)}3t^2 \\
\hline
\qquad\qquad - 16t^2 + 63t - 45 \\
\qquad\qquad - {}_{(+)}16t^2 + {}_{(-)}48t \\
\hline
\qquad\qquad\qquad 15t - 45 \\
\qquad\qquad\qquad {}_{(-)}15t - {}_{(+)}45 \\
\hline
\qquad\qquad\qquad\qquad 0
\end{array}
$$

Quotient (Q): $2t^3 - t^2 - 16t + 15$

Remainder (R): 0

Solution 2(3):

$$
\begin{array}{l}
\qquad\qquad t^3 - t^2 - 9t + 9 \\
2t - 5 \,\overline{\big)\, 2t^4 - 7t^3 - 13t^2 + 63t - 45} \\
\quad {}_{(-)}2t^4 - {}_{(+)}5t^3 \\
\hline
\qquad - 2t^3 - 13t^2 + 63t - 45 \\
\qquad - {}_{(+)}2t^3 + {}_{(-)}5t^2 \\
\hline
\qquad\qquad - 18t^2 + 63t - 45 \\
\qquad\qquad - {}_{(+)}18t^2 + {}_{(-)}45t \\
\hline
\qquad\qquad\qquad 18t - 45 \\
\qquad\qquad\qquad {}_{(-)}18t - {}_{(+)}45 \\
\hline
\qquad\qquad\qquad\qquad 0
\end{array}
$$

Quotient (Q): $t^3 - t^2 - 9t + 9$

Remainder (R): 0

Solution 2(4):

$$
\begin{array}{l}
\qquad\qquad 2t^3 - 13t^2 + 26t - 15 \\
t + 3 \,\overline{\big)\, 2t^4 - 7t^3 - 13t^2 + 63t - 45} \\
\quad {}_{(-)}2t^4 - {}_{(+)}6t^3 \\
\hline
\qquad - 13t^3 - 13t^2 + 63t - 45 \\
\qquad - {}_{(+)}13t^3 - {}_{(+)}39t^2 \\
\hline
\qquad\qquad 26t^2 + 63t - 45 \\
\qquad\qquad {}_{(-)}26t^2 + {}_{(-)}78t \\
\hline
\qquad\qquad\qquad - 15t - 45 \\
\qquad\qquad\qquad - {}_{(+)}15t - {}_{(+)}45 \\
\hline
\qquad\qquad\qquad\qquad 0
\end{array}
$$

Quotient (Q): $2t^3 - 13t^2 + 26t - 15$

Remainder (R): 0

Solution 2(5):

$$
\begin{array}{r|l}
 & t^3 - 5t^2 + t + 30 \\
\hline
2t + 3 & 2t^4 - 7t^3 - 13t^2 + 63t - 45 \\
 & {}_{(-)}2t^4 +{}_{(-)}3t^3 \\
\hline
 & -10t^3 - 13t^2 + 63t - 45 \\
 & -{}_{(+)}10t^3 - {}_{(+)}15t^2 \\
\hline
 & 2t^2 + 63t - 45 \\
 & {}_{(-)}2t^2 +{}_{(-)}3t \\
\hline
 & 60t - 45 \\
 & {}_{(-)}60t +{}_{(-)}90 \\
\hline
 & -135
\end{array}
$$

Quotient (Q): $t^3 - 5t^2 + t + 30$

Remainder (R): -135

Solution 3:

When $p(x) = x^4 – 4x^3 + 3x – 1$ is divided by $d(x) = x + 2$, the remainder is given by $p(-2)$.

Remainder(R) = $p(-2)$

$p(x) = x^4 – 4x^3 + 3x – 1$

$\therefore p(-2)$

$x^4 – 4x^3 + 3x – 1$

$= (-2)^4 – 4(-2)^3 + 3(-2) – 1$

$= 16 – 4(-8) – 6 – 1$

$= 16 + 32 – 6 – 1$

$= 41$

Thus, the remainder is 41.

Solution 4:

The quantity to be added to polynomial p(y) so that the resulting polynomial becomes divisible by y + 1 is the opposite of the remainder obtained on dividing polynomial p(y) by y + 1.

$$
\begin{array}{r|l}
 & 12y^2 - 51y + 101 \\
\hline
y + 1 & 12y^3 - 39y^2 + 50y + 97 \\
 & {}_{(-)}12y^3 + {}_{(-)}12y^2 \\
\hline
 & -51y^2 + 50y + 97 \\
 & -{}_{(+)}51y^2 - {}_{(+)}51y \\
\hline
 & 101y + 97 \\
 & {}_{(-)}101y +{}_{(-)}101 \\
\hline
 & -4
\end{array}
$$

Remainder(R) = -4

So, 4 should be added to polynomial $p(y) = 12y^3 – 39y^2 + 50y + 97$ so that the resulting polynomial is divisible by y + 1.

Solution 5:

The quantity to be subtracted from polynomial p(x) so that the resulting polynomial becomes divisible by $x + 3$ is the remainder obtained on dividing p(x) by $x + 3$.

$$
\begin{array}{r|l}
 & x^2 - 3x^2 + 9x - 27 \\
\hline
x+3 & x^4 + 85 \\
 & \underline{{}_{(-)}x^4 + {}_{(-)}3x^3} \\
 & -3x^3 + 85 \\
 & \underline{-{}_{(+)}3x^3 - {}_{(+)}9x^2} \\
 & 9x^2 + 85 \\
 & \underline{{}_{(-)}9x^2 + {}_{(-)}27x} \\
 & -27x + 85 \\
 & \underline{-{}_{(+)}27x - {}_{(+)}81} \\
 & 166
\end{array}
$$

Remainder(R) = 166

Thus, 166 should be subtracted from polynomial $p(x) = x^4 + 85$ so that the resulting polynomial is divisible by $x + 3$.

Solution 6:

The product of two polynomials is $x^3 - 8x - 12 + x^2$, i.e. $x^3 + x^2 - 8x - 12$ and one of polynomial is $x + 2$.

$$\text{Thus, the other polynomial} = \frac{\text{Product of two polynomials}}{\text{One polynomial}}$$

$$= \frac{x^3 + x^2 - 8x - 12}{x + 2}$$

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
\hline
x+2 & x^3 + x^2 - 8x - 12 \\
 & \underline{{}_{(-)}x^3 + {}_{(-)}2x^2} \\
 & -x^2 - 8x - 12 \\
 & \underline{-{}_{(+)}x^2 - {}_{(+)}2x} \\
 & -6x - 12 \\
 & \underline{-{}_{(+)}6x - {}_{(+)}12} \\
 & 0
\end{array}
$$

Thus, the other polynomial is $x^2 - x - 6$.

Solution 7(1):

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x^2+2 & x^4 + x^3 + 3x^2 + 2x + 2 \\
 & \underline{{}_{(-)}x^4 + {}_{(-)}2x^2} \\
 & x^3 + x^2 + 2x + 2 \\
 & \underline{{}_{(-)}x^3 + {}_{(-)}2x} \\
 & x^2 + 2 \\
 & \underline{{}_{(-)}x^2 + {}_{(-)}2} \\
 & 0
\end{array}
$$

$\therefore$ Remainder (R): 0

Solution 7(2):

$$
\begin{array}{r|l}
 & x - 18 \\
\hline
x^2 + 3x & x^3 - 15x^2 - 54x + 23 \\
 & {}_{(-)}x^3 + {}_{(-)}3x^2 \\
\hline
 & -18x^2 - 54x + 23 \\
 & -{}_{(+)}18x^2 - {}_{(+)}54x \\
\hline
 & 23
\end{array}
$$

$\therefore$ Remainder (R): 23

Solution 7(3):

$$
\begin{array}{r|l}
 & x^2 + 2x + 3 \\
\hline
x^2 + 2x + 3 & x^4 + 4x^3 + 10x^2 + 12x + 15 \\
 & {}_{(-)}x^4 + {}_{(-)}2x^3 + {}_{(-)}3x^2 \\
\hline
 & 2x^3 + 7x^2 + 12x + 15 \\
 & {}_{(-)}2x^3 + {}_{(-)}4x^2 + {}_{(-)}6x \\
\hline
 & 3x^2 + 6x + 15 \\
 & {}_{(-)}3x^2 + {}_{(-)}6x + {}_{(-)}9 \\
\hline
 & 6
\end{array}
$$

$\therefore$ Remainder (R): 6

Solution 8:

When the polynomial $p(x) = ax^5 - 23x^3 + 47x + 1$ is divided by $(x - 2)$, the remainder is given by p(2).

$\therefore$ Remainder = p(2)

$p(x) = ax^5 - 23x^3 + 47x + 1$

$\therefore$ p(2)

$= a(2)^5 - 23(2)^3 + 47(2) + 1$

$= 32a - 184 + 94 + 1$

$= 32a - 89$

But, the remainder is 7. (given)

$\therefore 32a - 89 = 7$

$\therefore 32a = 7 + 89$

$\therefore 32a = 96$

$\therefore a = 3$

Exercise 3.4:

Solution 1:

$(x - 1)$ is a factor of the polynomial p(x), if and only if the sum of all the coefficients of p(x) is zero.

Also, if p(1) = 0, then $(x - 1)$ is a factor of p(x).

1. $2x^3 - 3x^2 + 3x - 2$

   The sum of all the coefficients

   = 2 + (-3) + 3 + (-2)

   = 0

   Thus, $(x - 1)$ is a factor of $2x^3 - 3x^2 + 3x - 2$.
2. $4x^3 + x^4 - x + 1$

   The sum of all the coefficients

= 4 + 1 + (-1) + 1

≠ 0

Thus, (x – 1) is not a factor of $4x^3 + x^4 – x + 1$.

3. $5x^4 – 4x^3 – 2x + 1$

   The sum of all the coefficients

   = 5 + (-4) + (-2) + 1

   = 0

   Thus, (x – 1) is a factor of $5x^4 – 4x^3 – 2x + 1$.
4. $3x^3 + x^2 + x + 11$

   The sum of all the coefficients

   = 3 + 1 + 1 + 11

   = 16

   ≠ 0

   Thus, (x – 1) is not a factor of $3x^3 + x^2 + x + 11$.

Solution 2:

1. $p(x) = 21x^2 + 26x + 8$

   d(x) = 3x + 2 is a factor of p(x).

   $21x^2 + 26 + 8$

   $= 21x^2 + 14x + 12x + 8$

   (Splitting the middle term to get 3x + 2 as a factor)

   = 7x(3x + 2) + 4(3x + 2)

   = (3x + 2)(7x + 4)

   Thus, (7x + 4) is other factor of p(x).
2. $p(x) = x^3 + 10x^2 + 23x + 14$

   d(x) = x + 1 is a factor of p(x).

   $x^3 + 10x^2 + 23x + 14$

   $= x^3 + x^2 + 9x^2 + 9x + 14x + 14$

   (Splitting the terms to get x + 1 as a factor)

   $= x^2(x + 1) + 9x(x + 1) + 14(x + 1)$

   $= (x + 1)(x^2 + 9x +14)$

   Thus, $(x^2 + 9x +14)$ is other factor of p(x).
3. $p(x) = x^3 – 9x^2 + 20x – 12$

   d(x) = x – 6 is a factor of p(x).

   $x^3 – 9x^2 + 20x – 12$

   $= x^3 – 6x^2 – 3x^2 + 18x + 2x – 12$

   $= (x – 6)(x^2 + 3x + 2)$

   Thus, $(x^2 + 3x + 2)$ is other factor of p(x).

Solution 3:

If $p(x) = ax^3 + 3x^2 + 7x + 13$ is divided by (x + 3),

the remainder is given by p(-3).

Remainder = p(-3)

p(-3)

$= a(-3)^3 + 3(-3)^2 + 7(-3) + 13$

= 27a + 27 – 21 + 13

= 19 – 27a

But, the remainder is -8. (given)

∴ 19 – 27a = -8

∴ -27a = -8 – 19

$\therefore$ -27a = -27

$\therefore$ a = 1

Solution 4:

1. $3x^2 + 7x + 4$
   $= 3x^2 + 3x + 4x + 4$
   $= 3x(x + 1) + 4(x + 1)$
   $= (x + 1)(3x + 4)$
2. $15x^2 + 16x + 4$
   $= 15x^2 + 10x + 6x + 4$
   $= 5x(3x + 2) + 2(3x + 2)$
   $= (3x + 2)(5x + 2)$
3. $-21x^2 + 16x + 5$
   $= -(21x^2 - 16x - 5)$
   $= -(21x^2 - 21x + 5x - 5)$
   $= -[21x(x - 1) + 5(x - 1)]$
   $= -(x - 1)(21x + 5)$

Solution 5:

1. $p(x) = 3x^3 - 7x^2 + 5x - 1$
   The sum of all the coefficients
   $= 3 + (-7) + 5 + (-1) = 0$
   The sum of the coefficients of the odd powers of x
   $= 3 + 5 = 8$
   The sum of the coefficients of the even powers of x
   $= (-7) + (-1) = (-8) \neq 8$
   Thus, (x – 1) is a factor of p(x), but (x + 1) is not a
   actor of p(x).
2. $p(x) = 21x^3 + 16x^2 + 4x + 9$
   The sum of all the coefficients
   $= 21 + 16 + 4 + 9 = 50 \neq 0$
   The sum of the coefficients of the odd powers of x
   $= 21 + 4 = 25$
   The sum of the coefficients of the even powers of x
   $= 16 + 9 = 25$
   Thus, (x + 1) is a factor of p(x), but (x – 1) is not a
   actor of p(x).
3. $p(x) = 2x^4 - 3x^3 + 4x^2 - 5x + 2$
   The sum of all the coefficients
   $= 2 + (-3) + 4 + (-5) + 2$
   $= 8 - 8 = 0$
   The sum of the coefficients of the odd powers of x
   $= (-3) + (-5)$
   $= (-8)$
   The sum of the coefficients of the even powers of x
   $= 2 + 4 + 2$
   $= 8 \neq (-8)$
   Hence, (x – 1) is a factor of p(x), but (x + 1) is not a
   actor of p(x).
4. $p(x) = x^3 + 13x^2 + 32x + 20$

The sum of all the coefficients

$= 1 + 13 + 32 + 20$

$= 66 \neq 0$

The sum of the coefficients of the odd powers of x

$= 1 + 32$

$= 33$

The sum of the coefficients of the even powers of x

$= 13 + 20$

$= 33$

Thus, (x + 1) is a factor of p(x), but (x – 1) is not a actor of p(x).

Solution 6:

$p(x) = ax^4 - 7x^3 - 3x^2 - 2x - 8$ and $x - 4$ is a factor of p(x).

$\therefore p(4) = 0$

$\therefore a(4)^4 - 7(4)^3 - 3(4)^2 - 2(4) - 8 = 0$

$\therefore 256a - 448 - 48 - 8 - 8 = 0$

$\therefore 256a - 512 = 0$

$\therefore 256a = 512$

$\therefore a = 2$

Exercise 3.5

Solution 1(1):

$(x - 7)(x - 12)$

$= [x + (-7)][x + (-12)]$

$= x^2 + [(-7) + (-12)]x + (-7)(-12)$

$= x^2 - 19x + 84$

Solution 1(2):

$(5 - 4x)(7 - 4x)$

$= (-4x + 5)(-4x + 7)$

$= (-4x)^2 + (5 + 7)(-4x) + (5)(7)$

$= 16x^2 - 48x + 35$

Solution 1(3):

$$\left(x + \frac{3}{2}\right)\left(2x + \frac{5}{3}\right)$$

$$= x\left(2x + \frac{5}{3}\right) + \frac{3}{2}\left(2x + \frac{5}{3}\right)$$

$$= 2x^2 + \frac{5}{3}x + 3x + \frac{5}{2}$$

$$= 2x^2 + \left(\frac{5}{3} + 3\right)x + \frac{5}{2}$$

$$= 2x^2 + \frac{14}{3}x + \frac{5}{2}$$

$$= \frac{12x^2 + 28x + 15}{6}$$

$$= \frac{1}{6}(12x^2 + 28x + 15)$$

Solution 1(4):

$$\left(3x+\frac{3}{2}\right)\left(3x+\frac{5}{2}\right)$$
$$=(3x)^2+\left(\frac{3}{2}+\frac{5}{2}\right)(3x)+\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)$$
$$=9x^2+12x+\frac{15}{4}$$
$$=\frac{36x^2+48x+15}{4}$$
$$=\frac{1}{4}(36x^2+48x+15)$$

Solution 2:

1. 97×103
   $= (100 - 3)(100 + 3)$
   $= (100)^2 - (3)^2$
   $= 10000 - 9$
   $= 9991$
2. 57×63
   $= (60 - 3)(60 + 3)$
   $= (60)^2 - (3)^2$
   $= 3600 - 9$
   $= 3591$
3. 34×26
   $= (30 + 4)(30 - 4)$
   $= (30)^2 - (4)^2$
   $= 900 - 16$
   $= 884$

Solution 3(1):

$16x^2 - 40xy + 25y^2$
$= (4x)^2 - 2(4x)(5y) + (5y)^2$
$= (4x - 5y)^2$

Solution 3(2):

$$\frac{x^2}{9}+\frac{4xy}{15}+\frac{4y^2}{25}$$
$$=\left(\frac{x}{3}\right)^2+2\left(\frac{x}{3}\right)\left(\frac{2y}{5}\right)+\left(\frac{2y}{5}\right)^2$$
$$=\left(\frac{x}{3}+\frac{2y}{5}\right)^2$$

Solution 3(3):

$9a^2 + 25b^2 + 49c^2 - 30ab + 70bc - 42ac$
$= (3a)^2 + (-5b)^2 + (-7c)^2 + 2(3a)(-5b) + 2(-5b)(-7c) + 2(-7c)(3a)$
$= (3a - 5b - 7c)^2$
OR
$9a^2 + 25b^2 + 49c^2 - 30ab + 70bc - 42ac$
$= (-3a)^2 + (5b)^2 + (7c)^2 + 2(-3a)(5b) + 2(5b)(7c) + 2(7c)(-3a)$
$= (-3a + 5b + 7c)^2$

Solution 3(4):

$16a^4 - 625b^4$

$= (4a^2)^2 - (25b^2)^2$

$= (4a^2 + 25b^2)(4a^2 - 25b^2)$

$= (4a^2 + 25b^2)(2a + 5b)(2a - 5b)$

Solution 3(5):

$$\frac{8x^3}{27}+\frac{27y^3}{64}+\frac{64z^3}{125}-\frac{6}{5}xyz$$

$$=\left(\frac{2x}{3}\right)^3+\left(\frac{3y}{4}\right)^3+\left(\frac{4z}{5}\right)^3-3\left(\frac{2x}{3}\right)\left(\frac{3y}{4}\right)\left(\frac{4z}{5}\right)$$

$$=\left(\frac{2x}{3}+\frac{3y}{4}+\frac{4z}{5}\right)\left\{\left(\frac{2x}{3}\right)^2+\left(\frac{3y}{4}\right)^2+\left(\frac{4z}{5}\right)^2-\left(\frac{2x}{3}\right)\left(\frac{3y}{4}\right)-\left(\frac{3y}{4}\right)\left(\frac{4z}{5}\right)-\left(\frac{4z}{5}\right)\left(\frac{2x}{3}\right)\right\}$$

$$=\left(\frac{2x}{3}+\frac{3y}{4}+\frac{4z}{5}\right)\left(\frac{4x^2}{9}+\frac{9y^2}{16}+\frac{16z^2}{25}-\frac{xy}{2}-\frac{3yz}{5}-\frac{8zx}{15}\right)$$

Solution 3(6):

$125a^3 + 600a^2b + 960ab^2 + 512b^3$

$= 125a^3 + 512b^3 + 600a^2b + 960ab^2$

$= (5a)^3 + (8b)^3 + 120ab(5a + 8b)$

$= (5a)^3 + (8b)^3 + 3(5a)(8b)(5a + 8b)$

$= (5a + 8b)^3$

Solution 3(7):

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 36ab(4a - 3b)$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

Solution 4:

1. 105×102
   $= (100 + 5)(100 + 2)$
   $= (100)^2 + (5 + 2)100 + (5)(2)$
   $= 10000 + 700 + 10$
   $= 10710$
2. $(92)^2$
   $= (90 + 2)^2$
   $= (90)^2 + 2(90)(2) + (2)^2$
   $= 8100 + 360 + 4$
   $= 8464$
3. $(8)^3 - (4)^3$
   $= (8 - 4)^3 + 3(8)(4)(8 - 4)$
   $= (4)^3 + 3(32)(4)$
   $= 64 + 384$
   $= 448$

Solution 5:

$(-28)^3 + (15)^3 + (13)^3$

Let a = -28, b = 15 and c = 13

Then, a + b + c = (-28) + 15 + 13 = 0

Now, if a + b + c = 0 then $a^3 + b^3 + c^3 = 3abc$

$\therefore (-28)^3 + (15)^3 + (13)^3$

= 3(-28)(15)(13)

= (-84)(195)

= -16380